Chapter 6
Longitudunal (Panel) Evaluations Using DEA

6.1 Malmquist Index

Monitoring performance over time is essential in health care organizations. The Malmquist index is a method which provides an opportunity to compare the health care facility performance from one period to another. Such a tool was suggested first by Malmquist (1953), then developed as a productivity index by Caves, Christensen and Diewert (1982), and then further developed by Fare, Grosskopf and Lowell (1994) as the Malmquist-DEA performance measure.

The Malmquist DEA calculates DEA efficiency for the following input (or output) oriented CRS models:

[a] Calculating the frontier in time period-1 (time t) and comparing efficiency scores, $\theta_0^t(x_o^t, y_o^t)$, of health care organizations at period-1 (time t),
[b] Calculating the frontier in time period-2 (time t + 1) and comparing efficiency scores, $\theta_0^{t+1}(x_o^{t+1}, y_o^{t+1})$, of health care organizations at period 2 (time t + 1),
[c] Comparing efficiency scores of time period-1 (t), $\theta_0^t(x_o^{t+1}, y_o^{t+1})$, to frontier at time period-2 (t + 1), and
[d] Comparing efficiency scores of period-2 (t + 1), $\theta_0^{t+1}(x_o^t, y_o^t)$, to frontier at period-1 (t).

Malmquist efficiency is defined as the geometric mean of efficiency scores defined above:

$$M_o = \left[\frac{[a]Period - 1}{[c]Period - 1\ on\ Period - 2} \quad * \quad \frac{[d]Period - 2\ on\ Period - 1}{[b]Period - 2} \right]^{\frac{1}{2}} \tag{6.1}$$

or

$$M_o = \left[\frac{\theta_0^t(x_o^t, y_o^t)}{\theta_0^t(x_o^{t+1}, y_o^{t+1})} \frac{\theta_0^{t+1}(x_o^t, y_o^t)}{\theta_0^{t+1}(x_o^{t+1}, y_o^{t+1})} \right]^{\frac{1}{2}} \tag{6.2}$$

where M_o indicates the efficiency change between period-1(t) and period 2 (t + 1).

The efficiency change is observed as:

If $M_o > 1$, efficiency is decreased from period-1 to period-2.
If $M_o = 1$, no change in efficiency from period-1 to period-2.
If $M_o < 1$, efficiency is increased from period-1 to period-2.

An important feature of the DEA Malmquist index is that it can decompose the overall efficiency measure into two mutually exclusive components, one measuring change in technical efficiency (catching-up effect) and the other measuring change in technology (innovation). Since the Malmquist efficiency index is the product of these two components, the decomposition can be shown as:

$$M_o = \underset{\text{(EFFICIENCY CHANGE)}}{\frac{[a]Period-1}{[b]Period-2}} \quad * \quad \left[\frac{[b]Period-2}{[c]Period-1 \; on \; Period-2} \right.$$

$$\left. * \quad \underset{\text{(TECHNICAL CHANGE)}}{\frac{[d]Period-2 \; on \; Period-1}{[a]Period-1}} \right]^{\frac{1}{2}} \tag{6.3}$$

or

$$M_o = \frac{\theta_0^{t}(x_o^{t}, y_o^{t})}{\theta_0^{t+1}(x_o^{t+1}, y_o^{t+1})} \quad * \quad \left[\frac{\theta_0^{t+1}(x_o^{t+1}, y_o^{t+1})}{\theta_0^{t}(x_o^{t+1}, y_o^{t+1})} * \frac{\theta_0^{t+1}(x_o^{t}, y_o^{t})}{\theta_0^{t}(x_o^{t}, y_o^{t})} \right]^{\frac{1}{2}} \tag{6.4}$$

