1

Fundamental Issues in Cognitive Radio

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1.1 Introduction

The electromagnetic radio spectrum is a natural resource, the use of which by transmitters and receivers (transceivers) is licensed by government agencies. However, this resource is presently underutilized. In particular, if we were to scan the radio spectrum, including the revenue-rich urban areas, we would find that some frequency bands in the spectrum are unoccupied some of the time, some other frequency bands are only partially occupied, and the remaining frequency bands are heavily used. It is therefore not surprising to find that underutilization of the radio spectrum is being challenged on many fronts, including the Federal Communications Commission (FCC) in the United States of America.

Cognitive radio[1] offers a novel way of solving spectrum underutilization problems. It does so by sensing the radio environment with a twofold objective: identifying those subbands of the radio spectrum that are underutilized by the primary (i.e., legacy) users and providing the means for making those bands available for employment by unserviced secondary users. To achieve these goals in an autonomous manner, multiuser cognitive radio networks would have to be self-organized. Moreover, there would have to be a paradigm shift from transmitter-centric wireless communications to a new mode of operation that is receiver-centric, so as to maintain a limit on the interference produced by secondary user.

The underutilized frequency bands of the radio spectrum, owned by legally licensed (primary) users, are referred to as *spectrum holes*, which are formally defined as follows [1]:

[1] Cognitive radio is a constituent of the emerging discipline: *Cognitive Dynamic Systems*; see the point-of-view article in [2]. This discipline, motivated by the human brain, includes other constituents: *cognitive radar* and *cognitive immunity*. Unlike traditional radar, cognitive radar includes feedback from the receiver to the transmitter, resulting in immense benefits to radar performance. The purpose of cognitive immunity is to resist cyber attack in dynamic software systems.

> A spectrum hole is a band of frequencies assigned to a primary user, but at a particular time and specific geographic location, the band is not being utilized by that user.

The operation of cognitive radio hinges on the availability of spectrum holes. The identification and exploitation of spectrum holes presents technical challenges grouped under two categories, one rooted in computer software and the other rooted in signal-processing and communication technology. These technical challenges are further compounded by the fact that the spectrum holes come and go in a stochastic manner.

Much of the material presented in this article focuses on signal-processing and communication-theoretic aspects of cognitive radio. Specifically, the material is organized as follows. The notion of cognition is discussed in Sect. 1.2. Section 1.3 describes two complementary visions of cognitive radio, one addressing software architectural aspects of cognitive radio and the other addressing signal-processing and communication-theoretic aspects of the subject. Section 1.4 deals with radio-scene analysis, which encompasses the sensing of the radio environment and identifying the specific locations of spectrum holes in the radio spectrum. Section 1.5 deals with two related issues: channel-state estimation and predictive modeling, both of which are fundamental to efficient utilization of the radio spectrum and coherent detection of the information-bearing signal at a user's receiver. Information gathered by the receiver on its local environment is sent to the transmitter via a low bit-rate feedback channel, which is discussed in Sect. 1.6.

Up to this point in this chapter, the discussion is focused on issues relating largely to a single user (i.e., transmitter linked to its receiver). The rest of the chapter, beginning with Sect. 1.7, is devoted to self-organized multiuser cognitive radio networks, with emphasis on the complementary use of cooperation and competition. Section 1.8 discusses the function of dynamic spectrum management, where the use of orthogonal frequency-division multiplexing (OFDM) based on cooperative communication is advocated. Based on this encoding strategy, Sect. 1.9 describes a statistical model of cognitive radio networks, which sets the stage for formulation of the transmit-power control problem in Sect. 1.10. Section 1.11 views the multiuser cognitive radio network, operating in a non-cooperative manner, as a game-theoretic problem. Section 1.12 describes an iterative waterfilling algorithm for resolving the issue of transmit-power control, followed by Sect. 1.13 on the emergent behavior of cognitive radio networks. Section 1.14 briefly discusses a plan for distributed traffic coordination of cognitive radio users in an ad hoc network environment. Then the chapter concludes with some final remarks.

1.2 Cognition

In a way, it can be argued that cognitive radio draws its inspiration from cognitive science. The roots of *cognitive science* are intimately linked to two scientific meetings that were held in 1956 [3]:

- The Symposium on Information Theory, which was held at the Massachusetts Institute of Technology (MIT). That meeting was attended by leading authorities in the information and human sciences, including Allen Newell (computer scientist), the Nobel Laureate, Herbert Simon (political scientist and economist), and Noam Chomskey (linguist). As a result of that symposium, linguists began to theoretize about language, which was to be found subsequently in the theory of computers: the language of information processing.
- The Dartmouth Conference, which was held at Dartmouth College, New Hampshire. The conference was attended by the founding fathers of artificial intelligence, namely, John McCarthy, Marvin Minsky and Allen Newel. The goal of this second meeting was to think about intelligent machines. The Dartmouth Conference was also attended by Frank Rosenblatt (psychologist), the founder of (artificial) neural networks. At the conference, Rosenblatt described a novel method for supervised learning, which he called the perceptron.[2] However, interest in neural networks was short lived: in a monograph published in 1969, Minsky and Papert used mathematics to demonstrate that there are fundamental limits on what Rosenblatt's perceptron could compute. The Minsky–Papert monograph, coupled with a few other factors, contributed to the dampening of interest in neural networks in the 1970s. We had to wait for the pioneering contributions of John Hopfield on neurodynamic systems and Rumlehart, Hinton and Williams on supervised learning in the 1980s for the revival of research interest in neural networks.[3]

In a book entitled "The Computer and the Mind," Johnson-Laird [4] postulated the following tasks of a human mind:

- To perceive the world
- To learn, to remember and to control actions
- To think and create new ideas
- To control communication with others
- To create the experience of feelings, intentions and self-awareness

Johnson-Laird, a prominent psychologist and linguist, went on to argue that theories of the mind should be modeled in computational terms.

Much of what has been identified by Johnson-Laird as the mind's main tasks and their modeling in computation terms apply equally well to cognitive radio. Indeed, we can go on to offer the following definition for cognitive radio involving multiple users.

The cognitive radio network is an intelligent multiuser wireless communication system that embodies the following list of primary tasks:

- To perceive the radio environment (i.e., outside world) by empowering each user's receiver to sense the environment on continuous time

[2] The perceptron provided the inspiration for Widrow and Hoff to develop the least-mean-square (LMS) algorithm, which has established itself as the workhorse for adaptive filtering for close to 50 years.

[3] For a historical account of neural networks, see Haykin [5].

- To learn from the environment and adapt the performance of each transceiver to statistical variations in the incoming RF stimuli
- To facilitate communication between multiple users through cooperation in a self-organized manner
- To control the communication processes among competing users through the proper allocation of available resources
- To create the experience of intentions and self-awareness

The primary objective of all these tasks, performed in real time, is twofold:

- To provide highly reliable communication for all users
- To facilitate efficient utilization of the radio spectrum in a fair-minded way

1.3 Two Complementary Visions of Cognitive Radio

In the first doctoral dissertation on cognitive radio published in 2000, Joseph Mitola described how a cognitive radio could enhance the flexibility of personal wireless services through a new language called the radio knowledge representation language [6]. Mitola followed this dissertation with the publication of a book on cognitive radio architecture [7]. A distinctive feature of both publications is a *cognitive computer cycle*, which encapsulates the various actions expected from a cognitive radio, as depicted in Fig. 1.1. Through deployment of the right software control, it is envisioned that a cognitive radio could orient itself by establishing priorities, then create plans decide and finally take the appropriate action in response to sensing of the RF environment. As envisioned in Fig. 1.1, provisions are also made for the cognitive radio to do two things:

- Bypass the planning phase and go directly to the decision phase in the event of an urgent situation
- Bypass the two phases of planning and decision-making by proceeding immediately to the action phase in the event of an emergency.

In the first journal paper published in 2005, Simon Haykin presented detailed expositions of the signal-processing, adaptive and learning procedures that lie at the heart of cognitive radio [2]. In particular, the paper identifies three specific tasks:

1. *Radio-scene analysis* (RSA), which encompasses
   - Estimation of interference temperature of the radio environment localized around a user's receiver
   - Detection of spectrum holes
   - Predictive modeling of the environment.
2. *Channel identification*, which is needed for improved spectrum utilization and coherent detection of original information-bearing signal at the user's receiver.
3. *Dynamic spectrum management* (DSM) and *transmit-power control* (TPC), which culminates in decision-making and action taken by the user's transmitter in response to the analysis of RF stimuli picked up by the receiver.

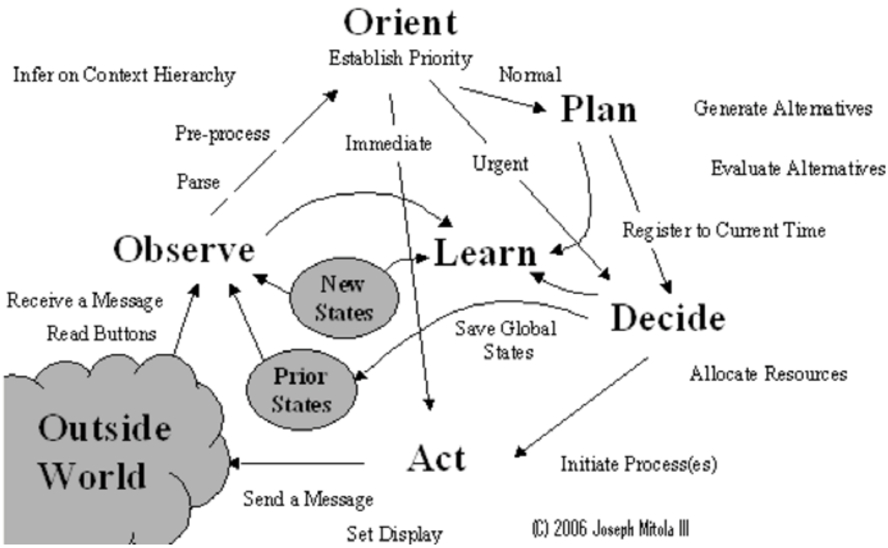

Fig. 1.1. Cognitive computer cycle. *(©2006 Joseph Mitola III. Reprinted, with permission from [7, p. 135]).*

Tasks (1) and (2) are performed in the receiver, and task (3) is performed in the transmitter, as depicted in the *cognitive signal-processing cycle* in Fig. 1.2; the depiction is presented in the context of a multiuser network.

For the transmitter to work harmoniously with the receiver,[4] there is an obvious need for a *feedback* channel connecting the receiver to the transmitter as shown in Fig. 1.2. Through the feedback channel, the receiver is enabled to convey to the transmitter two essential forms of information:

- Information on the performance of the forward link for adaptive modulation
- Information on the spectral state of the RF environment in the local neighborhood of the receiver

The cognitive radio is therefore, by necessity, an example of a *global closed-loop feedback control system.*

[4] Every node of the network is equipped with a transceiver (i.e., transmitter/receiver combination). Accordingly, the transmitting part of the node can analyze the radio scene in its local neighborhood, and thereby identify the spectrum holes available for use by the transmitter for communication with the receiver of some other node.

