Chapter 9
The Classical Cesium Standard

9.1 Definition of the Unit of Time

We will now take up the type of atomic clock that has been elevated to the status of the primary standard of time, displacing the historical role of astronomical observations in the definition of the unit of time, the second. In 1967 the 13th General Conference on Weights and Measures, attended by delegates from about 40 countries, signatories of the Treaty of the Meter, adopted a new definition of the international unit of time. At that conference there was overwhelming support to the idea that the time had come to replace the existing definition, based on the earth's orbital motion around the sun, by an atomic definition. The wording of the new definition is as follows: "The second is the duration of 9,192,631,770 periods of the radiation corresponding to the transition between the two hyperfine levels of the fundamental state of the atom of cesium-133." The ten-digit number assigned in the definition was chosen to agree with the then existing definition of the second, known as the "ephemeris second," which had been adopted in 1956. This latter definition was based on the length of the so-called tropical year, that is, the length of time for the earth to complete its orbit around the sun and return to a point where its axis again makes the same angle with respect to the earth–sun direction; it is the repetition period of the seasons. The obvious drawback to this definition is the practical one of not being available except through the intermediary of stable clocks that must be checked after the fact. But more importantly, a decade after its adoption it had become evident that the accuracy of atomic clocks, which had to be used to implement the ephemeris time, had reached the point where they had become *de facto* standards against which astronomical observations were compared.

9.2 Implementation of the Definition: The Cesium Standard

This new definition is based on the same type of microwave resonance as in the Rb standard, but because of some advantages in detail, the resonance chosen is in the heavier alkali atom, cesium. We should point out, however, that the labels Rb

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standard and Cs standard in common use do not refer merely to the species of atom used, but rather imply certain ways in which they attempt to extend the interaction time between the undisturbed atom and the resonant microwave field. In the common Rb standard we recall that a noble gas is used as a buffer to prevent the free flight of the Rb atoms to the walls of the cells, where their coherent response to the field would be interrupted, and the resonance thereby broadened. By contrast, in the cesium standard the atoms move freely as a beam in a chamber from which the air has been pumped out, that is, in a vacuum. (The use of the word "beam" is more than metaphorical; after all, a light beam can be looked on as a stream of photons.) There is, of course, no fundamental reason that precludes observing the Cs resonance in a diffusion cell by optical methods, or of observing the Rb resonance in an atomic beam machine; in fact, both possibilities have been explored in the past.

In the early development of these devices, they differed not only in the "containment" of the atoms, but also in the way the microwave resonance was made observable: The Rb clock detected resonance by optical hyperfine pumping using a "conventional" uhf-excited vapor lamp as a source, and the Cs standard used magnetic deflection as in the Stern–Gerlach experiment. We will describe in this chapter what might be justly called the classical Cs beam standard using magnetic deflection and reserve to a later chapter the laser-based systems.

Observing atoms in free flight ensures that they suffer only the desired interaction with the resonant field, and not with background particles or optical pumping radiation, both of which, we have seen, produce shifts in the resonance frequency. It is precisely this freedom from unpredictable frequency shifts that made the Cs standard uniquely suitable as a primary standard. Ideally, such a standard must make possible the faithful observation of the sharpest possible resonance with the highest possible signal-to-noise ratio, on a system insensitive to operating conditions. In fact, we can quantify this statement by recalling the result cited in Chapter 7 that for any resonator acting as a frequency reference, the uncertainty in finding the center frequency is $\Delta\nu/(S/N)$, where $\Delta\nu$ is the frequency width of the resonance, and S/N is the signal-to-noise ratio. A figure of merit that increases with decreasing uncertainty can therefore be defined as $F = (S/N)(\nu_0/\Delta\nu)$. In the case of the Cs standard, S/N is ultimately limited by shot noise due to the atomic nature of Cs and $S/N = \sqrt{n}$, where n is the number of atoms contributing to the resonance signal.

In the classical Cs standard the atoms undergoing the resonant transitions move *in vacuo* with thermal velocities, acted on by only a weak uniform magnetic field and the probing resonant microwave field. To deduce the "true" transition frequency of Cs at rest in zero magnetic field, free of interaction with a microwave field generator, involves deterministic or *systematic* corrections based on well-established theory. Thus as long as we believe that a cesium atom is a cesium atom no matter what its provenance, we have a universally reproducible standard. Of course, we can always speculate as to whether it is possible that the fundamental

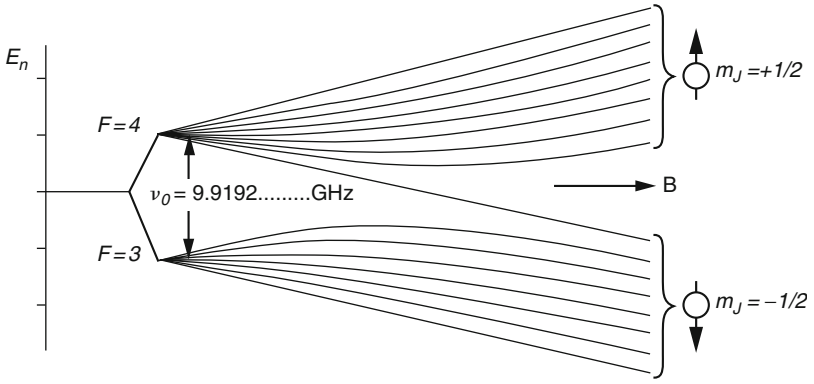

Figure 9.1 The energy of magnetic hyperfine states in cesium 133 as a function of an applied magnetic field

properties of atoms may be slowly evolving relative to a time scale established by other dynamical processes in the universe; however, this is of no practical concern.

The two hyperfine states between which the resonant frequency of transition defines the standard second are indicated in Figure 9.1, which shows the energies of all the hyperfine substates plotted as a function of the intensity of an external magnetic field. The cesium atom has only one stable isotope, mass 133, with nuclear spin $I = (7/2)h/2\pi$, which coupled with the outer electron spin $J = (1/2)h/2\pi$ yields according to quantum rules the following total angular momentum: $F = 4$ or $F = 3$ in units of $h/2\pi$. We saw in an earlier chapter that for magnetic field intensities near zero, the atoms in these two hyperfine states act like bar magnets, which however obey space quantization rules; that is, they can be observed to have only (in this case) integral values (in units of $h/2\pi$) for their components along the field axis. Thus an atom in the $F = 4$ state is further characterized by the magnetic quantum number m_F, giving the discrete components of the angular momentum, which can have only the integral values $+4, +3, +2, +1, 0, -1, -2, -3, -4$; and similarly for the $F = 3$ state. Near zero magnetic field intensity, the energies of the states with different m_F increase initially in a linear fashion with the field, with a gradient proportional to m_F, as would a bar magnet, to give us straight-line graphs. In particular, the substates having $m_F = 0$ have zero slope, and therefore a transition between them, the so called "0–0 transition" is not broadened by small field inhomogeneity. It is therefore chosen to define the second. As the magnetic field is made more intense, the energies of all but two of these states no longer increase in proportion to the intensity of the field; instead, the graphs start curving until for large field intensity they become grouped in two nearly parallel sets, as shown in Figure 9.1. The total angular momentum vector is no longer constant in time (because of the torque exerted by the field), and a different set of quantum numbers is required to specify the substates. In the limit, for very intense magnetic field, the electronic moment and the nuclear moment separately have constant components

along the field. The appropriate quantum description is in terms of quantum numbers giving the integral or half-integral components of each separately along the field axis. In the present case we have for $I = 7/2$ the following 8 possible components, with $m_I = +7/2, +5/2, +3/2, +1/2, -1/2, -3/2, -5/2, -7/2$, and for $J = 1/2$ only two possible components, $m_J = +1/2$, and $m_J = -1/2$. There are $8 \times 2 = 16$ possible combinations of m_I and m_J, the same number as we have in terms of F and m_F, where we had 9 substates with $F = 4$ and 7 substates with $F = 3$: $9 + 7 = 16$. This is as it should be, since increasing the field strength alone cannot generate new quantum states; it can only change their energy.

