Chapter 2

SPACE: STATIC AND DYNAMIC WORLD VIEWS

Abstract: The way in which we talk about geometrical objects is explored through several languages. The different way of talking in the Navajo language is extrapolated into a geometrical system. Different ways of navigation are then analysed and the chapter concludes with a discussion of philosophical links with the development of mathematics.

Keywords: geometry, space, coordinate systems, Tahitian, Navajo, Pacific navigation, Zeno's paradoxes

The search for evidence of different ways of talking mathematically was hardly systematic. It was necessary to gather information about languages that had not been part of the development of NUC-mathematics. The information needed was how these languages express mathematical ideas. What discourse is used to talk about the quantitative, relational, or spatial aspects of peoples' experiences in their general, everyday language, both spoken and written?

The problem of how to access these languages was left largely to chance. After Maori and Tahitian, the languages Euskera, Dhivehi, and Kankana-ey were chosen because I happened to have contact with speakers of these languages who were also mathematicians. Some other languages, including African and First Nation American ones, provided examples in existing literature.

There were two relevant books that contained information of the type I was looking for. In the 1960s Gay and Cole (1967) had written about the Kpelle people of Liberia, in particular describing their language of logic and the effect it had on their understanding of logical relations. In the 1980s Pinxten, van Dooren, Harvey, and Soberon (Pinxten, van Dooren, & Soberon, 1987) had built on an earlier anthropological study (Pinxten, van Dooren, & Harvey, 1983), and written about the geometrical language of the Navajo. This latter work tied in with the verbal numbers of Polynesian languages since, in Navajo, what we know as geometrical objects (circles, squares, lines, spheres) are expressed verbally.

1. WHAT ARE VERBAL SHAPES?

Verbal shapes? Let us think about this first in English. Shape is expressed in many ways. Consider the geometric idea of a square. In everyday language shapes are usually characteristics of something: a square piece of paper, a square table. But I can ask someone "to fold a piece of paper into a square", in the same way as I might ask them to fold it into a bird. The squareness is expressed like an object. Or I could ask them "to square the piece of paper" in the same way as I might ask them to screw it up. The squareness is expressed as an action.

Try another shape. We can say that a shape is a triangle (which makes the idea of a triangle into a thing), or that a shape is triangular (which makes this idea a characteristic of a thing), or that something is triangulating (which makes the idea into an action). Notice that the form of the word for the adjective and verb are clearly derived from the noun.

These three ways of speaking about a shape work for a square, a triangle, a circle (we could have a circular piece of paper, we could ask people to sit in a circle, the birds might circle the treetops), and for a line (planes may fly in a linear formation, we are asked to stand in a line, people line up at a ticket office). But in mathematical discourse, and especially with more complicated mathematical shapes, a shape is usually described as an object or as a characteristic. We can draw a pentagon, and something may be pentagonal, but it sounds clumsy to ask someone to "pentagonalise a piece of paper". Notice that in all these examples, the adjectives are either the same as, or derived from, the noun: square—square, circle—circular, line—linear, pentagon—pentagonal. The noun form is privileged in English; it seems to be the base concept in everyday language and in mathematical discourse. The derivations of these words are given in dictionaries as from nouns. It is, of course, possible to use any form, and even to construct odd but understandable forms ("decagonal"), however noun forms are more common, and sound better.

In Navajo the opposite is the case:

> A basic characteristic of the Navajo world view … is the fundamentally dynamic or active nature of the world and everything in it. … [This is a]

> basic perspectival difference from Western thought and language. (Pinxten, van Dooren, & Harvey, 1983, p. 15).

> … a cosmos composed of processes and events, as opposed to a cosmos composed of things and facts. (Witherspoon, 1977, p. 49).

