Chapter 3

QUANTITY: TRAPPING NUMBERS IN GRAMMATICAL NETS

Abstract: The grammar of numbers is explored through consideration of Maori, Kankana-ey, and Maldivian languages. This is used to discuss the ways in which we refer to numbers in English, and how that hides important mathematical ideas. The mathematical benefits or drawbacks of different languages are considered.

Keywords: Maori, Kankana-ey, Dhivehi, grammar of numbers

Chapters 1 and 2 have dealt with the way two spatial topics are described in different languages: defining position and finding your way. In this chapter we return to the topic of quantity in QRS-systems. (Remember that this is my code for a system for dealing with the quantitative, relational or spatial aspects of human life). Quantity involves number and measurement. The relationship between them is discussed later in Chapter 6. Initially, I want to look at numbers only, focussing on their grammar. The different bases of number systems have long been investigated and are not reviewed here (see, for example, Menninger, 1969; Lean, 1995).

My quest for other ways of talking about numbers began, it will be remembered, by the realisation that the Maori (and Tahitian) languages treated numbers in a way that was unusual for an English speaker. Before we examine this in detail, let us first think about the grammatical roles played by numbers in English, both in general discourse, and also when discussing mathematics.

Numbers are regarded, in English, to have their own grammatical category. However, in general, everyday discourse, they act more like adjectives than anything else: they seem to describe a characteristic. I could ask you to give me three pens, just as I might ask you to give me green pens. Threeness is a characteristic of the group of pens you are giving me, as is the fact that they are green.

In mathematical talk, numbers shift their grammatical nature. We discuss numbers as objects in themselves. We can say that five is a prime number, in the same way that we might say that a whale is a sea-going mammal. Numbers themselves have characteristics, for example primeness, or evenness, or divisibility. A number is often, grammatically, used as a noun.

Sometimes numbers are used in their adjectival sense and in their nominal sense in the same sentence. "Three fives are fifteen." The three is adjectival, the five and fifteen are nominal—the five is even made into a plural (there are three of them, just like you can have three hugs or three kisses). The important point is that all of this feels quite natural in English, we are not even aware of the different grammatical uses of number words, and we move between them quite easily depending on what we are trying to say.

This is not the case with all languages.

1. EMERGING NUMBERS: POLYNESIAN LANGUAGES

During the development of a Maori mathematical vocabulary, it was realised that numbers in old Maori (before European contact) were verbal in their grammar. There are still traces of the verbal use in modern Maori. The way that they are usually used in modern Maori is becoming increasingly like the way they are used in English. There is debate about whether this constitutes corruption or is evidence of a modern, living language. More on that later, let us first briefly review the verbal evidence.

In modern Maori grammars, as in English, numbers are regarded as having their own grammatical category. However, for the reasons outlined in the Introduction (the way a number statement is negated) and at the beginning of Chapter 1 (the use of particles with number words), this category is verbal in nature compared with the more adjectival English use.

A recent Grammar of the Maori language (Harlow, 2001) describes the verbal nature of numbers in Maori, focusing on their use with verbal particles. *E*, *ka*, *kua* and *i* are all tense markers, and kia indicates a wish or a command. All are used with numbers. Two examples are below. I have added my more verbal translation:

There are two houses in this street

(Maori translation):	*E rua*	*nga whare*	*kei roto I tenei rori*
(My translation):	Two-ing are	the houses	in the street
(Given translation):	The houses that are in the street are two in number		

Give me five pens

(Maori translation):	*Homai*	*kia rima*	*nga pene*
(My translation):	Give me	let them be fiv-ing	the pens
(Given translation):	Give me, let the pens be five in number		

Notice how the Grammar writer has tried to maintain the English grammatical role for the number words, but in doing so has distorted the way the sentence is constructed in Maori. It is actually consistent to think of the numbers as pure verbs.