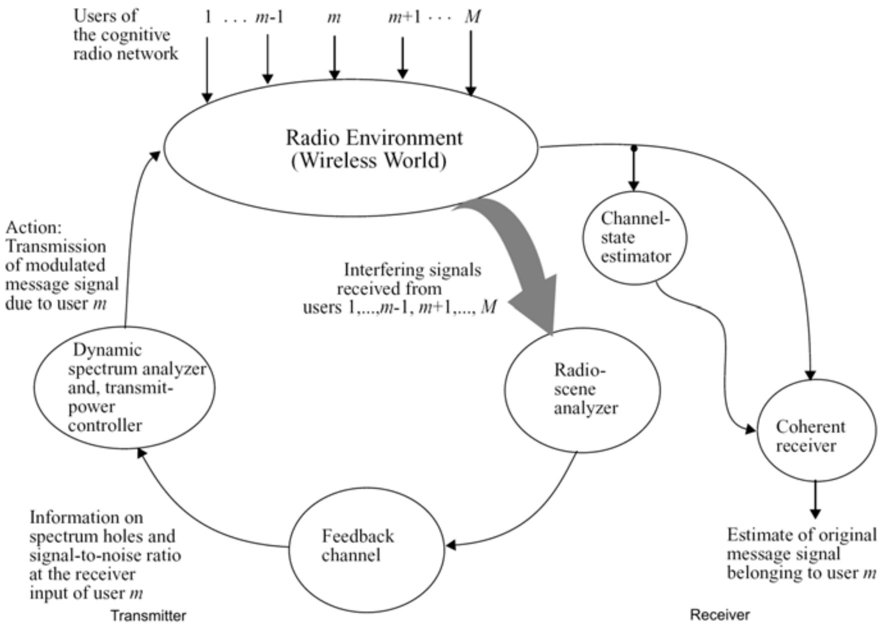

Fig. 1.2. Basic signal-processing cycle for user m in a cognitive radio network; the diagram also includes elements of the receiver of user m.

The pioneering contributions made by Mitola and Haykin are in fact complementary, with Mitola's contribution focusing on software computer aspects of cognitive radio, and Haykin's contribution focusing on signal-processing and communication-theoretic aspects of this exciting multidisciplinary subject.

One other relevant comment is in order. A broadly defined cognitive radio technology accommodates a *scale of differing degrees of implementation*. At one end of the scale, the user may simply find a spectrum hole and build its cognitive cycle around that hole. At the other end of the scale, the user may employ multiple implementation technologies to build its cognitive cycle around a wideband spectrum hole or a set of narrowband spectrum holes to provide the best expected performance in terms of spectrum management and transmit-power control, data rate, and reliable communication, and do all this in the most secure manner feasible.

1.4 Radio-Scene Analysis

With the background material covered in the previous three sections at hand, we are now ready to address the issues involved in radio-scene analysis (RSA). This section is organized as follows:

- We first describe the notion of interference temperature, followed by the issue of the non-stationary character of RF stimuli.

- Next, we describe the multitaper method as the preferred method for non-parametric estimation of the power spectrum of incoming RF stimuli.
- We then describe a spatio-temporal procedure for estimating the interference temperature across the prescribed frequency band.
- We next describe how the occupancy of the radio spectrum in its contiguous subbands is classified.
- Finally, the need for a predictive model describing the evolution of spectrum holes is addressed.

1.4.1 Interference Temperature

Currently, the wireless communication environment is *transmitter-centric*, in the sense that the transmitted power is designed to approach a prescribed noise floor at a certain distance from the transmitter. However, it is possible for the RF noise floor to rise due to the unpredictable appearance of new sources of interference, thereby causing a progressive degradation of the signal coverage. To guard against such a possibility, the FCC Spectrum Policy Task Force [8] has recommended a paradigm shift in interference assessment, that is, a shift away from largely fixed operations in the transmitter and toward *real-time interactions between the transmitter and receiver in an adaptive manner*. The recommendation is based on a new metric called the *interference temperature*,[5] which is intended to quantify and manage the sources of interference in a radio environment. Moreover, the specification of an *interference-temperature limit* provides a "worst-case" characterization of the RF environment in a particular frequency band and at a particular geographic location, where the receiver is expected to operate satisfactorily.

The FCC's recommendation is made with two key benefits in mind:

1. The interference temperature at a receiving antenna provides an accurate measure for the acceptable level of RF interference in the frequency band of interest; any transmission in that band is considered to be "harmful" if it would increase the noise floor above the interference-temperature limit.
2. Given a particular frequency band in which the interference temperature is not exceeded, that band could be made available to unserviced users; the interference-temperature limit would then serve as a "cap" placed on potential RF energy that could be introduced into that band. Logically, the licensed legacy users (i.e., primary owners of the radio spectrum) would be responsible for setting the interference-temperature limit.

[5] We may also introduce the concept of interference temperature density, which is defined as the interference temperature per capture area of the receiving antenna [9]. The interference temperature density could be made independent of the receiving antenna characteristics through the use of a reference antenna.

In a historical context, the notion of radio noise temperature is discussed in the literature in the context of microwave background, and also used in the study of solar radio bursts [10,11].

What about the unit for interference temperature? Following the well-known definition of equivalent noise temperature of a receiver [12,13], we may state that the interference temperature is measured in *degrees Kelvin*. Moreover, the interference-temperature limit, $T_{\max}$, multiplied by *Boltzmann's constant*, $\kappa = 1.3807 \times 10^{-23}$ Joules per degree Kelvin, yields the corresponding upper limit on permissible power spectral density in a frequency band of interest, and that density is measured in watts per hertz.

Summarizing, we may therefore say:

> Given an estimate of the power spectral density in a specific subband of the radio spectrum, we may determine the corresponding value of the interference temperature in that subband by dividing the estimate by Boltzmann's constant κ.

This statement emphasizes the need for reliable estimation of the power spectral density of the received RF signal.

1.4.2 Stochastic Approach for Dealing with Non-stationarity

The stimuli generated by radio emitters are *non-stationary spatio-temporal signals* in that their statistics depend on both time and space. Correspondingly, the passive task of radio-scene analysis involves space–time processing, which encompasses two adaptive, spectrally related functions, namely, estimation of the interference temperature and detection of spectrum holes, both of which are performed at a user's receiver.

Unfortunately, the statistical analysis of non-stationary signals, exemplified by RF stimuli, has had a rather mixed history. Although the general second-order theory of non-stationary signals was published during the 1940s by Loève [14,15], it has not been applied nearly as extensively as the theory of stationary processes published only slightly previously and independently by Wiener and Kolmogorov.

To account for the non-stationary behavior of a signal, we have to include time (implicitly or explicitly) in a statistical description of the signal. Given the desirability of working in the frequency domain for well-established reasons, we may include the effect of time by adopting a *time-frequency distribution* of the signal. During the last three decades, many papers have been published on various estimates of time–frequency distributions; see, for example [16] and the references cited therein. In most of this work, however, the signal is assumed to be deterministic. In addition, many of the proposed estimators of time–frequency distributions are constrained to match time and frequency marginal density conditions. However, the frequency marginal distribution is, except for a scaling factor, just the periodogram of the signal. At least since the early work of Lord Rayleigh [17], it has been known that *the periodogram is a badly biased and inconsistent estimator of the power spectrum*. We therefore do not consider matching marginal distributions to be important. Rather, we advocate a stochastic approach to time–frequency distributions which is rooted in the works of Loève [14,15] and Thomson [18].

For the stochastic approach, we may proceed in one of two ways:

1. The incoming RF stimuli are divided into a continuous sequence of successive sections (blocks), with each section being short enough to justify pseudo-stationarity and yet long enough to produce an accurate spectral estimate.
2. Time and frequency are considered jointly under the Loève transform.

Approach (1) is well suited for wireless communications by virtue of the fact that the transmitted signal is typically transmitted on a packet-by-packet basis; we may thus form each section from several adjacent packets, depending on the desired accuracy. In any event, we need a *non-parametric* method for spectral estimation that is both accurate and principled. For reasons that will become apparent in what follows, multitaper spectral estimation is considered to be the method of choice.

1.4.3 Multitaper Spectral Estimation

In the spectral estimation literature, it is well known that the estimation problem is made difficult by the *bias-variance dilemma*, which encompasses the interplay between two points:

- Bias of the power-spectrum estimate of a time series, due to the sidelobe leakage phenomenon, is reduced by tapering (i.e., *windowing*) the time series.
- The cost incurred by this improvement is an increase in variance of the estimate, which is due to the loss of information resulting from a reduction in the effective sample size.

How can we resolve this dilemma by mitigating the loss of information due to tapering? The answer to this fundamental question lies in the principled use of *multiple orthonormal tapers* (*windows*),[6] an idea that was first applied to spectral estimation by Thomson in 1982 [18]. The idea is embodied in the *multitaper spectral estimation procedure*.[7] Specifically, the procedure linearly expands the part of the time series in a fixed bandwidth $f-W$ to $f+W$ (centered on some frequency f) in a special family of sequences known as the *Slepian sequences*.[8] The remarkable property of Slepian sequences is that their Fourier transforms have the *maximal energy concentration* in the bandwidth $f-W$ to $f+W$ under a finite sample-size constraint. This property, in turn, allows us to trade spectral resolution for improved spectral characteristics,

[6] Another method for addressing the bias-variance dilemma involves dividing the time series into a set of possible overlapping sections, computing a periodogram for each tapered (windowed) section, and then averaging the resulting set of power spectral estimates, which is what is done in *Welch's method* [19]. However, unlike the principled use of multiple orthogonal tapers, Welch's method is rather ad hoc in its formulation.

[7] In the original paper by Thomson [18], the multitaper spectral estimation procedure is referred to as the *method of multiple windows*. For detailed descriptions of this procedure, see [18] and Chap. 7 of the book by Percival and Walden [20].

[8] The Slepian sequences are also known as *discrete prolate spheroidal sequences*. For detailed treatment of these sequences, see the original paper by Slepian [21], the appendix to Thomson's paper [18] and Chap. 8 of the book by Percival and Walden [22].

namely, reduced variance of the spectral estimate without compromising the bias of the estimate.

Given a time series $\{x_t\}_{t=1}^{N}$, representing the *baseband* version of the received RF signal with respect to the center frequency of the RF band under scrutiny, the multitaper spectral estimation procedure determines two things:

1. An orthonormal sequence of K *Slepian tapers* denoted by $\{w_t^{(k)}\}_{t=1}^{N}$
2. The associated *eigenspectra* defined by the Fourier transform

$$Y_k(f) = \sum_{t=1}^{N} w_t^{(k)} x(t) \mathrm{e}^{-\mathrm{j}2\pi f t}, \quad k = 0, 1, \ldots K-1. \tag{1.1}$$

The energy distributions of the eigenspectra are concentrated inside a *resolution bandwidth*, denoted by $2W$. The *time–bandwidth product*

$$p = 2NW \tag{1.2}$$

defines the *degrees of freedom* available for controlling the variance of the spectral estimator. The choice of parameters K and p provides a tradeoff between spectral resolution and variance.[9] A natural spectral estimate, based on the first few eigenspectra that exhibit the least sidelobe leakage, is given by [18]

$$\hat{S}(f) = \frac{\sum_{k=0}^{K-1} \lambda_k(f) \left|Y_k(f)\right|^2}{\sum_{k=0}^{K-1} \lambda_k(f)} \tag{1.3}$$

where λ_k is the eigenvalue associated with the kth eigenspectrum. The denominator in (1.3) makes the estimate $\hat{S}(f)$ unbiased.

The multitaper spectral estimator of (1.3) is intuitively appealing in the way it works: as the number of tapers, K, increases, the eigenvalues decrease, causing the eigenspectra to be more contaminated by leakage. But, the eigenvalues themselves counteract by reducing the weighting applied to higher leakage eigenspectra.

It is also noteworthy that in [24], Stoica and Sundin show that the multitaper spectral estimation procedure can be interpreted as an "approximation" of the *maximum-likelihood* power spectrum estimator. Moreover, they show that for wideband signals, the multitaper spectral estimation procedure is "nearly optimal" in the sense that it

[9] For an estimate of the variance of a multitaper spectral estimator, we may use a resampling technique called *jackknifing* [23]. The technique bypasses the need for finding an exact analytic expression for the probability distribution of the spectral estimator, which is impractical because time-series data (e.g., stimuli produced by the radio environment) are typically non-stationary, non-Gaussian, and frequently contain outliers. Moreover, it may be argued that the multitaper spectral estimation procedure results in nearly uncorrelated coefficients, which provides further justification for the use of jackknifing.

almost achieves the Cramèr–Rao bound for a non-parametric spectral estimator.[10] Most important, unlike the maximum-likelihood spectral estimator, the multitaper spectral estimator is computationally feasible.

