8
Old Babylonian Hand Tablets with Geometric Exercises

There are nine hand tablets with drawings of geometric figures in the Schøyen Collection. Five of the illustrated hand tablets are round, the other four are square or rectangular. The only text on the nine tablets consists of relative sexagesimal numbers or area numbers. Each tablet will be considered separately below, but in order to facilitate comparisons of format, size, and content, hand copies and conform transliterations of the tablets are grouped together in Figs. 8.1.1 - 2 and 8.2.2 below.

8.1. Triangles and Trapezoids

8.1 a. MS 3042. The Area of a Triangle

MS 3042 (Fig. 8.1.1, top) is a square clay tablet with a crude drawing of a triangle on the obverse, together with some numbers. The reverse is empty. The numbers 3 and 5 40 along the sides of the triangle indicate the lengths of the short side (Sum. sag 'front') and the long side, or the height (Sum. uš 'length', 'flank'). (Old Babylonian teachers did not bother to distinguish between the height and the long side of a triangle when teaching young students elementary geometry.) These sexagesimal numbers in relative place value notation have to be understood as meaning 3 (00) ninda and 5 40 ninda, respectively. Indeed, it is an easily observed, and almost universal rule that *the sides of geometric figures in Old Babylonian mathematical texts are of sizes appropriate for cultivated fields, measured in tens or sixties of the ninda. The areas of the figures are then, correspondingly, measured in multiples of the iku* (1 40 sq. ninda)*, the èše* (10 00 sq. ninda)*, and the bùr* (30 00 sq. ninda).

With the mentioned values for the lengths of the sides of the triangle, the area can be computed as

$$A = 5\ 40 \cdot 3(00)/2 \text{ sq. ninda} = 8\ 30(00) \text{ sq. ninda} = 17 \text{ bùr}.$$

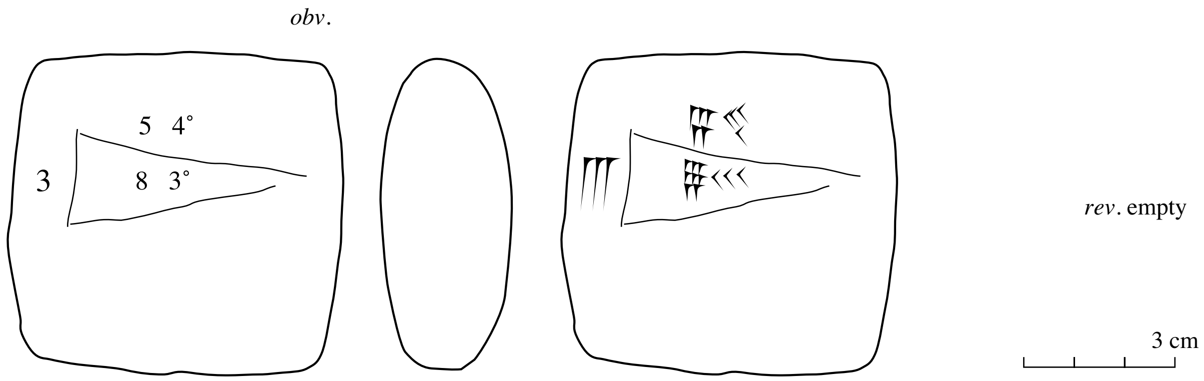
Hence, it is clear that the sexagesimal number 8 30 recorded in the interior of the triangle is the area of the triangle in relative place value notation. This placement of the value of the area is in agreement with what seems to be another general rule: *The area of a geometric figure is recorded in the interior of the figure, while the side lengths are recorded on the outside, along the sides they measure.*

8.1 b. MS 2107. The Area of a Trapezoid. An Almost Round Area Number

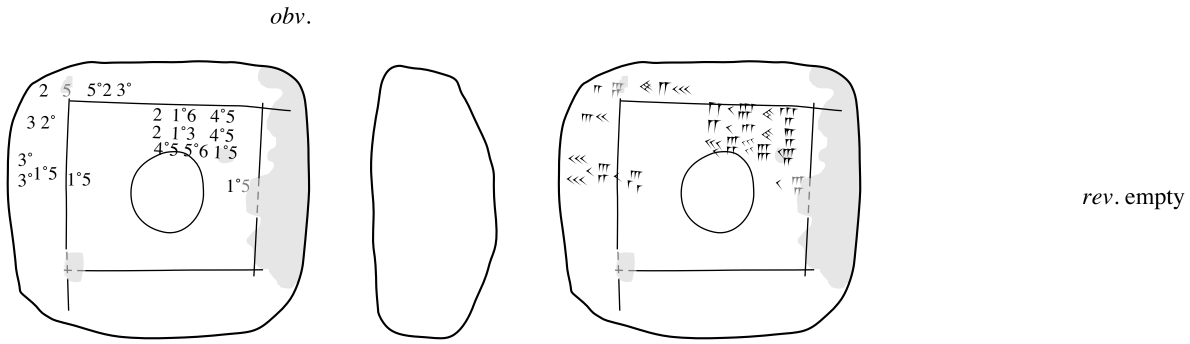
MS 2107 (Fig. 8.1.2, top) is a round hand tablet (a lentil) with a drawing of a trapezoid on the obverse, accompanied by some numbers. The reverse is empty. The recorded numbers are

30 for the upper front, 15 for the lower front, 3 30 for the long side or height, and 1 18 45 for the area.

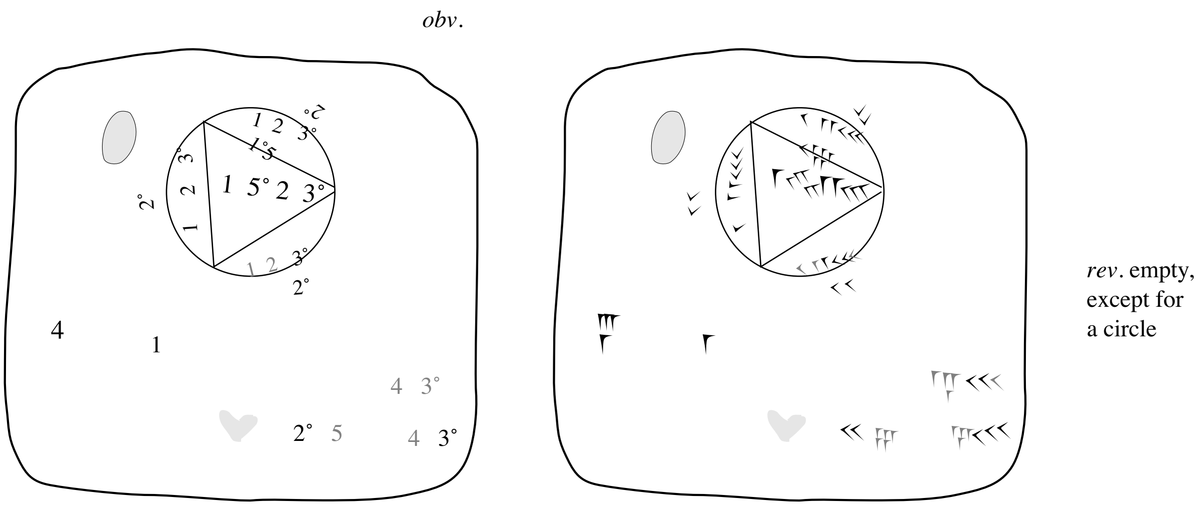
(In Old Babylonian cuneiform texts, the word an.ta 'upper' means to the left, and ki.ta 'lower' means to the right. This is because the direction of writing had changed from top-to-bottom to left-to-right, and when this happened the orientation of illustrating figures changed in the same way.)



MS 3042. Computation of the area of a triangle.

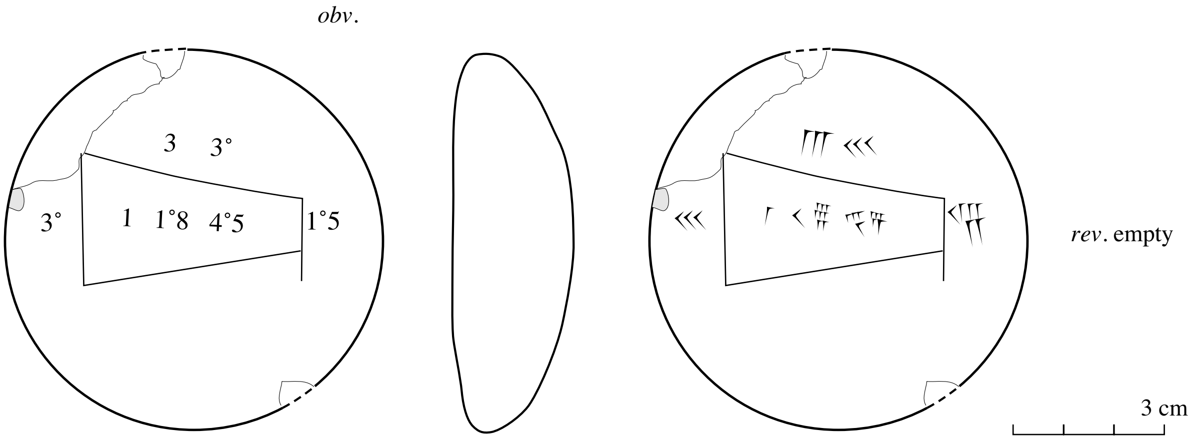


MS 2985. A problem for a circle inscribed in a square a certain distance away from the sides of the square.

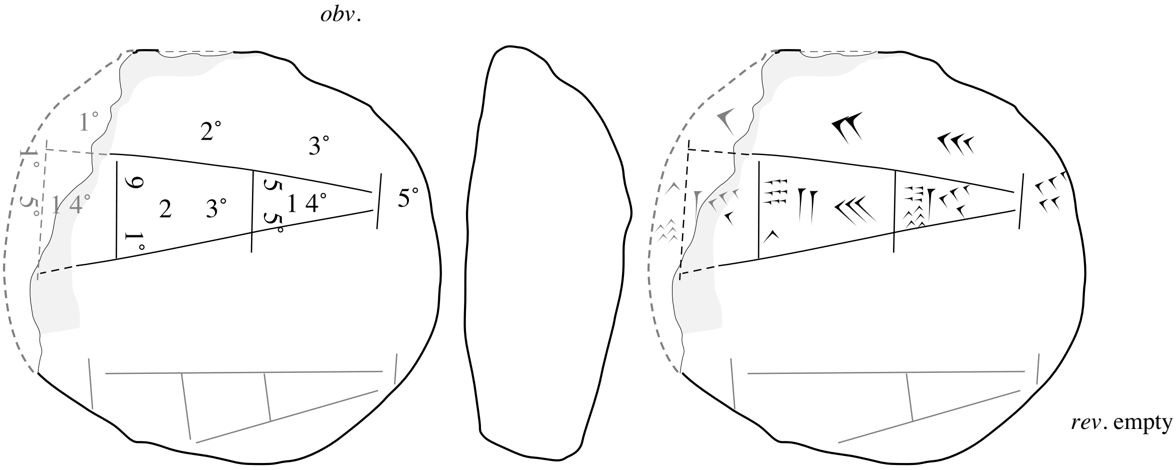


MS 3051. A problem for an equilateral triangle inscribed in a circle.

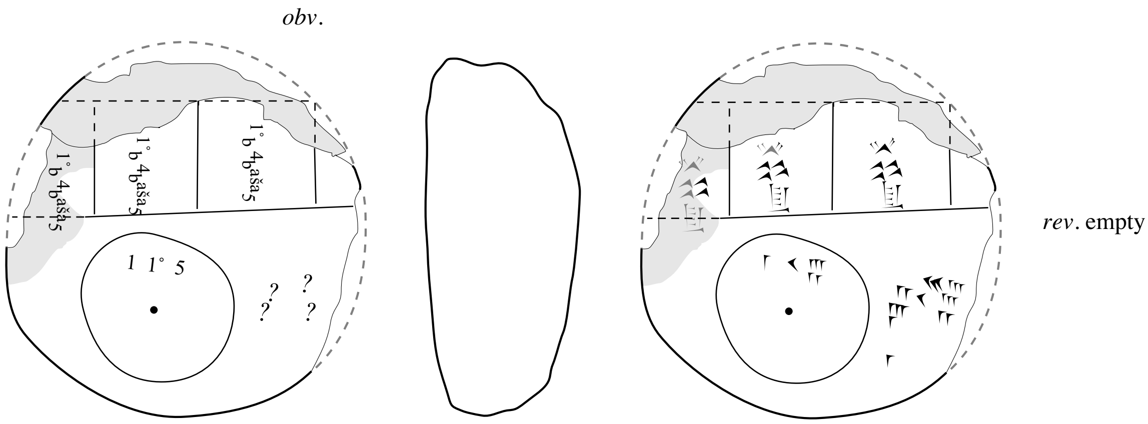
Fig. 8.1.1. Three square hand tablets with drawings of geometric figures and associated numerical data.



MS 2107. A trapezoid with an almost round area number.



MS 3908. A trapezoid divided into three stripe,s and a complete set of associated numerical parameters.



MS 3041. a) A rectangle divided into three equal parts, with area numbers. b) The area of a circle.

Fig. 8.1.2. Three round hand tablets with drawings of geometric figures and associated numerical data.

The area of the trapezoid can be computed as

A = 3 30 · (30 + 15)/2 sq. ninda = 3 30 · 22;30 sq. ninda = 1 18 45 sq. ninda = 2 bùr 1 èše 5 1/4 iku.

This computed value for the area of the trapezoid agrees with the number recorded inside the trapezoid. Thus, this geometric exercise, like the one considered above (MS 3042) does not look very exciting. However, a closer look at the numbers appearing in the exercise will tell a different story. On one hand, *the computed area number can be factorized in the following remarkable way*:

A = 3 30 · 22;30 = (1 + 1/6) · 3 00 · (1 + 1/8) · 20 = (1 +1/6) · (1 + 1/8) · 1 00 00.

At the same time, *the computed area number is close to a round area number*:

A = 2 bùr 1 èše 5 1/4 iku = 2 bùr 2 èše (– 3/4 iku) = appr. 2 bùr 2 èše = 1 20 00 sq. ninda

The deficit 3/4 iku = 1 15 sq. ninda is as little as 1/64 of 1 20 00 sq. ninda. This means that the computed area number is an almost round number of the kind considered in Friberg *AfO* 44/45 (1997/98) §§ 2-3. An almost round area number is an area number which is *close to a round area number and simultaneously equal to another round area number multiplied by one or two factors of the kind* (1 + 1/*n*)*, where n is a small regular sexagesimal integer*. Such almost round area numbers typically appear in proto-literate field texts as the result of an application of what may be called the "(proto-literate) field expansion procedure", the oldest known mathematical algorithm. (The "proto-literate period" in Mesopotamia is a convenient name for the centuries just before and after 3000 BC, a period to which can be dated the oldest known clay tablets, those inscribed with a predecessor of the cuneiform script.)

The proto-literate field expansion procedure seems to have been developed as a convenient geometric method to solve a problem of the following type by use of *successive approximations*:

To find the sides of a rectangle when the area and the ratio of the sides are given.

The way the method worked is best described by use of an explicit example.[1] Take, for instance, the following example with data borrowed from MS 2107:

Given a rectangle of area A = 2 bùr 2 èše = 1 20 00 sq. ninda, and with the front s equal to 1/9 of the length u.
Find the length and the front.

