10

Variation of Grid Inductance with Frequency

The variation of inductance with frequency in high performance power distribution grids is discussed in this chapter. As discussed in Chapter 5, the on-chip inductance affects the integrity of the power supply in high speed circuits. The frequency of the currents flowing through the power distribution networks in high speed ICs varies from quasi-DC low frequencies to tens of gigahertz. Thus, understanding the variation of the power grid inductance with frequency is important in order to built a robust and efficient power delivery system.

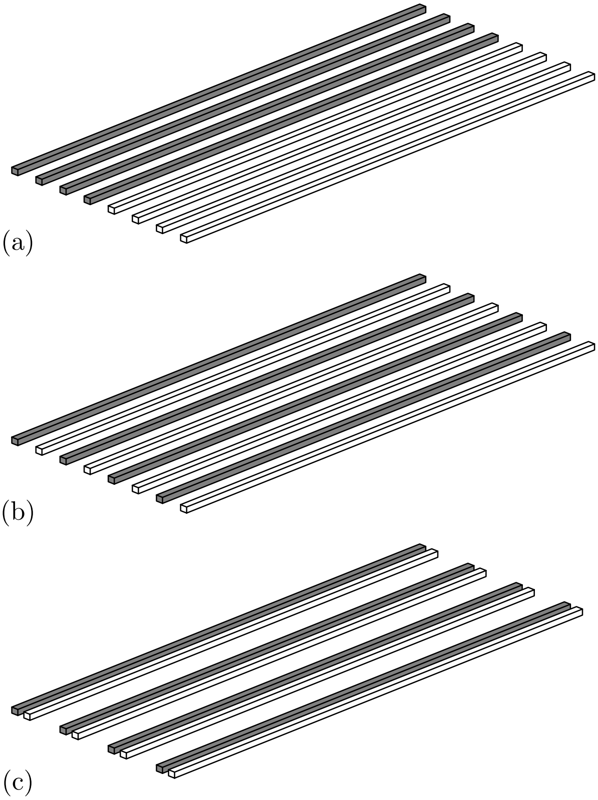
The chapter is organized as follows. A procedure for analyzing the inductance as a function of frequency is described in Section 10.1. The variation of the power grid inductance with frequency is discussed in Section 10.2. The chapter concludes with a summary.

10.1 Analysis approach

The variation of the grid inductance with frequency is investigated for the three types of power/ground grids: non-interdigitated, interdigitated, and paired. These types of grid structures are described in Section 9.3. The grid structures are depicted in Fig. 10.1.

The analysis is analogous to the procedure described in Section 9.3. The inductance extraction program FastHenry [67] is used to explore the inductive properties of these interconnect structures. A conductivity of $58\,\mathrm{S}/\mu\mathrm{m} \simeq (1.72\,\mu\Omega\cdot\mathrm{cm})^{-1}$ is assumed for the interconnect material.

The inductance of grids with alternating power and ground lines, *i.e.*, interdigitated and paired grids, behaves similarly to the grid resistance. With the width and pitch of the lines fixed, the inductance of



(a) (b) (c)

Fig. 10.1. Power/ground grid structures under investigation; (a) a non-interdigitated grid, (b) an interdigitated grid, the power lines are interdigitated with the ground lines, (c) a paired grid, the power and ground lines are in close pairs. The power lines are grey colored, the ground lines are white colored.

these grid types increases linearly with the grid length and decreases inversely linearly with the number of lines, as discussed in Chapter 9. Consequently, the inductive and resistive properties of interdigitated and paired grids with a specific line width and pitch can be conveniently expressed in terms of the sheet inductance $L_\square$, henrys per square, and the sheet resistance $R_\square$, ohms per square [276]. As with the sheet resistance, the sheet inductance is convenient since it is independent of a specific length and width of the grid; this quantity depends only on the pitch, width, and thickness of the grid lines. The impedance properties of interdigitated and paired grids can therefore be studied on structures with a limited number of lines. These results are readily scaled to larger structures, as described in Section 9.6.

The grid structures consist of ten lines, five power lines and five ground lines. The power and ground lines carry current in opposite directions, such that a grid forms a complete current loop. The grid lines are assumed to be 1 mm long and are placed on a 10 μm pitch (20 μm line pair pitch in paired grids). The specific grid length and the number of lines is not significant. As discussed in Section 9.6 and Chapter 3, the inductance scales linearly with the grid length and the number of lines, provided the line length to line separation ratio is high and the number of lines exceeds eight to ten. An analysis of the aforementioned grid structures has been performed for line widths W of 1 μm, 3 μm, and 5 μm. The line thickness is 1 μm. The line separation within power-ground pairs in paired grids is 1 μm.