1.4.4 Adaptive Modification of Multitaper Spectral Estimation

While the lower-order eigenspectra have excellent bias properties, there is some degradation as the order K increases toward the time–bandwidth product $2NW$. In [18], Thomson introduces a set of adaptive weights, denoted by $\{d_k(f)\}$, which downweight the higher order eigenspectra. Using a mean-squared error optimization procedure, the following formula for the weights is derived:

$$d_k(f) = \frac{\sqrt{\lambda_k} S(f)}{\lambda_k S(f) + \mathbf{E}[B_k(f)]}, \quad k = 0, 1, \ldots, K-1 \tag{1.4}$$

where $S(f)$ is the true power spectrum, $B_k(f)$ is the broadband bias of the kth eigenspectrum, and $\mathbf{E}$ is the expectation operator. Moreover,

$$\mathbf{E}[B_k(f)] \leq (1 - \lambda_k)\sigma^2, \quad k = 0, 1, \ldots, K-1 \tag{1.5}$$

where σ^2 is the *process variance* defined by

$$\sigma^2 = \frac{1}{N} \sum_{t=0}^{N-1} |x(t)|^2. \tag{1.6}$$

In order to compute the adaptive weights $d_k(f)$ using (1.4), we need to know the true spectrum $S(f)$. But if we did, then there would be no need to perform any spectrum estimation at all. Nevertheless, the formula of (1.4) is useful in setting up an *iterative procedure for computing the adaptive spectral estimator*

$$\hat{S}(f) = \frac{\displaystyle\sum_{k=0}^{K-1} |d_k(f)|^2 \hat{S}_k(f)}{\displaystyle\sum_{k=0}^{K-1} |d_k(f)|^2} \tag{1.7}$$

where

$$\hat{S}_k(f) = |Y_k(f)|^2, \quad k = 0, 1, \ldots, K-1. \tag{1.8}$$

Note that if we set $\{d_k(f)\}^2 = \lambda_k$ for all k, then the estimator of (1.7) reduces to that of (1.3).

[10] In [22], a comparative evaluation of the multitaper method (MTM) and maximum-likelihood (ML) method is presented for angle-of-arrival estimation in the presence of multipath. The results reported therein give consistent results for low grazing angles. The MTM is found to be slightly superior to ML, but the difference between them is not overwhelming.

Next, setting $S(f)$ equal to the spectrum estimator $\hat{S}_k(f)$ in (1.4), then substituting the new equation into (1.7) and collecting terms, we get (after simplifications)

$$\sum_{k=0}^{K-1} \frac{\lambda_k(\hat{S}(f) - \hat{S}_k(f))}{(\lambda_k \hat{S}(f) + \hat{B}_k(f))^2} = 0 \tag{1.9}$$

where $\hat{B}_k(f)$ is an estimate of the expectation $\mathbf{E}[B_k(f)]$. Using the upper bound of (1.5), we have

$$\hat{B}_k(f) = (1 - \lambda_k)\sigma^2, \quad k = 0, 1, \ldots, K-1. \tag{1.10}$$

We now have all that we need to solve for the null condition of (1.9) via the *recursion*

$$\hat{S}^{(j+1)}(f) = \left[\sum_{k=0}^{K-1} \frac{\lambda_k \hat{S}_k(f)}{(\lambda_k \hat{S}^{(j)}(f) + \hat{B}_k(f))^2}\right] \left[\sum_{k=0}^{K-1} \frac{\lambda_k}{(\lambda_k \hat{S}^{(j)}(f) + \hat{B}_k(f))^2}\right]^{-1} \tag{1.11}$$

where j denotes an iteration step. To initialize this recursion, we may set $S^{(j)}(0)$ equal to the average of the two lowest order eigenspectra. Convergence of the recursion is usually rapid, with successive spectral estimates differing by less than 5% in 5–20 iterations. For a more accurate (also more complex) estimate of $B_k(f)$, see [18,22]. In any event, the result obtained from (1.11) is substituted into (1.4) to obtain the desired weights, $d_k(f)$.

A useful by-product of this adaptive spectral estimation procedure is a *stability measure of the estimates*, given by

$$v(f) = 2 \sum_{k=0}^{K-1} |d_k(f)|^2 \tag{1.12}$$

which is the approximate number of degrees of freedom for the estimator $\hat{S}_k(f)$ expressed as a function of frequency f. If $\bar{v}$, denoting the average of $v(f)$ over frequency f, is significantly less than $2K$, then the result is an indication that either the window W is too small, or additional prewhitening of the time series $x(n)$ should be used.

The importance of *prewhitening* cannot be stressed enough for RF data. In essence, prewhitening reduces the dynamic range of the spectrum by filtering the data, prior to processing. The resulting residual spectrum is nearly flat or "white." In particular, leakage from strong components is reduced, so that the fine structure of weaker components is more likely to be resolved. In actual fact, most of the theory behind spectral estimation is smooth, almost white-like spectra to begin with, hence the need for "prewhitening" [22].

1.4.5 Summarizing Remarks I

1. Estimation of the power spectral density based on the multitaper method of (1.3) is said to be *incoherent*, because the kth magnitude spectrum $|Y_k(f)|^2$ ignores phase information for all k.

2. For the parameters needed to compute the multitaper spectral estimator (1.3), recommended values are:
   – Time-bandwidth product: $NW = 6$, possibly extending up to 10.
   – Number of Slepian tapers: $K = 10$, possibly extending up to 16.

   These values are needed, especially when the dynamic range of the RF data is large.

   As an illustrative example, in [25] describing the application of the multitaper method to radar sea-clutter classification, the number of available samples in each section of the radar data was relatively small, namely, 256. Reasonably good results were obtained using $NW = 6$ and $K = 10$ within each section.
3. If and when the number of tapers is increased toward the time–bandwidth product $2NW$, then the adaptive multitaper spectral estimator should be used.
4. Whenever possible, prewhitening of the data, prior to processing, should be applied.

1.4.6 Space–Time Processing

With cognitive radio being receiver-centric, it is necessary that the receiver be provided with a reliable spectral estimate of the interference temperature. We may satisfy this requirement by doing two things:

1. *Use the multitaper method to estimate the power spectrum of the interference temperature due to the cumulative distribution of both internal sources of noise and external sources of RF energy*. In light of the findings reported in [24], this estimate is near-optimal.
2. *Employ a large number of sensors to properly "sniff" the RF environment, wherever it is feasible*. The large number of sensors is needed to account for the spatial variation of the RF stimuli from one location to another.

The issue of multiple-sensor feasibility is raised under point (2) because of the diverse ways in which wireless communications could be deployed. For example, in an indoor building environment and communication between one building and another, it is feasible to employ a large number of sensors (i.e., antennas) placed at strategic locations in order to improve the reliability of interference-temperature estimation. On the other hand, in the case of an ordinary mobile unit with limited real estate, the interference-temperature estimation may have to be confined to a few sensors beamed at different directions.

Let M denote the total number of sensors deployed in the RF environment. Let $Y_k^{(m)}(f)$ denote the kth eigenspectrum computed by the mth sensor. We may then construct the M-by-K spatio-temporal complex-valued matrix [26]

$$\mathbf{A}(f) = \begin{bmatrix} a_1 Y_0^{(1)}(f) & a_1 Y_1^{(1)}(f) & \dots & a_1 Y_{K-1}^{(1)}(f) \\ a_2 Y_0^{(2)}(f) & a_2 Y_1^{(2)}(f) & \dots & a_2 Y_{K-1}^{(2)}(f) \\ \vdots & \vdots & & \vdots \\ a_M Y_0^{(M)}(f) & a_M Y_1^{(M)}(f) & \dots & a_M Y_{K-1}^{(M)}(f) \end{bmatrix} \tag{1.13}$$

where each row is produced using stimuli sensed at a different gridpoint, each column is computed using a different Slepian taper, and the $\{a_m\}_{m=1}^{M}$ represent variable coefficients accounting for relative areas of the gridpoints.

Each entry in the matrix $\mathbf{A}(f)$ is produced by two contributions, one due to additive ambient noise in the sensor and the other due to the interfering RF stimuli. Insofar as radio-scene analysis is concerned, however, the primary contribution of interest is that due to RF stimuli. An effective tool for denoising is the *singular value decomposition* (SVD), the application of which to the matrix $\mathbf{A}(f)$ yields the decomposition [27]

$$\mathbf{A}(f) = \sum_{k=0}^{K-1} \sigma_k(f)\mathbf{u}_k(f)\mathbf{v}_k^{\dagger}(f) \tag{1.14}$$

where $\sigma_k(f)$ is the kth *singular value* of matrix $\mathbf{A}(f)$, $\mathbf{u}_k(f)$ is the associated *left singular vector*, and $\mathbf{v}_k(f)$ is the associated *right singular vector*; the superscript $\dagger$ denotes Hermitian transposition. In analogy with principal components analysis, the decomposition of (1.14) may be viewed as one of *principal modulations* produced by the external RF stimuli. According to (1.14), the singular value $\sigma_k(f)$ scales the kth principal modulation of matrix $\mathbf{A}(f)$.

Forming the K-by-K matrix product $\mathbf{A}^{\dagger}(f)\mathbf{A}(f)$, we find that the entries on the main diagonal of this product, except for a scaling factor, represent the eigenspectrum due to each of the Slepian tapers, spatially averaged over the M sensors. Let the singular values of matrix $\mathbf{A}(f)$ be ordered $|\sigma_0(f)| \geq |\sigma_1(f)| \geq \ldots \geq |\sigma_{K-1}(f)| > 0$. The kth eigenvalue of $\mathbf{A}^{\dagger}(f)\mathbf{A}(f)$ is $|\sigma_k(f)|^2$. We may then make the following statements:

1. The eigenvalues are proportional to average power, expressed as a function of frequency f. In particular, the largest eigenvalue $|\sigma_0(f)|^2$, measured across the frequency band of interest, provides an estimate of the interference temperature in that band, except for a constant. This estimate would be improved by using a linear combination of the largest two or three eigenvalues: $|\sigma_k(f)|^2$, $k = 0, 1, 2$.
2. The left singular vectors $\mathbf{u}_k(f)$ for $k = 0, 1, \ldots, K-1$, provide information on the spatial distribution of the interferers. Most importantly, this information could be used for *wavenumber spectrum estimation* or *adaptive beamforming*; here, it is assumed that the number of sensors (i.e., spatial degrees of freedom) is large enough.
3. The right singular vectors $\mathbf{v}_k(f)$ for $k = 0, 1, \ldots, K-1$, provide the multitaper coefficients for the interferers' waveforms.

1.4.7 Summarizing Remarks II

In space–time processing, the spatial and temporal dimensions are distinct. The RF data therefore represent a multivariate time series, whose spectral structure is summed up in the matrix $\mathbf{A}(f)$ of (1.13). Accordingly, we can make the following statements:

1. The two-dimensional tapers of the time–space processor are the *tensor products* of the standard one-dimensional Slepian tapers.

2. The time–space processor is *coherent* and therefore richer in the extent of information it extracts from the RF environment. Specifically, it is capable of providing *joint* estimates of the interference temperature across a frequency band of interest and the angles-of-arrival of the interfering RF signals emitted by other users.
3. However, this rich source of information on the RF environment is obtained at the expense of a significant increase in computational complexity.

