Chapter 6

A NEVER-ENDING BRAID: THE DEVELOPMENT OF MATHEMATICS

Abstract: The evidence from language is brought together to discuss the nature of mathematics. Different conceptions of the way it develops are described, and the mechanisms that operate in its development are hypothesised. The Kama Sutra is invoked to illustrate the links between mathematics and society.

Keywords: philosophy of mathematics, history of mathematics, nature of mathematics

Douglas Hofstadter (1979) referred to mathematical thought as *The Eternal Golden Braid*. This book wove together mathematics in the form of the work of Kurt Gödel, graphic art as drawn by M. C. Escher, and music as epitomised in the Bach symphonies. The dominant theme of self-reference was played out through each of these human creations in such a way that the works that I knew (the mathematics and the graphic art) enlightened me on the work that I did not really understand (Bach's music). I had the feeling that it would be possible for any reader who knew well any two of the fields, to similarly reach an appreciation of the third.

Three different worlds dealt with the same theme differently but in depth, creating an image of a braid with the three strands weaving together and gaining strength from the existence of each other. They could never be the same, nor could one of them ever be encompassed by any other. Each creation had its own aspects that could not be adequately represented in the other: the abstract austerity of Gödel's mathematics, the emotional intensity of Bach's music, the aesthetic playfulness of Escher's etchings.

Hofstadter did not suggest the three pieces of work had the same origin, nor could I conceive how they might ever be completely amalgamated by some wider, more general activity. It reminded me of Hermann Hess' (2002) book *The Glass Bead Game*, and the imaginary "performance" of the maestro as he wove together literature, language, music, mathematics, art, dance, and other forms of cultural expression in a symphony of words, a picture of equations, and a poem of forms.

The idea of human creativity bringing together different forms in ever new combinations is a model that could be adopted for mathematics itself. Such a model is quite different from the commonly accepted idea that mathematics is one ever expanding stream, fed by tributaries that get encompassed by the main current in broader generalisation, higher levels of abstraction, or reorganisation of the components of mathematics.

But isn't it true that the mathematical stream is fed by its tributaries? When the mathematical community becomes aware of a new idea, it is accommodated into mathematics for the benefit of all. For example, when a mathematician became interested in the *kolam* patterns drawn by Indian women on their doorsteps (Ascher, 2002), and realised that the system represented there was not only mathematical but also contained some new mathematical ideas, he did not turn away from mathematics and work with the women to develop *kolam* further. He reinterpreted what he had seen using mathematical notation and wrote about it in a mathematical journal of an appropriate existing mathematical field (Siromoney, 1986; Siromoney & Sironmoney, 1987). In this case it was the mathematical aspects of computer science. The scientist was interested in the structure of "languages" used to describe drawings. He had worked with strings of symbols and how they could be used as a "language" for pictures. Watching women making *kolam* patterns he realised that another method for developing a language could be to create an array of symbols, as the women built an array first, before drawing their patterns. This was a new mathematical idea generated by the traditional craft.

Put another way, if there are other mathematical worlds as indicated by the evidence from language, why have they not been developed? Where are these other mathematical worlds? Could it be the case that mathematics as we know it is, in fact, universal; that it can express every abstract structure or system in our world? Perhaps the absence of other mathematical worlds implies that nothing useful could come of them that cannot be done equally well in mathematics as we know it?

I want to use the image of a braid to try to answer some of these questions, so let us look at another example to get a feel for this mathematical braid.

1. PACIFIC NAVIGATION: IS IT MATHEMATICS?

In Part I there is a short description of the navigation techniques of Pacific navigators, the way they used paths rather than positions, and the orientation system called *etak*. Another technique used by skilled navigators was the analysis of swells in the ocean.

One of the features of the mid-Pacific is that it is a relatively predictable environment. While there are storms and weather changes, many of these are seasonal, and most of the weather is fine. Thus the trade winds are both steady in force and direction, and navigators can use them for orientation. Similarly, the reliable fine weather means that clouds form over islands, and can be seen from a distance when the land is over the horizon, making a much bigger "target" for a navigator to aim at. These constant environmental features are also reflected in the ocean swells. Even swells caused by storms are constant over several days.

The reliability of ocean swells can be useful information for a seafarer. Swells are affected by the presence of islands, since swells change direction as they pass by. Surfers know this, the effect of land on swell direction is why good surfing is to be found off promontories: the swell bends as it rounds the promontory, creating a wave on which the break starts at one end and then runs along the length of the wave.

Thus the swells under your boat carry a lot of information if only you can read it: information about islands that are over the horizon; information about weather patterns; information about wind direction and strength. Pacific navigators used this information in quite systematic ways. The ancient navigation schools created models out of sticks and shells to teach their new navigators about swells, and every navigator learned what swells they could expect in different seasons, and how the swells would change, as they traversed each journey in their repertoire.