10.2 Discussion of inductance variation

The variation of grid inductance with frequency is presented and discussed in this section. Simple circuit models are discussed in Section 10.2.1 to provide insight into the variation of inductance with frequency. Based on this intuitive perspective, the data are analyzed and compared in Section 10.2.2.

10.2.1 Circuit models

As discussed in Section 2.2, there are two primary mechanisms that produce a significant decrease in the on-chip interconnect inductance with frequency, the proximity effect and multi-path current redistribution. The phenomenon underlying these mechanisms is, however, the same. Where several parallel paths with significantly different electrical properties are available for current flow, the current is distributed among the paths so as to minimize the total impedance. As the frequency increases, the circuit inductance changes from the low frequency limit, determined by the ratio of the resistance of the parallel current paths, to the high frequency value, determined by the inductance ratio of the current paths. At high signal frequencies, the inductive reactance dominates the interconnect impedance; therefore, the path of minimum inductance carries the largest share of the current, minimizing the overall impedance (see Fig. 2.10). Note that parallel current paths can be formed either by several physically distinct lines, as in multi-path current redistribution, or by different paths within the same line, as in the proximity effect, as shown in Fig. 10.2. A thick line can be thought of as being composed of multiple thin lines bundled together in parallel. The proximity effect in such a thick line can be considered as a special case of current redistribution among multiple thin lines forming a thick line.

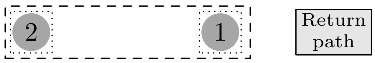


Fig. 10.2. A cross-sectional view of two parallel current paths (dark gray) sharing the same current return path (light gray). The path closest to the return path, path 1, has a lower inductance than the other path, path 2. The parallel paths can be either two physically distinct lines, as shown by the dotted line, or two different paths withing the same line, as shown by the dashed line.

Consider a simple case with two current paths with different inductive properties. The impedance characteristics are represented by the circuit diagram shown in Fig. 10.3, where the inductive coupling between the two paths is neglected for simplicity. Assume that $L_1 < L_2$ and $R_1 > R_2$.

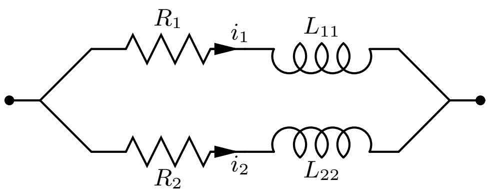


Fig. 10.3. A circuit model of two current paths with different inductive properties.

For the purpose of evaluating the variation of inductance with frequency, the electrical properties of the interconnect are characterized by the inductive time constant $\tau = L/R$. The impedance magnitude of these two paths is schematically shown in Fig. 10.4. The impedance of the first path is dominated by the inductive reactance above the frequency $f_1 = \frac{1}{2\pi}\frac{R_1}{L_1} = \frac{1}{2\pi\tau_1}$. The impedance of the second path is predominantly inductive above the frequency $f_2 = \frac{1}{2\pi}\frac{R_2}{L_2} = \frac{1}{2\pi\tau_2}$, $f_2 < f_1$. At low frequencies, *i.e.*, from DC to the frequency f_1, the ratio of the two impedances is constant. The effective inductance at low frequencies is therefore also constant, determining the low frequency inductance limit. At high frequencies, *i.e.*, frequencies exceeding f_2, the ratio of the impedances is also constant, determining the high frequency inductance limit, $\frac{L_1 L_2}{L_1+L_2}$. At intermediate frequencies from f_1 to f_2, the impedance ratio changes, resulting in a variation of the overall inductance from the low frequency limit to the high frequency limit. The frequency range of inductance variation is therefore determined by the two time constants, τ_1 and τ_2. The magnitude of the inductance variation depends upon both the difference between the time constants τ_1 and τ_2 and on the inductance ratio L_1/L_2. Analogously, in the case of multiple parallel current paths, the frequency range and the magnitude of the variation in inductance is determined by the minimum and maximum time constants as well as the difference in inductance among the paths.

