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Microeconomic Models for Dynamic Spectrum Management in Cognitive Radio Networks

Dusit Niyato and Ekram Hossain

Department of Electrical and Computer Engineering
University of Manitoba, R3T 5V6 Canada, Winnipeg
{tao,ekram}@ee.umanitoba.ca

14.1 Introduction

Software-defined radio technique [1] was invented to improve adaptability and flexibility of wireless transmission so that the network performance can be improved. Developed based on software-defined radio, "cognitive radio" has been identified as a new paradigm for designing next generation wireless networks. A cognitive radio transceiver has the ability to observe, learn, optimize, and adapt the transmission parameters (e.g., transmission power, modulation level) according to the ambient environment [2]. Also, with this agility of the cognitive radio transceiver, frequency spectrum can be shared among licensed and unlicensed services, i.e., the primary and secondary services, respectively, to improve the spectrum utilization. The basic components/processes to achieve adaptability of wireless transmission in cognitive radio are described below.

- *Observation process:* The observation process typically consists of measurement and noise reduction mechanisms. The radio transceiver can silently listen to the environment, or the special messages and signals are transmitted and measured to obtain information about the surrounding environment. Estimation techniques play an important role in the observation process [3].
- *Learning process:* This refers to the process of extracting useful information from collected data. A learning process utilizes data obtained from observation process, previous decisions and actions.
- *Planning and decision making process:* This refers to the process of using the knowledge obtained from learning to schedule and prepare for transmission in the future. If multiple choices of actions are available, a transceiver must decide to choose the best strategy to achieve the objective. This planning and decision making process will change the current state of the transceiver, and subsequently, the surrounding environment which is observed by all the users.
- *Action:* This refers to the process of responding to the environment. The action of a transceiver is controlled by the planning and decision making process.

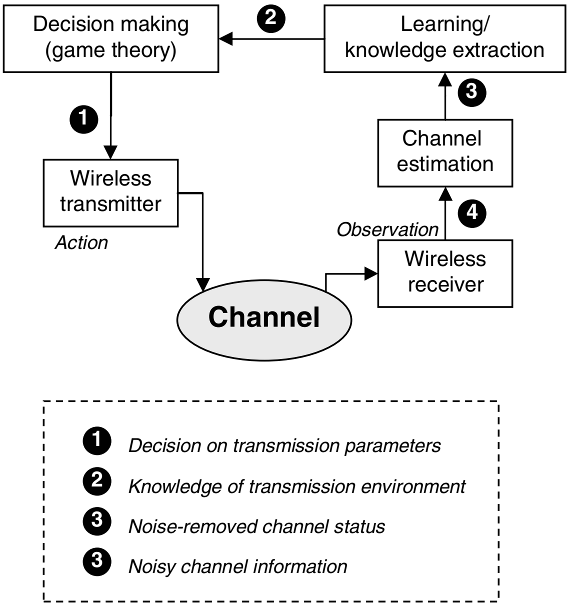

Fig. 14.1. Basic components and their interactions to achieve adaptivity in cognitive radio.

Figure 14.1 shows the interactions among these components.

Dynamic spectrum sharing is a challenging problem in cognitive radio network due to the requirement of "peaceful" co-existence of both licensed (primary) and unlicensed (secondary) users as well as the optimal utilization of radio spectrum. If the primary user cannot fully utilize the allocated spectrum, it results in spectrum hole which can be used by the secondary user(s) to improve the spectrum efficiency. In a scenario where the primary and the secondary services are provided by different operators, the secondary user(s) will require to pay the primary user(s) or service provider(s) for sharing the spectrum. Pricing is an important issue which affects dynamic spectrum sharing in cognitive radio networks. Channel allocation and spectrum sharing can be performed through the coordination of the service providers so that the spectrum owners can achieve their objectives. A negotiation protocol [4] is required for information exchange among the spectrum owners. The amount of shared bandwidth and the pricing should be determined such that the profit/utility of the service provider(s) is maximized while the quality of service (QoS) requirements of the the user(s) are satisfied. The dynamics of bandwidth sharing and pricing in a cognitive radio environment would depend on factors such as the number of primary users (or service providers), primary users' QoS requirements, and bandwidth demand of the secondary users.

14.2 Motivation, Contribution, and Organization of this Chapter

Game theory, widely used in microeconomics, can be effectively applied to address the problem of dynamic spectrum sharing, and in general, to the planning and decision-making process in a cognitive radio system. This is due to the fact that, in an environment where multiple agents interact with a view to achieve their own interests, in many cases, these objectives conflict with each other. Game theory can provide the basis to resolve this conflict so that all the agents are satisfied.

In this chapter, we investigate the problem of spectrum sharing and pricing in a cognitive radio environment using the game-theoretic *oligopoly model* from microeconomic theory. In microeconomics, oligopoly is defined as a situation where a small number of producers (i.e., oligopolists) dominate a particular market. In this market structure, producers compete with each other independently to achieve their objectives (i.e., maximize profit) by controlling the quantity or the price of the supplied commodity. The supply quantity and/or the price offered by one producer will affect the profit of other producers. For example, if one producer increases its supply to the market, the price for the entire market will decrease. As a result, profit of other producers tends to decrease.

Oligopoly is the more general case of a duopoly market (i.e., in duopoly market, the number of players is two), game theory can be used to analyze and predict the behavior of the producers. Each producer makes his decision independently, but the decision of one producer impacts the decision (i.e., profit) of other producers. The classical oligopoly models analyzed by game theory are *Cournot*, *Stackelberg*, and *Bertrand* game models. These models differ in market structure and competition. In particular, in Cournot model, producers compete in terms of quantity of supply to the market. All the producers make their decisions at the same time. On the other hand, in Stackelberg model, there are some producers (referred to as leaders) who are able to make decisions on the amount of supplied quantity before other producers (i.e., followers). Then, these followers make decision on the best amount of supplied quantity by taking into account the decision of the leader. Finally, in Bertrand model, all producers make decision simultaneously in terms of price. These different market structures result in different game formulations and also affect the behavior of the producers to achieve the best decision.

We demonstrate the applications of Cournot, Stackelberg, and Bertrand models of competition for spectrum/bandwidth sharing and pricing in cognitive wireless networks. Specifically, these three different models for oligopoly are applied to obtain the optimal size of spectrum/bandwidth sharing and the charging price. The oligopoly market models were well studied in economics. Also, they are computationally simple, and therefore, suitable for implementation in resource-limited software-defined radio transceiver.

The Cournot game model is used for the case where multiple secondary users share the spectrum/bandwidth with a primary user and the objective is to maximize the profit of the secondary users. Here, all secondary users can completely observe the strategies and the payoffs of other secondary users.

In the Bertrand model, several service providers (or primary users) compete with each other in terms of price to gain the highest profit under QoS constraints for the primary users. Here, the bandwidth demand of the secondary users is established based on a utility function which depends on the quality of transmission (i.e., channel quality) in the available spectrum. In addition, we consider *spectrum substitutability* which represents the ability of a secondary user to switch among the frequency spectra offered by different primary users.

Lastly, the Stackelberg leader-follower competition is used to model the problem of optimal sharing and pricing under elastic bandwidth demand from the secondary users. Numerical performance evaluation results are presented for these oligopoly competition models to show their efficacy in allocating radio resource in cognitive radio environments.

The rest of the chapter is organized as follows. Section 14.3 reviews the related works in the literature. The general characteristics of the three oligopoly competition models considered in this chapter are presented in Sect. 14.4. Section 14.5 presents the Cournot game model and its performance for dynamic spectrum sharing among multiple secondary users. The Bertrand game model for spectrum pricing under competition is presented in Sect. 14.6. Section 14.7 presents the Stackelberg game model for optimal pricing and sharing under elastic bandwidth demand. Then, the chapter is concluded.

14.3 Related Work

A partially observable Markov decision process (POMDP) was used for dynamic spectrum access in an ad hoc network [5]. An opportunistic spectrum access method was developed to allow secondary users to use the radio spectrum by using a decentralized cognitive medium access control (MAC) protocol. In the problem formulation, the state of the system was defined in terms of the availability of each channel and the action was defined as sensing and accessing the channel if available. The *reward* was defined as the amount of transmitted data. A heuristic algorithm was used to obtain the solution which was observed to be as good as the optimal algorithm but with much lower computational complexity.

In [6], a cognitive radio-based MAC layer scheduling algorithm was proposed for multihop wireless networks. An integer linear programming (ILP) model was formulated to solve the scheduling problem for time slot and channel allocation among the wireless nodes in the network. Also, to reduce the computational complexity, a distributed heuristic algorithm was devised to obtain the near optimal solution.

In [7], a pricing scheme for spectrum usage was presented where the price was described as a function of allocated spectrum, traffic intensity, and spectral efficiency of transmission. The pricing for spectral occupation under power constraints was obtained through an optimization formulation.

In [8], a game-theoretic adaptive channel allocation scheme was proposed to capture the selfish and the cooperative behaviors of the wireless nodes in the network.

The strategies of these players were defined in terms of channel selection. Two payoff calculation schemes were used both of which depend on the level of interference. Also, a no-regret learning algorithm was used to learn the historical actions of other players. It was shown that the solution of this game formulation converges to the deterministic Nash equilibrium strategy.

In [9], the convergence dynamics of different types of games in cognitive radio was studied (i.e., coordinated behavior, best-response, and better response for discounted repeated games, S-modular games, and potential games, respectively). Also, a game theory framework was proposed for distributed power control to achieve agility in spectrum usage in a cognitive radio network. The problem of competitive channel allocation among multiradio devices was considered in [10]. Noncooperative game theory was used to analyze the dynamics of channel allocation where the strategy of a user was defined in terms of channel allocation and the payoff was obtained through a utility function of transmission rate. An algorithm was presented to achieve a channel allocation configuration which was shown to be both Pareto-optimal and system-optimal.