The way the energy of the substates varies with the intensity of the magnetic field is of particular interest for us, since that energy constitutes the potential energy whose gradient determines the force with which the magnetic field acts on an atom to accelerate it. It follows that atoms in the group of substates whose energy increases with magnetic field will experience a force in the direction of decreasing field intensity, while conversely, atoms in the other group of substates will tend to move in the direction of increasing field intensity. Thus atoms acted on by nonuniform magnetic fields will not only execute the usual precessional motion but also experience a body force affecting the motion of their center of mass. There is a further essential point that must be made before we describe the beam machine in more detail: Atoms remain in the same quantum state as long as they move in a smoothly varying magnetic field without going through zero value, ensuring at all times that the time-varying field they see has negligible amplitude in the Fourier spectrum at the precession frequency. These facts are exploited in the atomic beam machines to deflect the atoms selectively according to their quantum state.

We have already been introduced to the idea of atomic beams, their formation and use in the study of magnetic resonance in free atoms and molecules. We have noted the culmination of that technique in the introduction by Ramsey (Ramsey, 1949) of the two separated field regions to induce transitions, which ultimately led to the adoption of the Cs standard as the primary one. The essential elements of a Cs beam machine using magnetic state selection are exemplified by the PTB (Physikalisch-Technische Bundesanstalt) standard designated as CS1, shown schematically in Figure 9.2. In a generic design, atoms from the source enter the strong magnetic field of the polarizer A-magnet, where because of a steep transverse gradient, atoms in the two groups having opposite energy-field dependence are deflected in opposite directions. By suitable beam stops, the atoms in the $F = 3$ group, including $m_F = 0$, can be removed, leaving only those in the other group with $F = 4$, among which are atoms in the desired $m_F = 0$ substate. These atoms leave the intense field of the polarizer magnet with greater number in the ($F = 4$, $m_F = 0$) state than in the ($F = 3, m_F = 0$) state, and *remain* in their respective quantum states as they continue to the much weaker, uniform C-field. If the oscillatory field applied there is *off* resonance with the desired quantum transition, they will again be deflected by the analyzer B-magnet in the same direction as in the polarizer and away from the detector. On the other hand, if the oscillatory field in the C-region is on resonance, some of the atoms in the ($F = 4, m_F = 0$) state will

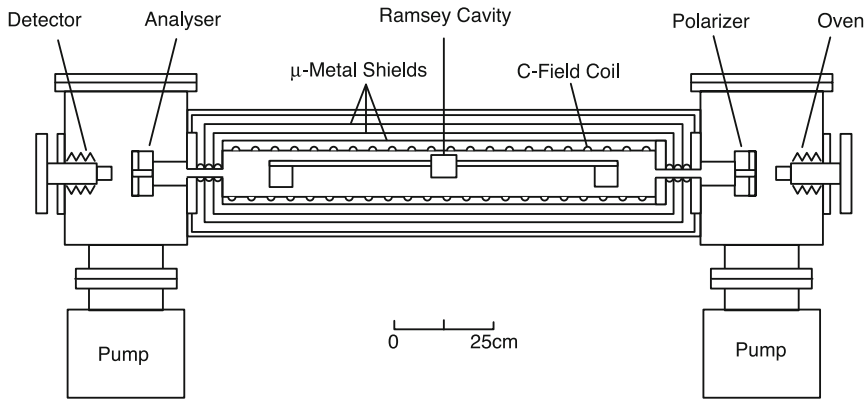

Figure 9.2 General layout of a Cs beam atomic standard as exemplified by the CS1 primary clock at PTB (A. Bauch, et al. 2000)

make the transition to the ($F = 3, m_F = 0$) state, which are deflected in the opposite direction by the analyser, towards the detector. This mode of design is called "flop-in," since only atoms that have made the desired transition are detected, to distinguish it from designs in which resonance leads to atoms being deflected away from the detector. In general a number of different configurations are possible; the choice is ultimately determined by considerations of signal-to-noise ratio.

9.3 The Physical Design

9.3.1 The Vacuum System

The entire space through which the atoms pass must be under high vacuum, and therefore a vacuum shell encloses that space, and suitable vacuum pumps and vacuum monitoring instrumentation must be provided. It happens that Cs has, for a metal, a relatively low melting point at 28.5°C and has an equilibrium vapor pressure as high as 10^{-3} Pa at 24°C. This dictates that a means must be provided to remove background Cs vapor, since that vapor density is comparable to that in the beam. In laboratory installations this formerly took the form of "cold traps," liquid containers forming part of the vacuum shell that are cooled by filling them with liquid nitrogen at −196°C. More commonly now, particularly in compact systems designed to be more or less portable, *getters* are used; these are materials onto whose surface the Cs either physically attaches in a process called adsorption, or with which it chemically combines, thereby removing it from the volume. In common vacuum practice molecular adsorbents such as carbon, or *zeolites*, which are alkali–metal aluminosilicates, are used. For a chemically reactive element such as Cs, any number of substances will serve as getters; a secondary criterion must

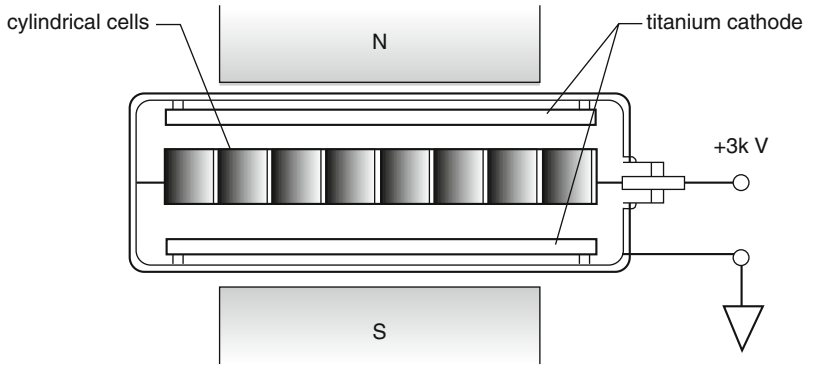

Figure 9.3 The field geometry of the titanium ion pump

be used to make a selection, such as low vapor pressure, temperature stability, and cost. Carbon surfaces are commonly placed at points where unwanted cesium atoms must be removed from the volume.

The element that occupies a unique position as a getter is titanium, either as a film deposited by evaporation from a titanium filament or as plates forming the negative electrodes in an electrical discharge. In the latter case the getter action is achieved by having the particles to be pumped impinge on the titanium surface as high-speed ions. The ions are formed in an electrical discharge made possible under very high vacuum conditions by the entrapment of electrons using a special electrode configuration in a strong magnetic field. This class of ion pump, illustrated in Figure 9.3, is effective in pumping all gases, including the noble gases. It has, since its introduction in the 1950s by Varian Associates, revolutionized vacuum technology, making it possible to reach the vacuum of outer space. It is universally used now to maintain the requisite high vacuum in Cs beam systems. The need to operate under high vacuum in a system whose length essentially determines the accuracy, largely dictates the physical size and aspect of the Cs standard, and in particular implies that the highest accuracy can be reached only in a fixed laboratory installation.

