Chapter 4

THE EVIDENCE FROM LANGUAGE

Abstract: The evidence from language in the preceding chapters is used to question common assumptions about mathematics. The links between mathematics and English language are explored through examples of the use of the words 'open' and 'normal'. The chapter concludes with a summary of the evidence presented so far.

Keywords: language and mathematics, open sets, normal distribution, abstraction, generalisation

In the first part of the book I presented some evidence from different languages together with some mind-games, and suggested that mathematics did not need to develop as it has done. We do not generally consider mathematics as one of several options. I must now, therefore, augment Part I by providing a coherent picture of mathematics that both explains how this can be so, and also fits with our experiences and perceptions of the subject.

Part II fills out the picture of mathematics, giving just such an account of how it originated, how it develops, and what it means. Chapter 5 starts with two examples of a different kind of evidence from language, and then reviews the implications of all the language evidence. In Chapters 6 and 7 the origins and development of mathematics are discussed respectively, and finally I address some philosophical issues. Part III examines educational implications.

1. TWO WORD STORIES: NORMAL AND OPEN

The first story concerns the word *normal*. This word first appeared in the English language in the 16^{th} or 17^{th} centuries, with a mathematical meaning. It occurs, for example, in *The English Euclide*, a translation into English of Euclid's text written in 1696. In that document it meant *right*, as in right-angled, or *rectangular*.

The origin of the word is Latin, starting with the word *norma*, which was the name for a carpenter's square, the pattern that a carpenter used for making exact, right-angled corners, or checking that posts were upright. Today a *norma* is called a set-square, and used in schools and graphic design as well as on building sites. From *norma* came the word *normalis*, meaning "made according to a carpenter's square" and, eventually, by the 15^{th} century, in late Latin, this word had come to mean "in conformity with the rule".

But this is not the end of the story. Someone who is normal is not just someone who conforms with the rules, they are someone like us—well, like me anyway. A normal programme is not the one that follows the rules, it is the one that occurs most frequently. "Most frequently" sounds like probability and statistics—and it is.

Through the 17^{th} and 18^{th} centuries the subject of probability emerged, originating in the interest in gambling in France by the mathematician Blaise Pascal (Hacking, 1975). Indeed, the word *probability* did not occur until 1657. Our word *normal* was still in use mainly as a mathematical term, but also, for example, in the French *école normale*, meaning "by the rule". The *école normale* were schools set up under the Republican foundation in 1794. Then, as late as 1892, *normal* got a new mathematical meaning. It was the name given to the probability distribution that occurs in nature, the Bell Curve as it is sometimes known (see Fig. 4-1).

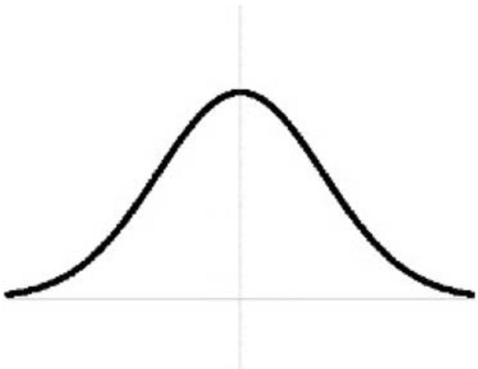

Figure 4-1. The Normal Curve

It was discovered that this distribution modelled what happened for many aspects of human and animal populations: height, weight, performance in mathematics tests, performance in intelligence tests. There seemed to be a general rule governing such data. The mathematical model was therefore named a *normal distribution*, and became a way of categorising. You could now find out whether you were close to the "norm", that is, average height. Young parents are familiar with the consequences. There are charts of baby weight against age, and lines drawn on the chart to indicate the percentiles (see Fig. 4-2).