In [11], the problem of dynamic spectrum access in open spectrum wireless networks was modeled by using a continuous-time Markov model. Also, a distributed algorithm modeled as a multiplayer game was proposed.

Oligopoly market model was used extensively to analyze the behavior of electricity market [13–15]. In the electricity market, there are several producers who generate electricity to supply to the load (i.e., consumers). In general, the producers have to compete with each other by adjusting the price/supplied power to the load to achieve the maximum profit. A Cournot game model was used to analyze the power bidding in electricity market [13]. Since the transmission line from generator to the load is capacity limited, there is a constraint on the transmission network which was considered in the model [14].

An oligopoly model was used to analyze and develop network resource allocation [16, 17]. In [16], the resource allocation problem in wired networks was formulated by using a Cournot model. In the considered system model, a user chooses the transmission rate and the links set the suitable price according to the marginal cost of the total rate allocation. In [17], a resource-trading mechanism for efficient distribution of large-volume contents in peer-to-peer networks was proposed. The objective of this mechanism was to maximize network capacity for higher revenue. The proposed mechanism was shown to be able to achieve Cournot equilibrium for resource-trading.

The problem of spectrum management and pricing can be formulated as an oligopoly market for which the product is the spectrum access opportunity (e.g., time, frequency, and code for time-division multiple access (TDMA), orthogonal frequency-division multiple access (OFDMA), and code-division multiple access (CDMA) networks, respectively). Game theory can be used to analyze the equilibrium of sharing and pricing so that all the service providers are satisfied with the solution.

14.4 Oligopoly Market Models

The general description of the game formulation for an oligopoly competition presented in microeconomic literature is as follows [24]:

- Players: The players of an oligopoly competition are the producers (oligopolists).
- Strategies: The strategy for each producer corresponds to the supplied quantity (for the Cournot and the Stackelberg models) and the offered price (for Bertrand model).
- Payoffs: The payoff for the producer is the profit which can be determined based on the inverse demand function and the strategies adopted by all the producers in the market.

To illustrate the oligopoly market models (i.e., Cournot, Bertrand, and Stackelberg), we consider a market with only two producers (i.e., duopoly), so that the decisions (i.e., strategies) of the producers and their impacts can be presented by a two-dimensional graph. However, the same approach can be applied to the case of more than two producers (i.e., oligopoly). In order to study these oligopoly models, a demand function is required. In this case, we consider a linear inverse demand function in which the price of the product is determined from the total amount of supply to the market. This function can be defined as $P(Q) = A - Q$, where P is the price for unit amount of supplied quantity, Q is the total amount of supplied quantity, and $A > 0$ is the parameter of the inverse demand function. This demand function is shown in Fig. 14.2.

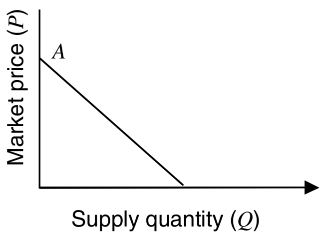

Fig. 14.2. Inverse demand function.

14.4.1 Cournot Competition

In Cournot competition, all producers who are the players of the game, make decisions (i.e., choose strategies) simultaneously on the amount of supplied quantity. Then, the total supplied quantity (i.e., aggregated supply) is used to determine the price which can be obtained from the given inverse demand function. The simplest case, to analyze this Cournot competition, assumes that all producers supply the same product, and therefore, there is no difference for the market to buy from a particular producer. Also, the cost of production for one unit of product is constant and is denoted by C and there is a fixed cost of production which is denoted by C_f. The objective of all of the producers is to maximize their profits by adjusting the supplied quantity to the market.

If Q_i and Q_j denote supply quantities from producer i and j respectively, the strategic form [12] of this game can be expressed as follows:

$$\begin{array}{c|c|c|c} & 0 & \cdots & Q_j^m \\ \hline 0 & (\pi_1(0,0), \pi_2(0,0)) & \cdots & (\pi_1(0,Q_j^m), \pi_2(0,Q_j^m)) \\ \hline 1 & (\pi_1(1,0), \pi_2(1,0)) & \cdots & (\pi_1(1,Q_j^m), \pi_2(1,Q_j^m)) \\ \hline \vdots & \vdots & \cdots & \vdots \\ \hline Q_i^m & (\pi_1(Q_i^m,0), \pi_2(Q_i^m,0)) & \cdots & (\pi_1(Q_i^m,Q_j^m), \pi_2(Q_i^m,Q_j^m)) \\ \hline \end{array}, \tag{14.1}$$

where $\pi_1(Q_i, Q_j)$ and $\pi_2(Q_i, Q_j)$ denote the profit functions of producers i and j, respectively. This strategic form shows the market for which the supplied quantity is an integer, and producer i chooses a strategy in the rows and producer j chooses a strategy in the columns.

The Nash equilibrium, which is the solution of the game, can be used to determine the decision of the producer. This Nash equilibrium will provide the optimal strategy for each of the players where all the players are rational. This rationality indicates that all the players are willing to maximize their payoffs.

To obtain Nash equilibrium, best response or reaction function is typically used. This best response function is the optimal strategy of one producer given the strategies of other producers. If Q_j denotes the given strategy of producer j, profit of producer i can be expressed as follows:

$$\pi_i(Q_i, Q_j) = P(Q_i + Q_j)Q_i - CQ_i - C_\mathrm{f} \tag{14.2}$$

$$= (A - Q_i - Q_j)Q_i - CQ_i - C_\mathrm{f}. \tag{14.3}$$

The response function is the strategy that maximizes this profit. By differentiating the profit with respect to the available strategy Q_i, the best response function is obtained as follows:

$$\frac{\partial \pi_i(Q_i, Q_j)}{\partial Q_i} = A - 2Q_i - Q_j - C \tag{14.4}$$

$$0 = A - 2Q_i - Q_j - C \tag{14.5}$$

$$Q_i^*(Q_j) = \frac{A - Q_j - C}{2}. \tag{14.6}$$

Similarly, the best response function of producer j is obtained as $Q_j^*(Q_i) = \frac{A-Q_i-C}{2}$. As an example, Fig. 14.3 shows profit of producer i when for $A = 10$, $C = 0.5$, and $C_\mathrm{f} = 1$. The best responses for which the maximum profit is achieved are also shown. We observe that if producer j increases its supply, profit of producer i decreases, and also the best response of producer i (in terms of supplied quantity)

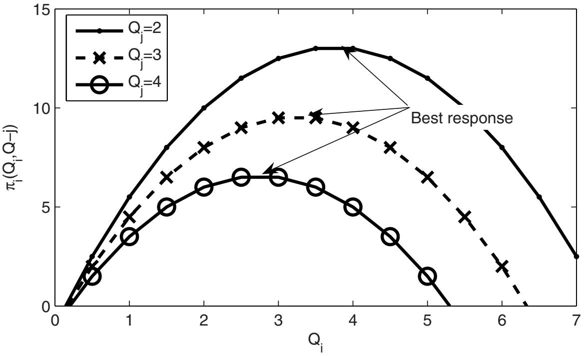

Fig. 14.3. Profit function.

decreases. This is due to the higher amount of total supply which results in lower market price. Therefore, the revenue becomes smaller while the cost remains the same.

The best response function of each of the producers given the other producer's strategy is shown in Fig. 14.4 for $C = 0.5$ and $C_{\mathrm{f}=1}$. The best response of one producer decreases as the other producer increases its supply. When the value of the parameter A in the inverse demand function increases, the best response function shifts to the larger supplied quantity since the price is higher at the same aggregated supplied quantity. The same effect is observed when the cost per unit of product increases. However, this best response is not affected by the fixed cost.

The solution of the Cournot game model (i.e., the Nash equilibrium) gives the optimal supplied quantity that maximizes the profits of the firms. The *Nash equilibrium* of a game is a strategy profile (list of strategies, one for each player) with the property that no player can increase his payoff by choosing a different action, given the other players' actions [12]. In the context of Cournot model, this Nash equilibrium can be expressed as follows:

$$Q_i^*(Q_j^*) = Q_j^*(Q_i^*). \tag{14.7}$$

The Nash equilibrium is graphically shown in Fig. 14.4. This equilibrium is the point where the best responses intersect with each other, and it can be expressed as follows:

$$(Q_i^*, Q_j^*) = \left(\frac{A-C}{3}, \frac{A-C}{3}\right). \tag{14.8}$$

Note that, the profits of both the producers are the same, which is consistent with the assumption that both have the same information and they make decisions simultaneously.

At this Nash equilibrium, none of the producers can have better profit without adjustment in the supplied quantity of another producer. For example, if producer i

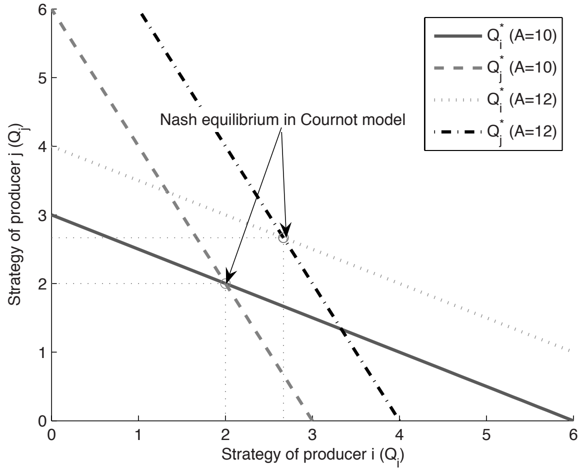

Fig. 14.4. Nash equilibrium in Cournot competition.

tries to increase its supplied quantity, the price will decrease. As a result, producer j must also increase its supplied quantity to gain higher profit. However, this will reduce the profit of both the producers. As a result, producer i is forced to reduce its supplied quantity. This process will repeat until the equilibrium is reached. This is referred to as the dynamic behavior of the Cournot model.