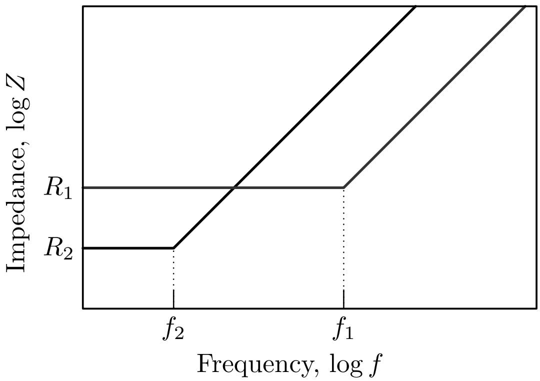


Fig. 10.4. Impedance magnitude versus frequency for two paths with dissimilar impedance characteristics.

The decrease in inductance begins when the inductive reactance $j\omega L$ of the path with the lowest R/L ratio becomes comparable to the path resistance R, $R \sim j\omega L$. The inductance, therefore, begins to decrease at a lower frequency if the minimum R/L ratio of the current paths is lower.

Due to this behavior, the proximity effect becomes significant at higher frequencies than multi-path current redistribution. Significant proximity effects occur in conductors containing current paths with significantly different inductive characteristics. That is, the inductive coupling of one edge of the line to the "return" current (*i.e.*, the current in the opposite direction) is substantially different from the inductive coupling of the other edge of the line to the same "return" current. In geometric terms, this characteristic means that the line width is larger than or comparable to the distance between the line and the return current. Consequently, the line with significant proximity effects is typically the immediate neighbor of the current return line. A narrower current loop is therefore formed with the current return path as compared to the other lines participating in the multi-path current redistribution. A smaller loop inductance L results in a higher R/L ratio. Referring to Fig. 2.10, current redistribution between paths one and two proceeds at frequencies lower than the onset frequency of the proximity effect in path one.

10.2.2 Analysis of inductance variation

The inductance of non-interdigitated grids versus signal frequency is shown in Fig. 10.5. At low frequencies, the forward and return currents are uniformly distributed among the lines. The two lines in the center of the grid form the smallest current loop while the lines at the periphery of the grid form wider current loops. The effective width of the current loop at low frequencies is relatively large, approximately half of the grid width. Non-interdigitated grids, therefore, have a relatively large inductance L and a low R/L ratio as compared to the other two grid types, interdigitated and paired. Consequently, the onset of a decrease in inductance occurs at a comparatively lower frequency, as illustrated in Figs. 10.5, 10.6, and 10.7. As the signal frequency increases, the current redistributes toward the center of the grid to decrease the grid inductance. Since the width of the grid is much larger than the width of the grid line, the decrease in inductance is primarily due to multi-path current redistribution among the different lines while current redistribution within the line cross sections (the proximity effect) is a secondary effect. The low frequency inductance of a non-interdigitated grid increases with grid width. As the grid width (*i.e.*, the number of lines) increases, the decrease in inductance with frequency becomes more significant and begins at a lower frequency [70].

In power grids with alternating power and ground lines (such as the interdigitated and paired grid structures illustrated in Figs. 10.1(b) and 10.1(c), respectively), each line has the same resistance and self inductance per length, and almost the same inductive coupling to the rest of the grid. As discussed in Chapter 11, long distance inductive coupling is

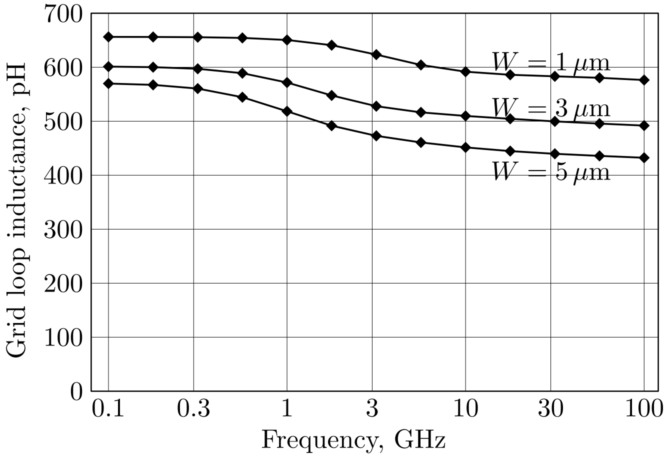


Fig. 10.5. Loop inductance of non-interdigitated grids versus signal frequency.

cancelled out in grids with a periodic structure, such that the lines are inductively coupled only to the immediate neighbors, making inductive coupling effectively a local phenomenon. As a result, the distribution of the current among the lines at low frequencies (where the current flows through the path of lowest resistance) practically coincides with the current distribution at high frequencies (where the current flows through the path of lowest inductance). That is, the line resistance has a negligible effect on the current distribution within the grid, *i.e.*, multi-path current redistribution is insignificant. Consequently, the decrease in inductance at high frequencies is caused primarily by the proximity effect which depends upon the line width, spacing, and material resistivity.