9.3.2 The Atomic Beam Source

The source of the Cs beam is a small constant temperature enclosure, the oven, in which the vapor density of the atoms is raised by heating a small quantity of the silvery metal to around 100°C. The Cs vapor from the oven passes through a collimator, or *effuser*, consisting often of a bundle of capillary tubes or finely crinkled metal foil forming a multichannel nozzle that is intended to cause atoms to emerge in as narrow a ribbon (or sometimes cone) as possible. The operating temperature is such that the vapor density is below the point where collisions between atoms

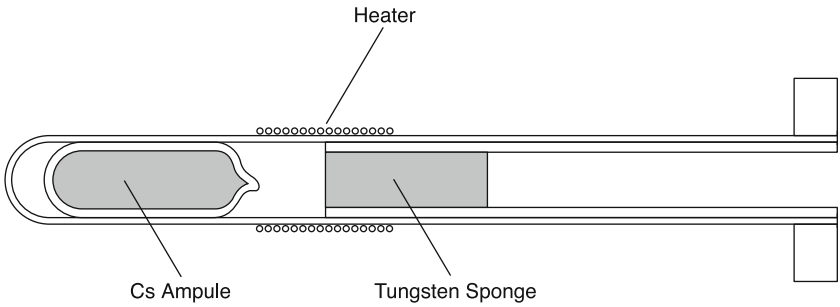

Figure 9.4 A recirculating Cs beam source. (Drullinger, et al. 1981)

can occur with significant probability within the collimator. Under this condition the movement of the atoms through it is described as *thermal effusion*, to distinguish it from the case in which the vapor density is very much higher, in which case the flow is called *hydrodynamic*, as in a gas jet. The most critical part of the source is obviously the collimator, and a great deal of care in the design and operation of the source must be taken to ensure that no buildup of Cs occurs in the collimator, causing fluctuations in the beam intensity. This requires that the temperature of the collimator be maintained sufficiently high, with the consequence that Cs atoms originating from the interior surfaces of the channels themselves add to the emitted beam. For this reason this type of source is sometimes referred to as a *bright-wall oven* to distinguish it from a less common design using an effuser made of Cs-adsorbing carbon, for example, which would be called a *dark-wall oven*.

In spite of all efforts in the design of the collimator to project a sharply narrow beam, it is inevitable in practice that a not inconsiderable amount of the cesium is sprayed out and lands uselessly on the first beam-defining aperture. This, of course, limits the useful life of the charge in the oven. In an attempt to overcome this limitation, *refluxing*, or re-circulating, ovens have been designed, in which the heat applied to the oven establishes a falling temperature gradient along a single collimator tube, reaching a value just above the melting point of cesium at the tip. The liquid cesium that would accumulate in the collimator and would be intolerable in a conventional bright-wall oven, is drawn back, in this design, into a reservoir filled with a tungsten sponge impregnated with cesium, by a clever use of capillary action. A form of re-circulating Cs oven is represented schematically in Figure 9.4.

9.3.3 The Polarizing and Analyzing Magnets

The powerful polarizing A-magnet has pole faces specially contoured in order to produce a steeply varying intensity from one point to another. The purpose of this magnet, we recall, is to act on the magnetic moments of the atoms to deflect them and thereby spatially separate them according to their magnetic quantum state.

There are essentially two different types of state-selecting magnets: focusing and non-focusing. The original Stern–Gerlach magnet and its later variants are non-focusing 2-pole magnets, with pole faces contoured to produce the steepest possible descent in the field intensity as we go from one pole to the other. Since not all atoms enter the field along precisely the same trajectory, clearly if the field gradient, and hence the force they experience is not the same at all points in the field, then the beam "profile," that is, the distribution of atoms over a cross section of the beam will be affected. The main object of magnet design is to produce a field gradient over the cross section of an atomic beam that if anything distorts the profile in a beneficial way that is, reduces natural divergence. One variant of the Stern–Gerlach magnet is shown in Figure 9.5.

There are two types of focusing magnets: the quadrupole with a 2-fold axis of symmetry, and the hexapole magnet with a 3-fold axis of symmetry. First let us dispose of the simpler quadrupole type of magnet, which has been exploited far more in the focusing of ion beams than neutral atomic beams. In the neighborhood of the magnet axis it can be shown that the field components are well approximated by $H_x = kx, H_y = -ky$, where x, y are coordinates referred to Cartesian axes X, Y chosen to bisect the north and south poles of the magnet. The resultant field is therefore $H = k(H_x{}^2 + H_y{}^2)^{1/2} = k(x^2 + y^2)^{1/2} = kr$, where k is a measure of the overall strength of the magnet and r is the radial distance from the axis to the field point. The motion of Cs atoms in such a field is complicated by the fact that their magnetic energy, which acts as potential energy analogous to the potential energy of an object moving under gravity, is not simply proportional to the magnetic field

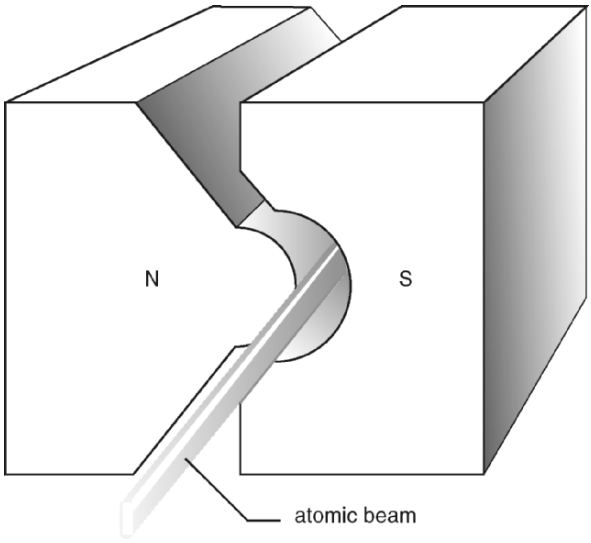

Figure 9.5 A constant gradient state selecting magnet

intensity, as it would be for a bar magnet, but rather is a nonlinear function of the field, given by the Breit–Rabi formula we have already encountered. It is as if we were dealing with the motion of a magnet whose strength varied from point to point according to the strength of the magnetic field it is passing through. We recall that at sufficiently high field intensities the plots of the energy versus field strength do tend to become linear, and moreover, in that limit the electronic and nuclear moments separately maintain a constant (quantized) angle with the direction of the magnetic field. In this high field limit, it follows that the force experienced by the atoms is simply proportional to the gradient in the magnetic field intensity, which is constant and in the radial direction. Under the assumed conditions, then, the atoms issuing from the source in the quantum states whose energy increases with field intensity would converge towards the axis, while the others would diverge away from it. The particle trajectory in a axial plane is similar to that of a particle falling under gravity.

In the same limit of high field intensity, the focusing properties of the important hexapole magnet, shown in Figure 9.6 are equally simple to predict. In this case the field in the neighborhood of the axis is approximated by $H_x = k(x^2 - y^2)$ and $H_y = -2kxy$, which lead to a resultant field $H = k(x^2 + y^2) = kr^2$ and a force that is radial and converging or diverging according to the same condition on atomic state cited above. In this case we see that the gradient of the field, and hence the force, is proportional to the distance from the axis, analogous to the force of an elastic spring. In fact, the radial motion will be a simple harmonic oscillation for atoms in one group of hyperfine states, and rapidly (exponentially) diverging from the axis for the other group. More will be said about the hexapole magnet in connection with the hydrogen maser in a later chapter.