1.4.8 Spectral Classification

In passively sensing the radio scene and thereby estimating the power spectra of incoming RF stimuli, we have a basis for classifying the spectra into three broadly defined types, as summarized here:

1. *Black spaces*, which are occupied by high-power "local" interferers some of the time.
2. *Gray spaces*, which are partially occupied by low-power interferers.
3. *White spaces*, which are free of RF interferers except for *ambient noise*, made up of natural and artificial forms of noise:
   - Broadband thermal noise produced by external physical phenomena such as solar radiation
   - Transient reflections from lightening, plasma (fluorescent) lights and aircraft
   - impulsive noise produced by ignitions, commutators and microwave appliances
   - thermal noise due to internal spontaneous fluctuations of electrons at the front end of individual receivers

White spaces (for sure) and gray spaces (to a lesser extent) are potential candidates for use by unserviced operators. Of course, black spaces are to be avoided whenever and wherever the RF emitters residing in them are switched ON. However, when at a particular geographic location those emitters are switched OFF and the black spaces assume the new role of "spectrum holes," cognitive radio provides the opportunity for creating significant "white spaces" by invoking its dynamic-coordination capability for spectrum sharing.

From the picture of the radio scene presented in this section, it is apparent that a *reliable strategy for the detection of spectrum holes* is of paramount importance to the design and practical implementation of cognitive radio systems. Moreover, the multitaper method combined with singular-value decomposition, hereafter referred to as the *MTM-SVD method*,[11] provides the method of choice for solving this detection problem by virtue of its accuracy and near-optimality.

[11] Mann and Park [26] discuss the application of the MTM-SVD method to the detection of oscillatory spatial-temporal signals in climate studies. They show that this new methodology avoids the weaknesses of traditional signal detection techniques. In particular, the methodology permits a faithful reconstruction of spatio-temporal patterns of narrowband signals in the presence of additive spatially correlated noise.

By repeated application of the MTM-SVD method to the RF stimuli at a particular geographic location and from one section of data to the next, a time–frequency distribution of that location is computed. The dimension of time is quantized into discrete intervals separated by the section duration. The dimension of frequency is also quantized into discrete intervals separated by resolution bandwidth of the multitaper spectral estimation procedure.

Let L denote the number of largest eigenvalues considered to play important roles in estimating the interference temperature, with $|\sigma_l(f,t)|^2$ denoting the lth largest eigenvalue produced by the section (block) of RF stimuli received at time t. Let N denote the number of frequency resolutions of width $\Delta f = 2W$, which occupy the frequency subband (space) under scrutiny. Then, setting the discrete frequency

$$f = f_{\text{low}} + v \cdot \Delta f, \quad v = 0, 1, \ldots, N-1$$

where f_{low} denotes the lowest end of a black, gray or white space, we may define the *decision statistic* for classifying the subbands as

$$D(t) = \sum_{l=0}^{L-1} \sum_{v=0}^{N-1} |\sigma_l(f_{\text{low}} + v \cdot \Delta f, t)|^2 \Delta f. \tag{1.15}$$

Let $D_{\min}$ denote the *minimum possible value* that could be assumed by the decision statistic $D(t)$ due to the ambient noise floor, and let $D_{\max}$ denote its *maximum permissible value* corresponding to the prescribed temperature limit. Let D_{av} denote the average value of $D(t)$, computed over a number of successive sections of the incoming RF signal. We may then classify the frequency subband (space) under scrutiny as follows:

- If $D_{\max} - \delta_1 \leq D_{\text{av}} \leq D_{\max}$, then the subband is said to be a black space.
- If $D_{\min} \leq D_{\text{av}} \leq D_{\min} + \delta_2$, then the subband is said to be a white space.
- Otherwise, the subband is declared to be a gray space.

The parameters δ_1 and δ_2 are chosen by the system designer, depending on how fine a spectral classification is described. Moreover, the specifications of $D_{\max}$ and $D_{\min}$ are *location-specific*. For example, if the spectral classification is performed in the basement of a building, then the spacing between $D_{\max}$ and $D_{\min}$ is expected to be significantly smaller than in an open environment.

1.4.9 Spatio-temporal Evolution of Spectrum Holes

From a cognitive radio user's viewpoint, the following pieces of information are needed:

1. The location of spectrum holes
2. The variance of the interference plus noise in each spectrum hole
3. The duration for which the spectrum hole is likely to be available for use

The MTM-SVD method addresses points (1) and (2). To address point (3), we need a predictive model of the evolution of spectrum holes over time, as discussed next.

The existence of spectrum holes is directly related to the primary user's traffic patterns, which can be of a deterministic or stochastic kind:

- *Deterministic traffic patterns* are attributed to television and AM/FM radio stations and/or air-traffic control radar and weather radar installations. The traffic patterns produced by these primary users are known on a daily basis, which makes their predictability a straightforward matter.
- *Stochastic traffic patterns* arise from wireless communication devices.

The availability of traffic patterns at different times of the day and different geographic locations, desirably provided by legacy users and/or government agencies, could form the basis of a *spatio-temporal prediction model* of traffic behavior. Such a model could make it possible to predict the duration of time for which spectrum holes are likely to be employable, thereby enhancing coexistence between legacy users and secondary users.

1.5 Extraction of Channel-State Information (CSI)

Section 1.4 on radio-scene analysis dealt with issues pertaining to spectral information on the radio environment, which is needed by the transmitter for efficient utilization of the radio spectrum. In this section, we deal with another function of the receiver, namely, the *extraction of channel-state information* (CSI), which is needed by a user's receiver for coherent detection of the transmitted information-bearing signal. This section is organized as follows:

- First, we set the stage for semi-blind training, which offers a compromise between two extreme approaches: differential detection for unsupervised transmission and pilot-assisted transmission for supervised training.
- Next, we describe a channel-tracking procedure that is basic to the semi-supervised training procedure.

1.5.1 Semi-supervised Training

To deal with the channel-state estimation problem, traditionally we have proceeded in one of two ways:

- *Differential detection*, which lends itself to implementation in a straightforward fashion, using M-ary phase modulation.
- *Pilot-assisted transmission*, which involves the periodic transmission of a pilot (training sequence) known to the receiver.

The use of differential detection offers robustness and simplicity of implementation, but at the expense of a significant degradation in the frame-error rate (FER) versus signal-to-noise ratio (SNR) performance of the receiver. On the other hand, pilot-assisted transmission (PAT) offers improved receiver performance, but the use of a pilot is wasteful in both transmit power and channel bandwidth, the very thing we should strive to avoid. What then do we do, if the receiver requires knowledge of CSI for efficient receiver performance? The answer to this fundamental question lies in the use of *semi-blind training* of the receiver, which distinguishes itself from the differential detection and PAT procedures in that the receiver has two modes of operations:

1. *Supervised training mode*. During this mode, the receiver acquires an estimate of the channel state, which is performed under the supervision of a short training sequence, consisting of fewer symbols than that required with PAT. As with PAT, the training sequence is known to the receiver. It is sent over the channel for a limited duration by the transmitter prior to the actual data-transmission session; the pilot transmission is repeated periodically.
2. *Tracking mode*. Once a reliable estimate of the channel state has been achieved, the training sequence is switched off, actual data transmission is initiated and the receiver is switched to the tracking mode; this mode of operation is performed in an unsupervised manner on a continuous basis during the course of data transmission.

1.5.2 Channel Tracking

The evolution of CSI with time is governed by a *state-space model* comprised of two equations:

1. *Process equation*. The state of a wireless link is defined as the *minimal set of data on the past behavior of the link that is needed to predict the future behavior of the link*. For the sake of generality, we consider a *multiple-input, multiple-output (MIMO) wireless link*[12] of a narrowband category. Let $h_{jk,t}$ denote the

[12] The use of a MIMO link offers several important advantages:

1. *Spatial degree of freedom*, defined by $N = \min\{N, L\}$, where N and L denote the numbers of transmit and receive antennas, respectively [28].
2. *Increased spectral efficiency*, which is asymptotically defined by

$$\lim_{N\to\infty} \frac{C(N)}{N} = \text{constant}$$

where $C(N)$ is the ergodic capacity of the link, expressed as a function of $L = N$. This asymptotic property provides the basis for a spectacular increase in spectral efficiency by increasing the number of transmit and receive antennas.
3. *Diversity*, which is asymptotically defined by

$$\lim_{\rho\to\infty} \frac{\log \mathrm{FER}(\rho)}{\log \rho} = -d_o$$

where d_o is the diversity order and $\mathrm{FER}(\rho)$ is the frame-error rate expressed as a function of the signal-to-noise ratio ρ.

These benefits (gained at the expense of increased complexity) commend the use of MIMO links for cognitive radio, all the more so considering the fact that the primary motivation channel coefficient from the kth transmit antenna to the jth receive antenna at time t, with $k = 1, 2, \ldots, N$ and $j = 1, 2, \ldots, L$. We may then describe the scalar form of the state equation as

$$h_{jk,t+1} = \sum_{l=0}^{M} (\beta_{l,t})\,(h_{jk,t-1}) + v_{jk,t} \tag{1.16}$$

where the $\beta_{l,t}$ are *time-varying autoregressive (AR) coefficients* and $v_{jk,t}$ is the corresponding *dynamic noise*, both at time t. The AR coefficients account for the *memory* of the channel due to the multipath phenomenon. The upper limit of summation in (1.16) namely, M, is the *model order*.

2. *Measurement equation.* The measurement equation for the MIMO wireless link, also in scalar form, is described by

$$y_{j,t} = \sum_{k=1}^{N} (s_{k,t})\,(h_{jk,t}) + n_{j,t} \quad \text{for} \quad j = 1, 2, \ldots, L \tag{1.17}$$

where $s_{k,t}$ is the *encoded symbol* transmitted by the kth antenna at time t, and $n_{j,t}$ is the corresponding *measurement noise* at the input of jth receive antenna at time t. The $y_{j,t}$ is the *signal observed* at the output of the jth antenna at time t.

The state-space model comprised of (1.16) and (1.17) is *linear*. The property of linearity is justified in light of the fact that the propagation of electromagnetic waves across a wireless link is governed by Maxwell's equations that are inherently linear.

What can we say about the AR coefficients, the dynamic noise, and measurement noise, which collectively characterize the state-space model of (1.16) and (1.17)? The answers to these questions determine the choice of an appropriate tracking strategy. In particular, we say the following:

1. *AR model*. A *Markov model* of order one offers simplicity and sufficient accuracy to model a Rayleigh-distributed time-varying channel.
2. *Noise processes*. The dynamic noise in the process equation is Gaussian, but the noise in the measurement equation is likely to be non-Gaussian due to the presence of impulsive noise generated in the radio environment. (The impulsive noise is attributed to different sources such as automobile engine noise in an outdoor environment and microwave devices in an indoor environment.)

Point (1) directly affects the design of the *predictive model*, which is an essential component of the channel tracker. Point (2) prompts the search for a tracker outside of the classical Kalman filters, whose theory is rooted in Gaussian statistics.

Two different channel-tracking procedures are described in [29] and [30]; herein, we briefly highlight the procedure described in [30].