It is easy to solve this problem by use of Old Babylonian metric algebra. In a typical application of the method of false value (Friberg, *RlA 7* Sec. 5.7 d), let the length initially have the false value 1 00. Then the front has the corresponding false value 1 00/9 = 6;40, and so the area of the rectangle has the false value 6 40. The true value 1 20 00 sq. ninda is equal to 6 40 multiplied by 12 sq. ninda. Hence, 12 sq. ninda is the square of the "correction factor" (the ratio between the correct and the false value of the length). The correction factor itself must then be equal to 2 · sqs. 3 ninda = appr. 3;30 ninda, since the Old Babylonian standard approximation to sqs. 3 was 1;45 (7/4). Therefore, the correct values of the length u and the front s are found to be

u = appr. 1 00 · 3;30 ninda = 3 30 ninda, and s = appr. 3 30 ninda/9 = 23;20 ninda.

The field expansion procedure solves the problem in a different way. The initial false value of the length is chosen as, say, the *round length number* u_1 = 3 00 ninda. The false value of the front is 1/9 of that, that is s_1 = 20 ninda. The corresponding false value of the area is the *round area number*

A_1 = 3 00 ninda· 20 ninda = 1 00 00 sq. ninda = 2 bùr. (Fig. 8.1.3 a)

Then the *deficit*, that is the difference between the wanted "true" area and the initial false area, is

$A - A_1$ = (1 20 00 – 1 00 00) sq. ninda = 20 00 sq. ninda = 1/3 of A_1.

The crucial idea in this situation is to *eliminate one half of this initial deficit* by expanding the length by 1/6 of its value, thereby expanding also the area of the rectangle by 1/6 of its value. Then the new values of the length and the area will be:

1. Cf. the discussion in Sec. 1.1 c above of the fourth multiplication exercise in **MS 3955**.

u_2 = (1 + 1/6) · 3 00 ninda = 3 30 ninda,
A_2 = (1 + 1/6) · 1 00 00 sq. ninda = 1 10 00 sq. ninda = 2 bùr 1 èše. (Fig. 8.1.3 b)

The new deficit is

$A - A_2$ = (1 20 00 – 1 10 00) sq. ninda = 10 00 sq. ninda = 1/7 of A_2.

This new deficit can be eliminated by *expanding the front by 1/7 of its value*. Since 7 is a non-regular sexagesimal number, a more appealing alternative is to expand the front by only 1/8 of its value. After this second expansion, the new values of the front and the area are

s_2 = (1 + 1/8) · 20 ninda = 22;30 ninda,
A_3 = (1 + 1/8) · 1 10 00 sq. ninda = 1 18 45 sq. ninda = 2 bùr 1 èše 5 1/4 iku. (Fig. 8.1.3 c)

Thus, the solution to the stated problem obtained by use of the field expansion procedure is

u_2 = (1 + 1/6) · 3 00 ninda = 3 30 ninda
s_2 = (1 + 1/8) · 20 ninda = 22;30 ninda,
A_3 = (1 + 1/8) · (1 + 1/6) · 1 00 00 sq. ninda = 1 18 45 sq. ninda = 2 bùr 2 èše – 1/2 1/4 iku.

This is a fairly good approximation to the correct solution. It can be compared with the approximate solution obtained by use of metric algebra and an approximation to sqs. 3, which is hardly better:

u = 1 00 · 2 · sqs. 3 ninda = appr. 3 30 ninda
s = 1/9 · u = 23;20 ninda,
$A = u \cdot s$ = 2 bùr 2 èše = 1 21 40 sq. ninda = 2 bùr 2 èše + 1 iku.

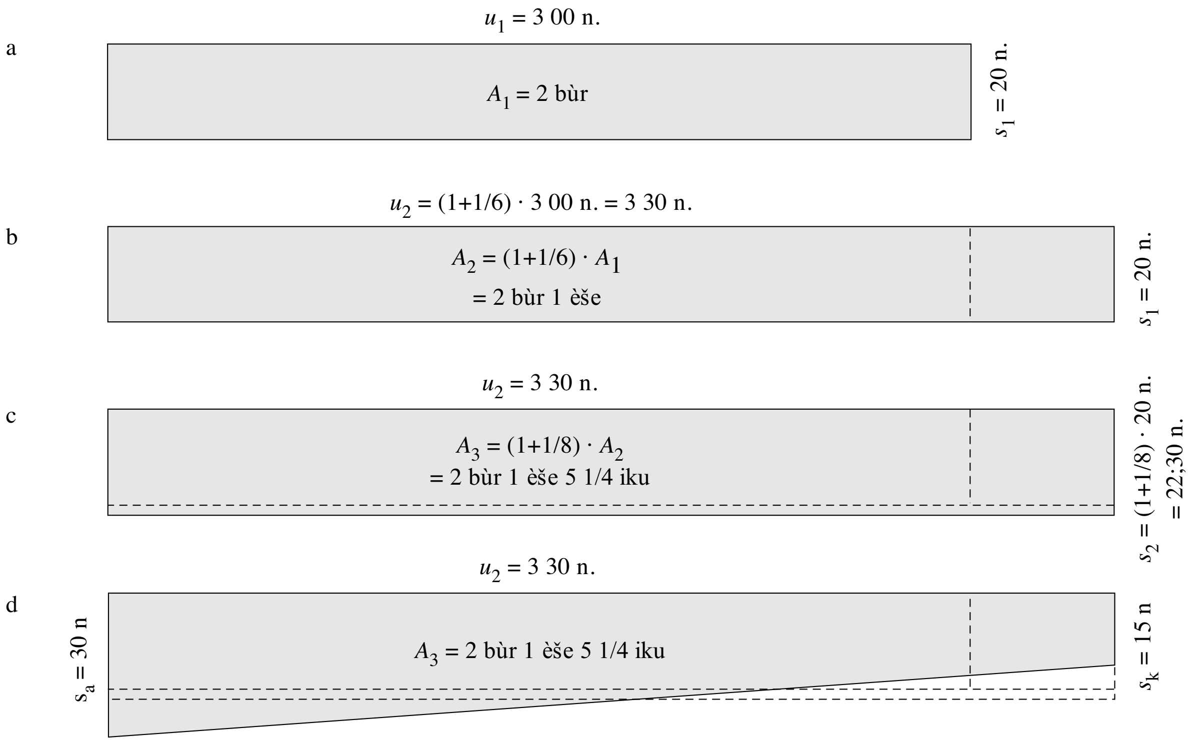


Fig. 8.1.3. MS 2107. An Old Babylonian application of the proto-literate field expansion procedure.

The geometric problem with the solution illustrated by the drawing and the numbers on MS 2107 is a more complex variant of the problem treated above. It may have been stated as follows:

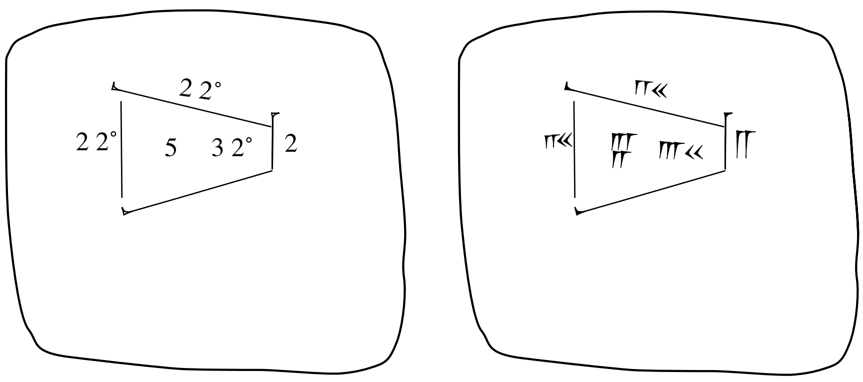
Given a trapezoid with the area A = 2 bùr 2 èše = 1 20 00 sq. ninda,
with the half-sum of the upper and lower fronts equal to 1/9 of the length u,
and with the upper front twice as long as the lower front.
Find the length and the front.

The solution of this more complex problem would require an additional step in the field expansion procedure, as shown in Fig. 8.1.3 d.

Remark: MS 2107 has been claimed to be from the same archive as MS 1844 (Fig. 7.4.2).

YBC 7290 (Fig. 8.1.4 below; *MCT*, 44) is a parallel to MS 2107. It is a rectangular hand tablet with a drawing of a trapezoid on the obverse, accompanied by numbers indicating the area 5 03 20, the length 2 20 and the upper and lower fronts 2 20 and 2 (00).

obv.



rev.: a small drawing of a trapezoid

Fig. 8.1.4. YBC 7290. A trapezoid with an almost round area number.

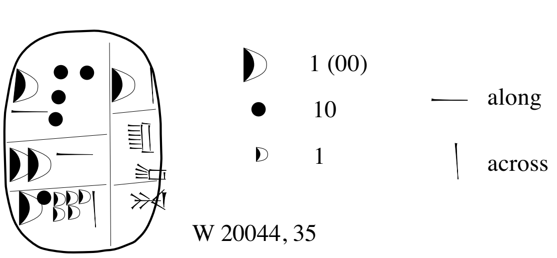
Just as in MS 2107, the area number in YBC 7290 is an almost round number, since

A = 5 03 20 sq. ninda = appr. 5 00 00 sq. ninda = 10 bùr, a round area number, and at the same time
A = (1 + 1/6) · 2 00 ninda · (1 + 1/12) · 2 00 ninda = (1 + 1/6) · (1 + 1/12) · 8 bùr.

8.1 c. Examples of Proto-Literate Field-Sides and Field-Area Texts

Here follows, for comparison, a brief discussion of the application of the field expansion procedure in a couple of proto-literate texts.

W 20044, 35 (Friberg, *AfO* 44/45 (1997/98), 10; Fig. 8.1.5 below) is one of several examples of proto-literate "field texts" documenting the use of the field expansion procedure.



u_1 = 1 40 (ninda), s_1 = 1 15 (ninda)
u_2 = 2 (00) (ninda), s_2 = 1 (00) (ninda)

A = 1 50 · 1 07;30 = 2 03 45 (sq. ninda)
= 1 14 1/4 iku = appr. 1 15 iku
and at the same time
A = (1 + 1/10) · (1 + 1/8) · 1 40 (00) (sq. ninda)
= (1 + 1/10) · (1 + 1/8) · 1 (00) iku

Fig. 8.1.5. A proto-literate field-sides text (c. 3200 BC). A quadrilateral with an almost round area

In this text are recorded four length numbers, apparently the lengths of the four sides of a quadrilateral, with vertical and horizontal lines close to the numbers, indicating that 1(géš) 40 and 2(géš) stand for the sides "along", while 1(géš) 15 and 1(géš) stand for the sides "across". The silently understood length unit is the ninda, as in Old Babylonian geometric texts. W 20044, 35 can be explained as an assignment. A student was probably expected to compute the area of a quadrilateral with the given sides, using the (inexact) "quadrilateral area rule", according to which the area of a quadrilateral is (approximately) equal to the half-sum of the sides along multiplied by the half-sum of the sides across. In the case of W 20044, 35, the result would be (in modern notations, using sexagesimal numbers in place value notation):

A = appr. (1 40 + 2 00)/2 n. · (1 15 + 1 00)/2 n. = 1 50 n. · 1 07;30 n. = 2 03 45 sq. n. = 1 14 1/4 iku = appr. 1 15 iku.

At the same time,

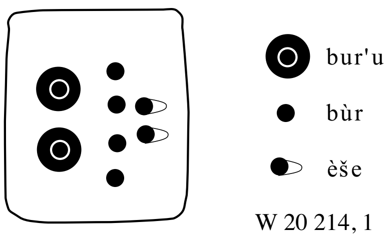
A = (1 + 1/10) · 1 40 n. · (1 + 1/8) · 1 00 n. = (1 + 1/10) · (1 + 1/8) · 1 00 iku.

Thus, clearly, the area is an almost round number, probably obtained as the result of an application of the field expansion procedure.

W 20214, 1 (Friberg, *AfO* 44/45 (1997/98), 9; Fig. 8.1.6 below) is one of a couple of proto-literate “area texts” also documenting the use of the field expansion procedure. In this text is recorded the area number

A = 2 bur'u 4 bùr 2 èše = 24 bùr 2 èše = 1 14 èše = 12 20 00 sq. ninda.

The recorded area number is close to the round area number 25 bùr = 1 15 èše = 12 30 00 sq. ninda. Therefore, it is reasonable to assume that this is another example of an almost round number.



A = 2 bur'u 4 bùr 2 èše = 24 bùr 2 èše =
appr. 25 bùr

Also:

A = appr. 24 bùr 2 èše 1 1/2 iku =
(1+1/10) · (1+1/8) · 20 bùr.

Fig. 8.1.6. A proto-literate field-area text (c. 3200 BC) with an almost round area number.

There is no obvious way to factorize the recorded number exactly in the way expected of an almost round number. Nevertheless, it is not difficult to see that the recorded number may be interpreted as an almost round number, rounded off to the nearest multiple of 1 èše. Indeed,

(1+1/10) · (1+1/8) · 10 00 00 sq. ninda = (1 + 1/10) · 5 00 n. · (1 + 1/8) · 2 00 n. =
5 30 n. · 2 15 n. = 12 22 30 sq. ninda = 24 bùr 2 èše 1 1/2 iku = appr. 24 bùr 2 èše.

8.1 d. MS 3908. A Trapezoid Divided into Three Stripes

MS 3908 (Fig. 8.1.2, middle) is a round hand tablet with a drawing of a trapezoid, accompanied by some numbers, on the obverse. There is also a weak outline of a second trapezoid on the obverse. The reverse is empty. The trapezoid with numbers is “three-striped”, that is, it is divided by two transversals, parallel with the upper and lower fronts of the trapezoid, into three stripes or sub-trapezoids. The lengths of the two transversals are given as 9 10 and 5 50, respectively. The areas of the middle and lower stripes are also given, as 2 30 and 1 40, respectively.

The upper stripe is damaged, and the numbers inscribed inside it and along its sides are not preserved. Nevertheless, it is not difficult to find a reasonable reconstruction of the lost numbers. The key observation is that the three “partial lengths” are [...], 20, 30, and that if the first number is reconstructed as [10], the three numbers will form an arithmetical progression with the sum 1 (00). Thus, the whole length of the trapezoid was probably chosen to be the round number 1 00 (ninda).

The inclination of the sides of a triangle is determined by what may be called the “growth rate” of the triangle, namely *the ratio of the front to the length*. Now, it is a generally observed convention in Old Babylonian geometry that *in every triangle the length is longer than the front*. Consequently *the growth rate of a triangle is always smaller than 1*.

The inclination of the sides of a trapezoid is similarly determined by *the growth rate of the trapezoid, defined as the ratio of the difference of the upper and lower fronts to the length. This growth rate, too, is always smaller than 1*. Therefore, if the length of the trapezoid in MS 3908 is supposed to be 1 00 (ninda), the preserved values 9 10, 5 50, and 50 for the transversals and the lower front (a decreasing sequence) must be interpreted as 9;10, 5;50, and ;50 (ninda), respectively. Consequently, the growth rates of the undamaged stripes or sub-trapezoids are

(9;10 – 5;50)/20 = 3;20/20 = ;10 for the middle stripe, (5;50 – ;50)/30 = 5/30 = ;10 for the lower stripe.

A naive application of a simple argument using similar triangles makes it clear that in every "striped trapezoid" like the one in the drawing on MS 3908, that is, in every trapezoid divided by a number of parallel transversals into several stripes or sub-trapezoids, *all the stripes have the same growth rate*. Many explicit examples show that this "common growth rate rule" was well known in Old Babylonian mathematics. That being the case, also the damaged upper stripe must have had the growth rate ;10. Therefore, the lost value of the upper front can be reconstructed as [10;50]. Indeed,

(10;50 – 9;10)/10 = 1;40/10 = ;10.