In the grammar of the language, this feature is expressed through verbs. This does not mean that the verbs can be considered as spatial terms themselves, rather the grammar of the language is such that a particular verb can only be used with a certain group of objects that have a particular spatial characteristic. So the geometrical reference is carried in the verb, rather than in the noun. For example, the idea of planeness (a flat expanse in two dimensions) is associated with the verb sikaad: tó sikaad = a layer of water spreading out; diih dikon tsin sikaad = a wooden floor spreading out (Pinxten, van Dooren, & Harvey, 1983, p. 93). It is not possible to use this verb to describe land, on the other hand, because land has a certain thickness, even though it does spread out.

The mathematician in me is intrigued by the idea of verbal expression of shapes. Could this make a difference mathematically? Does the way we think about the idea of triangularity affect what we understand about it? It was interesting to play the mind-game of what the study of shape might be like if it had developed verbally. How might geometry be different?

Let me be clear that this is my mind-game, not a Navajo mind-game. The way I am using the idea of "circle as an action" is my conception of that idea, not a Navajo one. For example, the idea of circular may be used to describe an object with a circular shape or outline. In Navajo this would be indicated by a verb, in English by an adjective. But I have taken the idea of circular as only an action: I am playing a mind-game where a circle is something you do, and I am using the verbal function of action from English, not from Navajo.

Imagine, then, that circularity is an action, not an object, thus we must talk about circling, not a circle. Working mathematically, it is necessary to make this idea more formal, that is, to explore the details of what makes the action exactly circular, and to distinguish it from actions that are not circular. I need to be able to define circling, to categorise different circlings, to describe the characteristics of circling, to know how circling is related to other shape-actions, and to understand how it changes.

2. BIRDS AND ORBITS

If a circle is an action, then it is necessary to imagine movement and not a static picture. One way to do this is to make yourself into the actor. Think of yourself as a bird circling a tree. For the moment, let us say that you are flying at a constant speed. Now, what is it about the way that you are flying that means you are flying in a circle (circling) and not a square (squaring) or an ellipse (ellipsing)? It is the fact that you are turning at a constant rate all the time. To be squaring, for example, you would fly straight, and then turn suddenly at the corners. So the defining feature of circling is what is called constant angular velocity.

What would be different if you were turning more quickly (but still at a constant rate)? You would be circling more tightly. Assuming constant speed, differently sized circlings are characterised by different angular velocities (that is, the rate at which you are turning).

If, on the other hand, we kept the angular velocity constant, we could then change the size of our circling by changing our speed: a greater speed would result in wider circling, a slower speed would make it tighter.

Constant speed, constant rate of turn.
If the speed of both birds is the same,
then the bird is turning more quickly in the smaller circling.
If the rate of turn of both birds is the same,
then the bird is flying more slowly in the smaller circling

Figure 2-1. Flying in smaller circles

That takes care of the size of the circling. What other characteristics of circling might we be interested in? Perhaps the length of time it takes to get back to where you started. Perhaps the way it is oriented to the ground; are you circling in a horizontal plane or is your circling tipping (like the fairground ride I know as an Octopus)? Or are you circling in a vertical plane like a Ferris Wheel?

In this geometry, how does circling relate to other shape-actions? Again let us imagine that we are a bird, flying at a constant speed and turning at a constant rate so that we are circling. Let us gradually change one of these variables: instead of turning at a constant rate, let us steadily increase the rate of turn, making us turn tighter and tighter. What does our path look like now? We would be spiralling inward. And if we steadily decreased the rate of turn as we are flying at constant speed, we would then be spiralling outward.

This means that circling is actually a special case of spiraling.

What happens if we are not turning at all, if the rate of turn is zero? Then, of course, we are flying along in a straight line (lining), thus lining is a special case of circling (rate of turn is also constant—but it is zero). Similarly, if we turn at an infinitely fast rate we will simply be staying at the same spot.