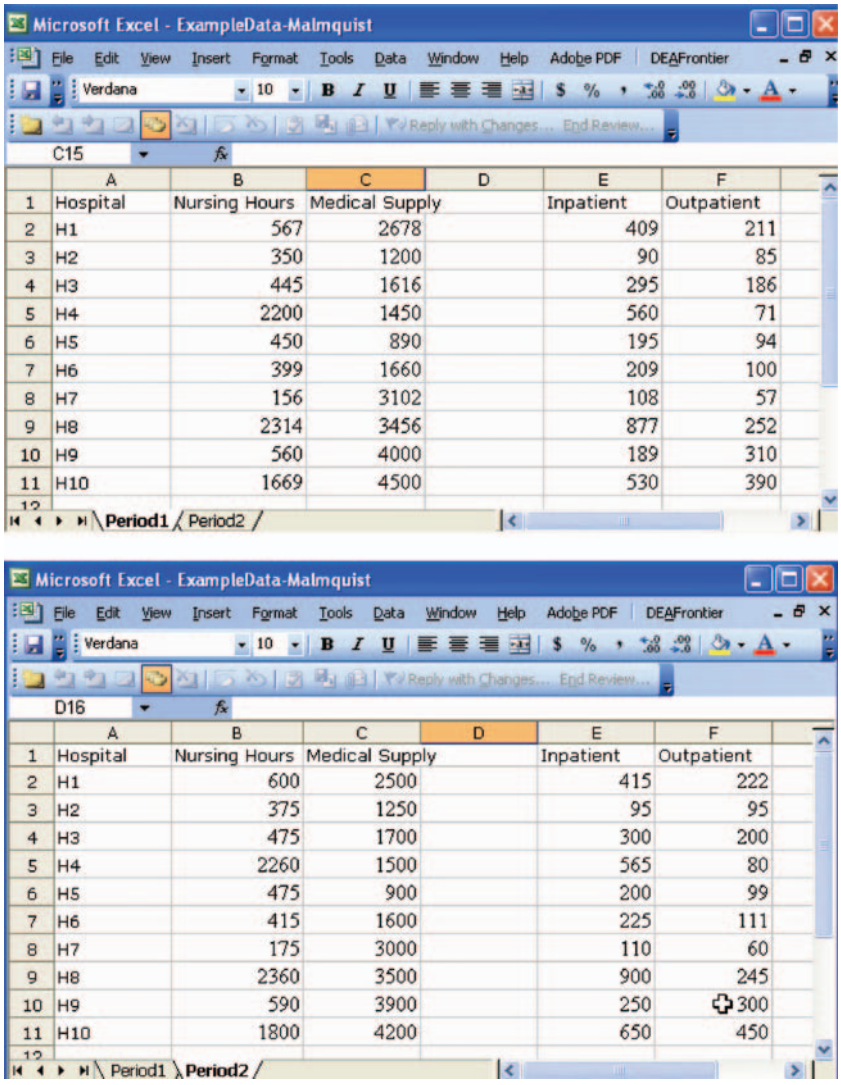
The efficiency component of the index (the first half) measures changes in technical efficiency from period t to period $t + 1$. That is, it measures how the units being examined have managed to catch up to the frontier. On the other hand, the technical component of the index (the second half) measures changes in the production frontier (i.e., a shift in best-practice technology) from period t to period $t + 1$. In an input-oriented evaluation, if the values of the Malmquist index and its components are less than 1, equal to 1, or greater than 1, they indicate progress, no change, or regress, respectively (Caves, Christensen and Diewert, 1982; Färe, Grosskopf, Lindgren, and Ross, 1994).

CRS output orientation can be handled similarly. However, for VRS the following constraint should be added to the model:

$$\sum_{j=1}^{n} \lambda_j = 1 \quad j = 1, \ldots n$$

6.2 Malmquist-DEA Efficiency Example

To illustrate the use of DEA based Malmquist index, we will use the ongoing example, in which we will consider the existing data belonging to period-1. Additional