However, the first problem is to be able to detect the swells. It is reported (Gladwin, 1970, p. 170–4; Kyselka, 1987; Lewis, 1975, p. 90–3; Thomas, 1987) that Pacific navigators could feel the swells coming from four different directions simultaneously—the most famous contemporary navigator Mau could detect five (Kyselka, 1987, p. 98). That is, the navigators could feel the way the boat moved (even from inside the cabin at night) and thereby distinguish the movement of swells coming from several directions at the same time. The problem of discriminating component waves from the total wave movement is easily describable using the language of mathematics, and is a familiar problem when the waves are all from the same direction—this is the field known as Fourier or Harmonic Analysis. There has been little work done on the problem of multi-directional wave analysis, but mathematicians have no difficulty discussing it and accepting it as a problem in mathematics. They can generalise from the one-dimensional problem to that of waves coming from several directions, conceptualising the difficulties of analysis, and identifying possible ways to get the solutions. The problem of two waves coming at right angles to each other can be solved computationally, using computers to get approximations. But the problem has not been solved for waves coming from four different directions, and no instruments have been developed that will quickly resolve a wave movement into four directional components.

Now let us imagine again. Think of all the mathematical and technological effort that went into the development of navigation: star, moon and sun position charts; sighting equipment; the accurate timepieces needed to make use of these sightings to determine latitude and longitude; and modern GPS (Global Positioning System) equipment. Imagine that all (even a good fraction of) that money and effort had been put into analysis of wave motion and developing technology to sense swells in the ocean. Perhaps, if this had happened, ships would now be equipped with such sensors, and would have computer systems that could resolve the information and detect changes in the size and directions of the swells under their hulls.

If such things had been developed, then captains would have another piece of navigation equipment—a piece that would be able to warn them of small islands, or icebergs, in their vicinity before they became visible to lookouts or radar. And if the Titanic had had such a piece of technology, then alarm bells might have been automatically triggered all over the ship well before she ripped her bottom out with such tragic results on an iceberg no-one had seen. Perhaps the lawyers of Star Line should start looking for who was responsible for shaping the course of mathematical development?

This story of unrealised mathematical development, however far-fetched, illustrates what might have happened if mathematicians had become interested in the systems of Pacific navigators. We can imagine that harmonic analysis would be much further advanced than it is. This is what happens. Mathematics absorbs good ideas, techniques, even symbol systems, and makes them part of the mainstream of the subject. The worth of the ideas are judged on mathematical grounds. But this is not a braid with independent strands woven together but retaining their individuality, this is a river with tributaries flowing in. However, we can reverse the situation.

There is another story, a real story, about Pacific navigation. In Hawaii there is a Polynesian Voyaging Society (<http://pvs.kcc.hawaii.edu/welcome.html>) that was established in 1973. There is another one in Tahiti. Many countries have established schools and courses in these navigation techniques. Ocean-going canoes are being built, both authentic replicas and modern versions, and are being sailed across the Pacific to take part in national celebrations, competitions, cultural exchanges, and on research voyages. Thor Heyerdahl's re-creation of a voyage from South America was the first that became well-known (Heyerdahl, 1958)—is it because he was a European, or is it just because he knew how to manipulate the media?

In these schools, on these boats, and as part of the curriculum for these courses, there is often mention of modern navigation techniques, use of modern equipment, and training in mathematical ideas. However these are used to enhance the development and activity of navigation derived from the original techniques. Ideas are co-opted, techniques are absorbed, mathematical systems are adapted to the necessities of Pacific navigation, and are judged useful or not according to its criteria. If this sounds like what is written two paragraphs back from the point of view of mathematics, then good. The parallel is exact.

It may be argued that what happens in Pacific navigation schools is not mathematics, it is navigation. Navigation uses mathematics, just like many applied sciences. A picture of a braid woven together with independent strands of its applications is easy to accept. If a collection of applied mathematics strands is all that is meant by the braid, then the history of mainstream mathematical development is not challenged. But the braid being argued for here is a braid of mathematical strands.

Remember, in the introduction, the difficulty with the word ‘mathematics’ was noted. Every time this word is used it conjures up connotations, based on personal experience, of school mathematics, university mathematics, mathematics as we know it now. We have been calling this NUC-mathematics. When producing an argument that involves a broadening of the concept of mathematics, there is a problem with how to express it. We need to escape the mindlock.

Let us return to the widened idea of mathematics, that of a QRS-system—a system developed to give meaning to the quantitative, relational, or spatial aspects of our world. Let us put some further requirements on a QRS-system, requirements that are usually associated with NUC-mathematics: reproducibility, levels of abstraction, generalisability, and symbolisation. Now look again at Pacific navigation.