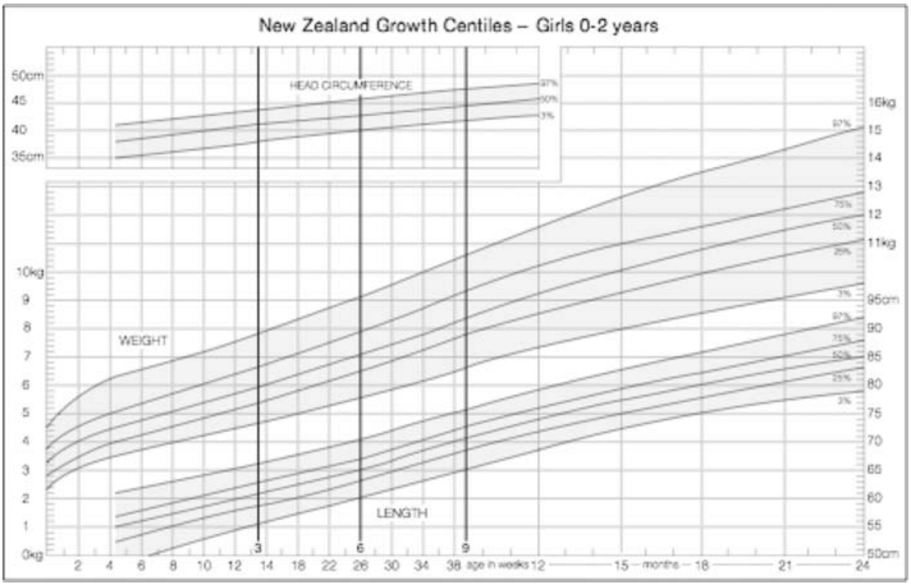

Figure 4-2. Baby Weight & Length Chart
(Reproduced with permission from the New Zealand Ministry of Health *WellChild Tamariki Ora Health Book*, Wellington: Ministry of Health, p. 70)

If your baby comes between these lines, then all is well with the world, grandparents are content, parents don't lie awake worrying about whether they're guilty of mal-nourishing or overfeeding their babies, and the social gathering at the playground is a proud display of ... of a *normal* baby.

That's right. Prior to 1926, when the word *normal* was first used to describe a population, there was no such thing as being normal. Babies just were. Some babies were different from others. Some people were different from others. A few people were a little odd, a little idiosyncratic sometimes, but that was accepted as how the world was. Only in the last 100 years has the concept of normality come to play a major role in how we talk and think about ourselves. As a result of being at the extreme end of the normal distribution, many healthy people have felt guilty about themselves, been put on medication, or been locked up in mental institutions. The sad part about all this is that the mathematics says there will always be some people at the extreme ends of any measurement that is made. Healthy people. Normal people. This is an example of mathematics affecting language and thought.

The second story is about the word *open*.

As a member of a research project I once attended an international conference of mathematical researchers in the field of topology. Topology is that part of mathematics that deals with the mathematical structure of concepts like nearness and continuity. Topologists concern themselves with what it means to say that one number is 'next to' another, for example. Or whether it is possible to have a small finite area that has a boundary that is infinitely long? (The answer is yes!)

Our team was investigating whether the languages that topologists speak affect the way they understand their very abstract subject (Barton, Lichtenberk, & Reilly, 2005; Barton & Reilly, 1999). Breakfast. I sit at a table with three topologists from three different countries. I ask how the name 'Open Set' came into existence. It does not matter, for this story, what an open set is, except to say that it is an absolutely fundamental concept in topology—one of the concepts on which everything else is built. After a short argument about who first used the term, I changed the question to which of the many meanings of the word 'open' was being used here.

"Ah," says the first topologist, "that is easy. Actually any word could have been used, so long as it had an opposite, since it is the relationship between an open set and a closed set that is what is important. Open/closed. Yin/yang. Black/white. It could have been any of these. It is the sense of complementarity that is being expressed."

"What?" queried the second. "I don't think so. The meaning of open in this context is the one used of an international border: anything can pass through, there is no well-defined restriction on what makes the border."

"Oh," mutters the third quietly (he was the junior member of the group), "I always thought that what was meant was the idea of without any boundary at all—like we refer to an open field, the open sea, or an open question."

Fortunately, not being a topologist, my view was neither expected nor important. Which was just as well, because I had imagined that the sense of open being referred to was that of a door. It can be open or shut, it depends what you want to do with it.

Why did the four of us hold four different views—was it language background, prior experiences, or the way we were taught? Is one right and the other three wrong? For the three who are research topologists, does it make a difference to the mathematics they do with open sets that they think about the meaning of open in different ways?

These three topologists each had a different understanding of the word naming the fundamental building block of their research field. It is difficult to imagine that this does not affect the way they research this highly conceptual area of mathematics. This story is an example of the potential for language to affect mathematics.

2. REVIEWING THE EVIDENCE

These stories do not prove anything, however they are further parts of the picture of the relationship between everyday language and mathematics. A picture of close ties between the two, of each affecting the development of the other, both in the past, and in the present.