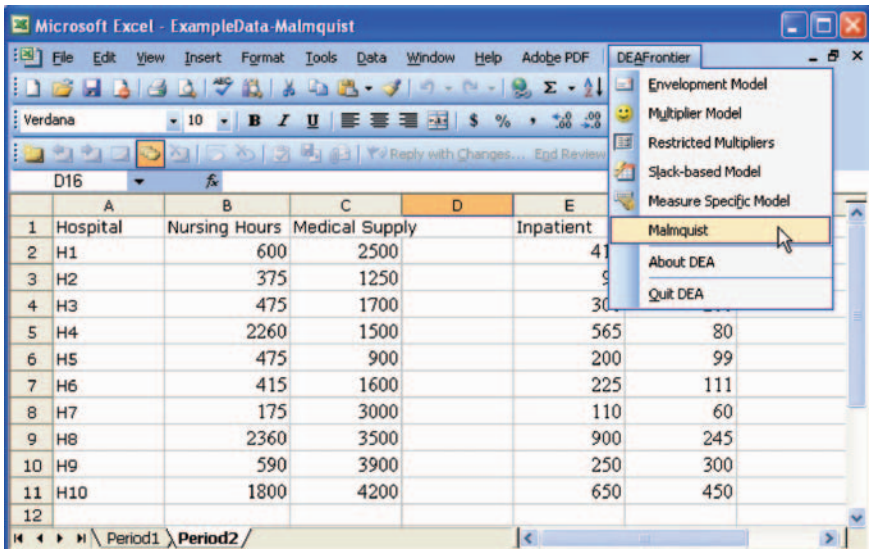


	A	B	C	D	E	F
1	Hospital	Nursing Hours	Medical Supply		Inpatient	Outpatient
2	H1	567	2678		409	211
3	H2	350	1200		90	85
4	H3	445	1616		295	186
5	H4	2200	1450		560	71
6	H5	450	890		195	94
7	H6	399	1660		209	100
8	H7	156	3102		108	57
9	H8	2314	3456		877	252
10	H9	560	4000		189	310
11	H10	1669	4500		530	390

	A	B	C	D	E	F
1	Hospital	Nursing Hours	Medical Supply		Inpatient	Outpatient
2	H1	600	2500		415	222
3	H2	375	1250		95	95
4	H3	475	1700		300	200
5	H4	2260	1500		565	80
6	H5	475	900		200	99
7	H6	415	1600		225	111
8	H7	175	3000		110	60
9	H8	2360	3500		900	245
10	H9	590	3900		250	300
11	H10	1800	4200		650	450

Fig. 6.1 Malmquist data for the example problem

data from the same hospitals was gathered from another time period (year) and labeled as period-2. The top part of Fig. 6.1 illustrates period-1 and the bottom part of Fig. 6.1 shows the data belonging period-2. As the reader can observe, the data setup is similar to the cross-sectional (single time period) version, however, for each period under consideration a new Excel sheet must be present. Health care managers and researchers can include more than two periods; however, the evaluation of Malmquist-DEA must be carried by choosing any two periods at a time.



	A	B	C	D	E	
1	Hospital	Nursing Hours	Medical Supply		Inpatient	
2	H1	600	2500		41	
3	H2	375	1250		9	
4	H3	475	1700		30	
5	H4	2260	1500		565	80
6	H5	475	900		200	99
7	H6	415	1600		225	111
8	H7	175	3000		110	60
9	H8	2360	3500		900	245
10	H9	590	3900		250	300
11	H10	1800	4200		650	450
12						

Fig. 6.2 Setup for Malmquist-DEA

To evaluate performance over time using Malmquist-DEA, select the Malmquist option from the DEAFrontier menu, as shown in Fig. 6.2. This will prompt another window for the selection of time periods from the available set. As shown in Fig. 6.3, our example contains only two periods; thus we choose both. In order to select the second period, the user should hold the Ctrl key then click into the designated box. Selection of the model orientation (input or output) completes the selection process, as shown in Fig. 6.3. Click OK to run the model.