David Turnbull (1991, p. 23), when considering Micronesian navigation, asks the question: “What is a navigation system”? Some characteristics mentioned are: it should be symbolic (and therefore transmittable); it should be manipulable (and therefore adaptable); it should be generalised (and therefore non-localised); and it should be open (and therefore innovating). Gladwin (1970) describes the system of navigation on Puluwat atoll. His (and others’) descriptions were further analysed by Hutchins (1983) in a way that made it clear that Turnbull’s characteristics are met. To quote Hutchins (1993, p. 223) “The Micronesian technique is elegant and effective. It is organised in a way that allows the navigator to solve in his head, problems that a Western navigator would not attempt without substantial technological support”.

Pacific navigation is not mathematics. Pacific navigation is not itself a QRS-system. But Pacific navigation does contain a QRS-system. Pacific navigation contains its own mathematics, a mathematics that is different in some fundamental ways from NUC-mathematics. For example, its criteria of accuracy are different (path accuracy is different from positional accuracy or distance), and its abstractions are different (path form is more important than map scale, and any scales may be time-based rather than length-based). We can discuss one set of criteria in terms of the other. We can transform the maps from one system to the other. That does not make them the same thing, nor can we assume that all features of the system are transformed intact.

The strand in the mathematical braid that carries the Pacific navigation QRS-system is smaller than that of NUC-mathematics. It is also wrapped inside a 'Pacific navigation' covering, but it is a mathematical braid nevertheless. The mathematics of standard, positional navigation remains as a fibre in the mainstream mathematical strand.

The picture of our mathematical braid is now one of a thick strand of NUC-mathematics woven with many smaller, braids that are disguised with other names. We have found a way of distinguishing different mathematics, no matter how limited in their application. Now let us look again at the main strand of NUC-mathematics. Is it what it seems?

2. A RIVER OR A BRAID?

When travelling to countries where you speak only a little of the language, or when talking to visitors who only speak a little of your language, a common response is to restrict the conversation to those things that are easily discussed, but about which there is likely to be common interest and agreement. As new grandparents spending six months in Spain, my wife and I became very competent at asking others, in Spanish, about their families: brothers and sisters, parents, children. If we were lucky and the people we met also had grandchildren, then we could hold a conversation that made us feel we could really speak Spanish, instead of the reality that we just had a minor facility in a couple of restricted areas. Always such conversations felt good, and left us smiling, and it wasn't just the remembered antics of Zephyr and Veronica. It was the joy of communication and shared common feeling.

Mathematics is a bit like this. That is to say, one of the mechanisms of mathematics is to focus on common features. It is natural that, when mathematicians talk, there is a tendency to talk about ideas that they have in common—we all do this, in every conversation. Even arguments depend on agreement on the topic and usually on the means of persuasion, although it does not always seem like it.

Some arguments do result from people talking past each other. These arguments are often unresolved, and usually lead to a feeling of dissatisfaction. Talking past each other can be cultural in origin.

My introduction to the phrase was in the title of a book for teachers (Metge, 1978) about cultural protocols and the misunderstandings they produce in classroom interactions. The result is alienation and isolation.

Since mathematics is formed and developed through communication, a consequence is that those parts of mathematics that get developed are those about which there is agreement. The areas of disagreement get dropped, or are only developed with difficulty. When research mathematicians come together in international communities, there are inevitably some difficulties of communication. Agreed symbolisms and definitions of mathematics make communication easier—but within a restricted domain. Here is the key point: that domain is restricted by the very agreements that make the communication possible. Where a definition is not agreed, or the nature of a named concept is different for different mathematicians, then we encounter talking past each other. Three examples of this have already been mentioned: non-standard analysis, the mathematics developed from Cauchy's concept of the continuum; the divergent paths of statistical analysis deriving from the two conceptions of probability; and Category Theory, the foundations of mathematics being written using functions, not sets.

What happens, however, is that these differences are, in some way, made invisible. There are several reasons for this, and several ways that it can happen. But the end result is the preservation of the sense that all mathematics is proceeding together in one large stream, a stream of different interests, but one stream nevertheless, with the happy family of mathematicians floating together along it. This may be what mathematicians feel, but below the surface, mathematics is made up of quite different ideas being developed, often interacting, and knowing of each other's existence, but conceptually different in important ways. Hence, the metaphor of a braid of many strands and fibres, is more appropriate than that of a river with tributaries.

One more important issue. The researchers in the international community of mathematicians are increasingly using only one language to communicate: English. It was noted above that mathematical communication is restricted by the agreements that make communication possible. One of those agreements is to use English. So mathematics is becoming increasingly restricted to the ideas that can be expressed in English, and mathematical development will increasingly be directed down paths that are privileged by English. This is not a new idea.

In the first half of last century, the linguist Benjamin Whorf wrote (1956, p. 244):

> ... but to restrict thinking to the patterns merely of English, and especially to those patterns which represent the acme of plainness in English, is to lose a power of thought which, once lost, can never be regained. It is the "plainest" English which contains the greatest number of unconscious assumptions about nature. ... Western culture has made, through language, a provisional analysis of reality and, without correctives, holds resolutely to that analysis as final.