Second language Maori speakers usually use the particle *e* in front of numbers, but often otherwise treat numbers as they are treated in English. In the mathematics classroom this is particularly true. When this happens the *e* makes no sense except that "it sounds right".

In order to make a number into a noun, it must be preceded by an article: *te* (the singular), *nga* (the plural), or *he* (a). So, to translate a mathematical sentence:

Five is a prime number

(Maori translation):	*He tau toitu*	*te rima*
(My translation):	A prime number (is)	the five

Thus, for Maori, having a mathematical discourse involves changing the grammar. In English the adjectival and the nominal use of 'three' do not involve a change to the word or its accompanying words—it is only a matter of word order. In Maori the change involves changing verbal particles (for example, *kia*) to an article (for example, *te*). This makes it sound strange to a native speaker.

2. NUMBERS TRAPPED AS ADJECTIVES: KANKANA-EY

The situation is more difficult for some other languages, where a conventional school-level mathematical discourse forces even greater alterations of accepted grammar. We will now look at the language Kankana-ey, spoken around Sagada in the mountainous regions of the northern Philippines.

Kankana-ey is a language that uses an unusual feature when putting an adjective with a noun. In linguistics this is called a ligature. This is a small word, in this case *ay*, that is put between an adjective and a noun. Thus you would say (the adjective comes first in Kankana-ey as in English):

tall children	=	*anandu ay ungung-a*
wide rice terrace	=	*nalawa ay payew*
white stone	=	*puraw ay bato*

This construction is also used with numbers:

four children	=	*epat ay ungung-a*

Compare the two sentences:

Do you have a raw banana?	=	*Ay wada nan maata ay baat?*
Do you have five bananas?	=	*Ay wada nan lima ay baat*

Thus, as far as the language is concerned, a number is grammatically fixed as a characteristic of something, like its colour or its dimensions. In fact the descriptive role of numbers is even stronger than that of some other adjectives. For example, when it is the existence of the characteristic that is being emphasised, then the structure changes for all characteristics except numbers, which is the only one that keeps the ligature:

The children are tall.	=	*Anandu nan ungung-a.*
The stone is white.	=	*Nan bato et puraw.*
There are four children.	=	*Wada nan epat ay ungung-a.*

It is possible, as in English, to construct sentences where the things being counted are suppressed but understood to be present. This grammatical feature is called ellipsis. For example the noun ‘people’ can be dropped in the sentence:

There are six people in the house, five are women	=	*Wada nan enem ay ipogaw sinan abong, babbai nan lima.*

Also it is possible to give just a number as an answer to a “How many?” question:

How many birds are in the tree?	=	*Kaat nan kuyat nan wada id kaiw?*
Five.		*Lima*

So far, the examples given are the same for English. Now consider:

Five pigs are too many.	=	*Adu unay nan lima ay boteg.*
	≈	Plenty too much are five pigs.
Five is too many	=	*Adu unay nan lima.*
	≈	Plenty too much are five.

Using numbers on their own can only happen when the object can be added to the sentence without change, as if it was always there in parentheses. Here is the difference: in English we can use numbers on their own by changing the grammar. Notice that in the example above the verb 'are' changes to 'is' when the noun is dropped. The effect of this is to make the number into a single object, as opposed to a characteristic.

The use of a ligature, *ay*, makes it more difficult than in English for the number word to act like a noun. It is trapped in its descriptive function. In schools in Sagada where Kankana-ey is used, there is noun-like usage of the number words in mathematical sentences:

Take away two from three.	=	*Kaanem nan dua isnan tulu.*
Remove the toy from the jar.	=	*Kaanem nan ay-ayam isnan gusi.*
Two is small compared to ten.	=	*Ban-ban-eg nan dua no nan simpoo.*
The stone is small compared to the tree.	=	*Ban-ban-eg nan bato no nan kaiw.*
Five is smaller than eight.	=	*Nan lima kitkittoy nu sin wao.*
Willy is shorter than Peter.	=	*Si Willy et ap-aptik nu si Peter.*

However this usage sounds very odd to a native speaker of Kankana-ey, whereas the structure sounds fine to an English-speaker. Kankana-ey is poorly suited to the mathematical use of number.