Rewriting (1.16) in matrix form, under the assumption of an AR model of order one, we have

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for cognitive radio is the attainment of improved spectral efficiency. Simply put, a MIMO wireless link is not a necessary ingredient for cognitive radio but a highly desirable one.

$$\mathbf{h}_{t+1} = \beta_{0,t}\mathbf{h}_t + \mathbf{v}_t \tag{1.18}$$

where $\mathbf{h}_t$ denotes the vector representation of the channel matrix $\mathbf{H}_t$ by stretching the columns of $\mathbf{H}_t$ one over another, and $\mathbf{v}_t$ denotes the corresponding vector representation of the dynamic noise. The key objective of the channel tracker is to estimate the update equation for posterior probability density function of the sequence

$$\mathbf{h}_{1:t} = \{\mathbf{h}_i\}_{i=1}^{t}$$

when we are given the entire set of measurements

$$\mathbf{y}_{1:t} = \{\mathbf{y}_i\}_{i=1}^{t}.$$

That is, the posterior density of the channel state is updated in accordance with the equation

$$p\left(\mathbf{h}_{1:t}|\mathbf{y}_{1:t}\right) = \frac{p(\mathbf{y}_t|\mathbf{h}_t)p(\mathbf{h}_t|\mathbf{h}_{t-1})}{p(\mathbf{y}_t|\mathbf{y}_{1:t-1})}p(\mathbf{h}_{1:t-1}|\mathbf{y}_{1:t-1}). \tag{1.19}$$

Reference [30] describes a novel procedure for computing this update equation, using a particle filter. The main idea of the procedure is to introduce a correction factor in the predicted estimate of the channel state, with the correction being based on an approximate *maximum-likelihood* (ML) estimate of the channel state. Specifically, the corrected channel estimate is defined by the convex combination of the old value of the channel state and the current maximum-likelihood estimate, as shown by

$$\mathbf{h}_{t|t-1}^{\mathcal{C}} = (1-\alpha)\mathbf{h}_{t|t-1} + \alpha\mathbf{h}_t^{\mathrm{ML}} \tag{1.20}$$

where α, lying in the range between zero and one, is a weight given to the confidence in the ML estimate; the value assigned to this weight depends on the signal-to-noise ratio (SNR) and the fading rate of the wireless environment. For example, at low SNR, we are less confident in the current estimation of the channel state and therefore a small value is assigned to α. Likewise, for a highly time-selective channel, we have less confidence in the ML estimate, in which case we also assign less weight to α. Choosing the "optimal" α is problem-specific and may therefore require the inclusion of an adaptive loop in the estimation procedure for online operation.

The motivation behind the convex-combined predictive channel estimate is to "guide" the particles in the tracking filter toward a high probability region of the density; as such, it may be viewed as a more refined approach than that taken in [29]. As the combined step in the state update incorporates recent measurements, the state space is efficiently exploited so as to improve the sampling efficiency. Indeed, in [30], Monte Carlo simulation results are presented for a radio environment assuming:

- The use of a frequency-flat time-selective channel based on the *Jakes model*
- The use of a *Middleton Class-A model* for an impulsive measurement noise

The simulations compare the performance of a wireless system using two channel trackers, one incorporating the approximate ML channel estimate in the particle filter to select *informative particles* as described herein and the other incorporating *gradient information* in the selection of the particles as described in [29]. The results of the simulation presented in [30] reveal that unlike the channel tracker based on gradient particle filtering, the asymptotic *performance gap* between the genie scenario (assuming that the channel state is known) and the corresponding scenario involving the use of the new channel tracker is essentially uniform for increasing SNR, which is desirably how it should be.

1.6 Feedback Channel

As pointed out previously, the primary motivation of cognitive radio is improved utilization of the radio spectrum, hence the requirement for identification of spectrum holes in the local neighborhood of a user's receiver. Having performed this function by the radio-scene analyzer in the receiver, we need a feedback channel to send relevant information of the receiver to the user's transmitter for appropriate action by that user. This information consists of two constituents:

- The center frequencies and bandwidths of the spectrum holes
- The combined variance of interference and thermal noise in each spectrum hole

Later on, we will also find that there is an additional role for the feedback channel:

- A measure of the signal-to-noise ratio at the output of the transmitter–receiver wireless link, which is needed by the adaptive modulator in the transmitter.

Rather than send the actual values of the various parameters identified here, the practical approach is to feed their respective *quantized* values back to the transmitter. To do this, a predetermined list of quantized values pertaining to the following parameters is kept in the receiver:

- Center frequencies and bandwidths of all possible spectrum holes
- Variance of interference plus noise in each possible spectrum hole
- Output signal-to-noise of the pertinent wireless link

Given such a list, the receiver picks the closest entries in the list that are less than the actual values of the parameters. In so doing, the bit rate of the feedback channel is minimized.

Putting it altogether, the feedback channel plays a fundamental role in the design and operation of cognitive radio. Indeed, we may go on to say that feedback is the *facilitator of intelligence*, without which the radio loses its cognitive capability.

1.7 Multiuser Cognitive Radio Networks

As it is with every other communication network, the deployment of a *cognitive radio network* can be justified in financial terms if, and only if, the network is utilized by a multiple users.

Mobile wireless communication networks are *centralized*, in that an infrastructure of base stations is deployed to route calls from one user to another. In contrast, for both civilian and military applications, it is desirable for cognitive radio networks to be *decentralized*. In other words, the network is configured in a *self-organized* manner [31,32], which makes it possible to dispense with the need for a costly pre-established infrastructure. With this objective in mind, the adoption of *ad hoc networks* [33,34] is the logical basis for cognitive radio networks.

From what we know about brain theory [35] and neural networks [5], self-organization builds on two basic mechanisms: *cooperation* and *competition*; these two mechanisms operate in a complementary manner so as to "bring order in the network out of disorder." In a similar sort of way, we may envision a self-organized cognitive radio network, in which cooperation and competition are purposely configured to perform complementary functions. Specifically:

- Cooperation is used to facilitate *communication* across the nodes of the network without any fixed infrastructure.
- Competition is used to provide *control* over the power transmitted from each individual node of the network to maintain the interference temperature at a receiving node below a prescribed limit.

In a cognitive radio network built on ad hoc wireless principles, the network is basically an association of nodes that cooperate. Insofar as network coordination is concerned, for example, we may simply require *each pair of neighboring nodes* be in direct communication. Thus, in the communication scenario, each node creates a *transmit–receive schedule*; the schedule is communicated to a nearest neighbor only when a source node's schedule and that of a neighboring node permit the source node to transmit the schedule and the neighboring node is able to receive it. In [36], it is shown that under reasonable assumptions, such a completely decentralized network can scale to an almost arbitrary number of nodes. It is therefore feasible to develop a dynamic frequency selection policy that supports utilization of the network by more users through a built-in cooperative mechanism. The capacity of wireless networks is discussed in [37,38].

Turning next to the benefit that could be gained from competition, it will be shown in Sect. 1.12 that by adopting a non-cooperative (i.e., competitive) game-theoretic approach, it is possible to design an efficient and effective transmit-power control policy. Most important, this policy does not require synchronization among the multiple users, thereby simplifying the design of the network.

1.8 Dynamic Spectrum Management

The primary motivation of cognitive radio is to improve utilization of the radio spectrum, subject to two requirements:

1. Secondary users of the spectrum's unoccupied subbands must coexist with the primary users.
2. Interference temperature at the receiver input of each user in the network does not exceed a prescribed limit.

Requirement (2) is considered later in Sect. 1.12. In this section, we address requirement (1).

We first note that by having the network operate in a decentralized cooperative manner, information-bearing signals could hop from one node of the network to a neighboring node, thereby facilitating communication across the entire network. Moreover, the spectrum holes come and go. Accordingly, we may formulate the dynamic spectrum management problem as follows:

> Given a set of spectrum holes detected by the radio-scene analyzer and whose composition is likely to change from one time instant to another, devise a decentralized dynamic spectrum management policy that enables secondary users to employ these spectrum holes without disruption to the primary users.

For the policy to be decentralized, we need a *random (probabilistic) multiple-access technique*. Here we have the choice between two protocols: *Aloha* and *carrier-sense multiple-access* (CSMA). For terrestrial networks, CSMA is the preferred choice [39].

In its simplest form, CSMA operates as follows:

1. If the wireless channel is sensed to be *idle* (i.e., a spectrum hole is available), the user transmits its packets.
2. If the channel is sensed to be *busy* (i.e., the spectrum hole has become occupied), the transmission of packets is scheduled for a later time according to a specified random distribution.
3. At the new point in time, the user senses the channel and repeats the algorithm.

If the transmissions were instantaneous, then collisions would occur in the CSMA protocol only if two users transmitted at exactly the same time; this should be a rare occurrence but nevertheless, it could happen.

In a modified form of CSMA called *carrier-sense multiple-access with collision avoidance* (CSMA/CA) each node of the network must inform other nodes in the network of the intent to transmit packets, and it is only then that transmission can take place. In so doing, packet collisions are prevented, because all nodes in the network have been made aware of packet transmission before it occurs. Such a protocol is indeed feasible by virtue of the cooperative communication built across the network.

1.8.1 Modulation Format

The next issue to be considered is the choice of a modulation format for the actual transmission of packets over the selected spectrum hole. For this purpose, we consider *orthogonal frequency-division multiplexing* as a method of choice [40,41]. We say so for the following reasons:

- OFDM is a bandwidth-efficient signaling scheme, which converts a difficult frequency-selective channel into a parallel collection of frequency-flat subchannels, whose subcarrier frequencies form an orthogonal set.

- Unlike ordinary frequency-division multiplexing (FDM), the spectra of the individually modulated subcarriers in OFDM overlap mutually and thereby optimally occupy the frequency response of the channel.
- The choice of OFDM fits perfectly into the design of the transmit-power controller.

Orthogonality of the subcarriers over the duration of a symbol is achieved by having the frequency spacing between them equal to the reciprocal of the symbol duration.

There are some other practical requirements that need to be satisfied:

1. The modulation format must be *impervious* to that of the primary user, so as not to violate the coexistence requirement.
2. The modulation format must be *adaptive*, so as to account for the time-varying conditions of the radio environment.
3. Due to channel noise and interference from other users, *reliable* communication must be maintained between the wireless link that connects the transmitting node to the receiving node.

To satisfy these requirements, we may use the *concatenated coding scheme* depicted in Fig. 1.3, where the concatenation of channel encoder and space–time encoder is performed on a symbol-by-symbol basis [40].

To explain, the data bits produced by the OFDM are first channel-coded by a *turbo convolutional encoder* [42], which is followed by a pseudo-random block interleaver. Next, the adaptive *quadrature amplitude modulator* (QAM) selects a mode of modulation from the set (for example):

- Binary phase-shift keying (BPSK)
- Quadrature phase-shift keying (QPSK)
- 16-QAM
- 64-QAM

This selection is made by the adaptive modulator in response to the quality of signal reception measured at the receiving node; in effect, *feedback* is needed between each pair of neighboring nodes in the network for adaptive modulation to be feasible.

Finally, the modulated symbol is space–time block encoded [43], with the encoding being performed in the frequency domain. Here, it is assumed that a set of

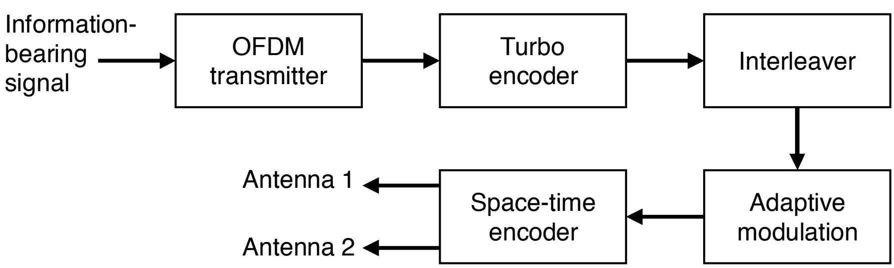

Fig. 1.3. Block diagram of adaptive OFDM transmitter.

adjacent subcarriers in the OFDM signal, belonging to the same space–time encoding block, have approximately the same signal-to-noise ratio. The space–time code, involving the use of multiple transmit as well as receive antennas, provides *diversity* to combat multipath; reliability of communication across the network is thereby further enhanced.