If we have a thought or understand a concept, it can be expressed in English or any other language. All languages are endlessly creative and adaptable, and once aware of mis-communication or nuances in ideas that are not expressed in a particular language, then it is possible to find a way to express what was missed. The point is that there are some thoughts that are unlikely to occur at all if only one language is used.

Perhaps this is more clearly seen in another development, the communication of mathematics over the web. There are many mathematical systems on the web: Matlab, Maple, Mathematica, for example. Mathematicians routinely use these systems to generate and explore hypotheses, to test ideas, and to communicate with each other. A recent development is the building of a mathematical language from very basic concepts, basic enough that all the different mathematical systems can be written using these concepts (Borwein, 1999). Once that has been achieved, all the systems can be linked together, and can communicate with each other. This basic language is intended to become *the* language of mathematics. Given what we have said, the danger is apparent. Only mathematical ideas that can be expressed in this language are likely to be developed—or, at the very least, mathematical ideas expressable in this language will be strongly privileged. Do the writers of the mathematical web language really believe that they can write a universal language that will accommodate all future mathematical ideas?

In the mathematics braid some strands are bigger than others, some strands merge with each other or split apart, some strands are disguised within non-mathematical coverings. But if we regard mathematics as QRS-systems, I argue that mathematics consists of parallel systems, not one consistent body. Ethnomathematics can be regarded as the study of the different fibres of mathematical knowledge.

Such an image calls into question the universal origin of mathematics. There is no reason to assume that, at the beginning of the braid, there was only one strand. Indeed, if we look at the current situation where there is a tendency towards convergence of ideas, the more likely scenario is that mathematics had multiple origins. Joseph's diagrams (1992, Figs. 1.1–1.4) of the very early development of mathematics expose the paucity of what he calls a Eurocentric model of the history of mathematics. His final picture details the plaiting of the mathematical braid in the early millennia of mathematical thought. An argument of this book is that increased communication amongst mathematicians leads not to a single stream, but to more complex plaiting of many braids.

I believe that it is important, for mathematics, for human development, and for mathematics education, that we start to focus on differences between strands as much as points of similarity. If mathematics is to continue to blossom, and to express all the things that human thought can achieve, then we must resist any convergence of what is investigated. To do that we need to understand more about how the restrictions occur. That is the next topic.

3. SNAPPING TO GRID AND OTHER MECHANISMS

Take a trip, if you will, to Hawai'i, renowned for its tourist hotels, beaches, pineapples, and big surf. Hawai'i was—still is—a centre of traditional Pacific navigation and sea-faring. Of course, for a sea-farer, winds are critical, and the trade winds, being so constant, are a good source of information and direction. Thus words associated with winds are going to be important. One such word is the word for *leeward*. In Hawaiian this is *lalo*. Given the north-east trade winds, this would be used for the south-eastern side of the islands.

Now, Hawaiian is a Polynesian language, and there are some simple transformations that generally apply to this family of languages when you move from one to the other. To move from Hawaiian to Maori, the 'l' becomes a 'r'. Thus *lalo* becomes *raro*. In Maori, *raro* means 'under' or 'north', particularly when associated with the wind. I cannot find any Maori word for *leeward*. Is there a relationship between the Hawaiian *lalo* (leeward) and the Maori word *raro* (north)?

In New Zealand, could the word *raro* have originally meant *leeward*? New Zealand is far enough into the southerly ocean that the dominant wind is the cold southerly or south-westerly. Thus leeward would be in the north or north-east. Or, perhaps, *raro* just had the other meaning of *under*. As noted before, the North Island of New Zealand is *Te Ika a Maui* (The Fish of Maui) and its head is at the bottom where Wellington now sits. That is why this region is known as *Te Upoko o te Ika*—The Head of the Fish. When you travel to the tail of the fish, that is the north-northwest part of the country, you go down. Under. *Raro*.

Whichever of these explanations is correct, *raro* meant either north-northeast or north-northwest, but referred to important characteristics of the geography of the country, not to due north.

When the Europeans arrived with their NSEW compass as a dominant reference, it seems likely that the word for the direction closest to north got adapted to due north. At this point one reference system transfers to another, and the language changes in response to a shift in spatial system. In contemporary dictionaries, *raro* means north. The phrase "snap-to-grid" is familiar to those who have tried to draw pictures in their Word documents on a computer. The lines automatically adjust to an invisible grid on the page, moving slightly from where you place them so that they join up exactly.

I wonder if the early attempts to create Maori word-lists also contain an example of this effect. Trinick (1999) reports that:

> In 1793, Lieutenant-Governor King of New South Wales, Australia visited the northern part of the North Island [of New Zealand] and collected information relating to the country and Maori. The information collected was published in Collin's *History of New South Wales* in 1804. The Maori numerals (pp 562) are misspelt but recognisable;
>
> 1: Ta-hie *(Tahi)* 2: **Du-o** *(Rua)* 3: Too-roo*(Toru)* ....