Once the model runs, the health care manager and researcher can view a file containing outputs in several spreadsheets. Naturally, the raw data from period-1 and period-2 are the essential parts of this file. The Malmquist-Index file shown in Fig. 6.4 displays the summary information for the Malmquist-DEA. The three columns of information display the results for each hospital, as shown in the formulation earlier in Sect. 6.

The reader can verify that "Malmquist Index=Efficiency Change * Frontier Shift" by multiplying the values in the last two columns of the report shown in Fig. 6.4. As discussed earlier, if $M_o > 1$, efficiency is decreased from period-1 to period-2; hence H1, H3, H4, H7 and H9 exhibit such a decrease. On the other hand, if $M_o < 1$, efficiency is increased from period-1 to period-2; hospitals H2, H5, H6, H8 and H10 all increased their efficiency between these two periods.

To further investigate the components of the Malmquist index, we can observe efficiency independently in each period. Fig. 6.5a,b show the independent efficiency evaluations of period-1 [a] and period-2 [b].

Using these independent evaluations to compare hospitals in Fig. 6.5a,b, we observe that inefficient hospitals H2, H6, and H10 increased their efficiency in the second period, while H7 decreased its efficiency score.

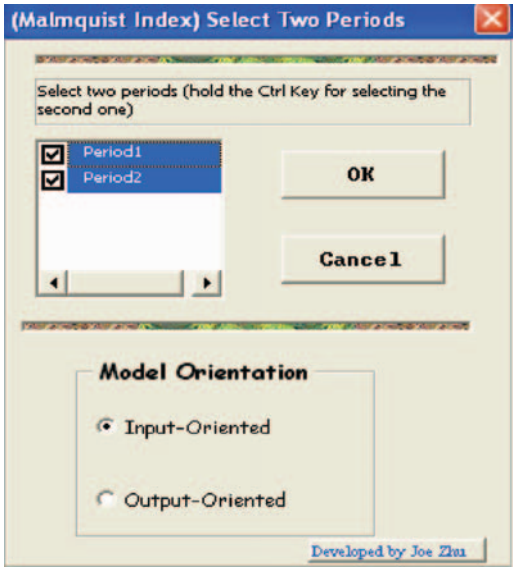


Fig. 6.3 Selection of periods and orientation

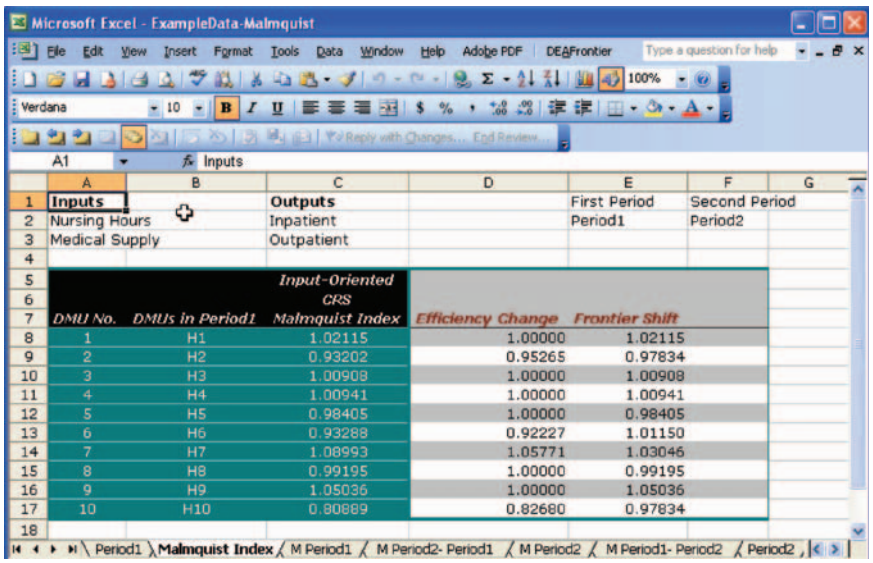
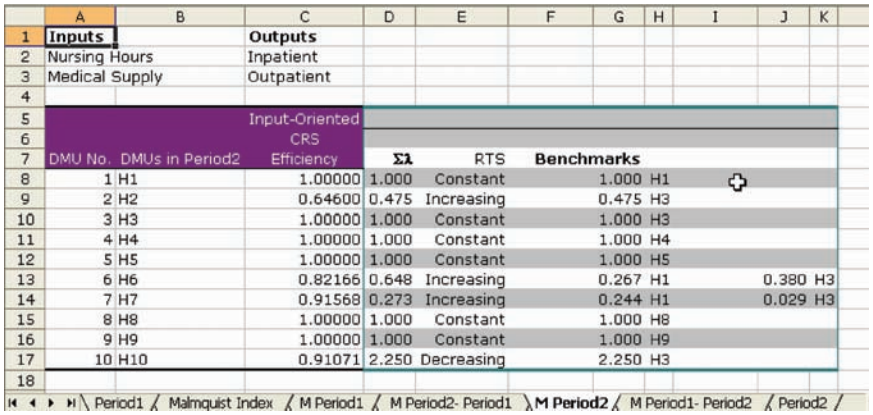