This situation is exemplified by paired grids, where multi-path current redistribution is insignificant and the proximity effect is more pronounced due to the small separation between adjacent power and ground lines. The loop inductance versus signal frequency for paired grids is shown in Fig. 10.6. The wider the line, the lower the frequency at which the onset of the proximity effect occurs and the larger the relative decrease in inductance [277], [278], as depicted in Fig. 10.6. Thus, the primary mechanism for a decrease in inductance in paired grids is the proximity effect.

The loop inductance versus signal frequency for interdigitated grids is shown in Fig. 10.7. As in paired grids, multi-path current

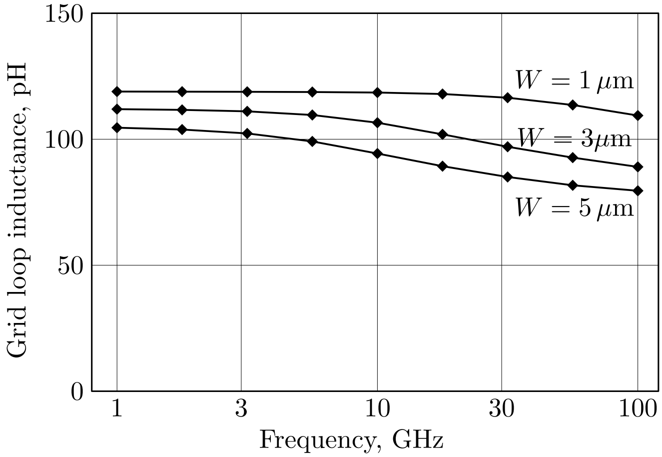


Fig. 10.6. Loop inductance of paired grids versus frequency.

redistribution is insignificant in interdigitated grids. However, the separation between grid lines is large as compared to the line width (unless the line width is comparable to the line pitch) and the proximity effect is, therefore, also insignificant [277], [278]. As shown in Fig. 10.7, the inductance of interdigitated grids is relatively constant with frequency, the decrease being limited to 10% to 12% of the low frequency inductance except for the case of very wide lines where the proximity effect becomes significant.

10.3 Summary

The variation of inductance with frequency in high performance power distribution grids is investigated in this chapter. The variation of inductance with frequency in three types of power grids is analyzed in terms of the mechanisms of inductance variation, as discussed in Section 2.2. These results support the design of area efficient and robust power distribution grids in high speed integrated circuits. The chapter results are summarized as follows.

- The inductance of power distribution grids decreases with increasing signal frequency

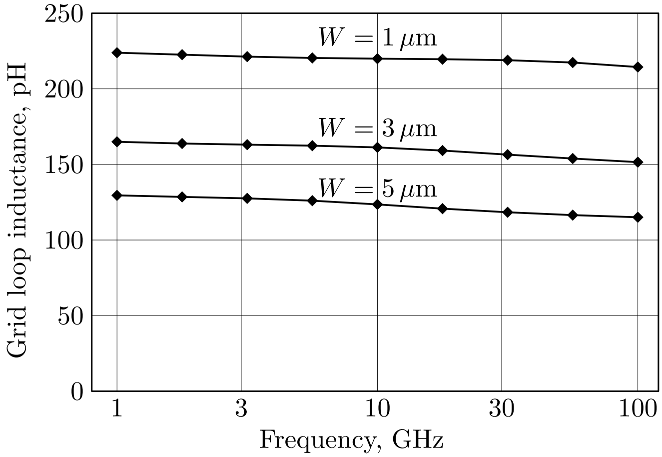


Fig. 10.7. Loop inductance of interdigitated grids versus frequency.

- The decrease in the inductance of non-interdigitated grids is primarily due to multi-path redistribution of the forward and return currents
- Multi-path current redistribution is greatly minimized in interdigitated and paired grids due to the periodic structure of these grids
- The smaller the separation between the power and ground lines and the wider the lines, the more significant the proximity effects become and the greater the relative decrease in inductance with frequency
- The wider the grid lines, the lower the frequency at which the onset of the decrease in inductance occurs