The accepted Maori words are in brackets. It is curious that the only sound that is clearly wrong is the 'o' on 'duo' (the Maori 'r' sound is very like a 'd'). Could this be an unconscious slip because Italian (and Latin, which, presumably the educated Governor would have known) have the word 'duo' for two?

"Snapping-to-grid" is one of the Universalising mechanisms by which mathematical development hides its differences or unifies itself.

Universalising mechanisms is the name I give to the ways in which mathematics normalises or links new ideas to the conventional mainstream, whatever their origins, (for further discussion see Barton, 1996, Section 4.3). If unification is successfully achieved, then there is no challenge to the rationality or correctness of existing mathematics. Rather, it enhances the subject by showing it to be, yet again, robust enough to accommodate new ideas, or even more richly intertwined.

During this process radical change may occur. Mathematical terms and concepts are continuously created, or may be re-created in the form of the old, but with new substance. Thus there is the appearance of old terms encompassing the new situations, when, in fact, new concepts are involved.

"Snapping-to-Grid" is a Universalising mechanism that is like colonisation. It transforms new ideas into existing terminology, thereby stripping them of their distinctive aspects, and, in particular, removing cultural characteristics. The ideas are acknowledged to be mathematics, but are not acknowledged to be mathematically new. The most common example is the way counting terminology in different cultures is transformed into direct equivalents of one, two, three, four, ... in the cardinal mathematical sense. The words may never have been used in this sense, as an example in addition to those in Chapter 3, in Burmese, the vast array of number classifiers (Burling, 1965) for use in different situations reduce to a single set for mathematical discourse.

The justification for such colonising is the principle that stripping of context is exactly what mathematics is about. Practices from other cultures are interesting only in so far as the 'real' mathematics can be found. What is forgotten in this justification is that mathematics has a context expressed through the language and symbolic conventions of its host culture. An effect of implying that any new ideas are merely reformulations of ideas already part of mathematics, is to maintain the source of the new ideas in an inferior position. Thus cultures that do not have counting words beyond 50, say, are demonstrated to be less mathematically sophisticated. Such notions lead to the idea of primitive cultures (Stigler & Baranes, 1988).

Another Universalising mechanism is subsumption. Subsuming mathematical ideas does not involve translation of the idea into new terminology, it relegates the idea to the status of an example. Like colonisation, the implication is that the idea is not new; unlike colonisation, the idea is not even regarded as mathematics itself, just as an example. Such examples are welcomed as interesting, and educationally illustrative, but they are not worthwhile in a mathematical sense. This is 'artefact' mathematics. An example of this mechanism is the identification of certain types of artistic decoration as mathematical. For example, in New Zealand, the Maori *kowhaiwhai* (rafter patterns from Maori Meeting houses) are recognised as mathematical strip patterns exhibiting symmetric groups and used in school publications to teach transformation geometry.

The result of this process does not necessarily remove the mathematical idea from its cultural context. On the contrary, the retention of its cultural surroundings is exactly what is required when subsumption occurs in an educational context. But the effect is to reinforce the idea that a cultural context can only be an example of mathematics, it is not mathematics itself. Any different, deep mathematical idea behind the artefact is now even less likely to be examined.

Yet complex ideas in mathematics can be found in cultural craft practices. The patterns formed in the weaving of Maori flax baskets (Pendergast, 1984, 1987) were also used to demonstrate mathematical groups and used in school resources (Knight, 1985). In doing this the conventional NUC-mathematics criteria of symmetry were used for classification. But the Maori names for these patterns form a different classification, grouping together patterns that are not easily recognised as similar in our eyes. However, to a weaver's eyes, the groups make sense: the classification depends on how the initial strands are set up. One group comes from strands set up as alternating white, black, white, black, white, black, white, and then different patterns made by different weaving; in another group the strands are white, white, black, white, black, white, black, white, black, white, white, … (see Fig 6-1).

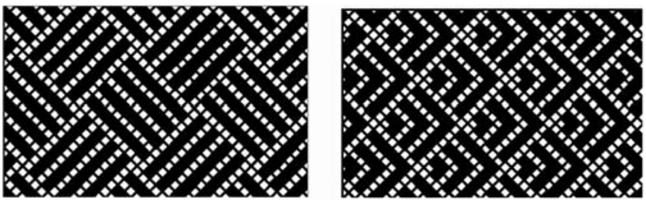

Figure 6-1. Weaving patterns from the same strand set-up

The two classification systems are not compatible, but are equally mathematical. I once used the ‘strand set-up’ classification as a talk on triple weaving patterns to a group of mathematicians, and discussed how two-colour patterns could transform into each other. The response? “Ah,” said a colleague, “your transformation group looks like a structure from hyperbolic geometry”.