14.4.2 Bertrand Competition

Different from Cournot and Stackelberg competitions in which the producers compete in terms of supplied quantity, in Bertrand competition the producers compete by adjusting the price of the product. Before supplying the product to the market, all producers make decision on the price and announce to the market. Then, based on the demand function, a consumer decides the quantity to buy from each producer. The objective of this competition is again to maximize the profit of the producers.

However, in this Bertrand competition, the solution depends mainly on the substitutability of the products. If the products from the different producers are identical, then they are said to be totally substitutable. On the other hand, if the products are different, the products may be partly substitutable or may be completely unsubstitutable. The basic model of Bertrand competition considers the cases of identical and totally different products as described below.

In the case of identical or homogeneous products, the products from all the producers are totally substitutable, i.e., buying from one producer is not different from buying from others. Therefore, the consumer will alway choose to buy from the producer offering the lowest price. Furthermore, the entire market will buy from that producer, and other firms will have zero profit. Studies have shown that there is a unique Nash equilibrium in which the price charged by all producers are identical. In particular, at the Nash equilibrium, the price is equal to the production cost. When one producer decreases the price, that producer will capture the entire market. As a result, other producers will try to decrease their price to gain positive profit. Any price which is larger than the production cost is not equilibrium since one producer can gain higher profit by reducing the price.

In the case of differentiated products, the demand functions for the products from the different producers are different. Therefore, the market could buy different quantities of products from different producers where the prices are different. To describe the Bertrand competition in case of differentiated products, we consider the following demand functions:

$$Q_i(P_i, P_j) = A - P_i + BP_j \tag{14.9}$$
$$Q_j(P_i, P_j) = A - P_j + BP_i \tag{14.10}$$

where A and B are constants. Here, B also represents the substitutability of the products. Similar to the previous model, to obtain the Nash equilibrium of the game, the best response of producer i (i.e., which maximizes its profit) can be derived as follows:

$$\pi_i(P_i, P_j) = P_i Q_i - CQ_i - C_f \tag{14.11}$$
$$= (A - P_i + BP_j)\, P_i - C\,(A - P_i + BP_j) - C_f. \tag{14.12}$$

Differentiating $\pi_i(P_i, P_j)$ with respect to P_i we obtain the best response as follows:

$$\frac{\partial \pi_i(P_i, P_j)}{\partial P_i} = A + BP_j - 2P_i + C = 0 \tag{14.13}$$
$$P_i^* = \frac{A + C + BP_j}{2}. \tag{14.14}$$

Similarly, the best response function of producer j is

$$P_j^* = \frac{A + C + BP_i}{2}. \tag{14.15}$$

The Nash equilibrium for the charging price can be found to be

$$(P_i^*, P_j^*) = \left(\frac{A + C}{2 - B}, \frac{A + C}{2 - B} \right). \tag{14.16}$$

The Nash equilibrium of the above Bertrand competition is shown in Fig. 14.5 for $A = 5$, $C = 0.5$, and $C_\mathrm{f} = 1$. Again, the Nash equilibrium is located at the point where the best responses of both the producers intersect with each other. Also, parameter B, which represents the substitutability of the products impacts the slope of the best response curves and hence the Nash equilibrium.

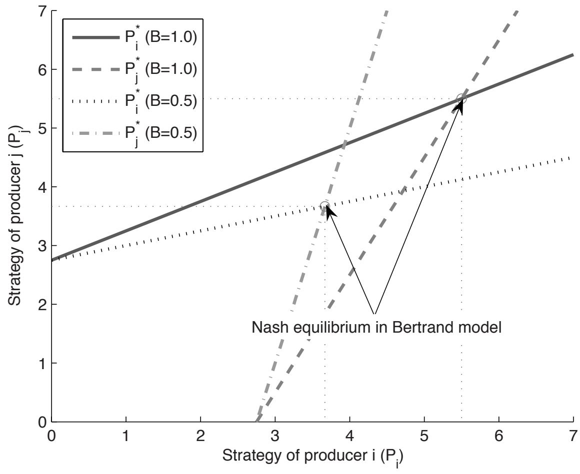

Fig. 14.5. Nash equilibrium for Bertrand competition.

14.4.3 Stackelberg Competition

In Stackelberg model, similar to Cournot model, the producers compete with each other in terms of supplied quantity. However, in the Stackelberg competition, there is at least one producer (referred to as the leader) who can commit the chosen strategy (i.e., supplied quantity) before other producers (referred to as the followers). An extensive form game [12] (as shown in Fig. 14.6) is used to present the Stackelberg competition model. This extensive form shows the sequence of decision making in which producer i is a leader and producer j is a follower. In this Stackelberg competition, since the leader will make the decision before the followers, the followers will choose their optimal strategy based on the observation from the leader. As a result, the solution of this game is a set of strategies where the profit of the leader is maximized for which the followers choose their best responses given the strategy of the leader.

In order to determine the equilibrium in a Stackelberg competition, *backward induction* is used. With backward induction, the best response of the follower is obtained at the last decision-making period. Again, the profit of the follower is computed from

$$\pi_j(Q_i, Q_j) = (A - Q_i - Q_j)Q_j - CQ_j - C_{\mathrm{f}}. \tag{14.17}$$

The best response of the follower is given as follows:

$$Q_j^*(Q_i) = \frac{A - Q_i - C}{2}. \tag{14.18}$$

Then, we backtrack to the decision of the leader. Here, the leader makes a decision based on the assumption that the follower will react with its optimal strategy

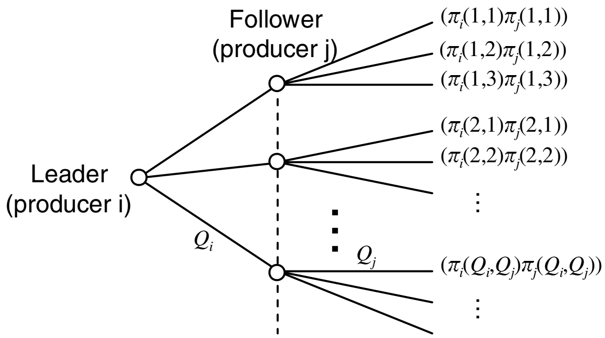

Fig. 14.6. Extensive form for the Stackelberg game.

(i.e., best response), and the objective of the leader is to maximize its profit. Therefore, we have

$$\pi_i(P_i, P_j) = (A - Q_i - Q_j)Q_i - CQ_i - C_f \tag{14.19}$$

$$= \left(A - Q_i - \frac{A - Q_i - C}{2}\right) Q_i - CQ_i - C_f. \tag{14.20}$$

Differentiating this profit function with respect to the strategy of the leader, which is Q_i, we obtain

$$\frac{\partial \pi_i(P_i, P_j)}{\partial P_i} = A - 2Q_i - \frac{A}{2} + Q_i + \frac{C}{2} - C \tag{14.21}$$

$$0 = \frac{A - C}{2} - Q_i \tag{14.22}$$

$$Q_i^* = \frac{A - C}{2}. \tag{14.23}$$

This is the subgame perfect Nash equilibrium or the optimal strategy for the leader if the leader can make a decision before the follower. Again, this optimal strategy for the leader will influence the decision of the follower. Based on the optimal strategy of the leader, the optimal strategy for the follower is

$$Q_j^* = \frac{A - C}{4}. \tag{14.24}$$

This Stackelberg equilibrium is graphically shown in Fig. 14.7, and it can be expressed mathematically as follows:

$$(Q_i^*, Q_j^*) = \left(\frac{A - C}{2}, \frac{A - C}{4}\right). \tag{14.25}$$

Note that, the optimal strategy of the leader is at the point where the leader predicts that the supplied quantity of a follower is zero. However, the follower will react with a non-zero supplied quantity.

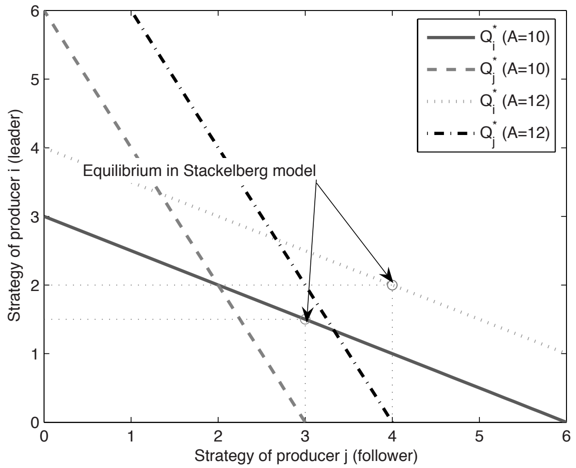

Fig. 14.7. Stackelberg equilibrium.

At the Stackelberg equilibrium, the leader will offer a larger amount of supplied quantity than that of a follower. Consequently, the profit of the leader is higher. This higher profit of the leader in a Stackelberg competition is also known as the *first-move advantage* in which the player of the game with the ability to make decision before other players will gain larger payoff.

In the following sections, we demonstrate the applications of the oligopoly market models to the spectrum/bandwidth sharing and pricing problem in cognitive radio networks. In particular, the three different oligopoly models described above are applied to obtain the optimal size of spectrum/bandwidth sharing and the charging price.