Since now all the lengths of the fronts, the transversals, and the partial lengths are known, the three "sub-areas" of the three-striped trapezoid can be computed. They are:

10 · (10;50 + 9;10)/2	= 10 · 20/2	= 1 40 (sq. ninda)	= 1 iku	for the upper stripe,
20 · (9;10 + 5;50)/2	= 20 · 15/2	= 2 30 (sq. ninda)	= 1 1/2 iku	for the middle stripe,
30 · (5;50 + ;50)/2	= 30 · 6;40/2	= 1 40 (sq. ninda)	= 1 iku	for the lower stripe.

The numbers 2 30 and 1 40 for the areas of the middle and lower stripes agree with the preserved numbers on MS 3908, while [1 40] for the upper stripe is a plausible reconstruction of a lost number.

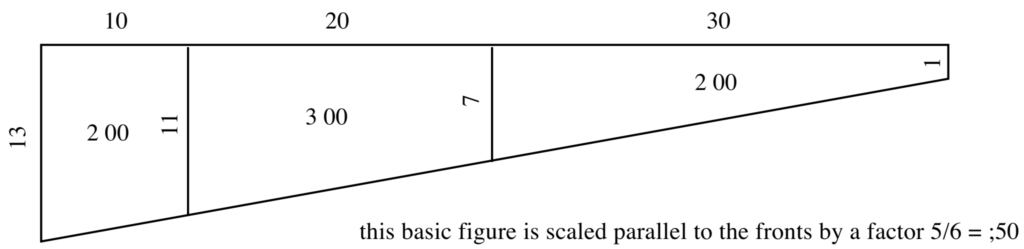


Fig. 8.1.7. The basic construction for the three-striped trapezoid in MS 3908.

Two important questions concerning MS 3908 remain to be answered. *How were the data for the three-striped trapezoid constructed, and what was the purpose of the text?* A first step towards an explanation of the construction of the data is the observation that all the lengths 10;50 and ;50 of the upper and lower fronts and 9;10 and 5;50 of the upper and lower transversals are multiples of ;50. It is very likely that a simpler set of data was first constructed in some way, and then the fronts and transversals of the three-striped trapezoid were scaled by the factor ;50 (= 5/6), obviously in order to make the areas of the upper and lower stripes equal to the round area number 1 40. sq. ninda = 1 iku. That original set of data is displayed in Fig. 8.1.7 above.

It is important to understand that all the numbers displayed in Fig. 8.1.7 cannot have been prescribed freely. As a matter of fact, they must have been chosen with great care in such a way that they would not come in conflict with the *three area equations* for the areas of the three stripes:

A_1	= 10 · (13 + 11)/2	= 10 · 12	= 2 00	the upper sub-area,
A_2	= 20 · (11 + 7)/2	= 20 · 9	= 3 00	the middle sub-area,
A_3	= 30 · (7 + 1)/2	= 30 · 4	= 2 00	the lower sub-area.

Neither must they conflict with the *common growth rate rule,* that is, with the *similarity equations*

$$r_1 = r_2 = r_3.$$

Actually,

r_1 = (13 – 11)/10 = 2/10 = ;12, r_2 = (11 – 7)/20 = 4/20= ;12, r_3 = (7 – 1)/30 = 6/30 = ;12.

What all this means is that the 10 parameters for the three stripes, consisting of 3 partial lengths, 2 fronts, 2 transversals, and 3 sub-areas, must satisfy 3 area equations and 2 similarity equations. Therefore, *only 5 of the 10 parameters for a three-striped trapezoid can be given arbitrary values*!

The most likely situation is that the five freely prescribed parameters in Fig. 8.1.8 below were the three partial lengths 10, 20, 30 and the two the sub-areas equal to 2 00. *The remaining five parameters were then computed as the solutions to the following system of 5 linear equations for 5 unknowns*:

$s_a + d_a$	$= 2\ 00 \cdot 2/10$	$= 24$	from the equation for the upper sub-area,
A_m	$= 20 \cdot (d_a + d_k)/2$		the equation for the middle sub-area,
$d_k + s_k$	$= 2\ 00 \cdot 2/30$	$= 8$	from the equation for the lower sub-area,
d_k	$= d_a - 2 \cdot (s_a - d_a)$		from the similarity equation for the upper and middle stripes,
s_k	$= d_k - 3 \cdot (s_a - d_a)$		from the similarity equation for the upper and lower stripes.

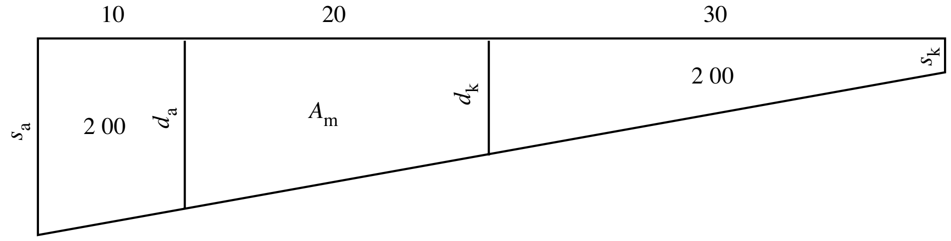


Fig. 8.1.8. The five unknowns for a three-striped trapezoid with five given parameters: the upper and lower fronts, the upper and lower transversals, and the middle sub-area.

It is, of course, impossible to know how the one who constructed the data for MS 3908 actually solved this system of linear equations. He may have proceeded *by trial and error*. Then, again, he may have proceeded *in a systematic way,* for instance in the following series of simple steps:

1)	d_a	$= 24 - s_a$	using the first area equation,
2)	d_k	$= 3\ d_a - 2\ s_a = 1\ 12 - 5\ s_a$	using the first similarity equation and step 1,
3)	s_k	$= 6\ d_a - 5\ s_a = 2\ 24 - 11\ s_a$	using the second similarity equation, step 2, and step 1,
4)	$d_k + s_k$	$= 3\ 36 - 16\ s_a$	using step 1 and step 2,
5)	$s_a = (3\ 36 - 8)/16 = 13$		using the third area equation and step 4,
6)	d_a	$= 11, d_k = 7, s_k = 1$	using steps 1, 2, and 3,
7)	$A_m = 3\ 00$		using the second area equation and step 6.

In whichever way it was done, it was quite an accomplishment to construct the data for the striped trapezoid figuring on MS 3908. As for the purpose of the text, it may be a student's answer to an assignment. There are many different kinds of problems that a teacher could design with departure from a striped trapezoid like the one on MS 3908, simply by erasing some of the parameters and asking the student to find them again.

8.1 e. Ash. 1922.168, IM 43996, Two More Texts with Striped Trapezoids or Triangles

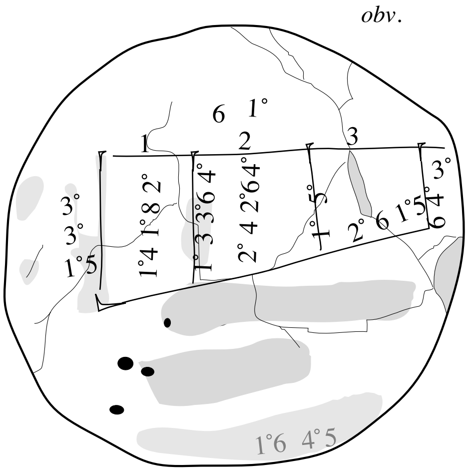
Although there is no known directly parallel text to MS 3908, there are several indirect parallels with striped trapezoids or triangles. Indeed, problems involving striped trapezoids or striped triangles was a popular topic in Old Babylonian mathematics. (See Friberg, *RlA 7* Sec. 5.4 i.) The most interesting example is **Str. 364** (*MKT 1*, 248 ff) with its many variations of problems for triangles with 2, 3, or 5 stripes (see Fig. 10.2.12 below). In the simplest cases, the stated problems lead to quadratic equations.

Ash. 1922.168 (Robson, *MMTC* (1999), 273-4) is round hand tablet with a drawing of a three-striped trapezoid and associated numbers. The construction of the striped trapezoid in this text is a rather close parallel to the construction of the striped trapezoid on MS 3908, but somewhat less sophisticated. The conform transliteration below, in Fig. 8.1.9, is based on Robson's hand copy.

The upper, middle, and lower lengths are 1 00, 2 00, and 3 00 (ninda) in this text, which means that they are of the same relative sizes as the corresponding values 10, 20, and 30 (ninda) in MS 3908. On the other hand, while apparently the *upper and lower areas* were arbitrarily given in MS 3908, it was probably the *upper and lower fronts*, 15 and 6;40 (ninda), that were arbitrarily given in Ash. 1922.168. With both the lengths and the fronts given, the upper and lower transversals can be computed directly by use of the common growth rate rule. First the growth rate is computed, it turns out to be $r = (15 - 6;40)/6\ 00 = ;01\ 23\ 20$. Next, the two transversals are computed as follows:

$$d_a = s_a - r \cdot u_a = 15 - ;01\ 23\ 20 \cdot 1\ 00 = 15 - 1;23\ 20 = 13;\ 36\ 40,$$
$$d_k = s_k + r \cdot u_k = 6;40 + ;01\ 23\ 20 \cdot 3\ 00 = 6;40 + 4;10 = 10;50.$$

After that, the computation of the three sub-areas is straightforward.



Given parameters (probably):

The upper, middle, and lower partial lengths:

1 00, 2 00, and 3 00 (ninda).

The upper and lower fronts:

15 and 6;40 (ninda).

(Above the trapezoid, the 10 following after 6 is the reciprocal of 6, the whole length of the trapezoid. The three numbers 30 possibly refer to the divisions by 2 in the three area equations.)

Fig. 8.1.9. Ash. 1922.168. A less sophisticated parallel text to MS 3908.

Another parallel text to MS 3908, concerned with a *triangle* rather than a trapezoid, is the square hand tablet **IM 43996** (Fig. 8.1.10; Bruins, *Sumer* 9 (1953); Friberg, *UL* (2005), Fig. 2.1.9).

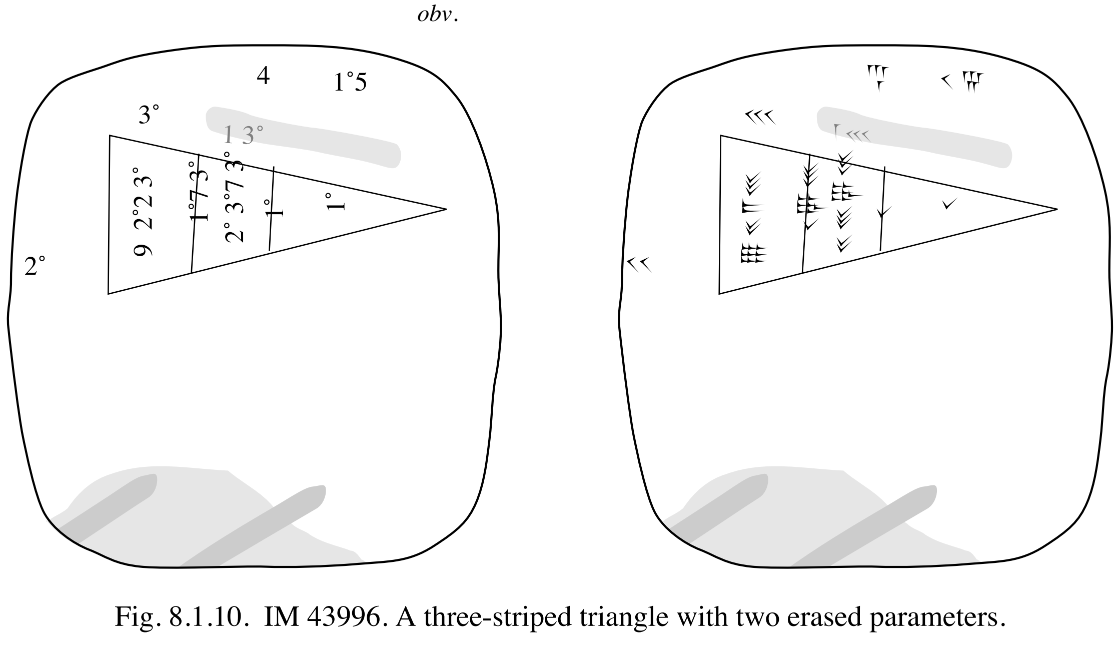


Fig. 8.1.10. IM 43996. A three-striped triangle with two erased parameters.

It is an assignment where the teacher apparently first gave the student the values of all the nine parameters for a three-striped triangle, then erased two of the values, asking the student to find them again. The text is interesting because of the way in which the values of the nine parameters must have been constructed. *In a three-striped triangle, the values for 4 of the 9 parameters can be chosen freely*. The remaining 5 values are determined by 3 area equations and 2 similarity equations.

It is likely that in the case of IM 43996 the teacher initially wanted to construct a striped triangle satisfying the following conditions:

The upper area and the lower area are both 1/2 of the middle area, the front is 20 (n.), and the upper length 30 (n.).

Then, since *the areas of similar triangles are proportional to the squares of their fronts* (a simple consequence of the common growth rate rule, with which Old Babylonian mathematicians were well acquainted), the front s and the two transversals d_a and d_k would have to satisfy the equations

$$\text{sq. } d_a - \text{sq. } d_k = 2 \text{ sq. } d_k, \quad \text{and} \quad \text{sq. } s - \text{sq. } d_a = \text{sq. } d_k.$$

Hence,

$$\text{sq. } d_a = 3 \text{ sq. } d_k, \quad \text{and} \quad \text{sq. } s = 4 \text{ sq. } d_k.$$

With the given value for the front, s = 20 (n.), it follows that

$$d_k = 1/2 \cdot s = 10 \text{ (ninda)} \quad \text{and} \quad d_a = \text{sqs. } 3 \cdot d_k = 10 \cdot \text{sqs. } 3 = \text{appr. } 10 \cdot 1;45 = 17;30.$$

With the given value for the upper front, u_a = 30 (n.), the growth rate is computed as follows

$$r = (20 - 17;30)/30 = 2;30/30 = ;05.$$

The two similarity equations then yield the values of the middle and lower lengths u_m and u_k:

$$u_m = (17;30 - 10)/r = 7;30 \cdot 12 = 1\ 30, \quad \text{and} \quad u_k = 10/r = 10 \cdot 12 = 2\ 00.$$

The last step of this construction is the straightforward computation of the areas of the three stripes. *Due to the inexactness of the approximate value used for the square side of 3*, the computed value for the upper area then is 9 22;30 instead of 10 00 and that of the middle area 20 37;30 instead of 20 00.

The problem posed to the student, with the computed values for the parameters, was *much easier* than the original construction of those values. Apparently, the student was asked to find the front s and the middle and lower lengths u_m and u_k when the areas of the three stripes were given as 9 22 30, 20 37 30, and 10, and when the two transversals were given as 17 30 and 10. All the student had to do was then to make use of the three area equations, one after the other.[2]

An interesting final example of a parallel to MS 3908 is the "geometric theme text" **MAH 16 055** (see the photo in Bruins, *Physis* 4 (1962)), a text with a series of drawings of five similar three-striped triangles. In the first of these triangles (Fig. 8.1.11), the upper and lower stripes both have the area 5 (00 sq. ninda). In the four following triangles, these areas are multiplied by 2, 3, 4, and 5, respectively, but in all the five triangles, the lengths are divided in three parts equal to 12, 12, and 36 (ninda).