The justification for subsumption is the principle that mathematics provides powerful ideas for solving a variety of situations. Therefore the identification of a known idea in a new situation, provides another opportunity to apply known results. Sociologically, subsumption has the effect of establishing status. If one idea is accepted as an example of another, then the example is relegated to a lower status, and its originating context is deprived of the intellectual credit.

A third Universalising mechanism I call appropriation. Appropriation of new ideas acknowledges the novelty in the ideas (unlike snapping-to-grid or subsumption), but assumes that they form part of an existing mathematical structure. This is done either by regarding the new idea as a new category in an existing hierarchy, or by creating a new generalisation under which existing mathematics and the new idea will both fit. In this process the mathematical concepts may change, for example ‘logic’ now includes multi-value logic although it originally only referred to Aristotelian logic. The appropriation effect becomes clear: it is assumed that Aristotelian logic provided the foundation for today’s logic, when it only provided the etymological origins. The investigation of swells as advanced Fourier Analysis could be another example of this.

The justification for appropriation is the assumption of generalisability: it is always possible to obtain a mathematical idea of greater generality to bring together previously unrelated concepts. Generalising usually involves greater abstraction, and the mechanism provides a way to reapply existing knowledge to new situations. The danger with this process is that once a generalisation has been made it is more difficult to perform a different generalisation. The assumption of universal structure mitigates against seeking other abstractions once the idea has been fitted into one satisfactory hierarchy. The sensation of a single universal structure is thereby enhanced.

4. REJECTION AND ISOLATION

The three Universalising mechanisms in which mathematics draws in ideas from other areas are complemented by ways in which it can reject other ideas if they do not fit with existing mathematical conventions by labelling them as something other than mathematics. I call processes of rejection Isolating mechanisms. The effect is to retain mathematics as a stable and 'true' field, not allowing other forms of 'truth' to be called mathematics. This can be successfully achieved because the arbiters of mathematics are mathematicians themselves. Society in general cannot tell mathematicians what their field is like.

The first Isolating mechanism is non-recognition, or the rejection of the idea as having anything to do with 'proper' mathematics. The people who have the power to define mathematics, for example, journal editors, appointment committees, or curriculum designers, place the new idea outside the borders of the field.

An example is the attitude of many mathematicians towards much of the mathematics originating in the East. Joseph (1992) and Berrgren (1990) have both documented the rejection of much of the mathematics from India, China, and medieval Islam as non-rigorous. The results are only accepted after they have been proven in an acceptable (Western) manner. When they are proven, or analysed, they carry the name of the Western mathematician who did the work: hence we have Pascal's Triangle and Pythagoras' Theorem when these ideas were known for centuries before Pascal and Pythagoras. Knowledge of Pascal's Triangle is attributed to Jia Xian who lived 600 years before Pascal (Stillwell, 1989, p. 136). Evidence of Pythagoras' Theorem can be found in the Chinese text *Chou Pei Suang Ching* that may date from 500 years before Pythagoras (Swetz & Kao, 1977, p. 14). One of the subtleties of this mechanism is the way that the number of new theorems is taken as the measure of mathematics achievement (Davis & Hersh, 1981, pp. 20–25). People who do not (or did not) prove theorems are therefore not mathematicians.

The justification for non-recognition is the importance of convention as a basis for mathematical knowledge. How can mathematicians be sure of their results if there is a variety of foundations for the acceptance of mathematics? Sociologically, it is only by establishing the boundaries of a discipline that those within it can control their own activities. However, making the contrary statement: "this IS mathematics", and exploring the possibilities which are thereby opened up can be interesting. For example, Marcia Ascher explores the set theoretic structure of inheritance patterns (Ascher, 1991, pp. 72–76), and investigates the mathematical ideas that could newly illuminate a game analysis, such as the Maori game Mu Torere (Ascher, 1991, pp. 97–108).

Dismissal is a second Isolating mechanism. It recognises the mathematical component of a new idea but makes it unworthy of consideration. The new idea may not be described in acceptable terms, in an appropriate forum, or by someone of the required status. The effect is to devalue the new idea. The justification for dismissal is the maintenance of standards, but sociologically it can be seen as legitimisation. Social systems regulate themselves in various ways, from formal regulations to sub-conscious peer-pressure.

A famous example is the rejection of Ramanujan's manuscripts (Hardy, 1978, Preface). These contained some of the best mathematics of the century, but had been previously rejected without comment by two notable English mathematicians of the time to whom they had been sent before Hardy recognised their worth.

Another Isolating mechanism is compartmentalisation. It recognises the mathematical nature of the new idea, but places it outside mathematics proper, into a related discipline or a new field. This mechanism often carries inferior connotations, for example the 'number crunching' label attached to numerical analysis (the mathem-atics of computer methods) in its early years. Mathematical computing is a good example of an area which has established itself sufficiently to now affect the nature of mathematics itself (Epstein & Levy, 1995).