14.5 A Cournot Game Formulation for Dynamic Spectrum Sharing among Multiple Secondary Users

In this section, we formulate the problem of spectrum sharing among the primary user[1] and multiple secondary users as an oligopoly market competition. The objective of this spectrum sharing is to maximize the profit of secondary users by utilizing the concept of equilibrium. A Cournot game model is formulated for the case where a secondary user is assumed to have the knowledge on the strategies and the payoffs of other secondary users.

[1] We use “primary/secondary service” and “primary/secondary user” interchangeably.

14.5.1 System Model and Assumptions

We consider a wireless system with a primary user and multiple secondary users (i.e., total number of secondary users is denoted by N) who want to share the spectrum allocated to the primary user (Fig. 14.8) [18]. In this case, the primary user is willing to share some portion of the spectrum (Q_i) with secondary user i. The primary user charges the secondary user for the spectrum at a rate of $c(b)$ per unit bandwidth, where b is the amount of available bandwidth that can be shared. After allocation, the secondary users transmit in the allocated spectrum using adaptive modulation to enhance the transmission performance.

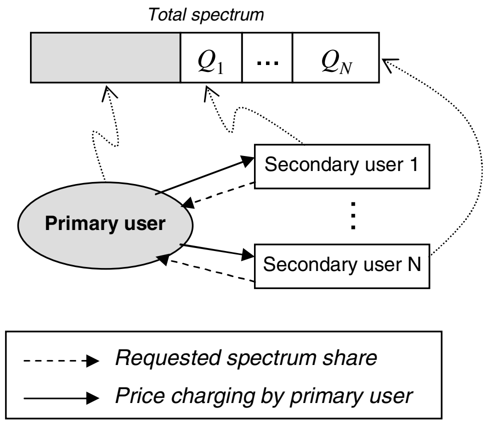

Fig. 14.8. System model for Cournot game spectrum sharing.

With adaptive modulation, the transmission rate can be dynamically adjusted based on the channel quality. For uncoded quadrature amplitude modulation (QAM) with square signal constellation (e.g., 4-QAM, 16-QAM) the bit-error-rate (BER) in single-input single-output Gaussian noise channel can be well approximated as follows [19]:

$$\mathrm{BER} \approx 0.2 \exp\left(\frac{-1.5\gamma}{(2^k - 1)}\right) \tag{14.26}$$

where γ is the SNR at the receiver and k is the spectral efficiency of the modulation scheme used. Without loss of generality, we assume that the spectral efficiency is a non-negative real number (which can be obtained given any BER). To guarantee the quality of transmission, BER must be maintained at the target level (i.e., $\mathrm{BER}_i^{\mathrm{tar}}$). Therefore, spectral efficiency of transmission for secondary user i can be obtained from

$$k_i = \log_2(1 + K\gamma_i) \tag{14.27}$$

where

$$K = \frac{1.5}{\ln\left(0.2/\mathrm{BER}_i^{\mathrm{tar}}\right)}. \tag{14.28}$$

We assume that the received SNR information is available at the transmitter by channel estimation. In short, for secondary user i, given the received SNR γ_i, target $\text{BER}_i^{\text{tar}}$, and assigned spectrum Q_i, the transmission rate (in bits per second) can be obtained.

The revenue of secondary user i is denoted by r_i per unit of achievable transmission rate. It is assumed that a secondary user can communicate with the primary user but not with any other secondary users. Therefore, the adaptation for spectrum sharing is performed between each of the secondary users and the primary user only.

14.5.2 Cournot Game Formulation

Based on the above system model, a Cournot game can be formulated as follows:

- Players: The players in this game are the secondary users.
- Strategies: The strategy of each of the players is the spectrum size requested (denoted by Q_i for secondary user i) which is nonnegative.
- Payoffs: The payoff for each player is the profit (i.e., revenue minus cost) of secondary user i (denoted by π_i) in sharing the spectrum with the primary user and the other secondary users.

Note that, the commodity of this oligopoly market is the frequency spectrum.

For the primary user, we assume that the pricing function used to charge the secondary users is given by

$$P(\mathbb{Q}) = x + y\left(\sum_j Q_j\right)^{\tau} \tag{14.29}$$

where x, y, and τ are nonnegative constants, $\tau \geq 1$, and $\mathbb{Q}$ denotes the set of strategies of all secondary users (i.e., $\mathbb{Q} = \{Q_1, \dots Q_N\}$). Let w denote the worth of the spectrum for the primary user. Then, the condition $P(\mathbb{Q}) > w \times \sum_j Q_j$ is necessary to ensure that the primary user is willing to share spectrum of size Q_j with the secondary users. Note that, the primary user charges all of the secondary users at the same price.

The revenue of secondary user i can be obtained from $r_i \times k_i \times Q_i$, while the cost of spectrum allocation is $Q_i P(\mathbb{Q})$. Therefore, the profit of the secondary user i can be obtained as follows:

$$\begin{aligned}
\pi_i(\mathbb{Q}) &= r_i k_i Q_i - Q_i P(\mathbb{Q}) && (14.30)\\
&= r_i k_i Q_i - Q_i\left(x + y\left(\sum_j Q_j\right)^{\tau}\right). && (14.31)
\end{aligned}$$

The marginal profit function for secondary user i can be obtained from

$$\frac{\partial \pi_i(\mathbb{Q})}{\partial Q_i} = r_i k_i - x - y\left(\sum_j Q_j\right)^{\tau} - y Q_i \tau \left(\sum_j Q_j\right)^{\tau-1}. \tag{14.32}$$

Let $\mathbb{Q}_{-i}$ denote the set of strategies adopted by all except secondary user i (i.e., $\mathbb{Q}_{-i} = \{Q_j | j = 1, \ldots, N; j \neq i\}$ and $\mathbb{Q} = \mathbb{Q}_{-i} \cup \{Q_i\}$). In this case, the optimal allocated spectrum size to one secondary user depends on the strategies of other secondary users. Therefore, Nash equilibrium is considered as the solution of the game to ensure that all secondary users are satisfied with the solution.

In this case, we obtain the Nash equilibrium by using the best response function which is the best strategy of one player given others' strategies. The best response function of secondary user i given the allocated spectrum size to other secondary users Q_j, where $j \neq i$, is defined as follows:

$$\mathcal{BR}_i(\mathbb{Q}_{-i}) = \arg\max_{Q_i} \pi_i(\mathbb{Q}_{-i} \cup \{Q_i\}). \tag{14.33}$$

The set $\mathbb{Q}^* = \{Q_1^*, \ldots Q_N^*\}$ denotes the Nash equilibrium of this game if

$$Q_i^* = \mathcal{BR}_i(\mathbb{Q}_{-i}^*), \quad \forall i \tag{14.34}$$

where $\mathbb{Q}_{-i}^*$ denotes the set of best responses for secondary users j for $j \neq i$. Mathematically, to obtain the Nash equilibrium, we have to solve the following set of equations:

$$\frac{\partial \pi_1(\mathbb{Q})}{\partial Q_1} = 0 = r_1 k_1 - x - y\left(\sum_j Q_j\right)^{\tau} - yQ_1\tau\left(\sum_j Q_j\right)^{\tau-1}$$
$$\vdots$$
$$\frac{\partial \pi_N(\mathbb{Q})}{\partial Q_N} = 0 = r_N k_N - x - y\left(\sum_j Q_j\right)^{\tau} - Q_N y\tau\left(\sum_j Q_j\right)^{\tau-1}.$$

14.5.3 Performance Evaluation

14.5.3.1 Parameter Setting

We consider a cognitive radio environment with a primary user and two secondary users sharing a frequency spectrum of size 15 MHz. The target BER for both the users is $\text{BER}_i^{\text{tar}} = 10^{-4}$. For the pricing function of primary user, we use $x = 0$ and $y = 1$, while τ is adjusted based on the evaluation scenario (e.g., $\tau = 1.0$), and the worth of spectrum for primary user is $w = 1$. The revenue of a secondary user per unit transmission rate is $r_i = 10, \ \forall i$.

14.5.3.2 Numerical Results

Figure 14.9 shows the best response of both secondary users in the Cournot game. The best response of each secondary user is a linear function of the other user's strategy. The Nash equilibrium is located at the point where the best responses of both

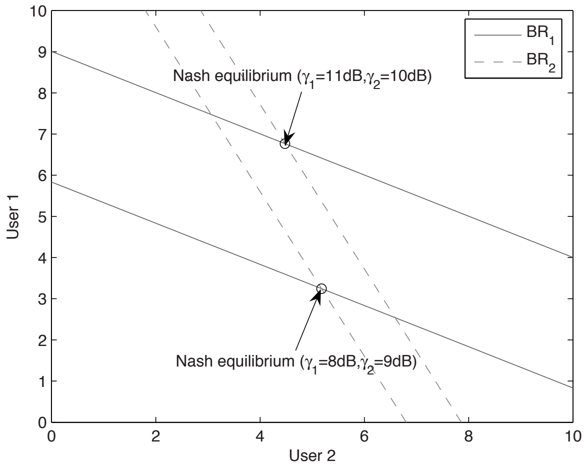

Fig. 14.9. Best responses and Nash equilibrium under different channel qualities.

the users intersect. We observe that the Nash equilibrium varies under different channel qualities. The adaptation of Nash equilibrium under different channel qualities is presented in Fig. 14.10. As expected, when the channel quality for a secondary user becomes better, the size of the spectrum allocated to that secondary users becomes larger. Also, we observe that the channel quality of one secondary user impacts the size of the allocated spectrum to other secondary user.