Since the field intensity tends to zero on the axis for both types of focusing magnets, beam stops must be used to eliminate atoms that would otherwise go through without state selection. Unfortunately, since the beam-forming effuser of the source commonly produces a beam profile that peaks on the axis, such a beam stop would seriously diminish the utilization efficiency of the Cs. A possible solution would be an off-axis ring-shaped source.

We should note one very important limitation of both focusing magnets: unlike the 2-pole magnet, there is no choice as to which states are focused and converge

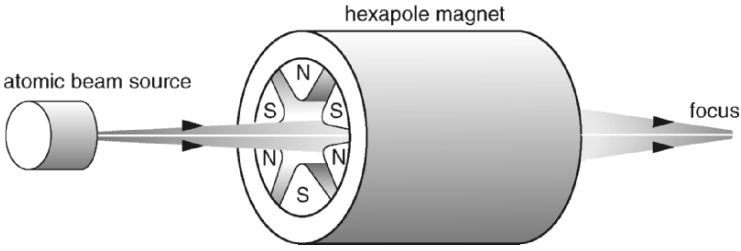

Figure 9.6 The hexapole atomic beam focusing magnet

towards the axis and which diverge away from it. In our case, atoms in the $F = 4$ state will always converge, and those in the other $F = 3$ state will diverge from the axis. This makes it impossible to have both the A- and B-magnets focusing in a "flop-in" design.

9.3.4 The Uniform C-Field

After leaving the intense field of the state-selecting A-magnet, the atoms must pass through a gradually decreasing intensity to the uniform C-field, without changing their quantum states. This requires that the time-varying field seen by a moving atom have negligible amplitude in the Fourier spectrum at the transition frequencies between the magnetic substates. Failure to meet this requirement leads to undesirable transitions between the magnetic substates, given the name *Majorana transitions*. Such transitions would cause *relaxation* between the desired $m_F = 0$ substates, defeating the state-selecting function of the magnet. The same situation is encountered in the subsequent transition from the C-field to the powerful analyzer B-magnet.

In the elongated C-field region, transitions between the two hyperfine states are resonantly induced by an oscillatory magnetic field. In this region the magnetic field must be relatively weak to take advantage of the first-order insensitivity of the energy of the $m_F = 0$ substates to magnetic field intensity in the neighborhood of zero field. On the other hand, the field must be intense enough to produce a sufficient separation among the m-substates, $m_F = 0, 1, 2, 3$, etc., so that the resonant field does not also cause field-dependent $\Delta m_F = \pm 1$ transitions; otherwise, field variations would further broaden the transition frequency. Needless to say, the C-field must be as uniform and stable as possible, and therefore magnetic shielding from extraneous magnetic fields is necessary. This is accomplished by enclosing the region with one or more thicknesses of high-permeability magnetic alloys such as mu-metal or supermalloy. A highly uniform magnetic field (the C-field) is produced typically by current flowing in a pair of rectangular coils placed symmetrically parallel to the beam; their separation is chosen to produce a constant field of the highest possible uniformity over the section of the atomic beam where the transitions are induced. Alternatively, an electromagnet with precisely machined plane parallel pole faces could in principle be used; however a properly designed coil system can realize adequate homogeneity of the field.

9.3.5 The Transition Field

As already indicated, the one refinement of the Cs beam resonance apparatus that put it in the class of a primary standard is the successive oscillatory field geometry introduced by Ramsey in 1949 for probing the atomic resonance. To appreciate this, we must go back to the Rabi field and examine the problems attendant upon the attempt to observe a microwave resonance in atoms traveling in a

beam with thermal velocities on the order of 250 meters per second (about 600 mph). These problems arise from the fact that the length of time the atoms interact with the resonant field is determined by the length of the field region. We recall that the frequency width of the resonance is increased as this time is made shorter; hence the length should be made as great as possible. In fact, the length must be on the order of one or two meters to yield resonance line widths small enough to be interesting for a frequency standard. If L is the length of the transition region and V the average thermal velocity of the atoms, then the average transit time is L/V, and the resonance line width is about $\Delta\nu \approx \frac{1}{2}(L/V) = V/(2L)$. Hence for $L = 1$ m and $V = 250$ m/s we find $\Delta\nu \approx 125$ Hz. This is a fundamental width, which can be derived simply from the Fourier spectrum of a pure oscillation of finite duration, as seen by any given atom. This oscillation starts from zero and rises to a constant amplitude for a finite period L/V while the atom is in the transition region, then falls again to zero; its Fourier spectrum is illustrated in Figure 9.7.

A serious consequence of the need to have an extended interaction region is the Doppler shift arising from the directed motion of the atoms though the oscillatory field. It may be thought that this may be overcome simply by using a stationary wave pattern in the interaction region. However, aside from the practical difficulty of ensuring a strictly stationary field, even if there were no net displacement of the resonance frequency, there would nevertheless be a broadening of the spectrum. This may be seen from the following argument: Since the wavelength of the microwave field is only about 3 cm, the atoms would pass through a field whose amplitude and phase vary periodically along their path; that is, they see a modulated field, the frequency of the modulation depending on their velocity. Such a modulation has a Fourier spectrum consisting of two equal sidebands separated from the center frequency by the Doppler frequency $(V/c)\nu_0$. We can reach this same conclusion by thinking of the standing wave as a superposition of two equal

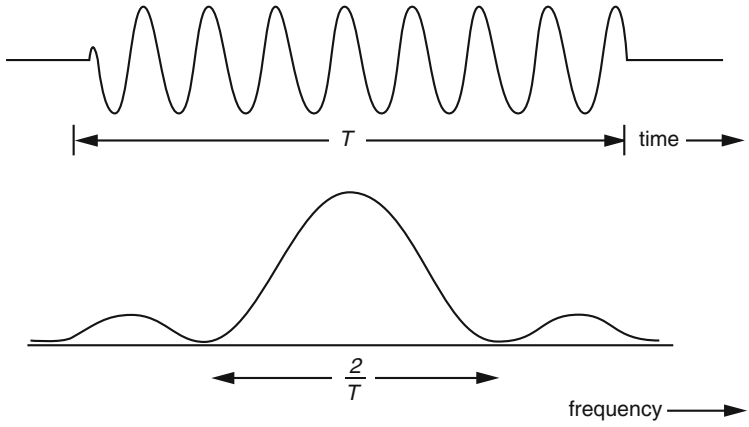

Figure 9.7 An oscillatory field of finite length and its Fourier (power) spectrum

waves traveling in opposite directions, as when a wave on the surface of water is reflected back on itself by a straight wall; each wave would have a Doppler shifted frequency in the direction opposite to the other. Since the atoms do not all have the same velocity but are distributed continuously over a wide range of velocity, characteristic of the temperature of the oven, the spectrum would consist of a line broadened out by the Doppler effect.

9.3.6 The Ramsey Separated Fields

These problems are removed, following Ramsey, by applying the resonant oscillatory field coherently (that is, with a definite phase relationship) in two separated narrow regions, one at the entrance and the other at the exit to the extended C-field transition region. Although the actual length of time a given atom interacts with the oscillatory field is thereby drastically reduced, it can be shown that since the fields in the two regions are in phase, the frequency width of the net response of the atoms traversing the whole transition region is determined by the much longer time the atoms spend in the intervening space. In order to ensure that the fields in the two regions maintain a constant phase relationship, a common microwave source is used, and the fields are symmetrically located at the ends of a single resonant microwave cavity, as shown in Figure 9.8. Thus the field in each narrow region can be limited to one with a single phase and nearly constant amplitude over the cross section of the beam.