Fig. 6.4 Summary of Malmquist-DEA results for the hospital example

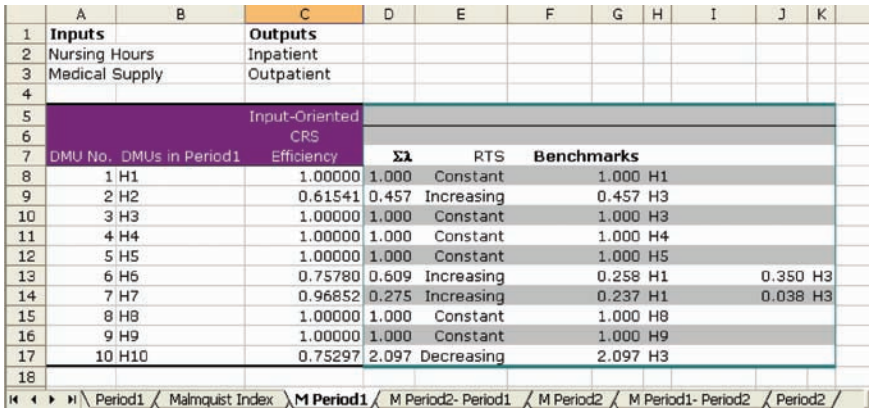
In order to calculate the Malmquist index shown by (6.4), we need to observe period-1 on period-2 and period-2 on period-1, where one period is under evaluation with respect to the other period, and the other period serves as reference. These are the [c] and [d] components of the formula. As shown in Fig. 6.6a "M period1-period2" indicates that period-2 is the reference set, and the Malmquist index for



	A	B	C	D	E	F	G	H	I	J	K
1	**Inputs**		**Outputs**								
2	Nursing Hours		Inpatient								
3	Medical Supply		Outpatient								
4											
5			Input-Oriented								
6			CRS								
7	DMU No.	DMUs in Period2	Efficiency	Σλ	RTS	**Benchmarks**					
8	1	H1	1.00000	1.000	Constant	1.000	H1				
9	2	H2	0.64600	0.475	Increasing	0.475	H3				
10	3	H3	1.00000	1.000	Constant	1.000	H3				
11	4	H4	1.00000	1.000	Constant	1.000	H4				
12	5	H5	1.00000	1.000	Constant	1.000	H5				
13	6	H6	0.82166	0.648	Increasing	0.267	H1			0.380	H3
14	7	H7	0.91568	0.273	Increasing	0.244	H1			0.029	H3
15	8	H8	1.00000	1.000	Constant	1.000	H8				
16	9	H9	1.00000	1.000	Constant	1.000	H9				
17	10	H10	0.91071	2.250	Decreasing	2.250	H3				
18											

Period1 / Malmquist Index / M Period1 / M Period2- Period1 / **M Period2** / M Period1- Period2 / Period2 /

Fig. 6.5 (**a**) Independent efficiency evaluation of period-1 [a]