1.9 Statistical Modeling of Cognitive Radio Networks

To set the stage for formulating the transmit-power control problem considered in the next section, we need a *statistical model* for cognitive radio networks. In what follows, we assume that the wireless channel linking one node in the network to a neighboring node is *frequency-selective*. As pointed out in the previous section, OFDM is well suited for dealing with such a channel, the use of which converts a frequency-selective channel into a set of frequency-flat subchannels whose individual subcarriers are ideally orthogonal to each other. In practice, however, we find that OFDM is sensitive to frequency offset in the channel, which arises because the subcarriers are inherently closely spaced in frequency, compared to channel bandwidth; consequently, the tolerable frequency offset is a small fraction of the channel bandwidth.[13]

Let f_0 denote the frequency spacing between adjacent subcarriers, and Δf denote the frequency offset. The baseband version of the OFDM signal radiated by the transmitter of a user labeled m is thus defined by

$$s(m,n) = c_n \exp[\mathrm{j}2\pi n(f_0 - \Delta f)] \tag{1.21}$$

where n denotes one of the N subcarriers in the OFDM signal, and c_n is the modulated amplitude of the nth subcarrier. To simplify the notation, we have omitted the dependence on time t in $s(m,n)$. Correspondingly, the signal picked up by the intended receiver of user m is given by[14]

$$y(m,n) = \sum_{k=1}^{M} g(m,k,n)s(k,n) + w(m,n) \tag{1.22}$$

where M is the total number of users. The term $g(m,k,n)$ is the combined effect of two factors:

- Propagation path loss from the transmitter of user k to the receiver of user m at subcarrier n; this loss also includes the effects of lognormal and Rayleigh fading phenomena.
- Subcarrier amplitude reduction due to the frequency offset Δf.

[13] Moose [44] describes an algorithm, based on maximum-likelihood estimation, for frequency offset correction.

[14] In the notation used herein, user m refers to the combination of a transmitter at one end of a wireless link and its intended receiver at the other end of the link.

The $w(m,n)$ denotes the zero-mean Gaussian thermal noise at the receiver input of user m on the nth subcarrier.

Next, we isolate the contribution due to $k = m$ in the summation term in (1.22) and rewrite that equation in the desired form:

$$y(m,n) = g(m,m,n)s(m,n) + \sum_{\substack{k=1 \\ k \neq m}}^{M} g(m,k,n)s(k,n) + w(m,n). \tag{1.23}$$

The first term on the right-hand side of (1.23) is due to user m acting alone. The second term is the *total interference* produced at the receiver of user m due to the signals transmitted by all the other users: $1, 2, \ldots, m-1, m+1, \ldots, M$; this interference is attributed to the frequency offset Δf as well as other imperfections in the network.

Let $P(m,n)$ denote the average power transmitted by user m on the nth subcarrier n, and $\sigma_w^2(m,n)$ denote the variance of zero-mean thermal noise $w(m,n)$. We may then express the *signal-to-interference plus noise ratio* (SINR) at the receiver input of user m on the nth subcarrier as

$$\begin{aligned} \text{SINR}(m,n) &= \frac{|g(m,m,n)|^2 P(m,n)}{\sum_{\substack{k=1 \\ k \neq m}}^{M} |g(m,k,n)|^2 |s(k,n)|^2 + \sigma_w^2(m,n)} \\ &= \frac{P(m,n)}{\sum_{\substack{k=1 \\ k \neq m}}^{M} \alpha(m,k,n)P(k,n) + v(m,n)} \end{aligned} \tag{1.24}$$

where, in the last line, the denominator is *normalized* with respect to the factor $|g(m,m,n)|^2$ that pertains completely to user m. Specifically, we have:

$$P(k,n) = |s(k,n)|^2 \tag{1.25}$$

$$\alpha(m,k,n) = \frac{|g(m,k,n)|^2}{|g(m,m,n)|^2} \tag{1.26}$$

and

$$v(m,n) = \frac{\sigma_w^2(m,n)}{|g(m,m,n)|^2}. \tag{1.27}$$

The numerator of (1.24) represents power transmission and reception by user m over a *direct lossless path*. The denominator of this equation represents the normalized value of the total interference plus noise measured at the receiver input of user m.

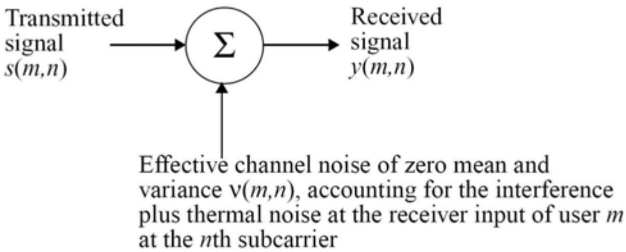

Fig. 1.4. Depiction of the equivalent additive noise model for user m operating on subcarrier n in the OFDM format.

Examination of this equation also leads us to make another important observation. Insofar as user m of the cognitive radio network is concerned, we may view the wireless channel connecting its receiver to the transmitter as the *equivalent of a single-user additive-noise channel*, as depicted in Fig. 1.4, where the noise variance refers to the variance of total interference plus thermal noise (i.e., the denominator of (1.24)). For analytic purposes, it is assumed that the channel noise in this figure is zero-mean Gaussian. It would be tempting to justify this assumption by recognizing the large number of users responsible for the overall interference, and therefore invoking the central limit theorem. Typically, however, we find that a few of the interferers are dominant and a large number of them are weak. Hence, in reality, the additive noise in the model of Fig. 1.4 may not be strictly Gaussian.

1.10 Formulation of the Transmit-Power Control Problem

Under the assumptions made in Sect. 1.9, we may now invoke *Shannon's* celebrated *information capacity theorem* for an additive Gaussian noise channel [45] to express the maximum achievable rate of data transmission over the wireless channel connecting the transmitter of user m to its receiver as follows:

$$R(m,n) = \log_2[1 + \text{SINR}(m,n)] \quad \text{bits per use of subchannel } n \tag{1.28}$$

where $\text{SINR}(m,n)$ is the signal-to-interference plus noise ratio defined in (1.24). The multiuser coding scheme needed to achieve the data-transmission rate $R(m,n)$ is implementable, since the only item that needs to be measured is the variance of the interference plus noise at the receiver input of user m for each n. In other words, from a practical perspective, no user in the cognitive radio network would need to identify the sources of interference or noise affecting its operation; rather, it is sufficient for the user to merely measure the variance of the overall interference plus thermal noise at its receiver input for each subcarrier frequency n. This measurement is the function of the radio-scene analyzer to undertake.

Consider then a non-cooperative multiuser cognitive radio network using OFDM for data transmission among its M users. The *transmit-power control problem* for this network may now be stated as follows:

Given:

1. a set of spectrum holes known to be adequate to support the data-transmission needs of M secondary users, and
2. measurements of the variance of interference plus noise at the receiver input at each of the N subcarriers of the OFDM for every user,

determine the transmit-power levels of the M secondary users so as to jointly maximize their data-transmission rates, subject to the constraint that the interference-temperature limits in the subfrequency bands occupied by the spectrum holes are not violated.

It may be tempting to suggest that the solution of this problem is attained by simply increasing the transmit-power level of each secondary user. However, increasing the transmit-power level of any one user has the undesirable effect of also increasing the levels of interference to which the receivers of all the other users are subjected. The conclusion to be drawn from this reality is that it does not make practical sense to represent the overall performance of the cognitive radio network by means of a single index of performance. Rather, we have to adopt a *tradeoff* among the data rates of all secondary users in some computationally tractable fashion.

Ideally, we would like to find a global solution to the constrained optimization of the joint set of data-transmission rates under study. Unfortunately, finding this *global* solution requires an exhaustive search through the space of possible power allocations for all M users, which is impractical for two reasons:

- The computational complexity needed to attain the global solution may assume a prohibitively high level.
- The time needed to find the solution could become unacceptably long.

To mitigate these practical difficulties, we relax the statement for global optimality by adopting *competitive optimality* as the criterion for solving the transmit-power control problem. Specifically, we now state:

> Given a multiuser non-cooperative cognitive radio network using OFDM as described above, optimize the performance of secondary user m, regardless of what all the other secondary users do, subject to the constraint that the interference-temperature limit at the receiver input of user m is not violated.

This formulation of the distributed transmit-power control problem leads to a solution that is of a local nature. Although, of course, the solution is suboptimum, it is not only insightful but also practically feasible. Most important, the *local* optimization envisioned here is a basic ingredient of self-organization.

To set the stage for presenting an iterative procedure (based on competitive optimality) for solving the transmit-power control problem, we find it informative to digress briefly to first think in terms of game theory.

1.11 The Multiuser Non-cooperative Cognitive Radio Network Viewed as a Game-Theoretic Problem

Game theory[15] is a well-established discipline; it deals with the mathematical modeling of practical situations, which involve the following ingredients:

- *Multiple players* who, by virtue of their responsibilities as decision-makers, are required to take specific *actions*.
- The actions may lead to consequences, which could be of mutual conflict to the players themselves.

The formulation of a mathematical framework for a non-cooperative game rests on three basic realities:

- *State space* that is the product of the individual players' states
- *State transitions* that are functions of *joint actions* taken by the players
- *Payoffs* to individual players that depend on joint actions as well

This framework is found in stochastic games [46], which, also occasionally appear under the name "Markov games" in the computer science literature.

A stochastic game is described by the five tuple $\{\mathcal{N}, \mathcal{S}, \overrightarrow{\mathcal{A}}, \mathcal{P}, \overrightarrow{\mathcal{R}}\}$, where

- $\mathcal{N}$ is a set of players, indexed $1, 2, \ldots, M$.
- $\mathcal{S}$ is a set of possible states.
- $\overrightarrow{\mathcal{A}}$ is the *joint-action space* defined by the product set $\overrightarrow{\mathcal{A}}_1 \times \overrightarrow{\mathcal{A}}_2 \times \ldots \times \overrightarrow{\mathcal{A}}_M$, where $\overrightarrow{\mathcal{A}}_m$ is the set of actions available to the mth player.
- $\mathcal{P}$ is a probabilistic transition function, an element of which for joint action a satisfies the condition

$$\sum_{s \in \mathcal{S}} \mathcal{P}_{ss'}^{a} = 1 \quad \text{for all } s' \in \mathcal{S} \text{ and } a \in \overrightarrow{\mathcal{A}}. \tag{1.29}$$

- $\overrightarrow{\mathcal{R}} = r_1 \times r_2 \times \ldots \times r_M$, where r_m is the payoff for the mth player and which is a function of the joint actions of all M players.

One other notational issue: the action of player $m \in \mathcal{M}$ is denoted by a_m, while the joint actions of the other $M-1$ players in the set $\mathcal{M}$ are denoted by a_{-m}.

Stochastic games are the *supersets* of two kinds of decision processes, namely, *Markov decision process* and matrix games, as illustrated in Fig. 1.5. A *Markov decision process* is a special case of a stochastic game with a single player, that is, $M = 1$. On the other hand, a *matrix game* is a special case of a stochastic game with a single state, that is, $|\mathcal{S}| = 1$.

[15] In a historical context, the formulation of game theory may be traced back to the pioneering work of John von Neumann in the 1930s, which culminated in the publication of the co-authored book entitled "Theory of Games and Economic Behavior" [47]. For modern treatments of game theory, see the books under [46,48]. Game theory is widely used in the study of economics [49]; it has also been applied in other areas such as machine learning [50] and neuroscience [51]. For the use of game theory in cognitive radio, see [52].

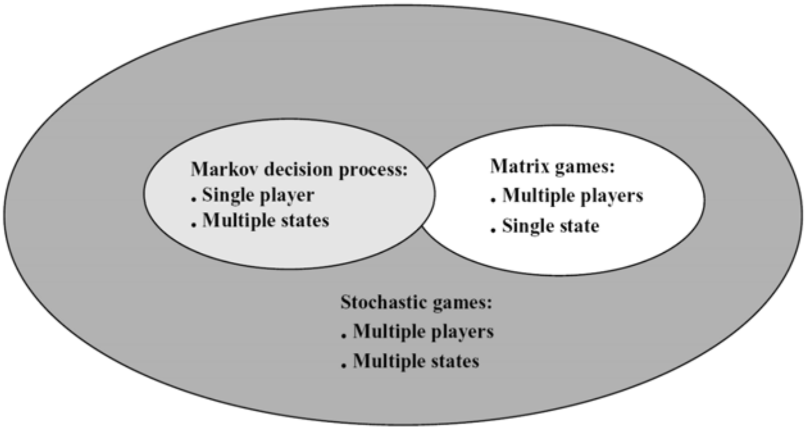

Fig. 1.5. Highlighting the differences between Markov decision processes, matrix games and stochastic games.