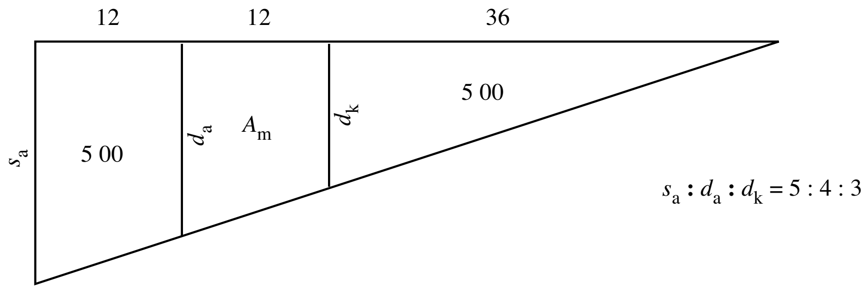


Fig. 8.1.11. The first of five similar three-striped triangles in the geometric table text MAH 16055.

The construction of the numerical values of the parameters for these three-striped triangles can be explained like this: Since the areas of similar triangles are proportional to the squares of their fronts, it follows that

If the upper and lower stripes in a three-striped triangle have the same area, then $\text{sq. } s - \text{sq. } d_a = \text{sq. } d_k$.

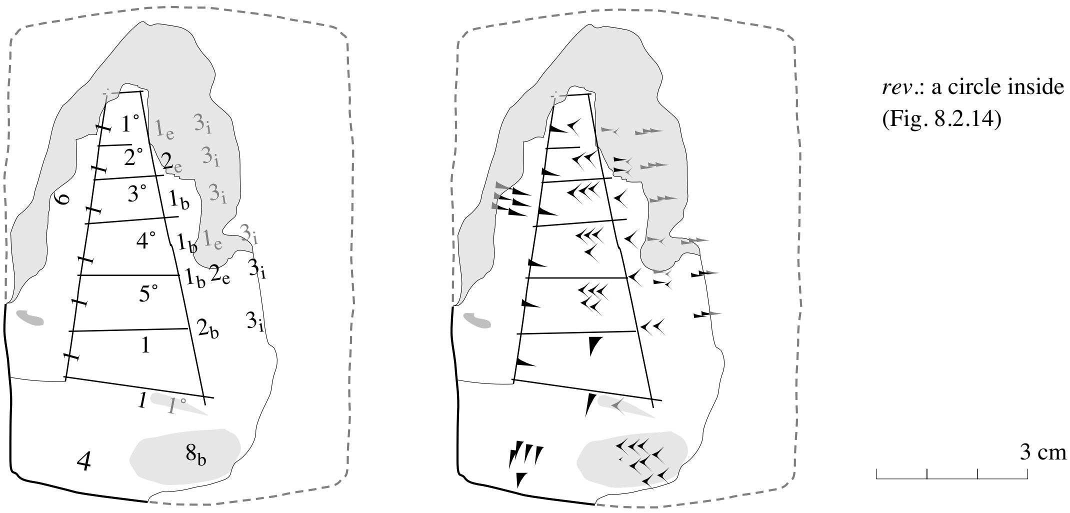
2. The number '4' inscribed near the upper edge of the obverse, together with its reciprocal '15', apparently confirms that the first step of the solution procedure was the observation that sqs. s = 4 sqs. d_k, from which it follows that sq. d_k = ;15 · sq. s = ;15 · 6 40 = 1 40, so that d_k = 10.

In other words, the front and the two transversals satisfy the indeterminate quadratic "Old Babylonian diagonal rule" (incorrectly known as the "Pythagorean equation"). The simplest and most well known solution *in integers* to that equation is, of course, the triple (5, 4, 3). Suppose, then, that the front and the two transversals have the relative values 5, 4, and 3. Then it follows from the two similarity equations that the upper, middle, and lower lengths have the relative values 5 – 4 = 1, 4 – 3 = 1, and 3. In particular, if the triangle is normalized so that its length is equal to 1 (00 ninda), then it follows that the three parts of the length have the values 12, 12, and 36 (ninda), as in MAH 16055.
Now, if the lower stripe has the area 5 (00 ninda), as in the first of the five drawings on MAH 16055, then the lower transversal must have the length 2 · 36/5 00 = 14;24 (ninda), *etc*.

8.1 f. MS 1938/2. An Arithmetical Progression of Stripes in a Trapezoid

MS 1938/2 (Fig. 8.1.12) is a fragment of a rectangular hand tablet. The curvature of the clay tablet, as seen from the side, makes it possible to reconstruct the original shape of the tablet. The tablet is inscribed on the obverse with the picture of a striped trapezoid, accompanied by sexagesimal numbers and area numbers. There is a drawing of a circle inside a hexagon (in Old Babylonian mathematics called a 'six-front') on the reverse.

obv.



rev.: a circle inside a 6-front (Fig. 8.2.14)

Fig. 8.1.12. MS 1938/2, *obv*. Six fields with their areas in an arithmetical progression.

The trapezoid on the obverse is divided by five evenly spaced transversals into six parallel stripes. The lengths of the two fronts and the five transversals form a decreasing arithmetical progression, proceeding in steps of 10 (ninda) from 1 10 to 10 (ninda). The the six partial lengths are all marked as '1', obviously meaning '1 00 ninda'. It is equally obvious that the number '6', written close to the left length of the trapezoid, stands for the value '6 00 ninda' of the whole length.

Alongside the trapezoid are written a series of area numbers, which can be assumed to stand for the areas of the six stripes, although the alignment of these area numbers is quite poor. The areas of the six stripes can be computed as follows:

A_1 = 1 00 · (1 10 + 1 00)/2 sq. ninda = 1 05 00 sq. ninda = 2 bùr 3 iku,
A_2 = 1 00 · (1 00 + 50)/2 sq. ninda = 55 00 sq. ninda = 1 bùr 2 èše 3 iku,
A_3 = 1 00 · (50 + 40)/2 sq. ninda = 45 00 sq. ninda = 1 bùr 1 èše 3 iku,
A_4 = 1 00 · (40 + 30)/2 sq. ninda = 35 00 sq. ninda = 1 bùr 3 iku,
A_5 = 1 00 · (30 + 20)/2 sq. ninda = 25 00 sq. ninda = 2 èše 3 iku,
A_6 = 1 00 · (20 + 10)/2 sq. ninda = 15 00 sq. ninda = 1 èše 3 iku.

These computed area numbers agree well with the preserved traces of area numbers on the clay tablet. (Cf. App.

4, Fig. A4.10.) It is easy to check that the sum of the six computed areas is equal to the area of the whole trapezoid, which can be computed as follows:

$$A = 6\ 00 \cdot (1\ 10 + 10)/2 \text{ sq. ninda} = 4\ 00\ 00 \text{ sq. ninda} = 8 \text{ bùr}.$$

The number 4, indicating this total area, is recorded in the lower left corner of the tablet. The eight oblique wedges inscribed to the right of the number 4, written in three layers of 3, 3, and 2 wedges, may stand for '8 bùr', although the normal way of writing '8 bùr' is in two layers of 4 plus 4 wedges.

Although MS 1938/2 with its drawing of a striped trapezoid and its series of area number looks like a field plan (cf. Ch. 5 above), it is more likely that the text is the answer to an assignment, possibly an inheritance problem for 'six brothers'.

Note: Normally trapezoids in drawings on Old Babylonian clay tablets are positioned with the long sides (the 'lengths') extending from left to right, and with the 'upper front' to the left and the 'lower front' to the right. However, judging from the way in which most of the numbers are written, the trapezoid on the obverse of MS 1983/2 may very well have been oriented in an unusual way, with what is normally the upper front facing downwards. MS 1983/2 is also unlike normal Old Babylonian hand tablets with geometric exercises in that the computed areas are expressed in terms of area numbers rather than sexagesimal numbers in relative place value notation. A possible explanation is that MS 1983/2 is a *Sumerian* rather than Old Babylonian geometric exercise!

For comparison is shown below a drawing of a five-striped trapezoid illustrating a mathematical problem text (of which almost nothing else is preserved) on the obverse of the fragment **IM 31248**.This a text from the site Ishchali, published by Bruins in *Sumer* 9 (1953). There is also a photo of the clay tablet in *Janus* 71 (1984). (The hand copy in Fig. 8.1.13 below is based on that photo.) The trapezoid is drawn totally out of scale, apparently in order to allow enough space to record all pertinent values. The areas of the five stripes are given as 1 57, 31, 1 09, 15, and 21, and the lengths of the four transversals as 33, 29, 17, 13. The upper front is 45, the lower front 1, and the length is divided into five parts with the lengths 3 (00), 1 (00), 3 (00), 1 (00), 3 (00). All recorded numbers are sexagesimal numbers in relative place value notation.

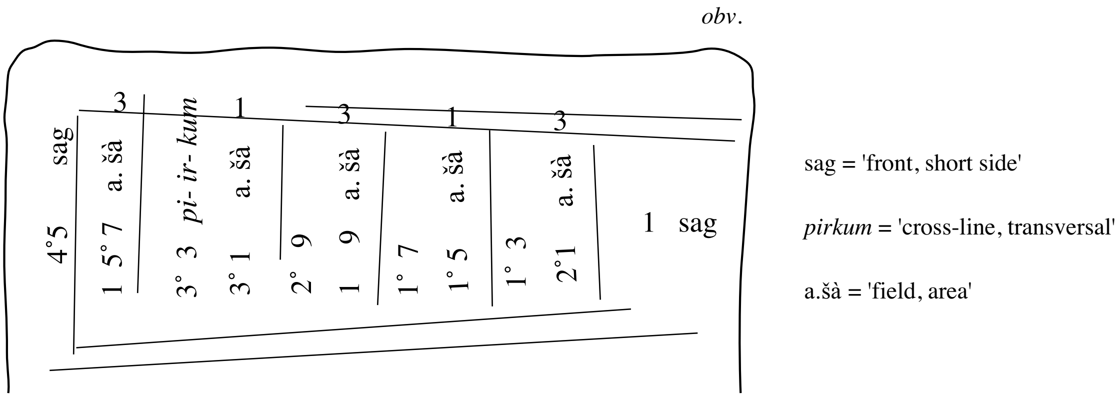


Fig. 8.1.13. IM 31248. A drawing of a five-striped trapezoid introducing an Old Babylonian problem text.

Since the text of the associated exercise is lost, there is no way of knowing what problem was illustrated by the drawing. Note, however, that the growth rate of the trapezoid is (45 – 1)/11 00 = 44/11 00 = 1/15 = ;04. Thus, the growth rate is a regular sexagesimal number, in spite of the fact that the length is a non-regular number.

8.1 g. MS 3853/2. A Doodle on the Reverse of a Single Multiplication Table

MS 3853/2 (Fig. 8.1.14) is a large fragment of a clay tablet with a single multiplication table on the obverse, head number 3 20 (the reciprocal of 36). On the reverse there is a geometric doodle, a symmetric triangle divided by three vertical and two horizontal straight lines. The triangle is also divided by several further straight lines, running obliquely downwards and to the right. The full details of the drawing are not visible any more due to damage to the surface of the clay tablet. It is not clear how the missing parts of the drawing should be reconstructed.

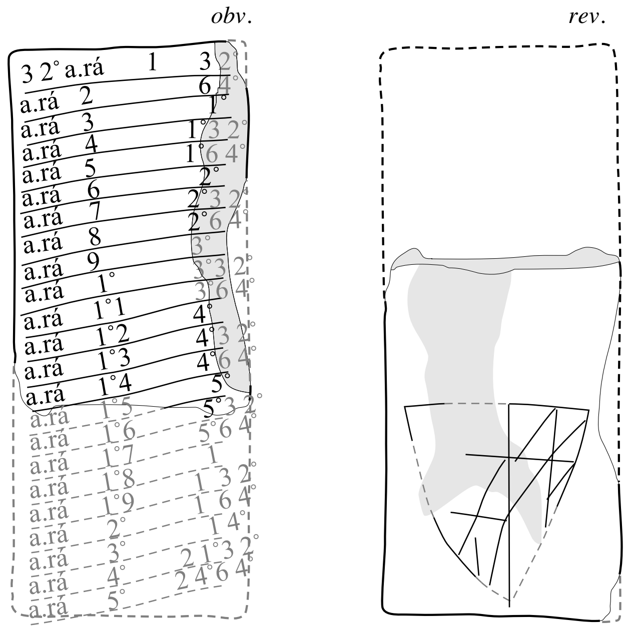


Fig. 8.1.14. MS 3853/2. A single multiplication table of type a with a geometric doodle on the reverse.

8.2. Figures Within Figures

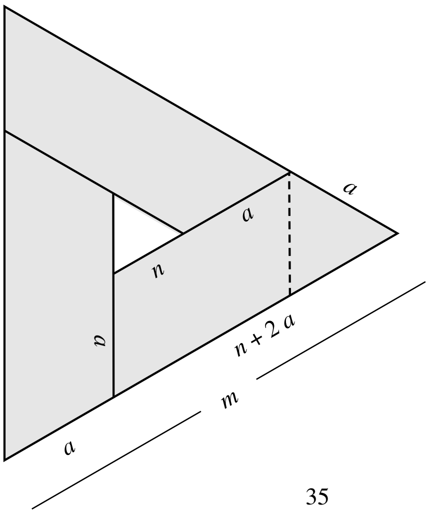
"Figures within figures" was a popular topic in Old Babylonian geometry (see Friberg, *RlA 7* (1990), Sec. 5.4 l.). In the Schøyen Collection, there are five hand tablets with drawings of figures within figures.They will be discussed separately below.

8.2 a. MS 2192. A Triangular Band between two Concentric Equilateral Triangles

MS 2192 (Fig. 8.2.2, top) is a round hand tablet (a lentil), with a relatively flat obverse but a strongly curved reverse. On the obverse of the lentil, there is a drawing of two "concentric" (and parallel) equilateral triangles. Each side of the inner triangle is extended in one direction. As a result, the space between the two concentric triangles, what may be called an "equilateral triangular band", is divided into a "chain" of three equal trapezoids. The lengths of all line segments in the figure are indicated by sexagesimal numbers in relative place value notation, without any indication of the unit of length, or of the absolute size of the numbers. The lentil is probably early Old Babylonian from southern Mesopotamia, since it makes use of variant number signs for 40.

The lengths indicated in the drawing can be interpreted as 10 ninda three times (the sides of the inner equilateral triangle), 43;20 ninda three times (the longer of the parallel sides in each of the three trapezoids), and 16;40 ninda six times (the non-parallel sides of the three trapezoids). Although that is not explicitly indicated in the drawing, the lengths of the shorter of the parallel sides in the three trapezoids are 10 + 16;40 = 26 40 (ninda), and the lengths of the sides of the outer equilateral triangle are all equal to 16;40 + 43;20 = 1 00 (ninda). The drawing is not true to scale (the inner triangle should be much smaller than the outer triangle).

The inscribed numbers can be explained as follows: Suppose that, initially, the lengths of the sides of the two concentric triangles were given as m = 1 00 and n = 10 (ninda), respectively. Let a be the length of the non-parallel sides of each of the three trapezoids. Then it is not difficult to see that the lengths of the two parallel sides of the trapezoids are 10 + a and 10 + 2 a, respectively, and that the length of the side of the outer equilateral triangle is 10 + 3 a. Since 10 + 3 a = 1 00 (ninda), it follows that a = 50/3 = 16;40 (ninda). Hence, the lengths of the parallel sides of the trapezoids are 10 + 16;40 = 26; 40 (ninda) and 10 + 2 · 16;40 = 43;20 (ninda), respectively.



m = 1 00 (ninda)
n = 10 (ninda)
$a = (m - n)/3$ = 16;40 (ninda)
$n + a$ = 26;40 (ninda)
$n + 2\,a$ = 43;20 (ninda)

$\{(n + a) + (n + 2\,a)\}/2$ = 35 (ninda)

sq. m – sq. n = 35 · sq. n

Fig. 8.2.1. Explanation of the numbers associated with the drawing on MS 2192.