So numbers in Kankana-ey are trapped in their descriptive, adjectival function. Numbers in Polynesian languages are trapped in their active, verbal function. Although mathematical discourse, and the use of numbers as objects, is possible in both languages, strange sounding distortions are necessary to make it happen.

As an aside, even English and French are slightly different in their grammar of number. The evidence is their expression of fractions. In English you can say "one and a half hours" or "one hour and a half", although the latter form is unusual. In French, only the second form is possible ("une heure et demie"). This form actually implies "one hour and a half hour". In French the mathematical phrase "une et demie" is understood as two numbers added together (1 + ½), as opposed to the

English "one and a half" meaning a single number, namely 1½. I am told that in Français Québécois 1½ can be said as a single number.

We have seen that some languages have verbal grammatical forms for number, and some have adjectival forms. The most common mathematical use is as a noun, and English allows this more easily than most languages. It is interesting to note that some American First Nation languages have a noun-like usage of number in their everyday discourse. Denny (1986) has written on the Ojibway language (which also has verbal numbers) and the Aivilingmiut language which has noun-like grammatical structures for numbers:

one	atausiq	This has no suffix and is a singular noun.
two	marruuk	This has the dual noun suffix –uk.
three	pingasut	This has the plural noun suffix –t. It translates as "a group of three".
	pingasuit	This has the adjectival suffix –uit. It translates as "three groups".

Hence *pingasut tuktuit* (three caribou) is actually literally translated as a three-group of caribou, or a caribou group-of-three. And *pingasuit tuktuit* is three groups of caribou.

We can imagine that a mathematical discourse involving abstract sets, might be grammatically straightforward in this language.

3. FUNCTIONING NUMBERS: DHIVEHI

What about Dhivehi, the language of The Maldives? In this language numbers are adjectives or nouns. We know they can be nouns because in Dhivehi nouns are declined, that is, the form of a noun changes when it performs different functions. The suffix on a noun indicates the case. Numbers use the suffix forms for indefinite, non-human nouns (see Table 3-1). Note that the base word for fifteen, *fanara*, is both "the fifteen" and also the form that can be used in the descriptive, adjectival sense: *fanara foiy* (fifteen books).

Dhivehi seems to be like English where number words can be used in a descriptive, adjectival way (as is most common in everyday talk), or as an object in a nominal way (as is most common in mathematics). However it is not quite as simple as that. We need to look more closely at what happens to numbers in English discourse.

Table 3-1. The Word for Fifteen in Dhivehi

Case	Object Word	Number Word	Example
Direct	*fotek*	*fanara*	*Tinek ehkuran **fanara** ehvarey ashaaraya.*
	(a) book	(a) fifteen	Three and **fifteen** are eighteen.
Dative	*fotek-aa*	*fanarayak*	*Hayek **fanarayak** ehkuray*
	to a book	to a fifteen	Add six **to fifteen**.
			*Tinek ehkuran baara ya ehvarey **fanarayak**.*
			Three and twelve are equal **to fifteen**.
Generative	*fotek-ge*	*fanaraige*	*Thireehakee **fanaraige** gunaeh.*
	of a book	of a fifteen	Thirty is a multiple **of fifteen.**
Instrumental	*foteku-n*	*fanarayakun*	*Tinek **fanarayakun** kendeema ehvarey baara ya.*
	from a book	from a fifteen	Three subtracted **from fifteen** equals twelve.
	by a book	by a fifteen	*Saalhees faheh **fanarayakun** gehleema ehvarey tinakaa.*
			Forty-five divided **by fifteen** equals three.
Locative	*foteku-ga*	*fanaraiga*	***Fanaraiga** innanee tin fahek.*
	in a book	in a fifteen	There are three fives **in fifteen**.