The resonant cavity, the microwave analogue of an echo chamber, is usually a section of rectangular wave-guide with 90-degree bends at its ends, where apertures are provided for the Cs beam to pass through the standing microwave field pattern. The cavity is terminated at the two ends by a short circuit and the atoms pass through an antinode of the standing microwave field pattern. In order to inhibit the leakage of the microwave field parallel to the atomic beam, small sections of waveguide are mounted on the beam apertures parallel to the beam. As in all resonant structures, the resonant modes, with their characteristic frequencies and

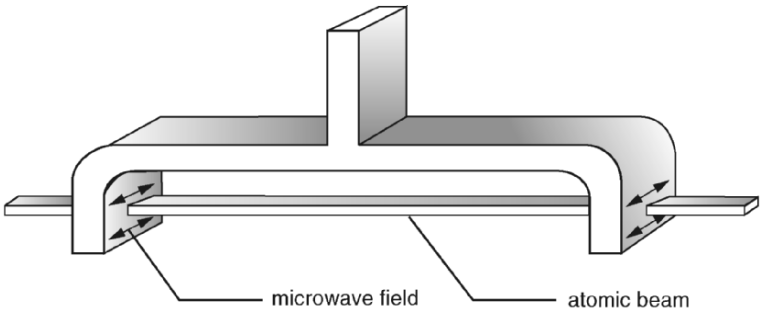

Figure 9.8 The Ramsey separated field atomic resonance cavity

field patterns, are determined by a formula relating the resonant frequencies to the dimensions of the cavity, a formula that contains three integers, the mode indices. A particular mode that might be used is designated as TE_{10n}, which represents what is called a *Transverse Electric* mode, that is, one in which the electric component of the electromagnetic wave is everywhere perpendicular to the length of the cavity. The indices 1, 0, n give the number of times the electric field passes through a maximum amplitude of oscillation as we go in the directions of the three principal dimensions of the wave-guide. Thus if the cross section of the cavity is a rectangle with sides A and B, where $A > B$, and the length is C, then the indices indicate that in this particular mode the field rises to one maximum in the middle of the A dimension, has no maximum (is constant) along B, and has n maxima along the length C. For the dimensions A, B, C to be compatible with this mode, it can be shown that the following condition must be satisfied: $C = n\lambda_g/2$, where $\lambda_g = \lambda_0/(1 - \lambda_0^2/4B^2)^{1/2}$ and λ_0 is the free-space wavelength of the microwaves. Thus in our case the wavelength of the microwaves resonant with the Cs transition is $\lambda_0 = 3.26$ cm; if we assume, for example, $B = 2.5$ cm, then $\lambda_g = 4.3$ cm and a choice of $n = 48$ would make the length of the cavity 103.2 cm, appropriate for a fixed installation. To be effective in inducing transitions between the substates $F = 4, m_F = 0$ and $F = 3, m_F = 0$, it is not enough for the frequency of the microwaves to satisfy the conservation of energy condition $h\nu = \Delta E_{hfs}$, where ΔE_{hfs}, represents the difference in energy between the two hyperfine states; the microwave field must also have the correct directional properties, that is, polarization. We recall that to observe magnetic resonance between Zeeman substates in which m_F increases or decreases by one, the field inducing transitions must have an angular momentum component along the constant field axis to satisfy the conservation of angular momentum law as it applies to the combined system of atom and radiation field. To have such a component of angular momentum, the radiation field must have a component rotating about the constant field axis. Similarly here, since there is no change in m_F, being zero before and after the transition, and since only one quantum of radiation is involved, it must have zero component of angular momentum along the constant field axis. This will be the case if the radiation field oscillates parallel to the constant field; this determines the relative orientation of the microwave cavity and the coils producing the constant field.

To help gain a broader perspective on the use of separated fields we should mention a closely parallel case in radio astronomy of the use of two separated radio antennas, as shown in Figure 9.9, to increase the angular resolution in observing distant sources. By maintaining a common phase reference for the receivers at the two antennas, the system's ability to distinguish neighboring sources, that is, its *resolving power*, is made to approach that of a much larger antenna having a diameter equal to the distance between the two small antennas. To see this we must recall that even if an antenna were perfectly parabolic, so that rays coming in parallel to its axis would geometrically converge to a point focus, physically the reflected wave pattern does not converge exactly to a point; it approaches this ideal only to the extent that the *aperture*, that is, the diameter of the antenna, is large compared

to the wavelength of the radio waves. The wave pattern near the geometric focus will be a series of maxima and minima resulting from the antenna cutting off the incoming wave at its outer rim, whence it spreads in a pattern dictated by interference from different parts of the aperture. The angular width of this diffraction pattern is set by the difference in the phase of an incoming wave across the aperture; the first minimum will occur when that difference in phase is on the order of 360°. The larger the aperture, the smaller will be the required increment in the direction of the incoming wave to produce that phase difference, and the greater the resolving power. We now see that by comparing the signals arriving at two widely separated antennas, a smaller difference in the angular position of distant sources is distinguishable because the longer base line magnifies the difference in the phase of the wave reaching the two antennas, as shown in Figure 9.9.

The same principle is used in the much older "stellar interferometer" of A.A. Michelson, of the velocity of light fame. In this, two optically flat mirrors are mounted some distance apart to receive starlight and the light reflected from them combined through precise optics to a common detector whose output depends on the relative phase between the two interfering reflected beams. By this interferometer Michelson was able to determine the (angular) diameter of stars that were smaller than could be resolved with telescopes available at the time.

Although in the years following its introduction in 1949 the principle of the separated field method has been applied in a variety ways in spectroscopy, it was originally developed to achieve greater accuracy in the measurement of atomic and molecular magnetic moments by the molecular-beam resonance method of I. Rabi. It is that application that provides the most visual explanation of the special properties of inducing transitions this way.

We recall that in an earlier discussion of magnetic resonance we described the gyroscope-like precession of an atomic angular momentum (with an associated

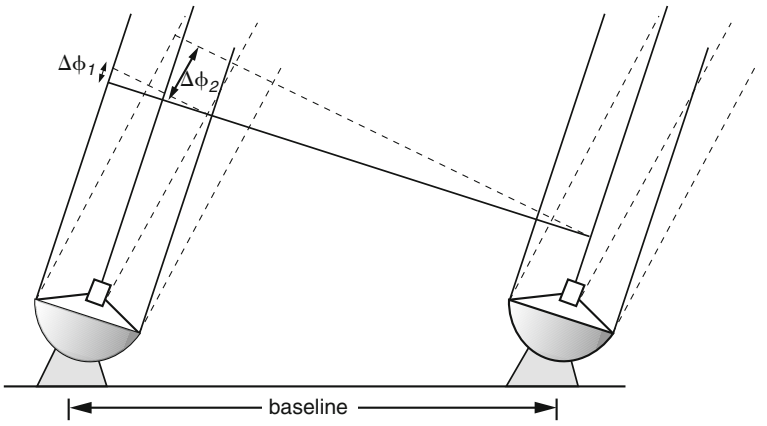

Figure 9.9 The use of separated antennae in radio astronomy to increase resolution

magnetic moment) about a static magnetic field, and how the application of a weak magnetic field oscillating at the frequency of precession will cause the axis of spin to tilt away from the static field so that the cone it sweeps out opens out to a larger apex angle. In the Ramsey arrangement, the atoms see an oscillating field in the first narrow transition region of sufficient strength to produce, for example, a 90-degree apex angle for atoms with the average thermal velocity. On leaving this first transition region, the atoms pass through a relatively long region free of any oscillating field, in which they continue to precess at the same frequency appropriate to the static uniform C-field. The atoms then enter the second narrow region, where they are again subject to an identical oscillatory magnetic field, which has a definite phase relationship with the first, usually the exact same phase. If the frequency of the oscillating fields is exactly the same as the average precession frequency appropriate to the static field, then the atoms will enter with the same phase as the field, and the direction of the spin axis will continue to tilt toward a cone angle of 180 degrees, corresponding to a complete reversal in the direction of the angular momentum. Note that the phase of the precessing moment relative to the oscillatory field will determine the direction and degree of tilt the latter produces; hence if the precession frequency in the C-field differs only slightly from the frequency of the oscillatory field, a large phase difference can develop in the intervening space, and the degree of tilt will be strongly reduced.