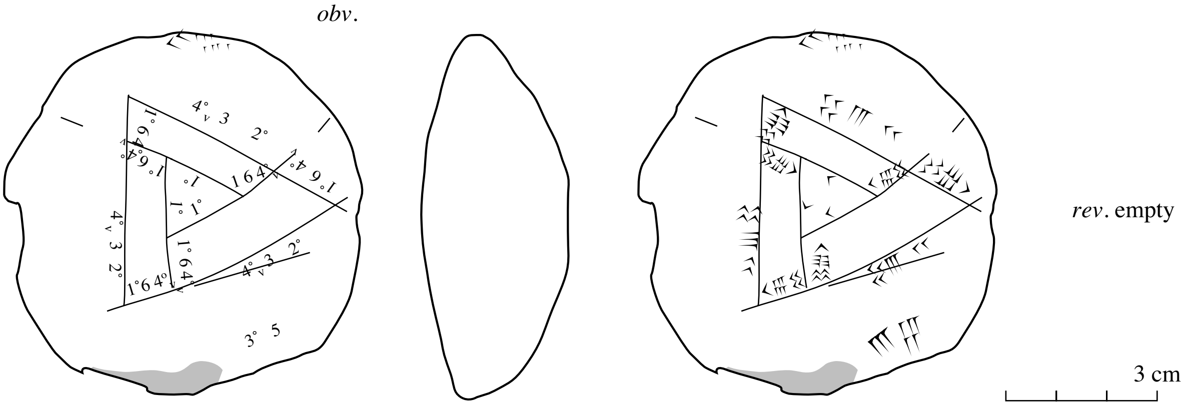
8.2 b. An Assignment: To Compute the Area between the Two Equilateral Triangles

An important sub-topic of the wide topic “figures within figures” was “concentric” figures, until now represented only by *concentric circles* and *concentric (and parallel) squares*. **MS 2192** is the only known Babylonian mathematical text dealing with *concentric (and parallel) equilateral triangles*. Actually, the only previously known instance of equilateral triangles being mentioned in an Old Babylonian mathematical text is an obscure entry in a table of constants (see below).[3] In several of the known examples of figures within figures, the consideration of concentric circles or squares was associated with the problem of computing the a.šà dal.ba.na ‘the field between’, by which is meant the area of the circular or square “band” bounded by given concentric figures. It is likely that MS 2192 with its drawing of two concentric equilateral triangles was *an assignment, the student’s task being to compute the area of the triangular band between the two triangles*. Compare the many mathematical lentils from Old Babylonian Ur with various arrays of numbers (discussed in Robson, *MMTC* (1999), App. 5, and Friberg, *RA* 94 (2000)), which may all be interpreted as assignments, with the student’s task being to fill in the missing details. Other examples of lentils with mathematical assignments are MS 1844 with the data for an inheritance problem (Fig. 7.4.2), MS 2268/19 with the data for a market rate problem (Fig. 7.2.2, top), MS 2107 with the area of a trapezoid (Fig. 8.1.2, top), and MS 3908 with a striped trapezoid (Fig. 8.1.2, middle).

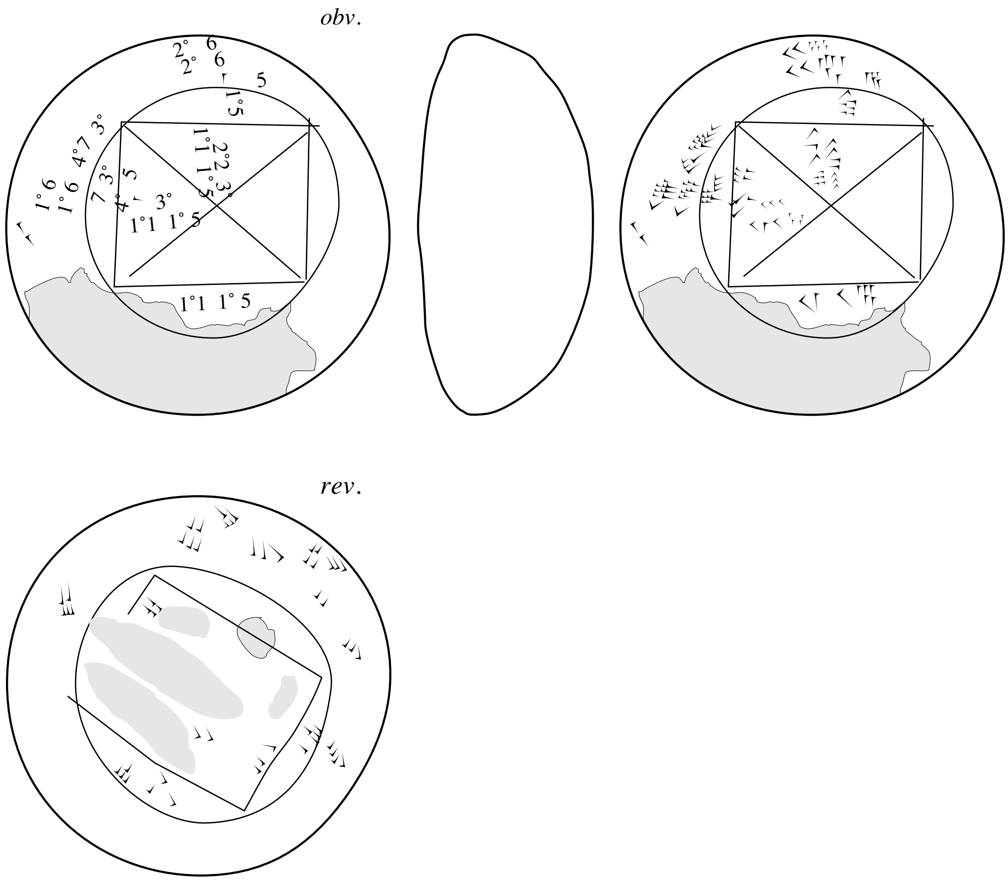
Trivially, the area of the band between two concentric circles, squares, or triangles can be computed directly as the difference between the areas of the outer and the inner figure. If one wants to use this simple rule in order to compute the area between two concentric equilateral triangles, one has to know first the area of an equilateral triangle with sides of given length. In modern terms, if the side of an equilateral triangle is s, then the height h of the triangle is sqs. 3 /2 · s, and the area is

$$A = h \cdot s/2 = \text{sqs. } 3/2 \cdot s \cdot s/2 = \text{sqs. } 3/4 \cdot \text{sq. } s.$$

3. Now, however, there is also the extremely interesting problem text **MS 3876** (see Fig. 11.3.1 below), which may be interpreted as a text dealing with equilateral triangles and regular icosahedrons. See also **MS 3051** (Fig. 8.1.1, bottom.)



MS 2192. Beginning of a computation of the area between two concentric equilateral triangles, expressed as the sum of the areas of three trapezoids.



MS 3050 *obv*. A square and its diagonals inscribed in a circle, and several groups of associated numbers.
rev. A square (or rectangle) inscribed in a circle, scattered numbers and erasures.

Fig. 8.2.2. Two round hand tablets with drawings of figures within figures.

A simple approximation to sqs. 3 is 1;45 (= 7/4). The corresponding approximation to sqs. 3 /4 is

sqs. 3 /4 = appr. ;26 15 (= 7/16).

This approximation is mentioned in the Old Babylonian table of constants G = IM 52916, obliquely referred to there as a constant for an equilateral triangle:

sag.kak-*kum ša sa-am-na-*[*tu na*]-*ás-ḫa*	A peg-head (triangle), that with an eighth torn out,	G *rev*. 7'
26 15 *i*-[*gi-gu-bu-šu*]	26 15 its constant.	

(See Robson, *MMTC* (1999), 40-41.) The obscure text refers to the fact that in an equilateral triangle, with the side *s* the height *h* and the area *A* can be computed as

$$h = \text{sqs. } 3/2 \cdot s = 1\ 45 / 2 \cdot s = 52;30 \cdot s = s - 1/8 \cdot s, \quad A = h \cdot s/2 = 26;15 \cdot \text{sq. } s.$$

Remark: In two successive exercises in the *Late* Babylonian large recombination text IM 23291, accompanied by drawings of equilateral triangles, the area of an equilateral triangle with the side 1 (00) is computed using two different approximations to the constant sqs. 3 /4. In the first exercise, the approximation used is ;26 15, in the second exercise it is ;26. (See Friberg, *BagM* 28 (1997), 285.) The reason why the Late Babylonian texts mentions two different approximate values for the same constant is probably that ;26 15 (in the first exercise) was the traditional value for sqs. 3 /4, borrowed from some Old Babylonian mathematical text, while ;26 (in the second exercise) was a new, more accurate value, the one normally used by Late Babylonian mathematicians. The Late Babylonian approximation to sqs. 3 /4 can be obtained in, for instance, the following way:

$$1\ 00 \cdot \text{sqs. } 3/4 = 15 \cdot \text{sqs. } 3 = \text{sqs. } (3 \cdot 3\ 45) = \text{sqs. } 11\ 15 = \text{appr. } 26, \quad \text{since} \quad \text{sq. } 26 = 11\ 16.$$

Thus, in the case of the *Old* Babylonian text MS 2192, the area of the triangular band between the two concentric equilateral triangles with the sides 1 00 and 10 could have been computed as

$$A(\text{band}) = A(\text{outer triangle}) - A(\text{inner triangle}) = \text{appr. } ;26\ 15 \cdot (\text{sq. } 1\ 00 - \text{sq. } 10) = ;26\ 15 \cdot 58\ 20 = 25\ 31;15.$$

However, this does not seem to be the method that the student who wrote MS 2192 was supposed to use. Apparently, his given task was instead to compute the area of the triangular band as the sum of the areas of the three trapezoids in the drawing. For the computation of the area of one of the trapezoids, he needed to know the height of the trapezoid. Now, it is easy to see that the height of the trapezoid is equal to the height of an equilateral triangle of side 16;40 (Fig. 8.2.1). Therefore,

$$A(\text{band}) = 3 \cdot A(\text{trapezoid}) = 3 \cdot \text{length of midline} \cdot \text{height}/2 = 3 \cdot 35 \cdot (16;40 - 2;05) = 25\ 31;15.$$

The result is, of course, the same as the difference between the areas of the outer and inner triangles.

The number '35' is inscribed on MS 2192 below the triangular band, near the edge. It is likely that this was the student's computation of the length of the midline of one of the trapezoids, the first step in his computation of the area of a trapezoid. Thus, it seems that MS 2192 is an *unfinished solution* to the problem of finding the area of the triangular band represented by the drawing.

Note: *The idea of computing the area of a triangular band as the area of a chain of trapezoids is a variation on the idea of computing the area of a square band as the area of a chain of four rectangles*. This is a simple idea, and it is likely that it was known by Old Babylonian mathematicians, although no cuneiform mathematical text has yet been found where this idea enters in an explicit way. There is, however, an interesting clay tablet from the site Tell Dhiba'i (Db_2-146 = **IM 67118**; Baqir *Sumer* 18 (1962)) in which the following problem is considered:

Given the diagonal (1 15) and the area (45 00) of a rectangle, find the sides of the rectangle.

Now there is also a parallel exercise, **MS 3971 § 2**, the whole text of which is reproduced in Sec. 10.1 b below. The explicit solution procedure is explained in Fig. 10.1.3.

It is well known (see, for instance, Høyrup, *LWS* (2002)) that Old Babylonian mathematicians relied on geometric models for their solutions of quadratic equations or systems of equations. A likely geometric model in the case of IM 67118 is the one illustrated to the left in Fig. 8.2.3 below, where the diagonals of the chains of rectangles forming a square ring are seen to form a square, the square on the diagonal. The basic idea of the solution procedure in IM 67118 is that if the diagonal d and the area A of a rectangle are given, and if q is the half-difference of the sides of the rectangle, then the area of the square with the side q can be computed as the area of the square with the side d minus 2 times the area A of the rectangle. (Actually, in Fig. 8.2.3, left, sq. q is seen to be equal to sq. d minus 4 times $A/2$.) In IM 67118, after the half-difference q has been computed, the square of the half-sum p is computed as the square of the half-difference q plus 4 times the area A of the rectangle.

In this connection, it is interesting to note that sq. q plus 2 times the area A is at the same time equal to sq. d (Fig. 8.2.3, left) and to sq. u + sq. s (Fig. 8.2.3, right). This is a simple and straightforward demonstration of the validity of the Old Babylonian diagonal rule, based exclusively on ideas that were well known in Old Babylonian mathematics.[4]

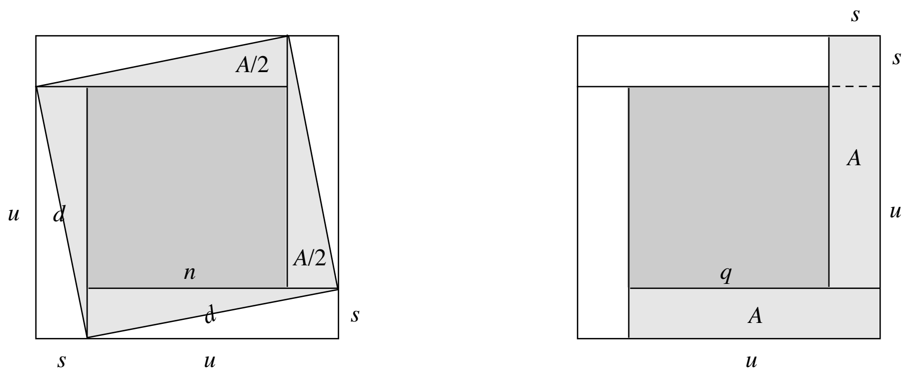


Fig. 8.2.3. Left: IM 67118. Finding the sides of a rectangle when the area and the diagonal are known. Right and left together: A possible Old Babylonian proof of the diagonal rule.