The position of the number word before the noun, like a colour or other descriptive word, makes numbers feel like adjectives: red trousers, denim trousers, five trousers. But when we use these in a sentence, we can get differences. Many sentences containing number words are constructed more like noun sentences. Compare the answers to the questions in Table 3-2 and the possible ways they could be answered.

Table 3-2. Number Questions in English (XX indicates an unacceptable form)

Noun Question	**Number Question**	**Adjective Question**	**Verb Question**
What is in the room?	**How many cats are in the room?**	**What are the cats in the room like?**	**What are the cats in the room doing?**
Cats.	Four.	Red.	Sleeping.
There are cats in the room.	There are four cats in the room.	There are red cats in the room.	There are sleeping cats in the room.
There are cats.	There are four.	XX There are red.	XX There are sleeping.
The cats are in the room.	XX The cats in the room are four.	The cats in the room are red.	The cats in the room are sleeping.
They are cats.	XX They are four.	They are red.	They are sleeping.

The similarity with nouns in the third format is because we are permitted to drop the noun, although it is implied. “There are four (cats)”. We cannot do this with other adjectives so easily. The final two formats show that using numbers in a sense that answers the implied question “How many?” requires us to use constructions that are different from adjectives, verbs, and nouns:

The trees on the hills are oaks.	No, they are pines.
The trees on the hill are green.	No they are grey.
The trees on the hill are waving in the wind.	No, they are still.
There are three trees on the hill.	No, there are four.

Numbers, in English, have their own grammar that is unlike the grammar of adjectives, verbs or nouns. In various situations their grammar is like the grammar of these other types of words, and this happens in both everyday discourse where they tend to be adjective-like, and in mathematical discourse where they are like nouns:

There are four birds sitting in a tree.	Adjective-like.
There are three boxes of ten bottles, making thirty bottles in all.	Adjective-like.
Add these three pens to those six, and there are nine altogether.	Adjective-like (with ‘pens’ implied).
Five is a prime number. (Cf. Green is a warm colour).	Noun-like.
Five is a factor of fifteen. (Cf. Green is the complement of red).	Noun-like.
Three times six is eighteen. (Cf. Yellow and blue make green.	Noun-like.
Three sixes are eighteen.	Adjective- and noun-like.

Now in Dhivehi, numbers can be used as adjectives or nouns, so it seems as though this will be well-suited to all the everyday and mathematical constructions above. However in Dhivehi the numbers have the different noun forms. This means not that they are *like* nouns, but that they *are* nouns, and this seriously affects some of the mathematical features of numbers.

First of all consider the difference in English, between “three fives” and “five threes”. This is short for saying “three groups of five” and “five groups of three”. We know that the total number of objects in these two agglomerations are equal (15), but the way they are structured are different. This is easier to see if we are talking about something in particular: a three-story apartment block with five apartments on

each floor is a very different building from a five-story apartment block with three apartments on each floor—although they are both buildings containing fifteen apartments.

In English we can express the multiplication of numbers in several ways:

3×5	5×3
Three fives.	Five threes.
Three multiplied by five.	Five multiplied by three.
Three times five.	Five times three.

In the first of these the number words differ, depending on which way round you say it. The plural form of 'fives' (that only occurs in mathematical talk, not in everyday talk) indicates that this is a 'group of five' and there are three of them, and vice versa. In the second example, the word 'by' indicates that the role of the three is different from the role of the five, despite the form of the words being the same. One number is the instrument of the multiplication of the other. The third example is actually similar, but does not look like it. The 'times' here refers to 'occasions of'. "Three occasions of five" or "three occurrences of five". In this form the 'of' indicates the different role played by each number, but it is suppressed in the conventional form.

In Dhivehi the number words change in the first and second cases because the different role played by the two numbers is embedded in the structure of the word. There is no equivalent to "three times five" in Dhivehi.