If the atoms all had precisely the same velocity, then there would exist a difference in frequency between the precession and the oscillatory field that will lead to a phase difference of exactly 360 degrees being developed between them in the space between the two transition regions; that is, the atoms would again enter the second region in phase with the field. In fact, the same would happen at frequencies leading to a phase difference of any multiple of 360 degrees. However, in reality, the atoms do not all have the same velocity, and these multiple "sidebands" occur at frequencies that depend on velocity, since a slow atom spends more time in the field and requires a smaller difference in frequency to develop the 360-degree phase difference than does a faster atom. The resonant frequency has the unique property of being independent of velocity; no phase difference can develop if the oscillatory field and the precession have the same frequency, no matter how long it takes an atom to reach the second transition region. Moreover, since there is a continuous distribution of velocity among the atoms, the sidebands form a continuum of reduced strength leaving a prominent central peak at exact resonance.

A quantitative analysis of the probability that an atom passing through the two separated field regions will emerge having made a transition to the other substate requires an exact quantum treatment of the problem, as was initially carried out by Ramsey. It would be inappropriate to attempt to reproduce that theory here; rather, we will try to gain some insight as to the shape of the resonance signal using a quantum result that is strictly valid only where the "perturbation" acting on an atom is weak. It is that the transition probability is proportional to the square of the Fourier amplitude of the field at the transition frequency. Although the perturbation of the atoms here is far from weak, nevertheless it serves to provide

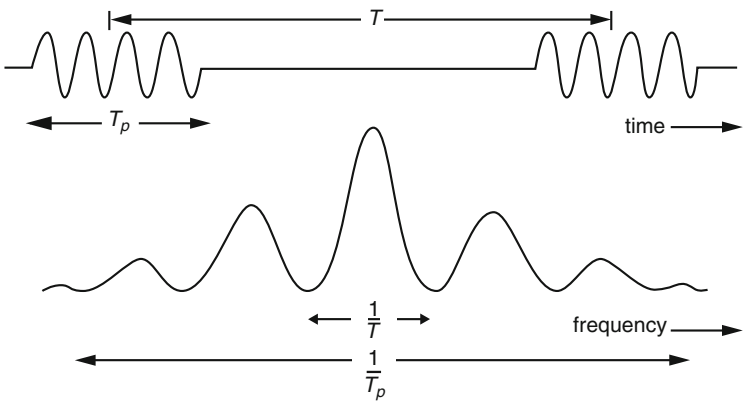

Figure 9.10 Field seen by an atom in the Ramsey cavity and its Fourier spectrum

some general basis for understanding the system. In any event, since the exact results are known for this case, there is little danger of being misled by an invalid approximation.

The time dependence of the assumed separated field is shown with its Fourier spectrum in Figure 9.10. If this is compared with the Fourier spectrum of an oscillatory field extending the full length of the C-field, we find that in fact the central maximum in the frequency spectrum is even narrower for the two separated fields than for the single extended one, a fact proved rigorously by Ramsey in a full quantum-mechanical treatment of the problem.

Since the duration of a given atom's interaction with the separated fields and the time spent between them depends on the atom's velocity, the signal produced by a beam consisting of a large number of atoms having a thermal distribution of velocities is obtained by summing over the contributions from individual atoms. This has been analyzed rigorously by Ramsey, including the effect of introducing phase differences between the two separated field regions; the result for zero phase difference is shown in Figure 9.11.

The Ramsey arrangement alleviates another problem: that of ensuring a sufficiently uniform and constant magnetic C-field over an extended space. This would clearly involve a complex array of compensating coils and impose severe tolerances on the mechanical and electrical parameters, and particularly the shielding from external magnetic fields, etc. Fortunately, the phase difference that accumulates between the atomic moments and the oscillatory field, as the atoms travel between the two transition regions, depends on the spatial average of the C-field taken over the path of the atoms. It is reasonable to expect that this average fluctuates from atom to atom far less than the field strength itself along the path of any given atom.

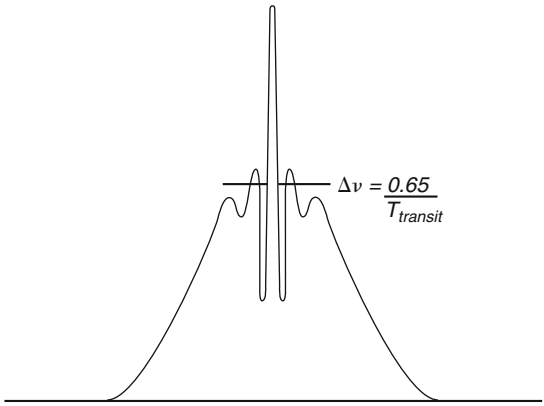

Figure 9.11 Theoretical signal shape for thermal atoms passing through Ramsey field (Ramsey, 1949)

9.4 Detection of Transitions

Next the atoms pass through the powerful analyzing B-magnet, which serves to analyze the magnetic states of the atoms, thereby monitoring the occurrence of the desired microwave transition by the change in the number of atoms reaching the final element, the detector. Since the greatest challenge in the design of an atomic beam machine is to achieve a high signal-to-noise ratio, which means, because of shot noise, a high beam intensity, it would seem advantageous to use focusing magnets for the A- and B-magnets. However, this presents a dilemma, since both magnets would focus atoms that are in the same state, so that ones that have made a transition in the C-region to the other state would diverge from the axis. If the detector is placed on the axis, then it would be exposed to the atoms that had *not* made a transition; to detect ones that *had* made a transition, it would have to accept atoms over an extended circular area. Neither option is particularly desirable, the first because the signal to noise ratio is compromised by the shot noise due to the larger number of atoms that have not made a transition, and the second because the increased area may incur greater noise from background Cs vapor.

The availability of an efficient low-noise Cs detector played a critical part in the development of the Cs beam resonance apparatus It is the so-called *hot wire detector* (which in fact is more typically a ribbon) which is based on the phenomenon of surface ionization of the alkali atom, in which a Cs atom impinging on certain pure metallic surfaces (which must be maintained at high temperature to prevent surface layers of adsorbed gases) loses an electron to the metal and emerges as a positively charged ion. The phenomenon is permitted by the energy conservation law for metals whose binding energy of an electron to the interior of the metal (the so-called *work function*) is greater than the binding of the outer electron in the Cs atom (3.87 electron volts). This is true of such metals as tungsten, niobium, molybdenum and

the alloy Pt-Ir. It is remarkable not only that the process occurs at all (the electron has to pass through a classically forbidden barrier to do it), but that it does so with a very high probability, so that very nearly all atoms reaching the surface of the metal become ions. The main difficulty in the early application of this type of detector was that other ions, particularly those of potassium, are also emitted. Thus for the highest possible signal-to-noise ratio, not only were the purest available materials used, but also a mass filter was incorporated into the detector design. The mass filter often took the simple form of a 60°-sector magnetic deflection type (now one would opt for a Paul RF quadrupole mass filter) with the output focused on the cathode of an electron multiplier capable of counting individual ions.