The same effect can be obtained from changing the speed, but keeping the rate of turn constant. If you fly at a faster constant speed, then the circle will be bigger—at infinite speeds you will fly in a straight line. If you fly at a slower constant speed, then the same rate of turn will make you fly in a smaller circle—and if you stop, of course, you will just turn on one point. So pointing (the action of being in one place) is a special case of circling.

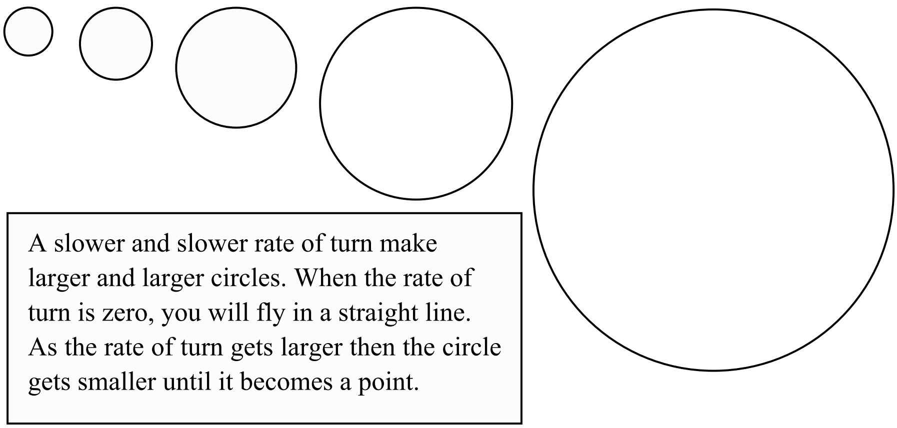

Figure 2-2. Flying in points and lines

In coordinate geometry, a circle is a special case of a family of curves known as conic sections that also include parabolas and ellipses. How would you fly in order to describe these shapes? Think about runners sprinting around a sports arena. This is not an ellipse, but it is similar, and will do while we think about what is happening to them. They are running at more or less constant speed, not turning at all as they go along the straights, and then leaning into the corners at each end. So the rate of turn is changing during the circuit: from zero, to turning, to zero again, to turning again. So it would be for an ellipse. In this shape there are no straights, so the rate of turn would never reduce to zero, but it would decrease, increase, decrease and increase again, all in a steady fashion.

Ellipsing can also be done by turning steadily, but changing your speed. Fly with a constant rate of turn, and then steadily increase your flying speed, then decrease it, then speed it up again, and decrease it again. The effect will be to elongate circling into ellipsing.

There is a situation that exemplifies ellipsing: that of planets orbiting around the sun. Think of yourself as the planet. What is happening? In fact it is a combination of the two situations we have been describing, since both the flying speed and the rate of turn are changing. As you approach the sun, the pull of gravity speeds you up and turns you towards that burning orb. But (fortunately) you are travelling too fast to become an Icarus, and you fly by. Now the sun is close and the pull of gravity is strongest, and you are forced to turn quite strongly in the direction of the sun. But your speed is such that you go right around the sun, and head back from whence you came. But now you are moving away from the sun, and again it starts to pull you back, slowing you down. But as you get well past it the pull gradually decreases. Nevertheless it is enough to slow you down, slower and slower, and to turn you around again. You are a long way away, turning slowly, and your speed is quite low. So low that that distant pull of the sun is enough to pull you back again for another approach. Uh-oh, here we go again.

The mathematics of this situation is well-known in conventional terms. But it is interesting to compare what is done in astronomy, and what might be done if the mathematician was on the planet (of course this is exactly the situation for artificial satellites that are thrown up into orbit around earth or the moon). What do astronomers do, when they think they have found a new heavenly body? They take observations of its position (with reference to the earth, sun, or centre of gravity of the solar system, and also using a reference plane). When they have enough observations over a great enough period of time, then they use these positions to fit an ellipse. If they have enough positions (theoretically three are enough to determine an ellipse, but in practice more are used to minimise the effect of measurement error), then the ellipse can be mapped accurately, and it is then checked against a data-base of ellipses of known heavenly bodies kept on a big computer in Harvard. New ellipse? Bingo. New heavenly body.