Figure 14.11 shows the impact of pricing function on the revenue of the primary user. When the value of the parameter τ increases, the primary user benefits from charging higher price to the secondary users. However, at a particular point (i.e., $\tau = 1.9$), the revenue gained by the primary user decreases since the price of the spectrum becomes too high and the secondary users request much smaller spectrum size. Therefore, the revenue from the secondary users increases at a rate smaller than the worth of spectrum to the primary users. Also, this result suggests that there is an optimal value for the pricing parameter τ which maximizes the revenue of the primary user.

14.6 Bertrand Game Model for Spectrum Pricing Under Competition

In this section, we consider a competitive situation for spectrum management where a few primary users offer spectrum access to the secondary users. For a primary user, the cost of sharing the frequency spectrum is modeled as a function of QoS degradation. The Nash equilibrium is considered as the optimal solution of this game.

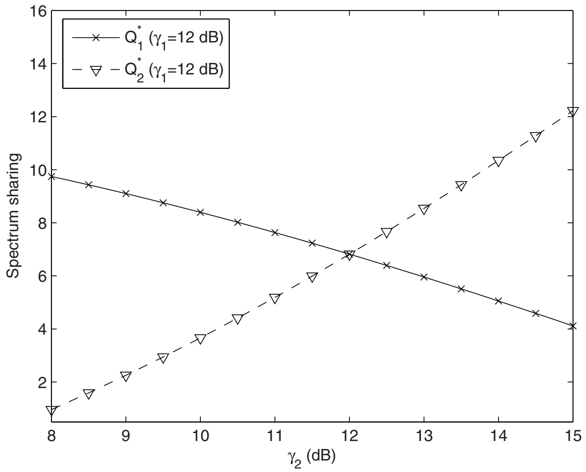

Fig. 14.10. Variation in Nash equilibrium of spectrum sharing under different channel qualities for user 2.

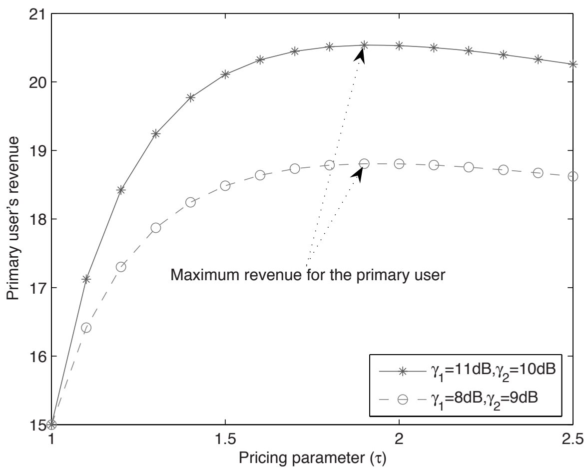

Fig. 14.11. Profit of primary user under different pricng parameters.

14.6.1 System Model and Assumptions

14.6.1.1 Primary and Secondary Users

We consider a wireless system with multiple primary users (total number of primary users is denoted by M) operating on the different frequency spectrum and a

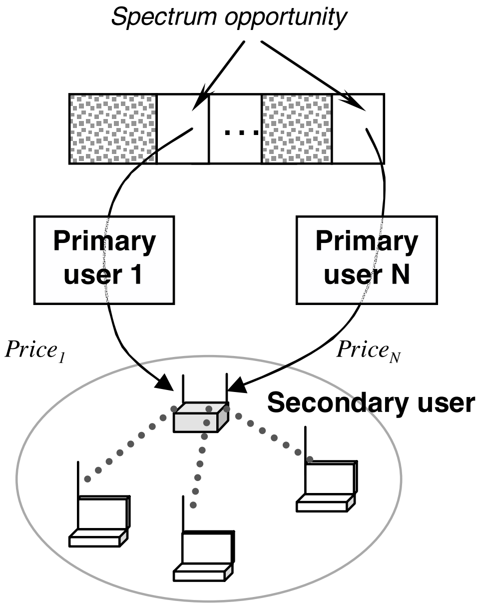

Fig. 14.12. System model of spectrum sharing and pricing for the Bertrand game model.

group of secondary users is willing to share these spectrum with the primary users (Fig. 14.12) [20]. In this case, primary user i wants to sell portions of the available bandwidth (e.g., time slots in a TDMA scheme) at price P_i (per unit bandwidth) to this group of secondary users. The spectrum can be shared by multiple terminals (i.e., secondary users) in which the base station (BS) or access point (AP) governs transmission in the allocated spectrum. The secondary users use adaptive modulation for transmissions in the allocated spectrum in a time-slotted manner. The spectral efficiency of the transmission for secondary user i is denoted by k_i. The spectrum demand of the secondary users depends on the transmission rate in the allocated frequency spectrum and the price charged by the primary user.

14.6.1.2 Cost Function of the Primary User

To develop a cost function, the QoS performance of the primary user needs to be considered. Degradation in the QoS performance of the primary user is expected if some portion of the frequency spectrum (i.e., in time domain or in frequency domain) is given to the secondary users. We consider the average delay as the QoS measure which is obtained for transmissions at the primary user based on an M/D/1 queueing model. Let λ_i denote the traffic arrival rate of the primary user, and $k_i^{(p)}(W_i - Q_i)$ denote the service rate, where $k_i^{(p)}$ and Q_i represent spectral efficiency of the wireless transmission by the primary user i, and the portion of frequency spectrum that is given to the secondary users. The average delay is defined as follows:

$$D_i(Q_i) = \frac{1}{2} \frac{\lambda_i}{\left(k_i^{(p)}(W_i - Q_i)\right)^2 - \lambda_i k_i^{(p)}(W_i - Q_i)} \tag{14.35}$$

where W_i is the total spectrum size of primary user i. The cost function can be simply defined as

$$C_i(Q_i) = dD_i(Q_i)\,, \tag{14.36}$$

where d is a constant.

14.6.1.3 Utility of Secondary User

To quantify the spectrum demand, we consider the utility gained by the secondary users (e.g., if the spectrum creates high utility, the demand is high). We adopt a commonly used utility function in the economics defined as follows [21]:

$$U(\mathbb{Q}) = \sum_{i=1}^{M} Q_i k_i^{(s)} - \frac{1}{2}\left(\sum_{i=1}^{M} Q_i^2 + 2\Delta \sum_{i \neq j} Q_i Q_j\right) + J \tag{14.37}$$

where $\mathbb{Q}$ is the set consisting of the size of the spectrum available from all the primary users, i.e., $\mathbb{Q} = \{Q_1, \ldots, Q_i, \ldots, Q_M\}$, and

$$J = -\sum_{i=1}^{M} P_i Q_i \tag{14.38}$$

where P_i is the price offered by primary user i. Note that, $k_i^{(s)}$ denotes the spectral efficiency for transmission by the secondary user (e.g., BS/AP in Fig. 14.12) operating on the frequency spectrum offered by the primary user i. This utility function takes the spectrum substitutability into account through parameter Δ. That is, if the secondary users use multi-interface radio, they can switch among the frequency spectra freely depending on the offered price. This spectrum substitutability parameter (i.e., $\Delta \in [-1.0, 1.0]$) is defined as follows. When $\Delta = 1.0$, the secondary user cannot switch among the frequency spectrum, while for $\Delta = 0.0$ the secondary user can switch among the operating frequency spectra freely.

When $\Delta < 0$, the spectrum sharing by the secondary user is complementary. That is, when the secondary user wants to share one frequency spectrum, it will be required to buy one or more additional spectrum simultaneously (e.g., one spectrum for uplink transmission and another for downlink transmission) from the same or different primary users.

To derive the demand function of the secondary user who operates on the spectrum offered by primary user i,[2] we differentiate $U(\mathbb{Q})$ with respect to Q_i and then the optimal value of Q_i can be obtained.

[2] For brevity, we call it spectrum i.

The demand function is defined as the size of shared spectrum that maximizes the utility of the secondary user given the prices offered by the primary user [21], that is,

$$Q_i = \frac{k_i^{(s)} - P_i - \Delta(k_j^{(s)} - P_j)}{1 - \Delta^2}. \tag{14.39}$$

14.6.2 Bertrand Game Model

Based on this system model, a Bertrand game can be formulated as follows:

- Players: The players are the primary users.
- Strategies: The strategy of each of the players is the price per unit of spectrum (denoted by P_i) which is nonnegative.
- Payoffs: The payoff for each player is the profit (i.e., revenue minus cost) of primary user i (denoted by π_i) in selling spectrum to the secondary user.

Based on the demand function in (14.39) and the cost function in (14.36), the profit of each primary user/service provider can be expressed as follows:

$$\pi_i(\mathbb{P}) = Q_i P_i - C_i(Q_i) \tag{14.40}$$

where $\mathbb{P}$ denotes the set of prices offered by all players in the game (i.e., $\mathbb{P} = \{P_1, \ldots, P_i, \ldots, P_M\}$).

Again, the Nash equilibrium is considered as the solution of this game, and it is obtained by using the best response function. The best response function of primary user i given the prices of other primary users P_j, where $j \neq i$, is defined as follows:

$$\mathcal{BR}_i\left(\mathbb{P}_{-i}\right) = \arg\max_{P_i} \pi_i(\mathbb{P}_{-i} \cup \{P_i\}) \tag{14.41}$$

where $\mathbb{P}_{-i}$ represents the set of prices offered by other players except player i (i.e., $\mathbb{P} = \mathbb{P}_{-i} \cup \{P_i\}$).