A historical example is the work of Florence Nightingale. No-one has ever recognised as a mathematical achievement her analysis of the causes of high mortality in field hospitals and maternity wards. It is now acknowledged to be a forerunner to the development of statistics as a discipline (Cohen, 1984), but at that time, such a field did not exist. Locating it now as statistics is partly to deny her work as mathematics.

Universalising and Isolating mechanisms not only occur as part of the colonial process when mathematical ideas from two cultures meet—as when Western reference systems dominated Pacific ones—but also operate internally within mathematics.

A request to a group of my mathematical colleagues to give me examples of Universalising resulted in each of them thinking of personal experiences where one thing they were working on or thinking about was suddenly "snapped to grid" or subsumed by another existing mathematical idea. The results are not always negative. The most famous example is Vaughan Jones' discovery of his knot invariants (now called the Jones polynomial). In the words of my colleague (Conder, Personal communication, 2006):

> He was working on aspects of subfactors of von Neumann algebras, and derived some equations associated with these, that turned out to look just like the braid relations from knot theory. Joan Birman and others helped him to "snap to the grid" of knot theory, and the rest is ... well ... history! This happens all the time, but usually not so spectacularly! [The 'history' in this case, was the Fields Medal, the mathematical equivalent of a Nobel Prize].

Lakatos (1976) talks about "monster-barring" as the way that mathematicians defend their proofs against counter-examples. This can be interpreted as a form of the Isolating mechanism non-recognition: the mathematician does not recognise the counter-example as relevant to the particular class of objects under discussion.

What has just been described are several ways in which the discipline of mathematics preserves the idea that it is a universal subject based on a single set of principles. This description is necessary if the argument of this book is to be accepted: if mathematics is to be seen as a braid of many strands, then it is necessary to explain why it has seemed like a river fed by tributaries.

I am not making a negative value-judgement, nor suggesting that mathematicians must start behaving differently. Rather, it is an attempt at a description of what happens. We need to recognise these processes if we are to fully understand the nature of our subject. Understanding what happens enables us to take another look at our field, to ask some other questions, and thereby consider other approaches to mathematical ideas that may be productive.

5. MATHEMATICS, SOCIETY & CULTURE

My Universalising and Isolating mechanisms are not the first attempt to describe what is happening in mathematics that explains its apparent universality. Others have written sociological accounts of mathematical knowledge, from Spengler, to Bloor, to Restivo (E.g. Spengler, 1926; Bloor, 1973, 1976, 1994; Restivo, 1983, 1992, 1993). Bloor, in particular, has attempted to use the anthropological theories of Mary Douglas to describe the Lakatos version of mathematical development (Bloor, 1978).

His programme has an even stronger aim than this. He seeks to break down the reification of mathematics as beyond sociological explanation (Bloor, 1973, p. 190), and to describe the mechanisms by which social and institutional circumstances (I would want to add cultural context) strongly determine the knowledge that scientists produce.

He focuses on mathematics and logic because this form of knowledge has been regarded as the most rational, a priori, and therefore the least likely to have sociological foundations. Bloor presents a number of examples of existing alternative forms of mathematical thought, and speculates on their social causes. For example (Bloor, 1976, pp. 125–9), he argues that the crises surrounding the development of calculus and the use of infinitesimals arose solely because the mathematicians attitudes to rigour had changed. The decline in rigour in the sixteenth century, in recognition of the practical results non-rigorous methods produced, actually allowed the infinitesimals to appear in calculations for the first time. The renewed interest in rigour in the nineteenth century produced a crises where there was not one before—and out of that crisis arose new mathematics.

Bloor also examines the historical process for the way in which it covers up variation, and concludes that the cumulative nature of mathematical development needs to be challenged. In responding to critics of his view (1976, pp. 179–83), he again makes the point that marginalisation of alternative mathematics' does not negate them, it just shows how a social cause creates an illusion of absolute knowledge.

Bloor's later work on mathematics (1983, Chpt. 5.; 1994) draws heavily on a Wittgensteinian analysis of the nature of mathematics to justify the idea that we construct conventions of meanings about numbers and relations as much as about words. The sociology of mathematics, in his view, aims to expose those conventions which have operated.

Davis (Davis, 1993, pp. 189ff) has also written about the relationship between mathematics and society. He argues that mathematics constitutes a way of thinking which is different from other ways, and that different ways of thinking need to be balanced in our society. For example, there has been a long literature concerning the use of mathematics in the social sciences. Kaplan (1960) gives an account of some early attempts—including a mathematical characterisation of sociology itself in which every social situation may be described by an equation. Catastrophe Theory, developments in Game Theory, and mathematical theories of politics all contribute to the mathematisation of social science. But it is not just the encroachment of mathematics into social life which is the subject of Davis' concern. He argues that computerisation, for example, has fundamentally changed our modes of thinking (Davis & Hersh, 1986).