Let us be clear about what this part of the book is trying to do, and what it is *not* trying to do. I am trying to paint a consistent picture of mathematics (its nature, its development, how it is connected to human thought) that fits with the evidence from language. What I am *not* trying to do is argue that all other views are wrong—although I will point out, in places, where the evidence from language contradicts some other conceptions of mathematics and its history.

For example, you will not find a denunciation of the Platonist conception of mathematics as an ideal world to be uncovered, nor of the formalist idea that mathematics is simply the setting up of rules and exploring their consequences. I just raise some questions about them. Nor will you find a challenge to the history of mathematics that sees the subject as a single river of development fed by tributaries of contributions from different mathematicians. Rather, I show that an alternative view of a braid of many fibres will also fit the evidence.

There is not room in this book (nor do I have the ability) to disband all other philosophical positions, nor to survey all the writing on the historiography and social influences on mathematics. Rather the book argues that there are some interesting things about the way different people talk mathematically, and that this suggests a picture of mathematics that is somewhat different from many accepted views. But this picture is consistent, and does "make sense" on the evidence available.

Now, let's summarise the evidence, as opposed to recounting anecdotes and flirting with the imagination.

First of all, everyday mathematical talk, that is, general language used to discuss quantity, relationships and space, can be quite different in different languages. For example, with respect to the grammar of quantity, the Polynesian verbal use of numbers, the Kankana-ey adjectival use, and the Dhivehi nominal use are significantly different from the English or Spanish use, a way of speaking that can move between adjectival-like and nominal-like.

Not only are there differences between different languages, but also everyday mathematical talk is changing within each language. For example, the modern Maori grammar of numbers is different from the Maori grammar of numbers before European contact. What causes the change is not clear, and there are likely to be many complex, interacting influences. (Although, in this case, there is evidence that the involvement in language development by those from another language background may have been significant. For example the involvement of missionaries in creating a written form, and the involvement of mathematicians taught in English in establishing a Maori mathematical discourse).

The third point is that the direction of change is towards more similarity. For example, as Dhivehi and Euskera are used in more technical mathematical areas, and as they are used in fields where English or French or Spanish are international mediums of communication, so Dhivehi and Euskera move towards grammatical forms, for example the grammar of numbers, that mirror those of the international language.

Note that these languages were not chosen for discussion *because* the grammar of number was different. It is not true that other non-Indo-European languages were studied, found to be similar to English, and then left out of this book. Each language I encountered had some feature of interest. This suggests that different and changing mathematical grammars are likely to be widespread phenomena.

The examples from the language of space gave evidence that there are linguistically related preferences, or predispositions, to see location and shapes in ways that are conceptually different. Furthermore, these concepts can give rise to formal systems that are different from NUC-mathematics, but perform some of the same functions. If it is true that geometry built up from the way humans conceived of the space and shapes around them, then it has been shown that geometry could have at least started differently, using different basic concepts, and that other graphical representations could have become more familiar.

The examples about the language of number similarly show that the way we describe quantity in NUC-mathematics is not universally familiar, but mirrors that of English and other Indo-European languages—the main languages of mathematical development.

The examples from the language of relationships confirm that categorisation and argumentation do not have universally applicable characteristics. In these examples, unlike those of the alternative geometries, it is not necessarily possible to map one system onto the other. The implication is that categorisation and argumentation are context dependent, and are, in our everyday world, to be judged on their utility within that context. The question for NUC-mathematics is whether it wishes to remain a context only ruled by one form of argumentation, or whether, as a discipline, it can become open to QRS-system investigations ruled by other forms of logic and categorisation?

Another aspect of the evidence is that mathematical processes like formalising, generalising, abstracting, or symbolising are all represented within the examples described. However, since it is everyday language we are talking about, many of the QRS-systems and their mathematical processes are embedded in particular activities, like navigating, weaving, land measurement, or resource allocation. (The study of the mathematical aspects of these systems is known as ethnomathematics (Ascher, 1991; Barton, 1996; Contreras, Morales, & Ramirez, 1998; Monteiro, 2002; Powell & Frankenstein, 1997)). We have seen that at least some of these concepts and systems can be extrapolated in a formal mathematical way to resemble elementary NUC-mathematics. The example of Action Geometry re-maps the relationship between some geometric objects, and appears to deal more easily with some geometric features and less easily with others.

A final note on this review of the evidence in Part I. After investigating mathematical talk in other languages I am left with questions about where the different conceptions came from? Are they linguistic accidents, or do they reflect different physical environments or social activities? Let us move on, then, to discuss the origins of mathematics.