However this form of detector design has been supplanted in recent years through advances in the fields of materials science and solid state electronics. Thus for example the machine designated as NBS-6 at the National Institute of Science and Technology NIST, Boulder, uses a double ribbon of Pt-Ir detector and low noise field-effect transistor preamplifiers, without a mass filter. This design is admissible because of a sufficiently high beam intensity, which simultaneously reduces the relative importance of impurities in the ribbons and the thermal (Johnson) noise that an electron multiplier is designed to overcome.

9.5 Frequency-Lock of Flywheel Oscillator to Cesium

Like the Rb standard, the Cs beam resonator is a passive device, which does not itself generate any microwave power but merely serves as a frequency reference. This reference can in principle be used in one of two ways: first, one may attempt to manually tune to the atomic resonance a frequency synthesized from the source under test, or more usefully, to automatically control the frequency of a *flywheel* oscillator by means of one or more servo loops to provide a convenient reference frequency output signal. This involves synthesizing from a stable radiofrequency source a microwave frequency that can be set with the utmost precision at the peak of the resonant response of the cesium atoms.

We will limit ourselves to the essentials of a Cs clock, in which a high-quality quartz oscillator, commonly operating at 5 MHz, has its frequency servo-controlled, so that a synthesized microwave frequency derived from it is locked to the center of the Cs resonance. The basic ingredients of such control circuitry have already been described as they apply to the Rb standard. We recall that in order to obtain an error signal for the servo-control, that is, a measure of how far, and in what direction, an applied microwave frequency is away from the center of the resonance curve, we begin by modulating the phase/frequency of the probing microwave field (or the C-field) at a frequency small compared with the frequency width of the atomic resonance, and far from any harmonic or subharmonic of frequencies commonly present in the environment. The form of modulation, once commonly sinusoidal, is more likely to be binary now with the probing frequency switched symmetrically between two values, in order to be

more naturally compatible with the digital synthesizer and processor of the cesium resonant response. In the case of analog modulation, the component of the output from the detector at the modulation frequency is zero when the modulation is symmetric about the center of the resonance curve (assuming it is symmetrical), and is of opposite phase on the two sides of the center. For the digital modulation the output would be ideally zero if the frequency is switched symmetrically about the resonance center. The detector output is connected to a synchronous demodulator whose output is the average product of the incoming signal and a reference signal of the same frequency. By using as reference the signal producing the phase/frequency modulation, we obtain an error signal that is a negative or positive voltage depending on whether the applied frequency is too low or too high. The optimum depth of modulation can be shown to be half the frequency width of the resonant response.

In the case of the Cs beam standard, the modulation of the phase/frequency poses a problem not encountered in the Rb standard: The transit time of the atoms between the two separated transition regions allows the phase of the microwave field (or C-field intensity) seen by the atoms to be different in the second region from what it was in the first. Now, such a difference is known to cause a shift in the observed resonance; however, over a modulation cycle, the shift oscillates symmetrically about zero and merely changes the effective width and phase of the modulation over the resonance curve. Furthermore, there is a delay between the time a given atom passes the second transition region and the time it reaches the detector; these and other possible sources of phase shift dictate that a compensating phase shifter be included to adjust the reference phase in the synchronous detector.

There are, not unexpectedly, numerous possible circuit designs for deriving a microwave field resonant with the Cs transition that is phase coherent with a 5 MHz quartz oscillator serving as a convenient frequency standard. The extent of the sophistication in the electronic design is obviously determined by the tolerable residual phase noise in the probing microwave signal applied to the Cs atoms. In recent years there has been rapid development of sophisticated digital synthesizer techniques to produce signals with extremely low phase modulation (PM) noise, and highly stable low-noise microwave generators, spurred by advances in the spectral resolution of atomic and ionic resonators.

There are broadly two approaches: a multiplication chain synthesis of the frequency to probe the Cs resonance based on a precision 5 MHz voltage controlled crystal oscillator (VCXO), or using the output of a low-noise microwave generator (for example a dielectric resonator oscillator (DRO)) and a direct digital synthesizer (DDS) to synthesize the Cs resonance frequency. An advanced example of a design that can be spacecraft qualified is shown in Figure 9.12.

The microwave source is a 6.4 GHz voltage controlled DRO whose output drives a regenerative divider that is expected to have low phase noise and low temperature sensitivity. Its outputs at 3.2 GHz and 9.6 GHz are separated by a diplexer (not shown in the figure). The 3.2 GHz signal is frequency divided, buffered and filtered in two stages to yield a 100 MHz reference frequency. A 50 MHz clock signal for the DDS is derived by dividing the 100 MHz reference by 2. The DDS

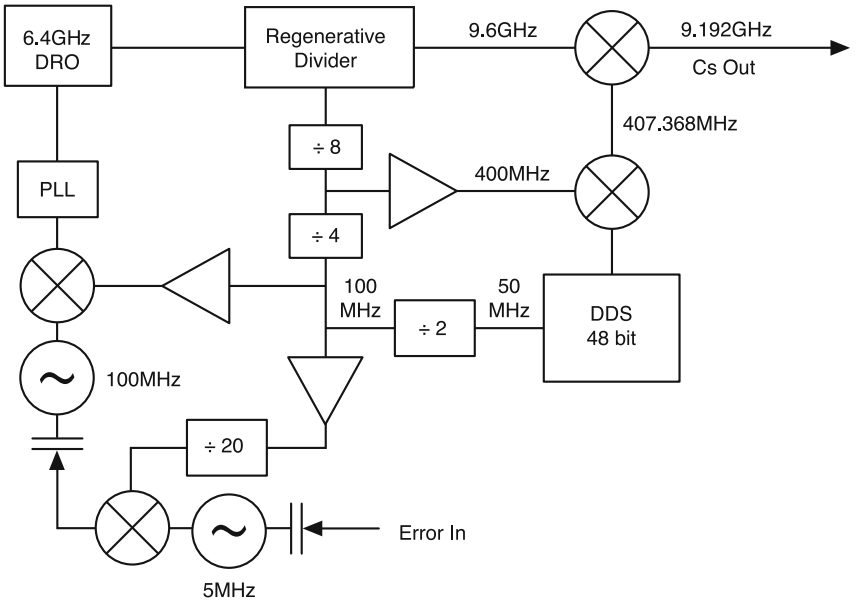

Figure 9.12 Simplified schematic diagram of the Cs frequency synthesizer intended for space applications (Gupta, et al. 2000)

output is a sine wave with a 2 μHz resolution and range of 20 MHz. It is designed to have phase continuity and low switching transients. The frequency output is set at 7.368 MHz and mixed with 400 MHz to obtain 407.368 MHz. This is finally mixed with the 9.6 GHz from the regenerative divider to obtain the Cs frequency of 9.192.... GHz. Since the use of the DRO avoids the use of high multiplication of frequencies, it has particularly low phase noise, and the DDS has a very fine resolution (frequency increments) in order to interrogate the resonance line with high precision. The output of the cesium beam detector is connected to the synchronous detector and integrator, and the servo loop is closed by connecting to the 5 MHz VCXO. In order for the VCXO to be phase-locked to exactly 5 MHz on the atomic time scale, it is necessary to allow for the various frequency offsets of the observed Cs resonance frequency from the intrinsic frequency of an isolated atom at rest. There are several such sources of deterministic frequency deviations, the more important of which are described in what follows. The deviations determine the setting of the DDS.