The set $\mathbb{P}^* = \{P_1^*, \ldots, P_M^*\}$ denotes the Nash equilibrium of this game if and only if

$$P_i^* = \mathcal{BR}_i(\mathbb{P}_{-i}^*), \quad \forall i \tag{14.42}$$

where $\mathbb{P}_{-i}^*$ denotes the set of best responses for player j for $j \neq i$. Mathematically, to obtain the Nash equilibrium, we have to solve the set of equations $\frac{\partial \pi_i(\mathbb{P})}{\partial P_i} = 0$ for all i where

$$\pi_i(\mathbb{P}) = P_i \frac{k_i^{(\mathrm{s})} - P_i - \Delta(k_j^{(\mathrm{s})} - P_j)}{1 - \Delta^2} - \frac{d\lambda_i}{2\left(W_i - Q_i\right)^2 - 2\lambda_i(W_i - Q_i)}.$$

We have to solve

$$0 = \frac{k_i^{(\mathrm{s})} - 2P_i - \Delta(k_j^{(\mathrm{s})} - P_j)}{1 - \Delta^2} + \frac{d\frac{\lambda_i}{1-\Delta^2}\left(4Q_i - \lambda_i\right)}{\left(2Q_i^2 - 2Q_i\lambda_i\right)^2} \tag{14.43}$$

where

$$Q_i = W_i - \frac{k_i^{(\mathrm{s})} - P_i - \Delta(k_j^{(\mathrm{s})} - P_j)}{1 - \Delta^2}. \tag{14.44}$$

14.6.3 Performance Evaluation

14.6.3.1 Parameter Setting

We consider a cognitive radio environment with two primary users and one secondary user/a set of secondary users (e.g., controlled by the the BS/AP in Fig. 14.12). The total frequency spectrum available to each primary user is 5 MHz. The target BER for the secondary user is $\text{BER}_i^{\text{tar}} = 10^{-4}$. Traffic arrival rate at a primary user is 1 Mbps, and we assume $d = 1$ for the cost function used by a primary user. The channel quality of the secondary user varies in the range of 10–20 dB.

14.6.3.2 Numerical Results

Figure 14.13 shows the demand function of the secondary user, and the revenue, cost, and profit of the first primary user under different pricing options. In this case, we set $\lambda_1 = 4$, $\gamma_1 = 15$ dB, $\gamma_2 = 18$ dB, $\Delta = 0.4$, $P_2 = 1$. As expected, when the first primary user increases the price, the secondary user demands a smaller spectrum size since the utility from the allocated spectrum decreases. Also, the cost for the primary user decreases since the secondary user demands smaller spectrum size. Therefore, the size of the remaining spectrum becomes bigger which results in smaller delay. However, the revenue and profit of the primary user first increase, and after a certain point it starts decreasing. Since at a small price the first primary user can sell a bigger spectrum size to the secondary user, the revenue and profit increase. In contrast, when the spectrum price becomes higher, a smaller amount of spectrum is sold to

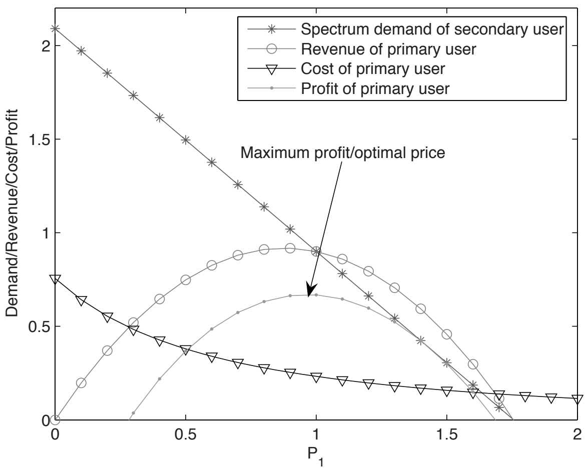

Fig. 14.13. Demand function of the secondary user, and revenue, cost, and profit of the first primary user.

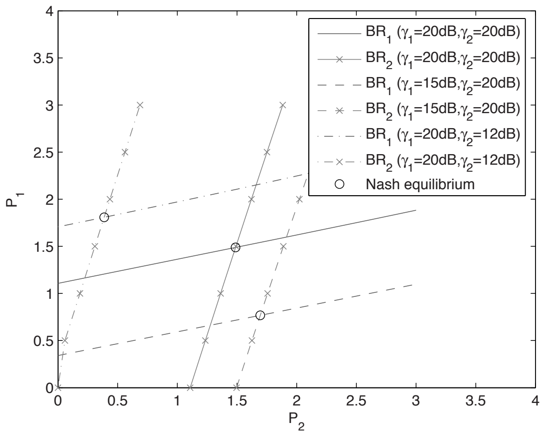

Fig. 14.14. Best response and Nash equilibrium under different channel quality (γ_1, γ_2).

the secondary user and this results in smaller revenue. We can observe that there is an optimal price for which the profit is maximized and this price is referred to as the best response of the corresponding primary user.

Then, the variations in the best response functions of both the primary users are shown in Fig. 14.14 under different channel quality (γ_1, γ_2) for the secondary user when $\Delta = 0.4$. As expected, when the channel quality becomes better, since the secondary user can transmit at a higher rate due to the adaptive modulation, the spectrum demand increases. As a result, the primary user can offer higher price. The Nash equilibrium is located at the point where the best response functions of both the primary users intersect.

Figure 14.15 shows the Nash equilibrium of the primary users under variations in channel quality when $\Delta = 0.4$. As expected, the price at the Nash equilibrium is higher for the spectrum with better channel quality. This is due to the larger demand (which is a function of utility) generated by the secondary user. Also, we observe that the channel quality of the spectrum offered by one player impacts the strategies adopted by the other player. When the demand for spectrum offered by one player changes, the other player must adapt the price to gain the highest profit.

Then, we investigate the impact of QoS requirements of the primary users on the the Nash equilibrium. Figure 14.16 shows the Nash equilibrium of the secondary user as functions of traffic arrival rate at the second primary user λ_2. In this case, $\gamma_1 = 15$ dB and $\gamma_2 = 18$ dB. Since this arrival rate λ_2 affects the cost of the second primary user in offering spectrum to the secondary user, at the Nash equilibrium the price offered by the second primary user increases significantly. When the traffic arrival rate increases, at the same spectrum size, traffic delay increases and the cost of primary user increases accordingly. However, this traffic arrival rate has only small impact on the price offered by the other player.

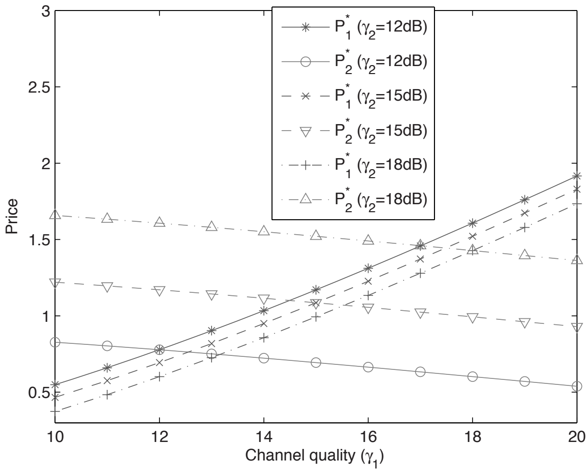

Fig. 14.15. Nash equilibrium under different channel qualities of the secondary user.

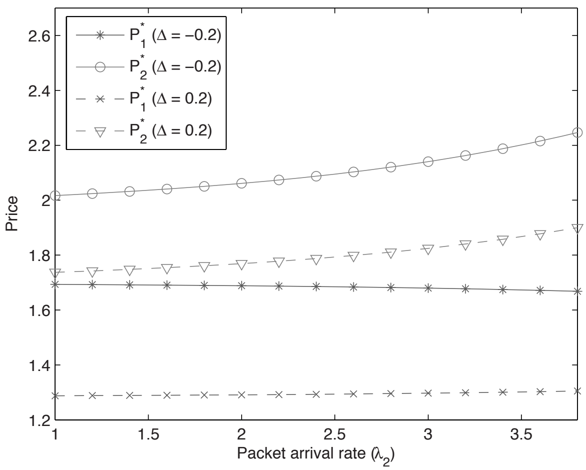

Fig. 14.16. Nash equilibrium under different traffic arrival rate of the second primary user.

14.7 Stackelberg Game Model for Optimal Pricing and Bandwidth Sharing Under Elastic Demand

In this section, we model the spectrum sharing problem between a primary user and multiple secondary users by a Stackelberg game model. The objective is to maximize the payoff of the service provider (leader) where the payoff considers price-elastic bandwidth demand of the secondary users. All the followers choose their best responses given the strategy of the leader.

14.7.1 System Model and Assumptions

The game model is described in the context of resource allocation/sharing in an integrated WiMAX/WiFi network where the WiMAX base stations (BSs) and the WiFi access points (APs)/routers are operated by different service providers. In the system model under consideration, the WiMAX BS charges the WiFi APs/routers for sharing the licensed WiMAX spectrum to provide mobile broadband Internet access to the WiFi clients. Each AP/router has a dual radio transceiver which can work by using both 802.11 and 802.16 interfaces. Traffic is transmitted from the BS using WiMAX radio interface and relayed through the WiFi AP/router using WiFi interface to the WiFi nodes.

The WiMAX subscriber stations (SSs) have fixed bandwidth demand, and therefore, subscribe at a flat rate to the WiMAX BS. On the other hand, the WiFi networks have elastic demand depending on the number of nodes and their preferences. Therefore, the WiMAX service provider charges the WiFi networks with adjustable pricing (i.e., P_1 and P_2 for WiFi router one and two, respectively, in Fig. 14.17). In this environment, the WiMAX and the WiFi service providers have to negotiate with each other to determine the optimal price such that their profits are maximized [22].