3×5	5×3
Tin fahek	Fas tinek
Tinek fahekun gunakururma	Fahek tinekun gunakuruma

An alternative way of saying "three fives" or "five three" uses two old words:

Tin fansa	Fas thirikhu

It seems as though these old forms of five (*fansa*) and three (*thirikhu*) mean something like "groups of five". They have been replaced in modern Dhivehi by *fahek* and *tinek*. But these are interesting also, because there is a choice between the definite and the indefinite forms. "The five" would be *fas*, which is never used in the nominal sense (only in the adjectival one), *fahek* means, literally,

"a five", that is, an example of five. Now to create an example of five, to illustrate it, there must be five somethings. So this form of the noun is a version of the implied context that was referred to above in relation to Kankana-ey: its origins at least are in situations where it is possible to add a noun afterwards: "There are six people in the house, five (people) are women." Perhaps this is why, when I asked my informant (a first language Dhivehi speaker) to translate "fifteen is a multiple of five", she responded that she was not sure about it. She had had no trouble with all the arithmetic phrases like "Three plus five is eight" in which additional nouns could more easily be added: "Three birds plus five birds are eight birds". And when I asked for her to translate a sentence that talked about an occurrence of the number five in an equation, "the five tells you the intercept on the axis", she responded that it was difficult.

4. CONGRUENCE OF LANGUAGE WITH MATHEMATICS

So what? Well this is an example of the difference between everyday language and mathematical language. The symbol form of multiplication (3×5) is not the same as the spoken form. The symbol form of multiplication refers only to number, the spoken form also refers to structure. Some people express this as saying that mathematics is about 'pure number'. If 'pure' number is being meant, then 3×5 is equal to 5×3, i.e. 15. The mathematical term for this reversible feature is commutativity, and it is an important feature of multiplication (and addition) of numbers.

Note that commutativity does not apply to subtraction or division ($3 - 5 \neq 5 - 3$ and $3/5 \neq 5/3$). In English, for those operations, we retain the prepositions that indicate the role of each number. This is most noticeable when we issue commands. We say "multiply 3 by 5" "add 3 to 5" (roles retained), but also "multiply 3 and 5" "add 3 and 5" (roles lost). For subtraction and division we can only say "subtract 3 from 5" or "divide 3 by 5" (role retained).

So what emerges is that, in this example, English is more aligned to the way things are expressed in mathematics. English, with its own grammar of number, allows us to express the operations of multiplication and addition in the way that they are intended to be understood

mathematically; Dhivehi, with numbers fixed as nouns, does not—or, it does not allow it quite so easily.

This is a good opportunity to note something further about the use of numbers in mathematics that is different from our use of numbers in everyday language. The mathematical use of numbers is strictly defined and highly conventional. We have given the example of multiplication, where the symbols 3 × 5 refer to the total (pure) number being represented and not the structure of the groupings. In mathematical language this is "three times five", and is commutative. We have already noted that this is different from any everyday use where actual objects are being discussed, where the structure is part of what is being communicated: 3 floors of 5 apartments; 3 × 5, a piece of timber may be described as 3 by 5 (often written 3 × 5) when what is being referred to is the shape and dimensions of the cross-section. Thus we say that mathematics is removed from reality, it represents the ideal.

Beware! Many have interpreted the use of the word "ideal" in this context as meaning that mathematics represents perfection, the ultimate in abstract thought. That is not what I mean, and nor do I think it is true. I mean that mathematics represents things that are ideas, they come from ideas, they are ideal. But there are many possible ideas, and perfection does not seem like an appropriate word to use for ideas. Ideas are just ideas.