Remark: *P.Cairo* J. E. 99127-30, 89137-43 is an Egyptian demotic mathematical papyrus from the 3rd c. BC, thus contemporary with Late Babylonian mathematical texts. It was published by Parker in *DMP* (1972). Exercises ## 34-35 in *P.Cairo* are parallels to the problem in IM 67118. One of these exercises, *P.Cairo* # 34 (Friberg, *UL* (2005), Sec. 3.1 i), asks for the sides of a rectangle with the diagonal d = 13 cubits, and the area A = 60 square cubits. The answer is given in a series of simple steps:

1) sq. $d + 2 \cdot A = 169 + 10 = 289$, sqs. 289 = 17
2) sq. $d - 2 \cdot A = 49$, sqs. 49 = 7
3) (17 – 7)/2 = 5 = the width of the rectangle
4) 17 – 5 = 12 = the length of the rectangle
5) verification: sq. 12 + sq. 5 = 144 + 25 = 169, sqs. 169 = 13 = the diagonal of the rectangle

An interesting interpretation of an obscure entry in the Old Babylonian table of constants BR (Bruins and Rutten, ***TMS* 3** (1961), entry 30) suggests that Old Babylonian mathematicians had a name for a “chain of right triangles” like the one formed by the light grey triangles in Fig. 8.2.3, left. Such a chain of rectangles superficially resembles the cuneiform number sign šàr (= 1 00 00). (There are, for instance, several examples of this number sign in the metrological text MS 3925 shown in Fig. 3.2.4. See also the factor diagram for system S in Fig. A4.1.) The entry in the table of constants is:

57 36 igi.gub *šà* šár 57 36, constant of the šár BR 30

The probable explanation, found by Vaiman in *VDI* (1963) is that the area of a chain of right triangles like the light grey chain in Fig. 8.2.3, left, is precisely 57 36 if the side of the chain (the diagonal of one of the right triangles) is normalized as 1 00, and if the sides of the right triangles are proportional to the triple 5, 4, 3. Then the sides of the triangles are 1 00, 48, 36, and it follows that the area of the chain of right triangles is equal to

4. See App. 8 below, and in particular Sec. A.8 f, for an up-to-date discussion of the role played by the diagonal rule in Old Babylonian mathematics.

the area of the oblique square with the side 1 00 minus the area of the straight square with the side 48 –36 = 12. More precisely, the area of this chain of right triangles is

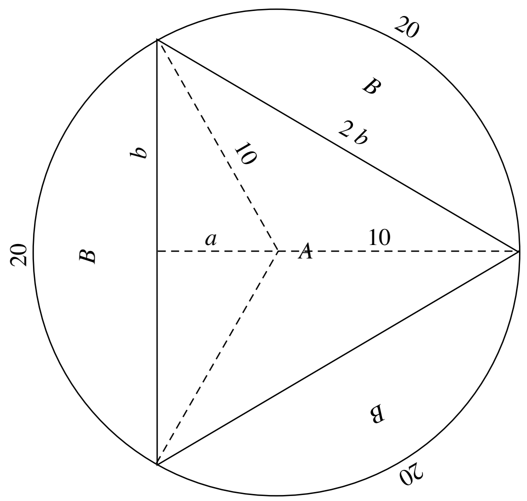
$$A = \text{sq. } 1\ 00 - \text{sq. } (48 - 36) = \text{sq. } 1\ 00 - \text{sq. } 12 = 1\ 00\ 00 - 2\ 24 = 57\ 36.$$

8.2 c. MS 3051. An Equilateral Triangle Inscribed in a Circle

MS 3051 (Fig. 8.1.1, bottom) is a somewhat large square hand tablet with a drawing and some numbers on the obverse. The reverse is empty, except for a weakly drawn circle. The drawing on the obverse represents an equilateral triangle inscribed in a circle. The equilateral triangle is oriented with one of its sides pointing to the left, in agreement with the observed general rule that in Old Babylonian geometric exercises *the 'front' of a triangle is always pointing to the left*. Compare with the orientation of the triangle in MS 3042 (Fig. 8.1.1, top). Compare also with the orientation of the trapezoids in MS 2107 (Fig. 8.1.2, top) and MS 3908 (Fig. 8.1.2, middle), which are oriented in agreement with the related Old Babylonian rule *that the upper front of a trapezoid is always pointing to the left*.

The drawing in MS 3051 is amazingly exact. The sides of the triangle are nearly equal. The circle passes through two of the three vertices of the triangle and passes close by the third vertex. It is clear that a compass must have been used in the construction of the figure, although there are no remaining traces of the point of the compass. It is also clear that the accurate construction of a figure of this kind would be difficult without a good understanding of basic geometric principles.

The equilateral triangle divides the circumference of the circle in three equal parts. In the drawing on MS 3051, they are all marked with the number '20', obviously meaning '20 ninda'. That means that the whole circumference of the circle is 1 00 (ninda). Now, as known from several Old Babylonian tables of constants and mathematical exercises, in Old Babylonian mathematical texts it is always assumed that *a circle of circumference a = 1 00 has the diameter d = a/3 = 20 and the area A = ;05 · sq. a = 5 00*.[5] Now, if the diameter is 20, the radius is 10. The height of the equilateral triangle is then equal to the radius 10 plus another piece, called *a* in Fig. 8.2.4 below.



$a/10 = b/(2\,b)$ and sq. b = sq. 10 – sq. a

$\Rightarrow$

$a = 5$, $b = 5 \cdot$ sqs. 3, $h = 10 + 5 = 15$

$2\,b = 2 \cdot 5 \cdot$ sqs. 3 = appr. $10 \cdot 1;45 = 17;30$

$A = h \cdot b = 15 \cdot 5 \cdot$ sqs. 3 = appr. $1\ 15 \cdot 1;45 = 2\ 11;15$

B = appr. $(5\ 00 - A)/3 = 2\ 48;45/3 = 56;15$

Fig. 8.2.4. Correct parameters for an equilateral triangle inscribed in a (normalized) circle.

An Old Babylonian mathematician could compute the values of *a* and *b* (= half the side of the equilateral triangle) by use of similarity, as follows. First,

$$a/10 = b/(2\,b) \;\Rightarrow\; a = 10/2 = 4.$$

5. **MS 3041** (Fig. 8.1.2, bottom) is a round hand tablet with two unrelated drawings. Above, there is a drawing of a rectangle divided into three equal parts, all with the area 14 bùr, written with traditional area numbers. Below, there is a drawing of a circle, and inside the circle the sexagesimal number 1 15, which can be explained as ;05 · sq. 30, that is the area of a circle of circumference 30 (ninda).

Knowing a, he could then compute b by use of the diagonal rule:

$$\text{sq. } b = \text{sq. } 10 - \text{sq. } 5 = 1\ 40 - 25 = 1\ 15 = 3 \cdot \text{sq. } 5.$$

Hence,

$$b = \text{sq. } 1\ 15 = 5 \cdot \text{sqs. } 3 = \text{appr. } 5 \cdot 1;45 = 8;45.$$

When the Old Babylonian mathematician had computed the values of a and b, he could easily compute also the side of the equilateral triangle, $2\,b = 10 \cdot$ sqs. 3 = appr. $10 \cdot 1;45 = 17;30$, as well as the area of the equilateral triangle, $A = 15 \cdot 17;30/2 = 2\ 11;15$. He could then also compute the area B of any of the three circular segments outside the equilateral triangle, since $A + 3 \cdot B$ = the area of the whole circle = appr. 5 00. Therefore, $B = (5\ 00 - 2\ 11;15)/3 = 2\ 48;45/3 = 56;15$.

Explicit arguments like the ones above can rarely be found in Old Babylonian mathematical texts. On the hand tablet MS 3051, there are recorded only the lengths of the three circular arcs, correctly given as '20', the area of the equilateral triangle, incorrectly given as '1 52 30', and the areas of the three circle segments, incorrectly given as '1 02 30'. In addition the height of the equilateral triangle is indicated by the number '15', misleadingly inscribed along one side of the triangle.

It is clear that the student who solved an assignment given to him by producing the hand tablet MS 3051 was guilty of some serious error. The numbers he actually recorded on his hand tablet can be analyzed as follows:

$$\text{“}A\text{”} = 1\ 52;30 = 15 \cdot 15\,/2, \quad \text{and} \quad \text{“}B\text{”} = 1\ 02;30 = (5\ 00 - 1\ 52;30)/3.$$

The mistake he made, absentmindedly thinking about more exciting things than his mathematical assignment, was to set the side of an equilateral triangle equal to the height of the triangle!

Remark: There are no parallels to MS 3051 in the known corpus of Old (or Late) Babylonian mathematics. There are, however, two parallel exercises in the Egyptian demotic mathematical text *P.Cairo*, mentioned above. One of these is *P.Cairo* # 36, where an equilateral triangle of side $s = 12$ 'divine cubits' is inscribed in a circle. The area of the circle is determined in a number of steps:

1) According to the diagonal rule, the height of the equilateral triangle is h = sqs. (sq. 12 – sq. 6) = sqs. 108 = appr. 10 1/3 1/20 1/120 d. c.
2) The area of the equilateral triangle is $A = 6 \cdot$ sqs. 108 d. c. = appr. 62 1/3 1/60 sq. d. c.
3) The height of a circle segment is k = 1/3 of sqs. 108 = appr. 3 1/3 1/10 1/60 1/120 1/180 d. c.
4) The area of a circle segment is B = appr. $k \cdot (s + k)/2$ = 26 5/6 1/10 sq. d. c.
5) The area of the circle is $A + 3\,B$ = appr. 143 1/10 1/20 sq. d. c.

To check the result, the area of the circle is then computed in a different way:

7) The diameter of the circle is $d = h + k$ = appr. 13 5/6 1/45 d. c.
8) The circumference of the circle is a = appr. $3 \cdot d$ = 41 1/2 1/15 d. c.
9) The area of the circle is A = appr. $a/3 \cdot a/4$ sq. d. c. = 143 5/6 1/10 1/30 sq. d. c.

This Late Egyptian parallel to the Old Babylonian mathematical exercise MS 3051 is particularly interesting because it also in other ways demonstrates *a close connection between an Egyptian demotic mathematical papyrus and Babylonian mathematics*. Here are some pertinent observations:

A. There are three drawings illustrating ***P.Cairo* # 36**, showing the equilateral triangle inscribed in a circle, the triangle with its height, and one of the three circle segments. See Fig. 8.2.5.

In these three drawings the triangles and the circle segment are standing upright on their bases. In the drawing in the middle, the height of the triangle is shown at right angles to the base. This is precisely the way in which equilateral triangles and their heights are drawn in the Late Babylonian large mathematical clay tablet **W 23291 § 4 a-c** (Friberg, *BagM* 28 (1997)). In Old Babylonian mathematical texts, on the other hand, triangles are always drawn with the 'front' pointing to the left. In Egyptian hieratic mathematical texts, like *P.Rhind* and *P.Moscow*, triangles are oriented with the short side pointing either to the left or to the right. Thus, in this respect, the demotic *P.Cairo* is closer related to Late Babylonian mathematics than to Old Babylonian or Egyptian hieratic mathematics.

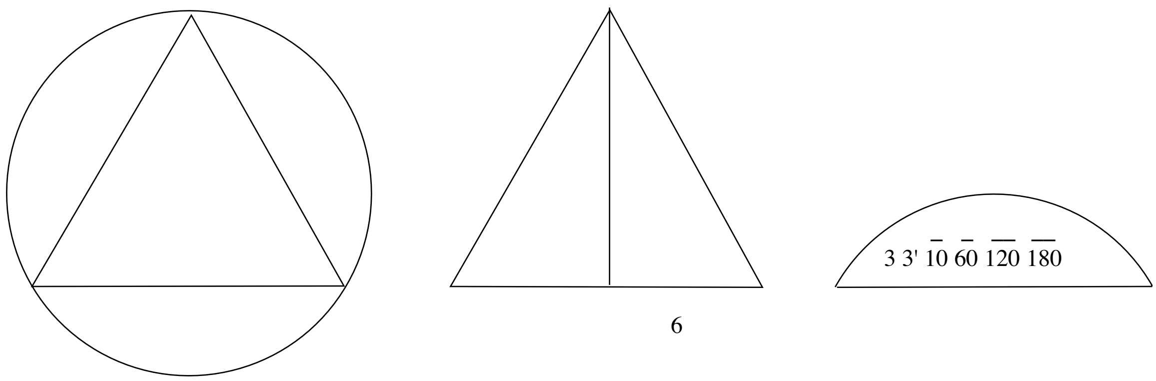


Fig. 8.2.5. Three drawings illustrating the demotic *P.Cairo* # 36.

B. In *P.Cairo* # 36, the circumference a of a circle is equal to $3 \cdot d$, where d is the diameter, and the area A is equal to $a/3 \cdot a/4$. These relations are essentially the same as in Babylonian mathematics, where $a = 3 \cdot d$ and A = ;05 · sq. a. (Note that ;05 = 1/12 = 1/3 · 1/4.) In the hieratic *P.Rhind*, on the other hand, the area of a circle is given by the totally different equation A = sq. $(d - d/9)$.

C. The calculations in *P.Cairo* # 36 have the appearance of being carried out in terms of “unit fractions” just like all calculations in Egyptian hieratic mathematical texts. Actually, however, the calculations were almost certainly carried out using Babylonian *sexagesimal arithmetic*. This is clear from the way in which the representations of numbers as sums of unit fractions use only *unit fractions that are regular sexagesimal numbers*. Here are sexagesimal interpretations of all the numbers appearing in *P.Cairo* # 36:

10 3' $\overline{20}$ $\overline{120}$	=	10 + ;20 + ;03 + ;00 30	=	10;23 30
62 3' $\overline{60}$	=	62 + ;20 + ;01	=	1 02;21
3 3' $\overline{10}$ $\overline{60}$ $\overline{120}$	=	3 + ;20 + ;06 + ;01 + ;00 30	=	3;27;30
26 6" $\overline{10}$	=	26 + ;50 + ;06	=	26;56
143 $\overline{10}$ $\overline{20}$	=	143 + ;06 + ;03	=	2 23;09
13 6" 45	=	13 + ;50 + ;01 20	=	13;51 20
41 2' $\overline{15}$	=	41 + ;30 + ;04	=	41;34
143 6" $\overline{10}$ $\overline{30}$	=	143 + ;50 + ;06 + ;02	=	2 23;58

An Old Babylonian text that is at least a partial parallel to MS 3051 is ***TMS* 1**, Bruins and Rutten, (1960), a fragment of a square hand tablet from Old Babylonian Susa. On the obverse of *TMS* 1 (Fig. 8.2.6, left), there is a drawing of what appears to be a “symmetric triangle” (in modern terms, an isosceles triangle) inscribed in a circle. The whole figure may or may not have been inscribed in a square, but that is of no importance. The inscription 50 uš ‘50, the length’ indicates that the two equal sides of the triangle measured 50 ninda. The double inscription ‘30’ and 2' sag ‘1/2 of the front’ indicates that the front of the triangle measured 2 · 30 = 1 00 ninda.

The height of the symmetric triangle is drawn as a straight line orthogonal to the front, and is accompanied by the inscription

[40 u]š sag.kak *ga-am-ru* *40*, the whole length of the peg-head (= triangle).

Note the confusing use of the same word to denote a long side of a triangle and the height of the same triangle. This is a common phenomenon in Old Babylonian geometric texts (and maybe the explanation for the fatal mistake in MS 3051, where the side of the equilateral triangle was thought to be equal to 15, where 15 was the computed length of the height of the triangle). The side of the symmetric triangle which is called ‘50, the length’ is actually the diagonal of a triangle with the sides 50, 40, 30.

The height h = 40 is called ‘the whole length’ because the center of the circle divides it in two parts. The length of one of these parts is indicated in the drawing as 8 45. The length of the radius is also given, as 31 15 uš, where in this case the use of the word uš ‘length’ is explained by the fact that the radius is the long side of the small right triangle with the sides 31 15, 30, 8 45 = 1 15 · (25, 24, 7).

The radius r and the short side q of the small right triangle were probably computed as the solution to a quadratic-linear system of equations of the following kind:

$$\text{sq. } r - \text{sq. } q = \text{sq. } 30 = 15\ 00$$
$$r + q = h = 40.$$

Such systems of equations were solved routinely by Old Babylonian mathematicians, probably with more or less silent reference to geometric arguments, instances of what may be called "metric algebra". (See, for instance, the many examples of metric algebra problems in the large text MS 5112 in Sec. 11.2 below.)

Just like the drawing on MS 30 51 of an equilateral triangle inscribed in a circle, so the drawing of a symmetric triangle inscribed in a circle on *TMS* 1 is amazingly accurate. It is also made to scale so that the proportions are correct. The whole construction shows, once again, that Old Babylonian mathematicians were familiar with the basic properties of triangles and circles.