We formulate the pricing problem as a Stackelberg game in which the profit of the WiMAX BS is maximized and also the WiFi routers are satisfied with the bandwidth sharing and pricing. The WiMAX BS is the major player in this game – the decision on bandwidth allocation by the base station to the subscriber stations influences the decision of the WiFi APs/routers. Therefore, we consider this as a Stackelberg leader-follower game in which the WiMAX BS and the WiFi APs/routers are the leader and the followers, respectively. The solution of this game, i.e., the Stackelberg equilibrium, can be obtained easily if the information of all service providers and customers are available.

14.7.2 Stackelberg Game Model

14.7.2.1 Revenue and Elastic Demand

The revenue of the WiMAX BS from the service provided to the SSs is a function of the corresponding QoS performance. On the other hand, the WiMAX BS charges different prices to the different WiFi APs/routers depending on the bandwidth demand from the WiFi clients. This type of pricing model is particularly suitable for an environment in which the SSs serve real-time traffic (e.g., those for real-time polling service (rtPS)), while the WiFi networks serve best-effort traffic.

For the SSs, the queueing delay is the QoS metric and the revenue of the WiMAX BS is expressed as

$$r^{(\mathrm{s})} = \sum_{i=1}^{N_{\mathrm{ss}}} \left(a_i - e_i D(\lambda_i, Q_i^{(\mathrm{s})}) \right) \tag{14.45}$$

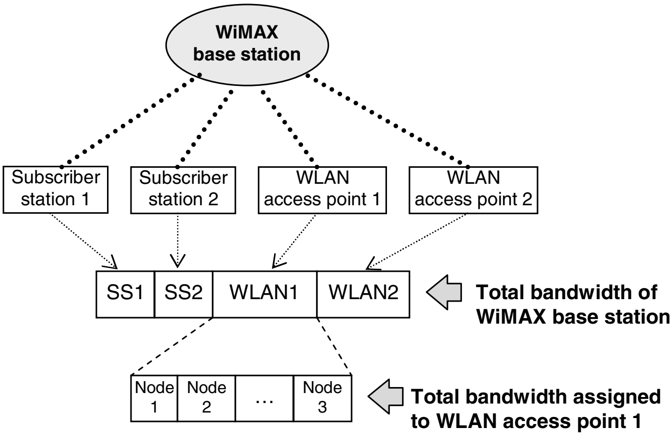

Fig. 14.17. An integrated WiMAX/WiFi network.

where a_i and e_i are constants (e.g., $a_i = 1$ and $e_i = 1$), $D(\lambda_i, Q_i^{(\mathrm{s})})$ is the queueing delay, λ_i is the traffic arrival rate at SS i (e.g., $\lambda_i = 0.2$ Mbps), $Q_i^{(\mathrm{s})}$ is the allocated bandwidth, and N_{ss} is the total number of SSs.

The bandwidth demand by a WiFi node depends on the price charged by the WiFi AP/router. We assume a linear demand function [23] which is expressed as follows:

$$\tilde{Q}_j = e_j - d_j P_k^{(\mathrm{wf})} \tag{14.46}$$

where $\tilde{Q}_j$ is the bandwidth demand of node j served by WiFi AP/router k, e_j and d_j are constants (e.g., $e_j = 2.0$ and $d_j = 0.4$), and $P_k^{(\mathrm{wf})}$ is the price charged at WiFi AP/router k. Therefore, the revenue of the WiFi network k is obtained from

$$r_k^{(\mathrm{wf})} = \sum_{j=1}^{N_k^{(\mathrm{wf})}} P_k^{(\mathrm{wf})} \tilde{Q}_j \tag{14.47}$$

and the cost is calculated from

$$C_k^{(\mathrm{wf})} = P_k^{(bs)} \sum_{j=1}^{N_k^{(\mathrm{wf})}} \tilde{Q}_j + F_k^{(\mathrm{wf})} \tag{14.48}$$

where $P_k^{(\mathrm{bs})}$ is the price charged by the WiMAX BS to the WiFi AP/router k, $N_k^{(\mathrm{wf})}$ is the number of WiFi nodes served by router k, and $F_k^{(\mathrm{wf})}$ denotes the fixed cost for WiFi router k. Note that, this demand function can be empirically obtained as in [23].

14.7.2.2 Stackelberg Game Formulation and the Equilibrium

We apply the Stackelberg game structure to obtain the equilibrium of bandwidth sharing and pricing between WiMAX and WiFi service providers. With the assumption that the WiMAX and the WiFi service providers are rational to maximize their profits, the game can be described as follows:

- The players: The WiMAX BS (i.e., leader) and the WiFi APs/routers (i.e., followers) are the players of this game.
- The strategies: For the WiMAX BS, the strategy is the price $P_k^{(\mathrm{bs})}$ charged to the WiFi APs and for a WiFi AP the strategy is the required bandwidth $Q_k^{(\mathrm{wf})} = \sum_{j=1}^{N_k^{(\mathrm{wf})}} \tilde{Q}_j$.
- The payoffs: For both the WiMAX BS and the WiFi APs/routers, the payoffs are the corresponding profits.

We first consider the payoff for a WiFi AP/router. Given the price charged by the WiMAX BS, $P_k^{(\mathrm{bs})}$, the profit of AP k is

$$\pi_k^{(\mathrm{wf})} = r_k^{(\mathrm{wf})} - C_k^{(\mathrm{wf})} \tag{14.49}$$

$$= \sum_{j=1}^{N_k^{(\mathrm{wf})}} P_k^{(\mathrm{wf})} \left(e_j - d_j P_k^{(\mathrm{wf})}\right) - P_k^{(\mathrm{bs})} \sum_{j=1}^{N_k^{(\mathrm{wf})}} \left(e_j - d_j P_k^{(\mathrm{wf})}\right) - F_k^{(\mathrm{wf})}. \tag{14.50}$$

Therefore, the optimal price charged to a WiFi node (i.e., $P_k^{(\mathrm{wf})}$) can be obtained by differentiating the profit function and then setting it to zero. Then, given price $P_k^{(\mathrm{wf})}$, the bandwidth demand for all WiFi nodes in hotspot k can be obtained. Based on the best response of the WiFi AP/router, the WiMAX BS can adjust the price $P_k^{(\mathrm{bs})}$ charged to router k to achieve the highest payoff. The payoff (i.e., profit) of the WiMAX BS can be defined as follows:

$$\pi^{(\mathrm{bs})} = r^{(\mathrm{s})} + \sum_{k=1}^{N_\mathrm{r}} r_k^{(\mathrm{wf})} \tag{14.51}$$

$$= \sum_{i=1}^{N_{ss}} \left(a_i - e_i D(\lambda_i, Q_i^{(\mathrm{s})})\right) + \sum_{k=1}^{N_\mathrm{r}} P_k^{(\mathrm{bs})} Q_k^{(\mathrm{wf})} \tag{14.52}$$

where N_r is the total number of WiFi APs/routers.

The Stackelberg equilibrium is defined as the strategy profile that maximizes the leader's payoff while the follower plays his/her best response [24]. We consider this equilibrium as the solution of the bandwidth sharing and pricing game to ensure that the profit of the WiMAX BS, which is the major player of this game, is maximized. In the case that all information on demand function are completely known, the equilibrium can be obtained easily by differentiating the profit function of the WiMAX BS and solving it for the price $P_k^{(\mathrm{bs})}$.

14.7.3 Performance Evaluation

14.7.3.1 Parameter Setting

We consider a single BS with multiple connections from SSs and WiFi APs/routers using the TDMA/TDD access mode based on single carrier modulation (e.g., WirelessMAN-SC). We consider downlink transmission from the WiMAX BS, and the frame size is assumed to be 5 ms. The total bandwidth of operation for the WiMAX BS is 20 MHz, and for transmission the BS uses QPSK modulation and a coding rate of 1/2.

14.7.3.2 Numerical Results

First, we show the bandwidth demand of the WiFi routers under different prices charged by the WiMAX BS (in Fig. 14.18). We consider the case of a homogeneous demand function for all WiFi nodes. This bandwidth demand represents the best response of the WiFi AP/router (i.e., follower) in the Stackelberg game formulation given the price charged by the WiMAX BS (i.e., leader). The best response for a WiFi AP/router can be obtained from the point at which the profit of the WiFi AP/router is maximized. The bandwidth demand decreases as the price increases since a WiFi router has to charge higher price to the WiFi nodes. As a result, the profit of the corresponding WiFi AP/router decreases. Also, as expected, when the number of WiFi nodes increases, the bandwidth demand of the WiFi AP/router increases.

Next, the profit of the WiMAX BS is shown in Fig. 14.19. Here, the WiFi routers serve 4 and 6 WiFi nodes, and the number of SSs is 10. The profit changes due to

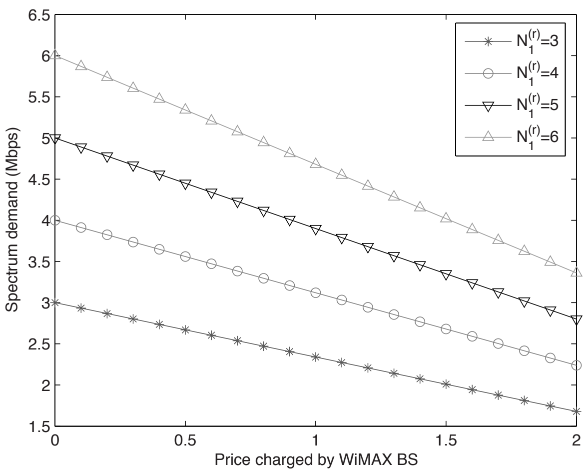

Fig. 14.18. Demand function which maximizes profit of the WiFi APs/routers.