For example, there is another use of "3 × 5" which is quite different from how we usually understand multiplication: this is when it is used to indicate the dimensions of a matrix. A matrix is an array of numbers, and, for most purposes, it is very important to distinguish between a 3 × 5 matrix and a 5 × 3 one (see Fig. 3-1).

$$\begin{pmatrix} 2 & 7 & 4 \\ 0 & -1 & 5 \\ 0 & 1 & -2 \\ -2 & 3 & 4 \\ 1 & -3 & 6 \end{pmatrix}_{5\times 3} \qquad \begin{pmatrix} 2 & 0 & 1 & 1 & -2 \\ -1 & 4 & 0 & 6 & 3 \\ 3 & 3 & -4 & 0 & 1 \end{pmatrix}_{3\times 5}$$

Figure 3-1. Matrices of Different Order

Here the structure is important. When "3 × 5" is read in this context, you should say "three by five"—the preposition is retained and the meaning is "three rows of five entries". This highlights the conventional use of symbols and words in mathematics: the meaning is precise, and natural language is co-opted to express this as best as possible. English is generally quite adaptable to this purpose.

What is the point here? Well, compared with Kankana-ey, Maori, or Dhivehi, English allows the movement of the use of numbers from everyday conversation mode to the mathematical mode quite easily. The grammatical structure of numbers allows them to be used in conjunction with a noun to describe how many, to be used as objects that can be talked about in isolation, and to be used in mathematical senses that are neither of these. This is not true in the other languages for different reasons: in Kankana-ey numbers are more fixed in their descriptive, adjectival mode; in Maori and Tahitian numbers are more fixed in their active, verbal mode; in Dhivehi the numbers are more fixed in their object, nominal mode.

It should be emphasised again that these are not immovable features of these languages. In English there are constructions or word-forms that force an adjectival use, or a verbal use, and so rob numbers of their mathematical features. In the other languages, the mathematical senses of numbers can be expressed, although it may sound a little odd. What is being noted is a privileging of English with respect to the mathematical use of numbers. It is easier, it is closer to NUC-mathematical discourse.

An English-speaker can more easily mathematise quantity into NUC-mathematics. There is no strangeness in the way of talking, so that mathematics-speak, where numbers are concepts to be played with, is natural: there is a congruence between this language and mathematics. Apart from the possible educational benefit of such congruence (and this is discussed in Part III), it is interesting to ask the question "Why does the congruence exist?" One possible answer is that it is simply chance, that English just happens to be more in line with mathematical talk, and therefore if you are an English speaker then mathematical talk will flow naturally. Alternatively, either mathematical ideas have developed the way they have because mathematics developed (and increasingly develops) through English (or Indo-European languages), or, alternatively, that English has developed in the way that it has because it evolved in close contact with mathematics.

I do not believe that it is coincidence that Indo-European languages just happened to be more consonant with mathematics than other languages. Given that mathematics as we know it today has the major parts of its history within an Indo-European environment, this congruence seems to be good evidence that mathematics is a human creation that is influenced by, and influences, other aspects of human creativity in the same environment. Mathematics and language evolved together. They have affected one another in the past, and they are influencing each other in the present.

This first part of the book has presented evidence from different languages of different mathematical conceptions that could lead to different mathematical systems of various kinds. It has also presented evidence that mathematics and language develop together. But developing mathematics is more complicated, of course, than just creating mathematical worlds through language. I do not mean to suggest that language comes first, and that it determines a mathematical world completely. What other factors shape a mathematical world, or, what else has made mathematics the way it is? How much is mathematics determined by the nature of the human mind? By accidents of history? By the needs of society? By already existing mathematics?

We are concerned about what the evidence from language tells us about the relationship between mathematics and human culture and the philosophical status of mathematics. Is it the same everywhere for everyone? What role does mathematics play in our society? How does it grow and what influences the directions of its development? More importantly, where might it be headed in the future?

The second part of the book addresses some of these questions while remaining mostly focused on language and mathematics. On the way, I will explain why mathematicians should be sued for the sinking of the Titanic, how mathematics can enhance your sex life, and why it is not your fault that you had problems adding fractions.