	A	B	C	D	E	F	G	H	I	J	K
1	**Inputs**		**Outputs**								
2	Nursing Hours		Inpatient								
3	Medical Supply		Outpatient								
4											
5			Input-Oriented								
6			CRS								
7	DMU No.	DMUs in Period1	Efficiency	Σλ	RTS	**Benchmarks**					
8	1	H1	1.00000	1.000	Constant	1.000	H1				
9	2	H2	0.61541	0.457	Increasing	0.457	H3				
10	3	H3	1.00000	1.000	Constant	1.000	H3				
11	4	H4	1.00000	1.000	Constant	1.000	H4				
12	5	H5	1.00000	1.000	Constant	1.000	H5				
13	6	H6	0.75780	0.609	Increasing	0.258	H1			0.350	H3
14	7	H7	0.96852	0.275	Increasing	0.237	H1			0.038	H3
15	8	H8	1.00000	1.000	Constant	1.000	H8				
16	9	H9	1.00000	1.000	Constant	1.000	H9				
17	10	H10	0.75297	2.097	Decreasing	2.097	H3				
18											

Period1 / Malmquist Index / **M Period1** / M Period2- Period1 / M Period2 / M Period1- Period2 / Period2 /

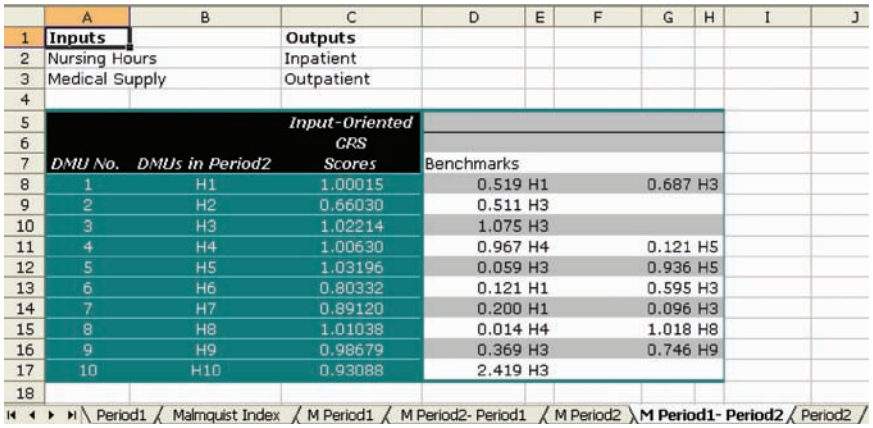
Fig. 6.5 (**b**) Independent efficiency evaluation of period-2 [b]

period-1 is under evaluation. Similarly, in Fig. 6.6b "M period2-period1" indicates that period-1 is the reference set, and the Malmquist index for period-2 is under evaluation.

To calculate the "Efficiency Change" and "Frontier Shift" components of the (6.3) or (6.4), we shall reorganize efficiency scores calculated from Fig. 6.5a, from Fig. 6.5b [b], from Fig. 6.6a [c], and from Fig. 6.6b [d]. Figure 6.7 displays the summary of these efficiency scores for each hospital in the respective columns, and also includes a summary of the Malmquist index, efficiency change and frontier shift from Fig. 6.4.

Now, if we customize (rewrite) the (6.4) for this example, let us say for hospital H6, then we get