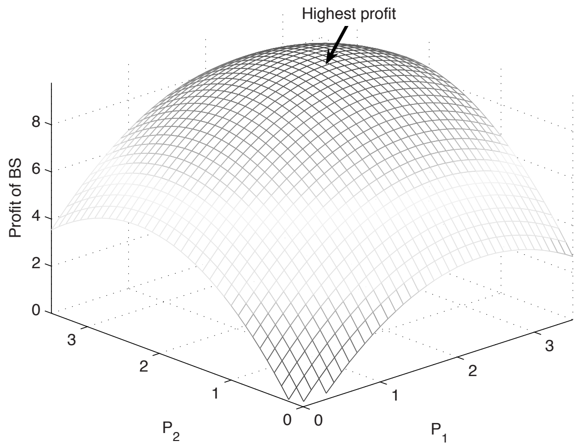

Fig. 14.19. Profit function of the WiMAX BS.

the different prices charged to the WiFi APs/routers, and also there is a point where the profit of the WiMAX BS is maximized. This point is the equilibrium of this Stackelberg game formulation. As is evident from Fig. 14.19, this profit, which is a function of price, is unimodel.

Figures 14.20a and b show optimal price for the WiMAX BS to charge the WiFi routers and the bandwidth to share with the WiFi routers. We consider the case when the first and the second WiFi AP/router serve 4 and 6 WiFi nodes, respectively. All the SSs have the same traffic arrival rate. Interestingly, even though the prices charged to the first and the second WiFi AP/router are formulated as different strategies in the game, at the equilibrium they are always equal. This implies that the WiMAX BS should charge the same price to the WiFi routers even though their bandwidth demands may be different. Also, as expected, when the traffic arrival rate increases, the WiMAX BS needs to increase the price charged to the WiFi routers to compensate the loss in revenue due to the degraded QoS performance (i.e., higher delay) for the SSs. Consequently, the bandwidth demand of both the WiFi APs/routers decreases. At the same price, bandwidth demand of the first WiFi router becomes smaller than that of the second router due to the smaller number of WiFi nodes.

Then, we vary the number of WiFi nodes served by router two and observe the price and the amount of bandwidth shared at the equilibrium (Fig. 14.21). We set the number of SSs to 16 and traffic arrival rate is assumed to be 0.5 Mbps. As expected, when the number of nodes increases, the bandwidth demand from the WiMAX BS increases. Consequently, the price charged to the WiFi APs/routers increases.

We observe that the bandwidth allocated to WiFi router two increases significantly while that to WiFi router one slightly decreases (which is due to the higher

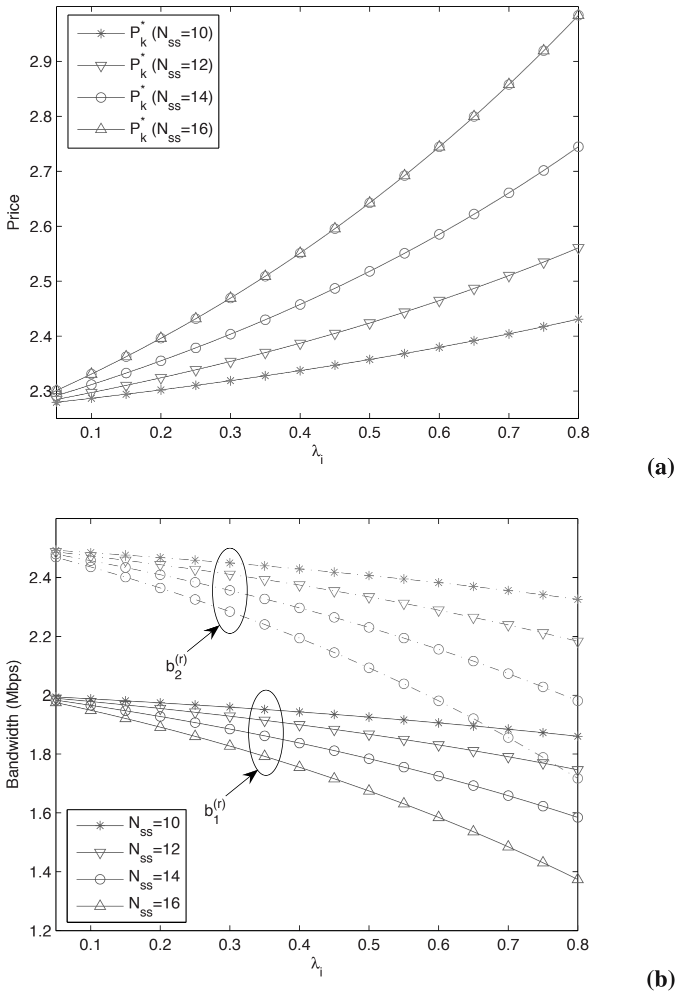

Fig. 14.20. (**a**) Price and (**b**) bandwidth sharing at the equilibrium under different traffic load at the subscriber stations.

price). We observe that with smaller number of SSs and lower traffic arrival rate (e.g., $\lambda_i = 0.1$ Mbps) the price does not change significantly. This is due to the fact that the WiMAX BS can take some bandwidth from the SSs (instead of taking bandwidth from other WiFi routers) with only slight degradation in their delay performances.

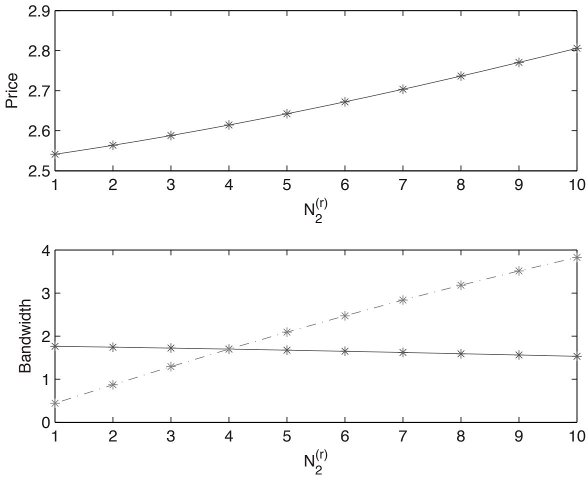

Fig. 14.21. Price and bandwidth sharing at the equilibrium under different number of WiFi nodes served by WiFi router two.

Conclusion

In this chapter, we have demonstrated the applications of oligopoly market models from microeconomics to solve the problem of spectrum/bandwidth sharing and pricing in cognitive radio environment. In microeconomics, oligopoly is used to describe a market situation which is composed of several producers, and the producers have their own interests to maximize their profits. We have considered three different oligopoly models, namely, *Cournot*, *Stackelberg*, and *Bertrand* models which have been analyzed by game theory techniques. In Cournot competition, producers compete in terms of supplied product quantity, and all of them make decisions simultaneously. In Stackelberg model, producers compete in terms of supplied quantity, but there are some producers who can make decision before the rest. In Bertrand competition, producers compete by varying the product price.

In the Cournot game model of bandwidth sharing, multiple secondary users (i.e., players) share the spectrum opportunity offered by a primary user. The Nash equilibrium of the game gives the bandwidth share for each secondary user such that the profits (or payoffs) for all the secondary users are maximized. Numerical performance evaluation results for this bandwidth sharing model in a cognitive wireless environment have been presented considering different channel qualities and prices (per unit bandwidth).

The Bertrand game model of bandwidth sharing and pricing considers multiple primary users (i.e., players) offering spectrum access to a secondary user and the objective is to maximize the profit of the primary users considering the degradation in QoS for the primary users due to spectrum sharing. The spectrum demand of the secondary user is obtained from its utility which is a function of spectrum price offered by the primary users and the corresponding channel quality such that the utility is maximized. The Nash equilibrium of the game is considered as the optimal solution of the game which gives the optimal price offered by the primary users. Numerical performance evaluation results have been presented under different channel qualities for the spectrum access by the secondary user, spectrum substitutability factor, and different traffic arrival rate at the primary users.

The Stackelberg game model of bandwidth sharing and pricing considers one leader (service provider) and multiple followers (secondary users) with time-varying demand function. The model has been used to obtain an optimal pricing scheme in an integrated WiMAX/WiFi network where the WiFi APs/routers share the WiMAX spectrum with the licensed WiMAX subscriber stations. The bandwidth demand of the WiFi APs/routers depends on the bandwidth demands by the WiFi nodes which depend on the price charged by the WiMAX BS. The Stackelberg equilibrium gives the optimal bandwidth share and the pricing for which the revenue of the service provider (i.e., leader) is maximized. The potential QoS degradation at the subscriber stations (i.e., licensed users) is considered while calculating the revenue for the service provider. Numerical performance evaluation results on price and bandwidth sharing at the equilibrium have been shown under different traffic load at the WiMAX subscriber stations and different number of WiFi nodes.

With oligopoly market model, the players in the game can adaptively adjust their strategies when only limited network information (e.g., the strategies adopted by other players, profit information) is available. Therefore, a learning algorithm would be required for the players to make their decisions effectively. Another possible research direction is the consideration of uncertainties in the model. In this case, the demand function can be random due to the time-varying traffic and QoS requirements of the users. Also, the payoff can be random due to the channel variation (e.g., fading). These issues should be addressed for competitive spectrum sharing in a practical cognitive radio environment.

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