1.11.1 Nash Equilibrium

In [53,54], John Nash focused his study of game theory on a class of games described as *non-cooperative, simultaneous-move, one-shot, finite games with complete information*, where

- "Simultaneous-move" means that each player picks a strategy without knowledge of the other players' strategies
- "One-shot" implies that the game is played once and once only
- "Finite game" refers to the fact the game involves a finite number of players, with each player taking only a finite number of possible actions

In the context of this background, Nash introduced the concept of an equilibrium of a game, which is defined as follows:

> A *Nash equilibrium* is defined as an action profile (i.e., vector of players' actions) in which each action is a *best response* to the actions of all the other players [53].

Consider, for example, a *multiple-access game* [55] involving two transmitters (i.e., players) p_1 and p_2 who respectively want to send data packets to their receivers r_1 and r_2 over a shared channel. In each time slot, each player can decide to transmit a packet or to remain quiet (i.e., not to transmit); these two actions are denoted by T and Q, respectively. Let c denote the *cost* incurred in the transmission of a packet, where $0 < c < 1$. With the channel being shared, transmissions by both players result in a collision, in which case, packets are lost. Thus, in strategic terms, the multiple-access is represented by Fig. 1.6. From this figure, it is apparent that the optimal solution to the multiple-access game is as follows:

p_1/p_2	Q	T
Q	(0,0)	(0, 1-c)
0	(1-c, 0)	(c, c)

Fig. 1.6. Tabular representation of the multiple-access game.

- If player p_1 decides to transmit, then the best response for player p_2 is to remain quiet.
- Conversely, if player p_2 decides to transmit, the best response for player p_1 is to remain quiet.

From this example, we see that Nash equilibrium is a *stable operating (*i.e., *equilibrium) point* in the sense that there is no incentive for any player involved in a finite game to change strategy, given that all the other players continue to follow the equilibrium policy. The important point to note here is that the Nash-equilibrium approach provides a powerful tool for modeling non-stationary processes. Simply put, it has had an enormous influence on the evolution of game theory by shifting its emphasis toward the study of equilibria as a *predictive concept*.

The Nash equilibrium features prominently in the study of game theory; indeed, it earned John Nash the Nobel Prize in Economics in 1994. This concept works perfectly well provided two assumptions are satisfied:

1. The players engaged in a game are all *rational*.
2. The underlying structure of the game is *common knowledge* to all the players.

Under these two assumptions, the Nash equilibrium offers an intuitively satisfying approach that predicts the equilibrium outcome of the game as follows. Any player, being "rational," will play a "best-response" strategy. Moreover, under the "common knowledge" assumption, this strategy is known to all the other players and, being rational, they will therefore play their own "best-response" strategies, which therefore leads the game to a Nash equilibrium.

1.12 Iterative Waterfilling Algorithm

Now that we understand the importance of the Nash equilibrium in the study of game theory, we can proceed with the solution to the transmit-power control problem in a non-cooperative multiuser cognitive radio network using OFDM. We begin with the statement:

> When users of such a network operate under the common knowledge that each user will follow the criterion of competitive optimality for maximizing its own data-transmission rate, subject to an interference-temperature constraint, the strategy so adopted will lead to a Nash equilibrium.

In information-theoretic terms, maximization of the data-transmission rate of each user in accordance with (1.28) over each subcarrier frequency of the user is similar to the idea of *waterfilling*. In the classical description of waterfilling [45], water is poured over the inverse of noise variance at each subcarrier frequency. In our situation, on the other hand, water is poured over the inverse of the combined interference plus noise. We may therefore modify the above statement as follows:

> If the users of a non-cooperative radio network perform "waterfilling" with respect to the combined variance of interference plus noise at each subcarrier frequency of the OFDM, subject to an interference-temperature constraint, then the network will reach a Nash equilibrium.

Although the multiuser solution produced by this strategy is *suboptimal*, it offers the practical virtue of eliminating the need for synchronization among users of the network insofar as transmit-power control is concerned. The fundamental question is: How do we perform the waterfilling procedure in an efficient manner?

This very question is addressed in the *iterative waterfilling algorithm* for multiuser data transmission systems. The algorithm was originally described in [56,57] in the context of discrete multitones (DMT); it is expanded on in [58]. Much of that theory is also applicable to frequency-selective channels using OFDM, since DMT and OFDM belong to the same family of multichannel transmission systems [13,59].

To simplify the presentation of the iterative waterfilling algorithm for a multiuser cognitive radio network using OFDM, we assume that each iteration of the algorithm starts with user 1 and ends with user M. Each iteration consists of an inner loop followed by an outer loop. In the inner loop of iteration j, say, each user maximizes its data transmission rate, subject to an interference-temperature constraint. In the outer loop of iteration j, the power allocation among the M users is adjusted up or down. The iterative waterfilling computation is terminated after a total of J iterations when a prescribed tolerance ϵ is attained.

Note also that at iteration j, the interference plus noise (IN) at the receiver input of receiver m at subcarrier frequency n has the fixed value [see the denominator of (1.24)]

$$\begin{aligned}\mathrm{IN}^{(j)}(m,n) = &\sum_{k=1}^{m-1} \alpha(m,k,n)P^{(j)}(k,n) \\ &+ \sum_{k=m+1}^{M} \alpha(m,k,n)P^{(j-1)}(k,n) + v^{(j)}(m,n). \qquad (1.30)\end{aligned}$$

The first summation term of (1.30) represents the normalized contributions made by users 1 to $m-1$ processed during the current iteration j, and the second summation term represents the normalized contributions made by users $m+1$ to M processed during the previous iteration $j-1$. With $\mathrm{IN}^{(j)}(m,n)$ fixed, it follows that placing a limit on the total interference temperature at the receiver input of user m is actually equivalent to the imposition of a corresponding limit on the total transmit power of user m.

We are now ready to describe the iterative waterfilling algorithm as follows:

1. *Initialization* $j = 0$
   Unless prior knowledge is available, the power distribution across the users, $m = 1, 2, \ldots, M$, is set equal to zero.
2. *Inner loop: iteration* $j = 1, 2, ...$
   In this iteration, user m maximizes its total data transmission rate through waterfilling, subject to a total power constraint. In mathematical terms, for iteration j we write:

$$\text{Maximize} \quad R^{(j)}(m) = \sum_{n=1}^{N} \log_2 \left(1 + \frac{P^{(j)}(m,n)}{\text{IN}^{(j)}(m,n)}\right)$$

$$\text{subject to the constraint} \sum_{n=1}^{N} P^{(j)}(m,n) \le \bar{P}(m) \tag{1.31}$$

   where the permissible transmitter power $\bar{P}(m)$ is determined as follows: the total power measured at the receiver input of user m, summing the contributions due to its own transmission and ambient noise plus those due to the remaining $M-1$ interferers, is defined by:

$$P_{\text{total}}(m) = \sum_{n=1}^{N} \left(\sum_{k=1}^{M} |g(m,m,n)|^2 P(k,n) + \sigma_w^2(m,n) \right). \tag{1.32}$$

   Given that the interference-temperature limit $T_{\max}$ must not be exceeded by user m, we require:

$$P_{\text{total}}(m) \le \kappa T_{\max} B_m \tag{1.33}$$

   where κ is Boltzmann's constant and B_m is the bandwidth of the spectrum hole being occupied by user m. Using the definition of (1.27) and (1.28) and recognizing that $\alpha(m,m,n) = 1$ for all n, we write:

$$\sum_{n=1}^{N} P(m,n) \le \bar{P}(m) \tag{1.34}$$

   where $\bar{P}(m)$ is defined by:

$$\bar{P}(m) = \frac{\kappa T_{\max} B_m}{|g(m,m,n)|^2} - \sum_{n=1}^{N} \left(\sum_{\substack{k=1 \\ k \ne m}}^{M} \alpha(m,k,n) P(k,n) + v(m,n) \right). \tag{1.35}$$

   Here it is presumed that the spectrum hole being occupied by user m is , at least, partially filled to permit $\bar{P}(m)$ to assume a positive value. Bearing in mind that cognitive radio is receiver-centric, the determination of $\bar{P}(m)$ requires knowledge of two quantities for user m:

(i) *Total interference plus noise* measured at its own receiver input.
(ii) The *path loss* $|g(m, m, n)|^2$ from its transmitter to the receiver.

The measurement of item (i) is performed at the receiver and supplied to the respective transmitter via the feedback channel. The calculation of item (ii) is performed by the transmitter itself, knowing how far away its own receiver is from it.

The above-stated constrained maximization problem is a *convex optimization problem*, coupled across the set of N subcarrier frequencies. It may therefore be solved using dual decomposition [60]. Specifically, we first set up the Lagrangian

$$L^{(j)}(m) = \sum_{n=1}^{N} \log_2 \left(1 + \frac{P^{(j)}(m,n)}{\mathrm{IN}^{(j)}(m,n)}\right) - \lambda^{(j)}(m) \left(\sum_{n=1}^{N} P^{(j)}(m,n) - \bar{P}(m)\right) \tag{1.36}$$

where $\lambda^{(j)}(m)$ is the Lagrangian multiplier for user m at iteration j. Next, invoking the orthogonality property of the OFDM subcarriers, the convex optimization problem is decomposed into N suboptimization problems, as shown by

$$\text{Maximize} \quad \log_2 \left(1 + \frac{P^{(j)}(m,n)}{\mathrm{IN}^{(j)}(m,n)}\right) - \lambda^{(j)}(m) P^{(j)}(m,n) \tag{1.37}$$
$$\text{for} \quad n = 1, 2, \ldots, N.$$

Solutions of this optimization are obtained by waterfilling [45]. A subgradient search is used to find the optimal value of the Lagrange multiplier $\lambda^{(j)}(m)$ for each user m; this optimal value is denoted by $\lambda^{*(j)}(m)$.

3. *Outer loop: iteration* $j = 1, 2, ...$
After the inner loop of iteration j is completed, the power allocation among the M users is adjusted. Specifically, for user m the optimal power

$$P^{*(j)}(m,n) = \left(\frac{1}{\lambda^{*(j)}(m)} - \mathrm{IN}^{(j)}(m,n)\right)^{\dagger} \tag{1.38}$$

is computed, such that the total power constraint

$$\sum_{n=1}^{N} P^{*(j)}(m,n) = \bar{P}(m)$$

is satisfied.

4. *Confirmation step*
After the power adjustments for the M users have been made, the condition

$$\sum_{m=1}^{M} \sum_{n=1}^{N} |P^{(j)}(m,n) - P^{(j-1)}(m,n)| < \epsilon \tag{1.39}$$

is checked for the prescribed tolerance ϵ at iteration j. If this *tolerable condition* is satisfied, the computation is terminated at $j = J$. Otherwise, the iterative process (encompassing both the inner and outer loops) is repeated.

1.12.1 Robustification of the Algorithm

In describing the iterative waterfilling algorithm, we have made a fundamental assumption:

- The normalized parameter $\alpha(m,k,n)$, denoting the combined effect of (1) propagation-path loss from the transmitter of user k to the receiver of user m at subcarrier n, and (2) frequency offset in the OFDM, is maintained constant throughout the entire sequence of iterations $j = 1, 2, ..., J$ of the algorithm.