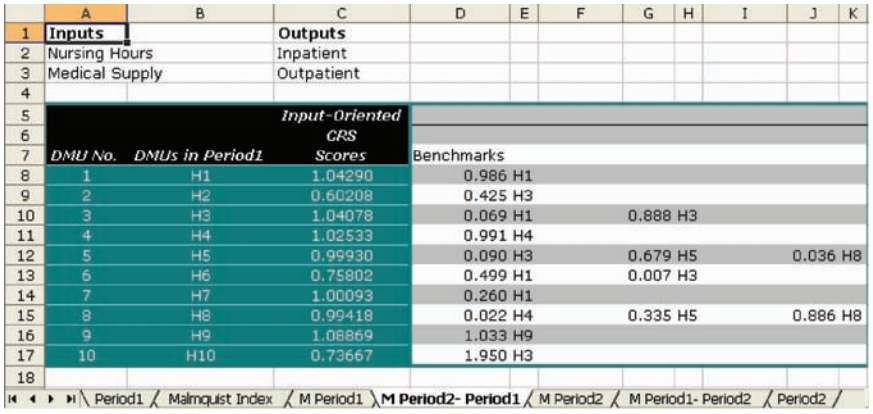
$$M_6 = \frac{\theta_6^1(x_6^1, y_6^1)}{\theta_6^2(x_6^2, y_6^2)} \quad * \quad \left[\frac{\theta_6^2(x_6^2, y_6^2)}{\theta_6^1(x_6^2, y_6^2)} \quad * \quad \frac{\theta_6^2(x_6^1, y_6^1)}{\theta_6^t(x_6^t, y_6^t)}\right]^{\frac{1}{2}}$$



	A	B	C	D	E	F	G	H	I	J
1	**Inputs**		**Outputs**							
2	Nursing Hours		Inpatient							
3	Medical Supply		Outpatient							
4										
5			*Input-Oriented*							
6			*CRS*							
7	*DMU No.*	*DMUs in Period2*	*Scores*	Benchmarks						
8	1	H1	1.00015	0.519 H1			0.687 H3			
9	2	H2	0.66030	0.511 H3						
10	3	H3	1.02214	1.075 H3						
11	4	H4	1.00630	0.967 H4			0.121 H5			
12	5	H5	1.03196	0.059 H3			0.936 H5			
13	6	H6	0.80332	0.121 H1			0.595 H3			
14	7	H7	0.89120	0.200 H1			0.096 H3			
15	8	H8	1.01038	0.014 H4			1.018 H8			
16	9	H9	0.98679	0.369 H3			0.746 H9			
17	10	H10	0.93088	2.419 H3						
18										

Period1 / Malmquist Index / M Period1 / M Period2- Period1 / M Period2 / **M Period1- Period2** / Period2

Fig. 6.6 (**a**) Malmquist index period-1, period-2 is reference [c]



	A	B	C	D	E	F	G	H	I	J	K
1	**Inputs**		**Outputs**								
2	Nursing Hours		Inpatient								
3	Medical Supply		Outpatient								
4											
5			*Input-Oriented*								
6			*CRS*								
7	*DMU No.*	*DMUs in Period1*	*Scores*	Benchmarks							
8	1	H1	1.04290	0.986 H1							
9	2	H2	0.60208	0.425 H3							
10	3	H3	1.04078	0.069 H1			0.888 H3				
11	4	H4	1.02533	0.991 H4							
12	5	H5	0.99930	0.090 H3			0.679 H5			0.036 H8	
13	6	H6	0.75802	0.499 H1			0.007 H3				
14	7	H7	1.00093	0.260 H1							
15	8	H8	0.99418	0.022 H4			0.335 H5			0.886 H8	
16	9	H9	1.08869	1.033 H9							
17	10	H10	0.73667	1.950 H3							
18											

Period1 / Malmquist Index / M Period1 / **M Period2- Period1** / M Period2 / M Period1- Period2 / Period2

Fig. 6.6 (**b**) Malmquist index period-2, period-1 is reference [d]