For Davis, the balance of mathematical versus other types of thinking is to be achieved through education, hence (Davis, 1993, p. 190):

> I should like to argue that mathematics instruction should, over the next generation, be *radically* changed. It should be moved up from subject-oriented instruction to instruction in what the mathematical structures and processes mean in their own terms and what they mean when they form a basis on which civilization conducts its own affairs. .... [This requires] the teacher to become an interpreter and a critic of the mathematical processes and of the way these processes interact with knowledge as a database.

He sums up:

> If mathematics is a language, it is time to put an end to overconcentration on its grammar and to study the "literature" that mathematics has created and to interpret that literature.

Davis' makes a convincing case for this consequence of a sociological view of mathematics. The case is even more persuasive if a world view description of mathematics is correct. If there are alternative mathematical languages which may be enculturated in any education system, it is imperative that every society produces the means to question these ways of thought, and to make informed choices about how dominant they are to become. This theme is developed further in the next section.

But finally, to finish this chapter, and before we turn to the philosophical implications of these ideas, let us take a small diversion into the world of the Kama Sutra and discuss the issue of mathematics in society.

We are familiar with the uses of mathematics in science, technology, economics, and industry. Mathematics as an applied science seems to provide the *raison d'etre* for the investment and effort that societies spend on mathematical development. Yet many mathematicians claim that the real reason for studying the subject is its own joys (Hardy, 1941), and David Singmaster's Chronology of Recreational Mathematics (2006) goes back three thousand years. The unique attraction for mathematics and the role it can play is best illustrated by the (truly) unexpurgated version of the Kama Sutra.

There are, unfortunately, no fully unexpurgated versions of the Kama Sutra in English. All translations have an important chapter omitted. Why? Too lascivious? Well you might think so if mathematics was your passion. These are hot mathematics problems. Mathematics problems? As the introduction to the Kama Sutra? Yes, indeed. There exist Sanscrit manuscripts which make reference to mathematics problems in the Kama Sutra, problems couched in the most delicate language and using sexual imagery. We have examples of similar problems from Aryabhata (c. 800AD), Mahavira (c. 850AD), and Bhaskara II (c. 1150AD).

One problem from Bhaskara II concerns a bee that falls into its lover's lotus flower, which closes upon him. Upon asking her to let him out, she responds that he must first solve the mathematics problem (the translation below is George Joseph's (1992) adaptation of Colebrook's original translation):

> From a swarm of bees, a number equalling the square root of half the total number of bees flew out to the lotus flowers. Soon after, 8/9 of the total swarm went to the same place. A male bee enticed by the fragrance of the lotus flew into it. But when it was inside the night fell, the lotus closed and the bee was caught inside. To its buzz, its consort responded anxiously from outside. Oh my beloved! How many bees are there in the swarm?

Here is a problem from Mahavira's *Ganitasarasamgraha* (again the translation is an adaptation by George Joseph (1992), this time from the original translation by Rangacharya):

> One night in spring, a young lady was lovingly happy with her husband on the floor of a big mansion, white like the moon and situated in a pleasure garden with trees bent with flowers and fruits. The whole place was resonant with the sweet sounds of parrots, cuckoos and of bees which flew around intoxicated with the honey from the plentiful flowers. In the course of a "love quarrel" between the husband and wife, the lady's necklace came undone and the pearls scattered all around. One third of the pearls reached the maid-servant who was sitting nearby; one sixth fell on the bed; one half of what remained (and one half of what remained thereafter and again one half of what remained thereafter and so on, counting six times in all) were scattered everywhere. On the broken necklace it was found that there were 1161 pearls left. Oh my love, tell me the total number of pearls on the necklace.

Now what were these problems for? What part did problems like these have in a sex manual? The answer lies in the social context. At that time in India, the high society for whom the book was written was extremely well-educated in mathematics. Solving mathematical problems was a pleasure and delight that was part of the social scene. It could perhaps be compared with cryptic crosswords for some people nowadays.

So, what happens when a couple meet together after a long hard day at the office? Do they leap straight into bed? No, that would hardly be a romantic and sensitive way to behave. First it is necessary to reconnect as people, and what better way than to engage together in some gentle recreational activity, like, well, like solving a mathematics problem together. And if the mathematics problem is written in a suggestive way that might lead you on to more intimate things, so much the better.

We know these problems are not to be taken too seriously. The answers, for example, are not realistic. The answer to the Pearl Necklace problem is 148 608. That is a lot of pearls to count when there are better things to do. Joseph calls this a fantasy necklace and notes the fascination for very large numbers at that time—the content is more abstract than the erotic context suggests.

Mathematics and its role in society? There are clearly more possibilities than we ever dreamed about.