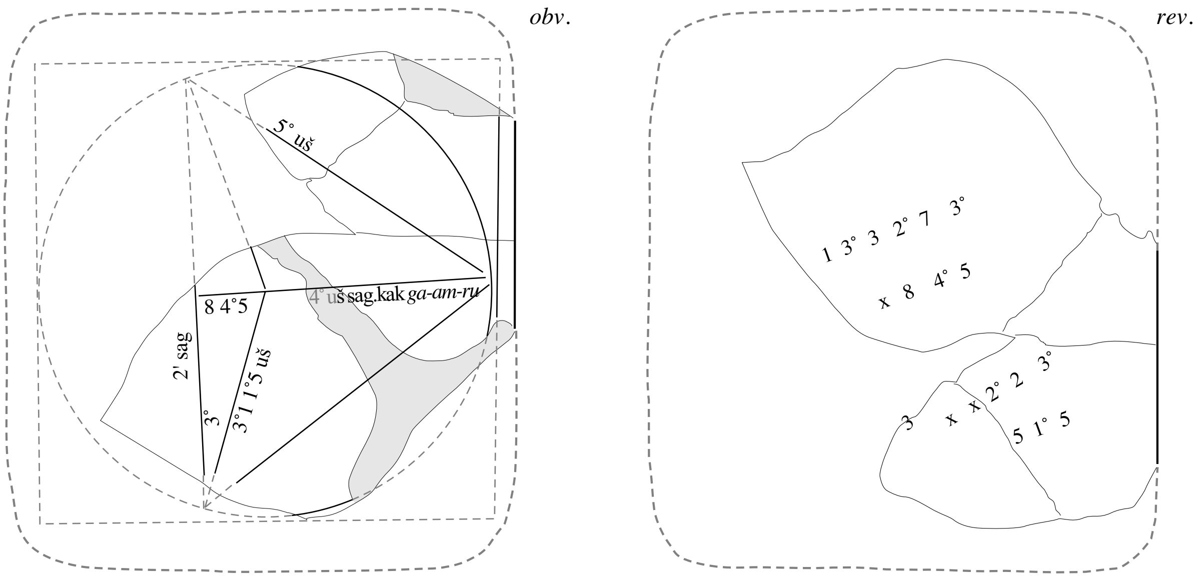


Fig. 8.2.6. *TMS* 1. A symmetric triangle inscribed in a circle. (Conform transliteration based on a photo.)

8.2 d. MS 3050. A Square with Diagonals, Inscribed in a Circle

MS 3050 (Fig. 8.2.2, bottom) is a round hand tablet with drawings and numbers on both the obverse and the reverse. The drawing on the obverse depicts a square with diagonals, inscribed in a circle. (The drawing on the reverse is similar but partly erased.) It is not easy to make sense of the numbers associated with the drawing on the obverse. Several of the numbers are carelessly written, and their positioning does not seem to relate closely to what they stand for. Anyway, the numbers inscribed on the obverse appear to be the following:

45, 26? (twice), 22 30, 16 40?, 16?, 15, 11 15 (three times), 7 30 (twice?), 5.

Several of these numbers can be explained by assuming that *the diameter* of the circle in the drawing on the obverse is 1 00 (ninda), as in Fig. 8.2.7 below.

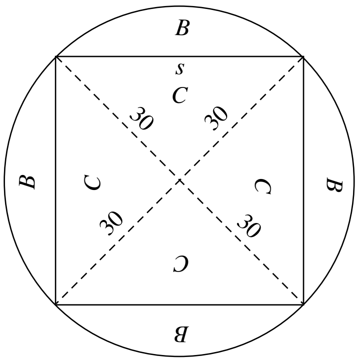
If the diameter of the circle is 1 00, then the diagonal of the square is also 1 00, and it follows that the area $2 \cdot C$ of half the square is equal to sq. 30 = 15 00. This can be shown through a simple (and well known) argument where the two quarters of the square with diagonal 1 00 are seen to form together a square with the side 30. Consequently, the area C of the quarter-square is equal to 7 30.

Again, if the diameter of the circle is 1 00, then the circumference a of the circle is equal to appr. 3 00, and the area $4 \cdot (B + C)$ of the circle is appr. 5 00 · sq. 3 = 45 00. The area of the semicircle is then $2 \cdot (B + C)$ = 22 30, and the area of the quarter circle $1 \cdot (B + C)$ is 11 15.

Thus, some of the values recorded more or less at random on the obverse of MS 3050 seem to be

45 (00)	the area of the circle with the diameter 1 00
22 30	the area of the semi-circle
11 15	the area of the quarter-circle
15 (00)	the area of the half-square
7 30	the area of the quarter-square

The area *B* of any one of the four circle segments is not computed, and the side of the square is not computed, either. Furthermore, there is not much that can be said about the drawing and the numbers on the reverse. Thus, it seems that the dawdling student who wrote this text did not finish his work. Note also that there are no obvious explanations for the numbers 26?, 16 40?, 16?, and 5, unless, of course, 5 is the constant for the area of a circle.



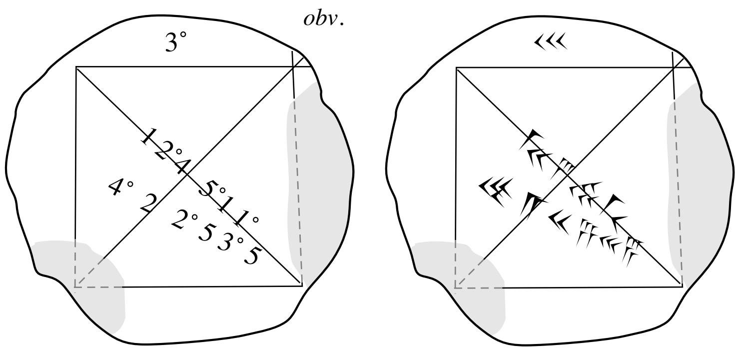
$2 \cdot C$ = sq. 30 = **15 00**
$1 \cdot C$ = **7 30**

$4 \cdot (B + C)$ = ;05 · sq. (3 · 1 00) = **45 00**
$2 \cdot (B + C)$ = **22 30**
$1 \cdot (B + C)$ = **11 15**

B = 11 15 – 7 30 = 3 45

Fig. 8.2.7. Computation of some parameters for a square inscribed in a circle of diameter 1 00.

A distant parallel to MS 3050 is the well known round hand tablet **YBC 7289**, published by Neugebauer and Sachs in *MCT* (1945), 42. It has on the obverse a drawing of a square with its diagonals. There are three recorded numbers, which can be interpreted as 30 for the side of the square, 1; 24 51 10 for an excellent approximation to sqs. 2 (see Friberg, *BagM* 28 (1997), § 8 c), and 42;25 35 for the diagonal of the square. (Note that the hand copy of YBC 7289 in *MCT* is not correctly oriented. According to the observed conventions of Old Babylonian mathematics, a square should always be drawn with one of its sides facing to the left, as in Fig. 8.2.8 below, which is based on a photo of the clay tablet.)



rev. : a drawing of a rectangle with a diagonal of length 10

sqs. 2 = appr. 1;24 51 10

Fig. 8.2.8. YBC 7289. A square with its diagonals. An excellent approximation to sqs. 2.

Remark: Closer parallels to MS 3050 are two exercises in the Egyptian demotic mathematical text *P.Cairo*, already mentioned twice above. One of these is *P.Cairo* # 37 (Friberg, *UL* (2005), Sec. 3.1 k), where a square is inscribed in a circle of diameter d = 30 cubits and area A = 675 (= 3/4 · sq. 30) sq. cubits. Here is a brief presentation of the computations in that exercise, with all Egyptian sums of unit fractions appearing in the text converted into equivalent sexagesimal numbers:

1) The area of the inscribed square with diagonal 30 is $B = 30 \cdot 30/2 = 450$.
2) The side of the square is s = sqs. 450 = appr. 21 1/5 1/60 (21;13).
3) Verification: sq. 21 1/5 1/60 = (appr.) 450. (In sexagesimal numbers: sq. 21;13 = 7 30;08 49.)
4) The height of one of the four circle segments is $p = (30 - 21\ 1/5\ 1/60)/2 = 4\ 1/3\ 1/20\ 1/120$ (4;23 30).
5) The area of one of the four circle segments is C = appr. $(s + p)/2 \cdot p$ = (sic!) 56 1/4 (56;15).
6) Verification: The area of the circle is $B + 4 \cdot C = 450 + 4 \cdot 56\ 1/4 = 675$.

It is interesting that the student who made this computation cheated in step 5. He obviously *counted backwards from the expected answer* to get the exact value 56;15 for the area of a circle segment. This kind of cheating appears occasionally also in Old Babylonian mathematical texts

Remark: The attempted computations in MS 3051 and MS 3050 of the areas of circle segments cut off from a circle by an inscribed equilateral triangle or an inscribed square were not very successful. Nevertheless, it is known that at Old Babylonian mathematicians *could* compute correctly the areas of such circle segments. This is obvious in view of the following six entries from the Old Babylonian table of constants **BR = *TMS* 3**, Bruins and Rutten (1961):

13 20	igi.gub	*šà*	a.šà še	13 20	constant	of	a barley-corn field	BR 16
56 40	dal	*šà*	a.šà še	56 40	transversal	of	a barley-corn field	BR 17
23 20	*pi-ir-ku*	*šà*	a.šà še	23 20	cross-line	of	a barley-corn field	BR 18
16 52 30	igi.gub	*šà*	igi.gu$_4$	16 52 30	constant	of	an ox-eye	BR 19
52 30	dal	*šà*	igi.gu$_4$	52 30	transversal	of	an ox-eye	BR 20
30	*pi-ir-ku*	*šà*	igi.gu$_4$	30	cross-line	of	an ox-eye	BR 21

What is going on here is explained in detail in the commentary to problem # 1 in the Late Babylonian mathematical recombination text W 32291-x (Friberg, Hunger, and Al-Rawi, *BagM* 21 (1990).) The "barley-corn field" is an oval figure composed of two circle segments glued together, in the case when the circle segments are those cut off from a circle by an inscribed square. Similarly, the "ox-eye" is an oval figure composed of two circle segments joined together, in the case when the circle segments are those cut off from a circle by an inscribed equilateral triangle. In both cases, the figures are evidently thought of as normalized in the sense that they are bounded by circular "arcs" of length $a = 1$ (00). The "constant" of such a figure is its area, the "transversal" d is its long diameter (the common base of the joined circular segments), and the "cross-line" p is its short diameter (orthogonal to the transversal).

It can be left to the reader to check that the 6 constants BR 16-21 are correctly computed. Briefly, the first three can be explained as follows, with sqs. 2 = appr. 1;25 (17/12):

$A = 2 \cdot 1/4 \cdot (A_{\text{circle}} - A_{\text{square}})$ = appr. $2 \cdot 1/4 \cdot$ (;05 · sq. $(4 \cdot a)$ – 1/2 · sq. (1;20 a)) = ;13 20 (2/9) · sq. a
d = sqs. 2 · ;40 · a = appr. 1;25 · ;40 · a = ;56 40 (17/18) · a
$p = 2 \cdot 1/3 \cdot (2 -$ sqs. 2$) \cdot a$ = appr. ;23 20 (7/18) · a.

Similarly, the last three can be explained as follows, with sqs. 3 = appr. 1;45 (7/4):

$A = 2 \cdot 1/3 \cdot (A_{\text{circle}} - A_{\text{triangle}})$ = appr. $2 \cdot 1/3 \cdot$ (;05 · sq. $(3 \cdot a)$ – 3/16 · 1;45 · sq. a) = ;16 52 30 (27/64) · sq. a
d = sqs. 3 · 1/2 · a = appr. ;52 30 (7/8) · sq. a
p = appr. $2 \cdot (1/2 - 1/4) \cdot a$ = ;30 (1/2) · a.

8.2 e. MS 2985 A Circle in the Middle of a Square

MS 2985 (Fig. 8.1.1, middle) is a square hand tablet with a drawing and numbers on the obverse. The reverse is empty. The drawing shows a square and circle inside it, a distance away from the sides of the square. The numbers are written in an inexperienced hand and are difficult to read. They seem also to be placed more or less at random on the obverse, without any obvious connection to the drawing. For this reason, it would have been difficult to find a meaningful interpretation of MS 2985 if it had not been for the existence of a couple of partially parallel texts.

The first of these partial parallels is **YBC 7359**, a square hand tablet with drawings on the obverse and the reverse of a couple of "concentric squares" with associated numbers. The drawing on the obverse is clearly a teacher's neat model, while the drawing on the reverse is a student's clumsy copy. The conform transliteration in Fig. 8.2.9 below is based on photos of the clay tablet published by Nemet-Nejat, *UOS* (2002), 275-276.

Nemet-Nejat correctly observes (*op. cit.*, 262) that the inscribed numbers are related to each other in various ways but can offer no further explanation of the text, probably because she interprets the figures as rectangles. However, a clue to the meaning of the text is the observation that the number 9 inscribed within the inner figure is the square of 3, while the number 1 40 inscribed above the outer figure is the square of 10. (The numbers 10 and 3 themselves are inscribed to the left of the drawings on the obverse and the reverse, respectively.) Therefore, it seems reasonable to start an analysis of the text with the assumption that the two figures are a couple of concentric squares with the sides 3 and 10, respectively. That this is a correct assumption becomes clear when it turns out that then the area of the region between the squares is

$$\text{sq. } 10 - \text{sq. } 3 = 1\ 40 - 9 = 1\ 31.$$

The number 1 31 is inscribed between the squares, to the left, in the drawings on both the obverse and the reverse. Furthermore, the distance between the two squares can be computed as

$$(10 - 3)/2 = 7/2 = 3;30.$$

The number 3 30 is recorded in both drawings, between the outer and the inner square, above and below the inner square.

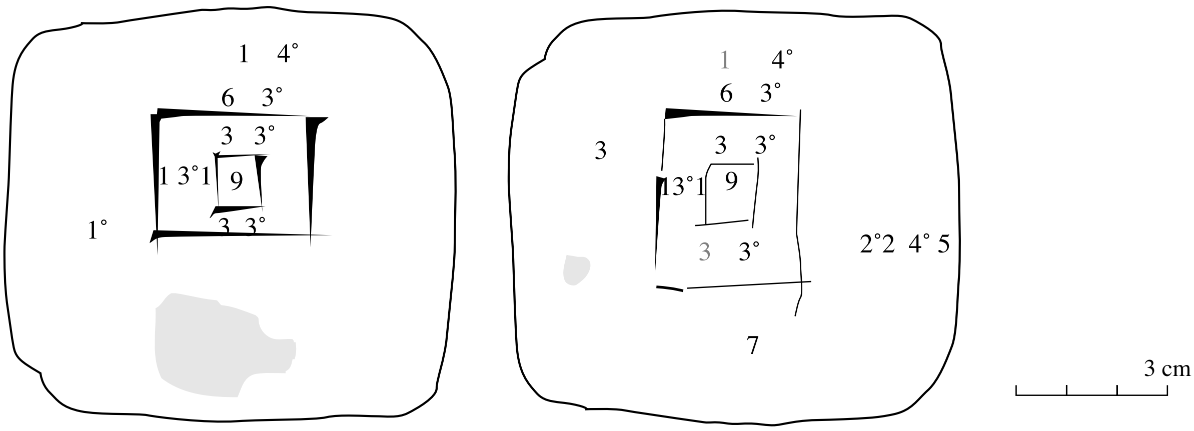


Fig. 8.2.9. YBC 7359. A teacher's and a student's drawings of concentric squares.

It is likely that the drawings were meant to illustrate a metric algebra problem, which may have been of the following form:

> The area *A* between two (concentric) squares is 1 31. The distance *d* between the squares is 3;30.
> Find the sides and the areas of the squares.