Now, in Action Geometry we have used speed and rate of turn as the basic elements, not position. If the mathematician was on the planet, then rather than determine position relative to some reference point as the basis for calculations, they might rather use the speed and rate of turn as the basis for predicting where they were going.

What about other shapes. Can a square be an action? There are all those sharp corners. It is here that we see more clearly the differences in the items of interest between Action Geometry and conventional (Static) Geometry. If you are travelling in a square, then you must either stop and turn, or turn infinitely quickly, at the corners. The way you trace the shape becomes a combination of flying speed and turning speed. Also, the time taken on each side is important (if a constant flying speed is assumed). Of course it is possible to describe any shape at all using either Action Geometry or Static Geometry. Notice, however, that shapes without sharp corners are more easily described in Action Geometry. Action geometry would privilege such smoothly curved shapes, but would have a difficult time describing the constructions of Euclidean geometry.

Seymour Pappert's computer environment LOGO (often known as Turtle Geometry) appears to be a mix of Static and Action geometry (Abelson & diSessa, 1980). In this environment the screen becomes a field on which the icon (originally represented as a turtle) can be made to move. The original version enabled the user to move forward or back a given length, or to turn a given angle. This uses the idea of movement as its base, but still characterises movement as going from one point to another. A true Action geometry environment would allow the user to adjust speed and rate of turn along a continuous path, not iterate a number of small positional movements to make a path.

3. EUROPEAN AND PACIFIC NAVIGATION

A parallel exists between the two geometries being described and two ways of conceptualising navigation. The different conceptual systems possible for navigation first came to my attention when I read about the navigation techniques of Pacific peoples (Gladwin, 1970;

Hutchins, 1983; Irwin, 1997; Kyselka, 1987; Lewis, 1975; Thomas, 1987; Turnbull, 1991). The basis of their navigation is to determine where they are on their journey, not their exact position.

Consider traditional navigation as it developed through European navigators. From the early art of "way-finding" (Collinder, 1954), a system evolved that required a number of sightings of the sun or stars, and measurements of time, so that position could be accurately located on a map. The history of this development, and the technological effort and expenditure that went into it, is described in Sobel's book Longitude (1995). One way of thinking about this is to imagine that a grid has been constructed upon the world and the position of places of interest are known with respect to this grid. Thus if you can locate your position on the grid, then you know your position in relation to the places you came from or are going to. This system is now developed to such an extent that using satellite GPS (Global Positioning Systems), a hand-held computer will give you a read-out of your position to within a metre. I have friends who, in thick fog, sailed out of a narrow gap between two rocky outcrops using only such equipment and their charts. If you use this system then your aim is to be constantly aware of your position, and of how far you are from known critical points.

Notice that this system relies on a reference system that has been created by humans. The original references were features of the real world (headlands, islands, reefs), but the latitude and longitude grid that has developed from these is artificial.

Now consider traditional Pacific navigation. The experienced navigators have the equivalent of charts in their minds, but these are not position charts, they are a set of features and signs that indicate the path that they will travel. This path is not always a straight line, rather it goes from landmark—or, rather, sea-mark—to sea-mark. For example they are likely to know more about the direction of their destination and how long it will take to get there, than how far away they are from it. In a well-documented experiment a navigator did two return journeys from Hawaii to Tahiti in a replica Polynesian double canoe, and travelled along the same dog-leg shaped path each time (Kyselka, 1987). Indeed, on one occasion, the following tracking ship with modern navigation aids, lost all power and had to reply on the canoe to reach its destination. Sea-paths do not, however, always cover the same ground: they depend on weather, seasons and sea conditions.