This assumption is highly likely to be violated in practice, particularly when dealing with a *rapidly changing wireless channel*. It could also be aggravated by variations in the frequency offset Δf with time. The implication of these realities is that the multiuser cognitive radio problem should be modeled as a *partially observable Markov decision process*. For a possible cure, we could mitigate the effect of these sources of uncertainty by including a *signal-to-noise ratio gap* in formulating the data-transmission rate of each user $m = 1, 2, \ldots, M$. In effect, this gap is chosen large enough to provide reliable communication under practical operating conditions of the multiuser cognitive radio environment. Let the signal-to-noise ratio gap be denoted by Γ. We then rewrite the information capacity formula of (1.28) in the expanded form

$$R(m,n) = \log_2\left[1 + \frac{\mathrm{SINR}(m,n)}{\Gamma}\right] \quad \text{bits per use of subchannel } n \qquad (1.40)$$

which applies to all users $m = 1, 2, \ldots, M$ and subcarrier frequencies $n = 1, 2, \ldots, N$. Accordingly, the iterative waterfilling procedure is modified in a corresponding way.

1.12.2 Summarizing Remarks

Based on the criterion of competitive optimality, the iterative waterfilling algorithm is *user-centric* and therefore a *selfish*, *greedy*, and *sub-optimal algorithm* for solving the transmit-power control problem in a multiuser cognitive radio network using OFDM. Nevertheless, practical virtues of the algorithm include:

- The algorithm functions in a self-organized manner, thereby making it possible for the network to assume an ad hoc structure.
- It avoids the need for communication links (i.e., synchronization) among the multiple users, thereby significantly simplifying the design of the network.
- By using convex optimization [60], the algorithm tends to converge relatively rapidly to a Nash equilibrium; however, once this stable condition is reached, no user is permitted to change its transmit-power control policy unilaterally.
- Computational complexity of the algorithm is relatively low, being on the order of two numbers: the number of secondary users and the number of spectrum holes available for utilization.

1.13 Emergent Behavior of Cognitive Radio Networks

In light of the material presented in the preceding sections, we may characterize the multiuser cognitive radio network as a *complex*, *stochastic* and *time-varying feedback control system* that exhibits the following unique combination of attributes (among others): partial observability, adaptivity, learning, self-organization, cooperation, competition and exploitation. Given this characterization, we may wonder about the emergent behavior of a cognitive radio environment by virtue of what we know on two relevant fields: *self-organizing systems* and *evolutionary games*.

First, we note that the emergent behavior of a cognitive radio environment, viewed as a self-organized network is influenced by the *degree of coupling* that may exist between the actions of different users (i.e., transmitter–receiver linkages) operating in the network. The coupling may have the effect of *amplifying* local perturbations in a manner analogous with *Hebb's postulate of learning*, which accounts for self-amplification in self-organizing systems [5]. Clearly, if they are left unchecked, the amplifications of local perturbations would ultimately lead to *instability*. From the study of self-organizing systems, we also know that competition among the constituents of such a system can act as a stabilizing force [5]. By the same token, we expect that competition among the cognitive radio users for limited resources (e.g., transmitted power) may have the influence of a *stabilizer*, provided, of course, that the competition is carried out on the basis of the common application of the competitive optimality criterion by all the users. However, the tendency of one or more users to exploit the limited resources for selfish interests may drive the network into instability and possibly a chaotic state.[16]

For additional insight, we next look to evolutionary games. The idea of evolutionary games, developed for the study of ecological biology, was first introduced by Maynard Smith in 1974. In his landmark work [62,63], Maynard Smith wondered whether the theory of games could serve as a tool for modeling conflicts in a population of animals. In specific terms, two critical insights into the emergence of so-called *evolutionary stable strategies* were presented by Maynard Smith, as succinctly summarized in [51,63]:

- The animals' behavior is stochastic and unpredictable, when it is viewed at the microscopic level of actions taken by individual animals.
- The theory of games provides a plausible basis for explaining the complex and unpredictable patterns of the animals' behavior.

[16] The traditional method of studying the stability of a time-varying feedback control system is to apply the Lyapunov stability theory [61]. To apply this theory, we need to formulate a Lyapunov function for a multiuser cognitive radio network, which can be a hard task to do. The problem is complicated further by the stochastic nature of the network. For these reasons, we advocate the approach described in this section on evolutionary games.

Two key issues are raised here:

1. *Complexity*.[17] The emergent behavior of an evolutionary game may be *complex*, in the sense that a change in one or more of the parameters in the underlying dynamics of the game can produce a dramatic change in behavior. Note that the dynamics must be nonlinear for complex behavior to be possible.
2. *Unpredictability*. Game theory does not require that animals be fundamentally unpredictable. Rather, it merely requires that the individual behavior of each animal be *unpredictable with respect to its opponents*.

From this brief discussion on evolutionary games, we may conjecture that the emergent behavior of a multiuser cognitive radio network is explained by the unpredictable action of each user, as seen individually by the other users (i.e., opponents).

1.13.1 State of the World

In light of the conflicting influences of cooperation, competition and exploitation on the emergent behavior of a cognitive radio environment, we may identify two possible end-results for the state of the (wireless) world [64]:

1. *Positive emergent behavior*, which is characterized by *order*, and therefore a harmonious and efficient utilization of the radio spectrum by all primary and secondary users of the cognitive radio. (The positive emergent behavior may be likened to Maynard Smith's evolutionary stable strategy).
2. *Negative emergent behavior*, which is characterized by *disorder*, and therefore a culmination of traffic jams, chaos,[18] and therefore unused radio spectrum.

From a practical perspective, what we therefore need are, first, a reliable criterion for the early detection of negative emergent behavior (i.e., disorder) and, second, corrective measures for dealing with this undesirable behavior. With regards to the first issue, we recognize that cognition, in a sense, is an exercise in assigning probabilities to possible behavioral responses, in light of which we may say the following. In the case of positive emergent behavior, predictions are possible with nearly complete confidence. On the other hand, in the case of negative emergent behavior, predictions

[17] The new sciences of complexity (whose birth was assisted by the Santa Fe Institute, New Mexico) may well occupy much of the intellectual activities in the twenty-first century [64–67]. In the context of complexity, it is perhaps less ambiguous to speak of complex behavior rather than complex systems [68]. A non-linear dynamic system may be complex in computational terms, but it is incapable of exhibiting complex behavior. By the same token, a non-linear system can be simple in computational terms, but its underlying dynamics are rich enough to produce complex behavior.

[18] The possibility of characterizing negative emergent behavior as a chaotic phenomenon needs some explanation. Idealized chaos theory is based on the premise that dynamic noise in the state-space model (describing the phenomenon of interest) is zero. However, it is unlikely that this highly restrictive condition is satisfied by real-life physical phenomena. So, the proper thing to say is that it is feasible for a negative emergent behavior to be *stochastic chaotic* [69].

are made with less confidence. There is therefore the need to formulate a likelihood function based on predictability as a criterion for the onset of negative emergent behavior. The key question is how to do it effectively and efficiently?

Given a multiuser non-cooperative cognitive radio network based on OFDM and designed along the lines described in Sect. 1.13 on iterative waterfilling, we know the following: when all the users of the network use competitive optimality as their common criterion to satisfy their individual transmit-power control requirements, the network will reach a Nash equilibrium, that is, an orderly behavior throughout the network. On the other hand, when any of the users exploit the limited resources (i.e., transmitted power and spectrum holes) for selfish interest, there is the likelihood that the network will assume a disorderly behavior. It would therefore seem logical to look to the Nash equilibrium as the basis for designing a maximum-likelihood processor capable of detecting the emergence of disorderly behavior in the network; recall that the Nash equilibrium is a prediction concept.

To summarize, what we are advocating here is an expansion of the game-theoretic viewpoint of multiuser cognitive radio networks to embrace evolutionary games as described originally by Maynard-Smith. By so doing, we may be able to quantify the predictability of individual users' behavior. In particular, the expansion could facilitate the design and development of a maximum-likelihood processor for detecting the onset of the disorderly utilization of limited resources in the network due to the misbehavior of one or more users.

1.14 Distributed Traffic Coordination in Cognitive Radio Networks

The material presented up to this point in this chapter has focused on signal-processing and communication-theoretic issues relating to the identification of spectrum holes, the extraction of channel-state information, dynamic spectrum management, and transmit-power control. With the emphasis on a self-organized ad hoc network as the structure for building a cognitive radio network, we need a protocol for the distributed traffic coordination of secondary users of the network in such an environment. Needless to say, the development of this protocol is a challenging task.

Basically, the issue to be addressed is summed up as follows:

> In a self-organized and decentralized cognitive radio network, how can we establish the dissemination of control traffic signals between neighboring secondary users of the network, which is rapid, robust and efficient?

The requirement that the dissemination of control traffic signals be *rapid* is essential, because the secondary user could be faced with a limited duration of time for which spectrum holes are likely to be available. The dissemination has to be robust with respect to external attack not only for reasons of security but also to prevent disruptions in network use due to traffic congestion. Lastly, it has to be *efficient* so as to minimize the use of energy and computing resources.

For self-organized coordination among neighboring secondary users to be feasible, we expect two provisions:

1. Each user has knowledge of all the spectrum holes that are locally available.
2. Neighboring users have reasonably similar views of their respective spectral scenes, so as to guarantee the availability of common wireless channels.

Given these two provisions, which are provided by the radio-scene analyzer, we may then envision a *self-organized traffic-coordination* protocol that proceeds as follows [70]:

1. By exploiting the availability of similar spectrum holes, each local group of neighboring users forms a *mini-multihop network* with a common coordination channel. This could be achieved through broadcasting a beacon and recursive voting procedure, whereby the channel with the highest level of connectivity is selected by all the users in the mini-multihop network as the common coordination channel.
2. Through the availability of one or more spectrum holes common to adjoining mini-multihop networks, communication across the cognitive radio network is established.
3. Through eavesdropping on coordination messages from "bridge" users, a new user may join an existing mini-multihop network and thereby quickly subscribe to appropriate channels.

In a loose sense, point 1 of the procedure described herein is similar to what goes on in the formation of a self-organizing map in neural networks [5].

For an alternative solution to the traffic-coordination problem, one may look to the use of an out-of-band licensed channel as the dedicated common channel. In [70], Zhao, Zheng and Yang present simulation results that compare the performance of the self-organized coordination approach against an approach based on the dedicated common channel; the results presented therein appear to show that the self-organized approach outperforms the dedicated common channel approach, in terms of both throughput and processing delay.

Conclusion

Cognitive radio holds the promise of a new and exciting frontier in wireless communications. Most importantly, the development of an orderly dynamic spectrum-sharing process will make it possible to improve the utilization of radio spectrum under constantly changing user conditions. For the spectrum-sharing process to become a reality, two basic issues have to be in place:

1. There has to be a paradigm shift in wireless communications from transmitter-centricity to receiver-centricity, which, in turn, means that interference power at the receiver rather than transmitted power at the transmitter is regulated.

2. A new generation of wireless communication systems is developed, in which awareness of the radio environment and the ability to adapt to the environment and learn from it feature prominently.

Specifically, from a signal-processing and communication-theoretic perspective, we need to develop new algorithms that operate satisfactorily and in a robust manner in a wireless communications environment to perform the following functions:

- Identification of spectrum holes for employment by secondary users
- Channel-state estimation for improved utilization of the radio spectrum
- Adaptive modulation format that is impervious not only to the modulation format of the primary user but also to varying received signal-to-noise conditions
- Transmit-power control to support the transmission needs of multiple users
- Development of a decentralized radio network that is efficient in the use of resources and effective in performance
- Detection of the onset of instability whenever the network is misused
- Coordination of distributed traffic in the network.

The ideas and algorithms described in this chapter (building on [1]) should be viewed as starting points for a long road ahead, which will occupy the ingenuity and extensive research and development efforts of numerous researchers.

This immense effort is justified, given the potential of cognitive radio to make a significant difference to wireless communications; hence the reference to it as a "disruptive, but unobtrusive technology." In the final analysis, however, the key issue that will shape the evolution of cognitive radio in the course of time, be that for civilian or military applications, is trust. By this we mean, trust by users of cognitive radio, and trust by all other users who could be interfered with. For this trust to be a reality, cognitive radio will not only have to improve spectrum utilization but also do so in a robust, reliable and affordable manner.

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