	M Period 1 [a]	M Period 2 [b]	M Period1-Period2 [c]	Period2-Period-1 [d]			
DMU No.	Input-Oriented CRS Efficiency	Input-Oriented CRS Efficiency	*Input-Oriented CRS Scores*	*Input-Oriented CRS Scores*	*CRS Malmquist Index*	*Efficiency Change*	*Frontier Shift*
1	1.00000	1.00000	1.00015	1.04290	1.02115	1.00000	1.02115
2	0.61541	0.64600	0.66030	0.60208	0.93202	0.95265	0.97834
3	1.00000	1.00000	1.02214	1.04078	1.00908	1.00000	1.00908
4	1.00000	1.00000	1.00630	1.02533	1.00941	1.00000	1.00941
5	1.00000	1.00000	1.03196	0.99930	0.98405	1.00000	0.98405
6	0.75780	0.82166	0.80332	0.75802	0.93288	0.92227	1.01150
7	0.96852	0.91568	0.89120	1.00093	1.08993	1.05771	1.03046
8	1.00000	1.00000	1.01038	0.99418	0.99195	1.00000	0.99195
9	1.00000	1.00000	0.98679	1.08869	1.05036	1.00000	1.05036
10	0.75297	0.91071	0.93088	0.73667	0.80889	0.82680	0.97834

Fig. 6.7 Summary of efficiency scores

DMU No.	A [a]/[c]	B [d]/[b]	√A*B M_o	C [a]/[b] Efficiency Change	M_o/C Frontier Shift	D [b]/[c]	E [d]/[a]	√D * E Frontier Shift
1	0.99985	1.042902	1.02115	1.0000	1.02115	0.99985	1.04290	1.02115
2	0.93202	0.932018	0.93202	0.9527	0.97834	0.97834	0.97834	0.97834
3	0.97834	1.040778	1.00908	1.0000	1.00908	0.97834	1.04078	1.00908
4	0.99374	1.025328	1.00941	1.0000	1.00941	0.99374	1.02533	1.00941
5	0.96903	0.999296	0.98405	1.0000	0.98405	0.96903	0.99930	0.98405
6	0.94333	0.922543	0.93288	0.9223	1.01150	1.02283	1.00029	1.01150
7	1.08676	1.093097	1.08993	1.0577	1.03046	1.02747	1.03346	1.03046
8	0.98972	0.994179	0.99195	1.0000	0.99195	0.98972	0.99418	0.99195
9	1.01338	1.088690	1.05036	1.0000	1.05036	1.01338	1.08869	1.05036
10	0.80889	0.808889	0.80889	0.8268	0.97834	0.97834	0.97834	0.97834

Fig. 6.8 Detailed calculations of Malmquist-DEA index

and, substituting the respective efficiency values, θ_6^*, from Fig. 6.7, we obtain:

$$M_6 = \frac{0.75780}{0.82166} * \left[\frac{0.82166}{0.80332} * \frac{0.75802}{0.75780}\right]^{\frac{1}{2}}$$

$$M_6 = 0.92227 * [1.022825 * 1.000291]^{\frac{1}{2}}$$

$$M_6 = 0.92227 * [1.023123]^{\frac{1}{2}}$$

$$M_6 = 0.92227 * 1.01150$$

$$M_6 = 0.93288$$

Figure 6.8 shows the correspondence of these calculated scores for all ten hospitals (DMUs). The reader can observe that hospital H6's Malmquist index, M_6, is 0.93288 as shown in column M_0 in Fig. 6.8. The components of this index, efficiency change and frontier shift values, were also obtained while calculating M_6 as 0.92227 and 1.01150, respectively.

Independent calculation of the frontier shift is also demonstrated in Fig. 6.8, in columns D and E, where the square root of the cross product of this calculation yields the frontier shift.

It should be noted that when more than two periods involved in the evaluation, one can perform Malmquist index for any pair of periods given that periods are identified properly on Excel worksheets. Ozgen and Ozcan (2004) study demonstrated seven year evaluation of performance for dialysis centers using Malmquist index (see Chap. 13, Sect. 13.2 for further information).

6.3 Summary

This chapter demonstrated the longitudinal evaluations of performance using the Malmquist-DEA index. In doing so, we can identify changes in efficiency from one

period to another, but can also determine whether this change is due to pure efficiency improvement and/or due to technological changes in service delivery, such as medical innovations, which caused a shift in the efficiency frontier. As health care organizations adopt many new technologies, frontier change is expected, provided there is a long enough duration lag to capture this effect.