Such a means of travel is, of course, very common for people travelling by land. If I drive from my city to another three hours away, there is no need for a map. There are many signs to tell me that I am on the right path, and as I become familiar with the journey. I will note landmarks and sights along the way, and I will have my favourite stopping places, from where I will know how long it is until I reach my destination. At any point I may not even know in which direction my destination lies, but, nevertheless, I am confident that I am "on track". In this system it is important to know, first, that you are on the right path, and secondly, how long it will take to reach other critical places on the path.

The difference between what we can call Position Navigation and Path Navigation can be illustrated by two ways in which movement is characterised. In the televised animations that accompany America's Cup yacht race coverage, the speed of the yachts is visually represented by a trail of dots behind each boat. These dots are created for the animation from the highly accurate GPS equipment on board by recording the position at regular time intervals. If the dots are close together, that means that the boat is going slowly, if they are spreading out, then the boat is increasing speed, and so on. Speed represented by position.

Compare this with the idea of etak (Akimichi, 1985; Gladwin, 1970, Chpt. 5; Gunn, 1970), one of the conceptual formulations of travel of the Pacific navigators. When a canoe is moving along its path, then we can imagine that there is an island ahead that we need to pass by (let us say to the right of it). As we pass by, this island will appear to move from nearly directly in front, to ahead but on the left, to abeam on the left, to behind and to the side, to nearly directly behind. It is as if the island moves while the boat stays still. This idea is etak, and Pacific navigators use it to describe islands or features that cannot be seen (perhaps because they are over the horizon) as indicators of how well they are travelling down their path. Motion is thus represented by changing bearings of sea-marks.

What is the correspondence between Position Navigation and Path Navigation and Static and Action geometries? Position navigation focuses on reference points and distances, using them to find the bearing that must be travelled. Path navigation focuses on pathways and speed, using them to find the direction of the next sea-mark. The first has static references, the second has active ones.

The examples of planetary orbits and navigation illustrate different ways of conceptualising space. One way uses the basic idea of static position with reference to an origin. Another way has movement through the space as the base idea. Each way of seeing makes some things easy and other things complicated. In the study of space that is part of NUC-mathematics, objects and position are treated first, and movement (speed and turn) is a more complicated idea that is treated later. This section has tried to illustrate that it is possible to begin a study of space using movement, and then think about position at a later time.

Note that there is nothing that has been mentioned about Action geometry that cannot be described in terms of conventional Static geometry. The reverse is also true. We can do Static geometry in terms of Action geometry, or vice versa. (Note that describing the orthocentre of a triangle would be complicated in Action geometry and easy in Static geometry, and the opposite is true for describing a changing spiral). The point is that we do not do this, or, we tend not to, certainly not at first. This is because some things are easier, or more natural, than others, depending on which geometry you are using. This is not an unusual idea. In conventional geometry we have several systems, for example the Cartesian coordinates and Polar coordinates mentioned above, and we use the system that is easiest for what we are trying to do: the Cartesian systems for straight lines and some curves, the Polar system for circles and other curves. However it should not surprise us that the systems that are in common use are not all the systems available.

4. LINKING THE LINGUISTIC AND MATHEMATICAL SYSTEMS

We are predisposed to see space using particular basic ideas. It is suggested that part of the reason that NUC-mathematics is the way it is results from the linguistic and cultural orientation of those who developed it. Western thought is culturally and linguistically predisposed to reference and position, whereas, for example, the Navajo one is predisposed to action and movement. Let us again use the word privilege to describe what is happening: languages, as the expression of cultures, privilege different ways of thinking about shapes and space.

The investigation starting from the dynamic Navajo world view has given us something more substantial than the flight of imagination based on differences in the few words in Tahitian used to describe the position of an object. Now we are talking about a whole way of understanding shape and the potential geometrical world that that creates: a world with different base concepts, with different foci of attention, with different relations and contexts, with different applications.