9.6 Corrections to the Observed Cs Frequency

Prior to being adopted as the basis for the definition of the second, exhaustive studies had been conducted to identify and analyze possible sources of systematic errors in the observation of the Cs resonance. These, we recall, are persistent errors, arising from usually subtle factors, affecting the frequency at which the Cs resonance is observed in a deterministic way, rather than as an unpredictable random fluctuation. It is a primary quest of those responsible for the establishment of physical standards to seek out all conceivable sources of these errors and to attempt to correct them. Having a number of systematic errors does not in itself detract from the acceptability of a standard, provided that they are well understood and calculable. However, the discovery of previously unsuspected sources would obviously be disastrous. In order to gain some appreciation of the complexity of establishing that what is measured *is* in fact what is called for in the definition of the standard, we list in what follows some important corrections.

9.6.1 Magnetic C-Field

The transition frequency in weak magnetic fields, ν, is given by

$$\langle \nu \rangle_C = \nu_0 + 427 \langle B^2 \rangle_C, \tag{9.1}$$

where $\langle\,\rangle_C$ represents the average taken over the transition region of the C-field expressed in gauss. This equation, of course, applies only to Cs and specifically to our hyperfine transition. Typically, B is on the order of 50 milligauss, giving a correction of around 1 Hz. Since it is generally much easier to measure B rather than B^2, some error is incurred if $\langle B \rangle_C^2$ is used in equation 9.1, and B is not exactly uniform.

9.6.2 Unequal Phases of Ramsey Fields

If the phases of the oscillating fields in the two separated transition regions of the Ramsey arrangement are not exactly the same, the detector output will not be at its maximum at resonance. To estimate the displacement in frequency, we note that a 180° difference in phase reduces the detector output to its first minimum, that is, corresponding to a shift in frequency equal to the width of the resonance. If the latter is, say, 125 Hz, a 1° difference in phase would cause a shift in frequency of about $125/180 = 0.7$ Hz, which is not negligible in the context of a frequency standard. This asymmetry in phase and other asymmetries in the apparatus with respect to a reversal of the beam direction may have any number of causes; for example, inertial forces acting on the atoms or strains in the mechanical structure due to acceleration, such as might be experienced in a spacecraft. Where the environment dictates it, or where the utmost accuracy is sought, as in large standards laboratory installations, the effects of such asymmetries are corrected by providing

for Cs beams to traverse the apparatus in opposite directions. Any spurious frequency shift due to asymmetry will reverse direction, and a corrected frequency can be obtained as the average of the two frequencies at which the signal is at its maximum. To provide for the reversal of the atomic beam without opening up the vacuum system obviously requires that both an oven and detector be provided at both ends of the machine, and the ability to move parts and relocate them precisely under vacuum using bellows or sliding seals.

9.6.3 Relativistic Doppler Shift

As we discussed in Chapter 7, the Doppler effect as manifested by electromagnetic waves is not accurately described by classical theory; the formula that is in accord with the principles of relativity cannot differ between situations where only the frame of reference is different. We saw that Einstein's theory yields the following:

$$\nu = \sqrt{\frac{1 - \frac{V}{c}}{1 + \frac{V}{c}}}\nu_0. \tag{9.2}$$

In the present case, V/c is only on the order of 10^{-6}, and so in a power series expansion in V/c, terms beyond the second power are negligible; thus the Doppler correction to the frequency is given by

$$\nu - \nu_0 = -\frac{V}{c}\nu_0 + \frac{1}{2}\frac{V^2}{c^2}\nu_0 + \cdots . \tag{9.3}$$

The first term on the right, which involves V/c to the first power and is the dominant effect, agrees with classical theory and is called the linear Doppler effect. We have already seen how the Ramsey separated field technique circumvents this linear effect; however, at a level of accuracy on the order of 1 part in 10^{12}, the second term, involving $(V/c)^2$, becomes significant. The first thing we note about this second-order Doppler effect is that the shift does not change if the sign of V is changed; that is, it is the same whether the source and observer are approaching or receding from each other. Secondly, we note that as "seen" by the moving atoms, the microwave frequency is higher than would be observed if the atoms were not moving. Therefore, if a moving atom "sees" a microwave field that is resonant with its quantum transition, that same field would be below resonance for a stationary atom; that is, the observed frequency is lower than the "proper frequency" of the Cs transition.

9.6.4 Spectral Impurity of Microwave Field

An ideal standard would have a microwave field whose spectrum consists of a single, infinitely narrow line at a frequency that can be controlled to lock on to the

maximum of the Cs resonance curve. In reality, the microwave field has a distribution of frequencies determined by the microwave oscillator and frequency synthesizer from which it is derived. The potential contributors to this distribution are transients originating in the synthesizer as well as discrete sidebands spaced at intervals of 60 Hz, the commercial power frequency, due to modulation of the crystal oscillator frequency by the ubiquitous AC fields and possibly residual ripple on its DC power supply. This latter source of spectral impurity is aggravated by high orders of frequency multiplication, since the relative amplitude of the sidebands can be shown to increase with the order of multiplication. Serious error is incurred if the sideband amplitude distribution is not symmetrical about the central (unmodulated) frequency. Presumably, proper shielding and the use of battery power would largely eliminate this problem. Of course, just to be able to analyze the spectrum of microwaves at a frequency around 9 GHz with a resolution in the sub-Hertz range is no mean challenge, but such is required nowadays to match the spectral resolution attained in atomic resonances.

9.6.5 Neighboring Transitions

We recall that in addition to the desired $(F = 4, m_F = 0) - (F = 3, m_F = 0)$ transition, there are atoms in the beam in neighboring $m_F = \pm 1$ states that contribute to the signal by making magnetic-field-dependent transitions in which the magnetic quantum number m_F changes by one unit. While the application of a uniform magnetic field in the transition region will separate these transitions from the desired one, there will nevertheless remain a finite probability of their contributing to the signal. Ideally, these transitions have an amplitude distribution that is symmetrical about the center of the desired one; however, in reality it can happen that the way the atomic trajectories fit within the magnet geometries leads to an asymmetric overlap between the desired transition and its neighbors. The consequence is a signal intensity distribution that is distorted, and whose maximum is displaced with respect to the true resonance frequency.

9.6.6 Residual Linear Doppler Effect

This arises from failure to meet the ideal conditions under which the linear Doppler effect is eliminated in the Ramsey cavity. In particular, if there is an asymmetrical flow of microwave power from the source to the two ends of the cavity through which the atoms pass, the resonance signal will be broadened asymmetrically and the maximum will be shifted. If we examine closely the microwave field at the shorted ends of the Ramsey cavity, we see that in the presence of power loss in the walls, the quasi-stationary field pattern can be analyzed into counter-traveling components of slightly different amplitudes. The (small) transverse component of velocity of the Cs atoms passing through the field will lead to a small *linear*

Doppler shift; but because the two traveling components are of different amplitudes, the transition probabilities at the two Doppler-shifted frequencies will not be equal, leading to an asymmetry in the resonance signal profile and a shift in the maximum. The presence of such a shift and other possible power-related effects can be ascertained by varying the microwave power.

9.6.7 Final Word

The listing of so many possible sources of systematic error must not be allowed to leave the impression that this type of atomic clock is fraught with uncertainties. Quite the contrary, the sources listed and the many more subtle effects not listed merely show the exhaustive degree of scrutiny to which this standard has been subjected. As a primary standard, of course, the achievement of the highest possible accuracy and reproducibility requires that this be done.

Since the 1970s, when the classical beam machines described in this chapter had reached a high degree of development and general acceptance, thanks to the laser there have been fundamental developments that have radically changed the design of Cs standards, a subject we will take up in a later chapter.