At the end of the previous chapter, we noted that the Chinese philosophy of Yin and Yang, and the logic of dialectics, each provided philosophies with more than one origin in contrast to Descartes egocentric theories. It was suggested that there might be parallels with graphical representation being developed through a single-origin model based on Descartes ideas, with double-origin models being relatively ignored in Western mathematics.

The parallel can be extended to the idea of Static and Dynamic geometries. The idea of constant change is at the heart both of the theory of Yin and Yang, and of dialectics. Western philosophy, on the other hand, developed through the Greeks. Rotman (1987, p. 62) writes:

> [The Greeks were] logically persuaded that change and plurality, however much they seem real to us, must be illusions. … Parmenides, and more famously his disciple Zeno, gave many arguments defending his unitary static cosmos. Those that survive are principally in the form of paradoxes which forced their interlocutors into accepting that the ideas of motion and plurality were inherently contradictory and incoherent, and were therefore, by a reductio ad absurdum argument, not real.
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> Zeno's celebrated paradoxes … had a profound affect on the structure of Greek thought—on its mathematics no less than its theology and cosmology. …
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> … In terms of definition, [the Greeks] denied any role to motion. All their objects of Greek mathematical thought such as numbers, ratios, points, figures, and so on, were characterized as wholly static fixed entities so that, for example, the figure of a circle was defined as the locus of points equidistant from some given point and not as the path of a moving point.
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> … The Zeno-Parmenides interdiction of motion … engendered within Greek mathematics … an attachment to visually concrete icons which influenced mathematics from the time of Euclid to the Renaissance (and beyond: a version of Parmenidean stasis is central to the dominant present-day conception of mathematics in which mathematicians are supposed to apprehend eternal truths about entities –'structures' – in an unchanging, timeless, static, extra-human world).

Of course Parmenides' and Zeno's paradoxes can be rewritten to make the opposite conclusion. Consider the paradox of the arrow:

> If everything when it occupies an equal space is at rest, and if that which is in locomotion is always occupying such a space at any moment, the flying arrow is therefore motionless. (Aristotle *Physics* VI:9, 239b5).

That is, time is made up of indivisible moments, in each of which the arrow occupies a space that is just the size of itself. Hence it is motionless. But let us pretend that we are in a world based on motion, then the argument might go like this:

> Arrows move from bow to target, and in any time interval, no matter how small, they traverse a length. Since they are always traversing, arrows never occupy any position.

The conclusion of the paradox is that the arrow never occupies space.

What is interesting is that the resolution of the paradox (in modern terms, the way we define instantaneous velocity), is to calculate the speed over a small distance, and to define as instantaneous what happens as these distances get smaller and smaller—although they never actually become a single point. This resolution never gives up the idea of point: position is the basic tool we have to define our world. The paradox could, however, be resolved by defining position as the path traversed by the head of the arrow as the time interval gets shorter and shorter, without ever requiring time to be reduced to an instant. This is equally as satisfactory (or unsatisfactory) as the conventional resolution.

The other area of Western thought identified as being initially dominated by a single origin perspective was that of anthropology. As with philosophy, the modern concepts of culture are dynamic and take account of cultural development. For example, Welsch's (1999, p. 202) transculturalism in which:

> The basic task is not to be conceived as an understanding of foreign cultures, but as an interaction with foreignness. Understanding may be helpful, but is never sufficient alone, it has to enhance progress in interaction.

The questioning of egocentricity and stasis in Western thought is taking place in philosophy and anthropology. This book is doing it in mathematics.

In Part II of this book we look more closely at how mathematical worlds might be created by language, and the consequences of this. But before that we turn to other aspects of mathematical systems. Are the examples of different ways of talking about quantity and relationships, similar to those we have described about space? Numbers, it turns out in the next chapter, might seem like the simplest idea, but in fact they have caused an awful lot of trouble. And, in Chapter 4, we look at examples from other languages of people making sense of relationships of various kinds. How are categorisations made, how do we explain human relationships, and how do we create logical arguments using these relationships?