It was known (cf. Fig. 8.2.3 above) that the region between two concentric squares with the sides p and q, respectively, can be divided into four rectangles, all four with the long side $(p + q)/2$ and the short side $(p - q)/2 = s$. Therefore, the area A of the region between the squares, a "square band" with the middle length u, can be expressed as

$$A = 4\ u \cdot s, \quad \text{where} \quad u = (p + q)/2.$$

If both A and s are known, this equation can be used to determine the value of $(p + q)/2$. Thus, with $A = 1\ 31$ and $s = 3{,}30$ as in YBC 7350, p and q can be found as the solutions to the equations

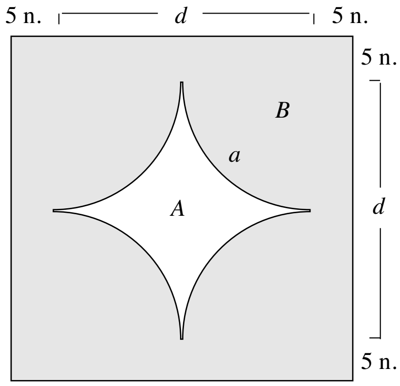
$$(p + q)/2 = A/4 \cdot 1/s = 22;45/3;30 = 6;30, \quad (p - q)/2 = s = 3;30.$$

Note that the number $A/4 = 22\ 45$ is inscribed to the right of the drawing on the reverse of YBC 7359, and that the number 6 30 appears above the drawings on both sides of the clay tablet. Hence,

$$p = 6;30 + 3;\ 30 = 10, \quad \text{and} \quad q = 6;30 - 3;30 = 3.$$

Another partial parallel to MS 2985 is the fragment ***TMS* 21** (Bruins and Rutten (1961)), a difficult problem text that was only recently explained by Muroi in *SCIAMVS* 1 (2000). According to Muroi, the problem is concerned with an *apsammikku,* a 'sound-hole', or what may be called a "concave square" (see Friberg, *RlA 7* (1990) Sec. 5.4 g). A concave square is the figure bounded by four circular arcs which remains when four touching quarter-circles have been removed from the four corners of a square.

In ***TMS* 21 a** (Fig. 8.2.10), a concave square is inscribed in the middle of a square, at a distance of 5 ninda from all the four sides of the square. The a.šà dal.ba.na, the 'field between', bounded on one side by the square and on the other side by the concave square, is given as 35 (00 sq. ninda).



d = 1;20 a

A = ;26 40 sq. a, B = 35 00 sq. ninda

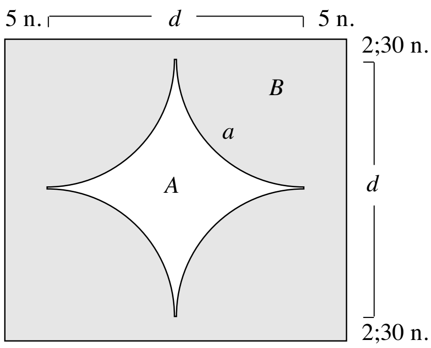
Equation:

sq. (d + 2 · 5 ninda) = A + B

Solution:

a = 30 ninda, the sides of the square are 50 ninda.

Fig. 8.2.10. *TMS* 21 a. A concave square inscribed in a square, 5 ninda away from the sides of the square.



d = 1;20 a

A = ;26 40 sq. a, B = 30 50 sq. ninda

Equation:

(d + 2 · 5 ninda) · (d + 2 · 2;30 ninda) = A + B

Solution:

a = 30 ninda, the sides of the rectangle are 50 and 45 ninda.

Fig. 8.2.11. *TMS* 21 b. A concave square inscribed in a rectangle, 5 n. from the front, 2;30 n. from the length.

The following basic parameters for an *apsammikku* are given in the table of constants BR:[6]

26 40	igi.gub	*šà*	*a-pu-sà-am-mi-ki*	26 40	constant	of	a sound-hole	BR 22
1 20	bar.dà	*šà*	*a-pu-sà-mi-ki*	1 20	cross-over (diagonal)	of	a sound-hole	BR 23
33 20	*pi-ir-ku*	*šà*	*a-pu-sà-mi-ki*	33 20	cross-line	of	a sound-hole	BR 24

The dimensions of the square and the concave square are determined in the following way:

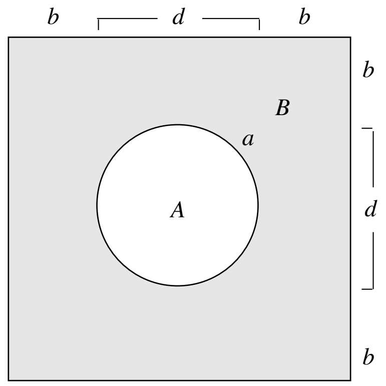
In a normalized concave square, the arc a = 1 00, the diameter d = 1 20, and the area A = 26 40.
In an arbitrary concave square with arc a, the diameter d = 1;20 · a, and the area A = ;26 40 · sq. a.
Equation: sq. (1;20 · a + 2 · 5 ninda) – ;26 40 · sq. a = 35 00 sq. ninda.
The (positive) solution to this quadratic equation is a = 30 ninda.
Hence, d = 1;20 · a = 40 ninda, and the side of the square is d + 2 · 5 ninda = 50 ninda.

***TMS* 21 b** (Fig. 8.2.11) is even less well preserved than *TMS* 21 a, but what remains of the text seems to suggest that in this second exercise the object considered was a concave square inscribed in a *rectangle*, 2 1/2

6. The reading bar.dà, as a variant spelling for Sum. bar.da 'cross-bar', was suggested by Muroi in *HSJ* 2 (1992). Note that the cuneiform sign bar is an upright wedge crossed over orthogonally by a horizontal wedge.

ninda away from the front (the short side) of the rectangle, and 5 ninda away from the length (the long side) of the rectangle.

It is likely that the drawing on MS 2985 is meant to illustrate a similar problem for a circle inscribed in a square a given distance away from the sides of the square. If that is so, then the situation can be described by the diagram in Fig. 8.2.12 below, which mimics the diagram in Fig. 8.2.10.



d = ;20 a

A = ;05 sq. a

Equation:

sq. ($d + 2 \cdot b$) – A = B = 39 36;33 45 (?).

Solution:

d = 22;30 ninda, b = 15 ninda, the sides of the square = 52;30 ninda.

Fig. 8.2.12. MS 2985. A circle inscribed in a square, a distance b away from the sides of the square.

Now, according to Old Babylonian mathematical conventions, the main parameter of a circle is its 'arc' a, the circumference of the circle. The diameter d and the area A of the circle are given by the following equations:

$$d = ;20 \cdot a \ (= a/3), \quad A = ;05 \cdot \text{sq. } a = ;45 \cdot \text{sq. } d.$$

Therefore, if both the area B between the circle and the square and the distance b are given, then the value a of the arc of the circle can be found as the solution to the following quadratic equation:

$$\text{sq. } (;20 \cdot a + 2 \cdot b) - ;05 \cdot \text{sq. } a = B.$$

An equation like this would be given a geometric interpretation by Old Babylonian mathematicians and would then routinely be solved through a geometric procedure equivalent to our "completion of the square". Translated to modern symbolic notations, the essential steps of that geometric procedure, in the present case, would be the following: First, since sq. ;20 = ;06 40 and ;06 40 – ;05 = ;01 40, the quadratic equation above can be reduced to:

$$;01\ 40 \cdot \text{sq. } a + 1;20 \cdot b \cdot a = B - \text{sq. } (2 \cdot b).$$

Next, since ;01 40 = sq. ;10, this equation can be further reduced to

$$\text{sq. } (;10 \cdot a + 4 \cdot b) = B + 12 \cdot \text{sq. } b.$$

And so on.

If this is really what is going on in the case of MS 2985 (see again Fig. 8.1.1, middle), then it should be possible to identify some of the numbers recorded on the obverse of MS 2985 as square numbers. There is one obvious candidate, since the numbers near the left side of the square are 30, 30, and 15, clearly a notation meaning that sq. 30 = 15 (00). Another likely candidate is the number 45 56 15 inscribed just above the circle, since 45 56;15 = sq. 52;30.

Now, suppose, for instance, that the following identification can be made:

$$45\ 56;15 \text{ sq. ninda} = \text{sq. } (;20 \cdot a + 2 \cdot b) = \text{the area of the square containing the circle.}$$

It then follows that

$$d + 2 \cdot b = ;20 \cdot a + 2 \cdot b = \text{sqs. } 45\ 56\ 15 \text{ sq. ninda} = 52;30 \text{ ninda} = \text{the side of the square.}$$

As a matter of fact, the value 52 30 appears to be inscribed over the upper left corner of the square.

Furthermore, the value of b can probably be identified with the number 15 which is inscribed both to the left and to the right of the circle, inside the square, in the drawing on the obverse of MS 2985. Now, it follows from the suggested identifications that $d + 2 \cdot b = d + 30 = 52;30$ ninda, and that

$$d = 22;30 \text{ ninda}.$$

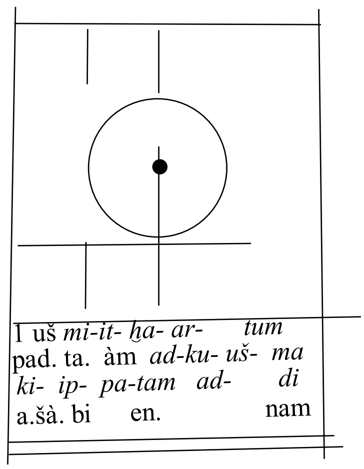
Note that this means that $2 \cdot d = 3 \cdot b$. The indicated values for d and b imply that

$$a = 3 \cdot d = 1\ 07;30 \text{ ninda}, \quad \text{and} \quad B = \text{sq. } 52;30 - ;05 \cdot \text{sq. } 1\ 07;30 = 45\ 56;15 - 6\ 19;41\ 15 = 39\ 36;33\ 45.$$

A final possible identification is that the numbers 30, 30, 15 to the left of the square in the drawing stand for the computation sq. $(2 \cdot b)$ = sq. 30 = 15 (ninda).

Note: Although the suggested interpretation of MS 2985 appears to be quite plausible, it is disturbing that it does not take any account of four of the numbers recorded on the clay tablet (25 and 3 20 to the left of the square and 2 16 45 and 2 13 45 right above 45 56 15). The reason may be that the proposed interpretation is not correct, or it may be that the student who wrote the text made some mistakes in his calculations (although it is not clear what those mistakes can have been).

Maybe it is an unwarranted assumption that MS 2985 is related to *TMS* 21 b, the problem for a concave square in a square, which leads to a quadratic equation. An alternative, simpler explanation of MS 2985 is that the drawing was associated with the problem of finding the area between the circle and the square, when the side of the square and the distance from the circle to the sides of the square are given. This is a problem that is known from the third example on the well known large geometric theme text BM 15285. (See, most recently, the excellent hand copy of the whole text in Robson, *MMTC* (1999), App. 2.)



BM 15285, col. *i*, pr. 3:
1 uš (= 1 00 ninda) the equalside (= square).
A bit(?) each way I thrust (inwards),
an arc (= circle) I drew.
Its area is what?

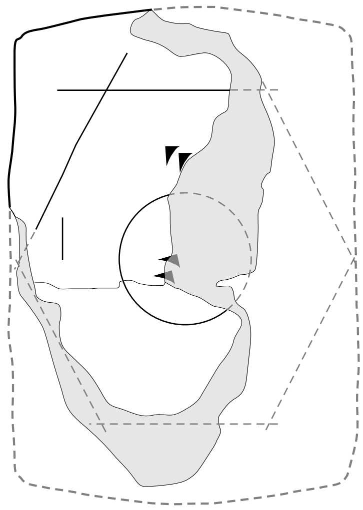
Fig. 8.2.13. One of (probably) 41 illustrated problems on the large geometric theme text BM 15 285.

There are ten columns on BM 15285, five on the obverse and five on the reverse, with four or five small exercises in each column. The exercises are all of a common format, always with a drawing of a square divided into smaller pieces by straight lines and circular arcs, and under each subdivided square there is always a caption with a brief text specifying the construction of the small pieces, naming them, and asking for their areas. In col. *i*, the third problem (Fig. 8.2.13 above) is illustrated by a drawing of a circle inscribed in a square, a distance away from the sides of the square. The exact meaning of the text is not clear, but it seems to say that the side of the square is 1 00 ninda and that the student is asked to thrust inwards from the side of the square equally much on each side and then draw the circle. His task then is, as in all exercises on BM 15285, to compute the areas of all parts of the subdivided square.

8.2 f. MS 1938/2. A Circle in the Middle of a 6-Front

MS 1938/2 is a fragment of a rectangular clay tablet, with unrelated drawings on its obverse and reverse. On the obverse, there is a drawing of a six-striped trapezoid with associated numbers (see Fig. 8.1.12). On the reverse (Fig. 8.2.14 below) there is a badly damaged drawing, which may be interpreted as what remains of a drawing originally depicting a a regular hexagon, with a circle inside it a certain distance away from the sides of the hexagon.

rev.



obv.: a 6-striped trapezoid (Fig. 8.1.12)

Fig. 8.2.14. MS 1938/2, *rev*.: A circle inscribed in a regular 6-front, some distance away from the sides.

This is an assignment of the same type as the one with a circle inside a square on MS 2985 (Figs. 8.1.1 and 12). Now, if the drawing on MS 1938/2 is connected with a problem where the area between a regular hexagon and a circle is one of the given parameters, then the one who designed the problem must have known the area of a regular hexagon. The following three entries in the Old Babylonian table of constants **BR =** ***TMS*** **3** demonstrate that, indeed, Old Babylonian mathematicians were familiar with regular polygons and had methods they used to compute the areas of such polygons:

1 40	igi.gub	šà	sag.5	1 40	the constant	of a	5-front	BR 26
2 37 30	igi.gub	šà	sag.6	2 37 30	the constant	of a	6-front	BR 27
3 41	igi.gub	šà	sag.7	3 41	the constant	of a	7-front	BR 28

The area of a *normalized regular hexagon*, with the length of each side equal to 1 00, can be computed as *the sum of the areas of six normalized equilateral triangles*. Explicitly,

A_6 = 6 · (sqs. 3/ 4) · sq. 1 00 00 = appr. 6 · ;26 15 · 1 00 00 = 6 · 26 15 = 2 37 30.

Thus, the entry in line 27 of BR = *TMS* 3 is the Old Babylonian approximation to the area of a (normalized) regular hexagon. Apparently, '6-front' was the Babylonian name for a regular hexagon.

It is just as easy to compute the area of a *normalized regular pentagon*. If the side of the pentagon is 1 00, then the circumference of the circumscribed circle is approximately equal to 5 00. Consequently, the diameter of the circle is approximately equal to 1/3 · 5 00 = 1 40. The area of the regular pentagon can therefore be computed as the sum of the areas of five symmetric triangles with one side equal to 100 and the other two equal to 50 (the radius of the circumscribed circle). The height in each triangle is easily computed and is equal to 40. Hence, the area of a normalized regular pentagon is:

A_5 = appr. 5 · 30 · 40 = 5 · 20 00 = 1 40 00.

Thus, the entry in line 26 of *TMS* 3, is the Old Babylonian approximation to the area of a (normalized) regular pentagon, called a '5-front'.

Similarly, in the case of a *normalized regular heptagon*, the circumference of the circumscribed circle is approximately equal to 7 00. The diameter is then approximately equal to 1/3 · 7 00 = 2 20, so that the radius will be 1 10. The height can then be computed as

h_7 = sqs. (sq. 1 10 – sq. 30) = sqs. 1 06 40 = appr. 1 00 + 6 40/2 00 = 1 03;20.

Hence, the area of a normalized heptagon is:

$$A_7 = \text{appr. } 7 \cdot 30 \cdot 1\ 03;20 = 7 \cdot 31;40 = 3\ 41;40 = \text{appr. } 3\ 41.$$

Thus, the entry in line 28 of *TMS* 3, is the Old Babylonian approximation for the area of a (normalized) regular heptagon, called a '7-front'.

***TMS* 2** (Fig. 8.2.15 below) is a square hand tablet with drawings of a 6-front on the obverse and a 7-front on the reverse. Circumscribed circles appear to have been drawn by use of a compass as an aid for the construction, then erased when they were no longer needed. Only vague traces of the circles are now remaining.

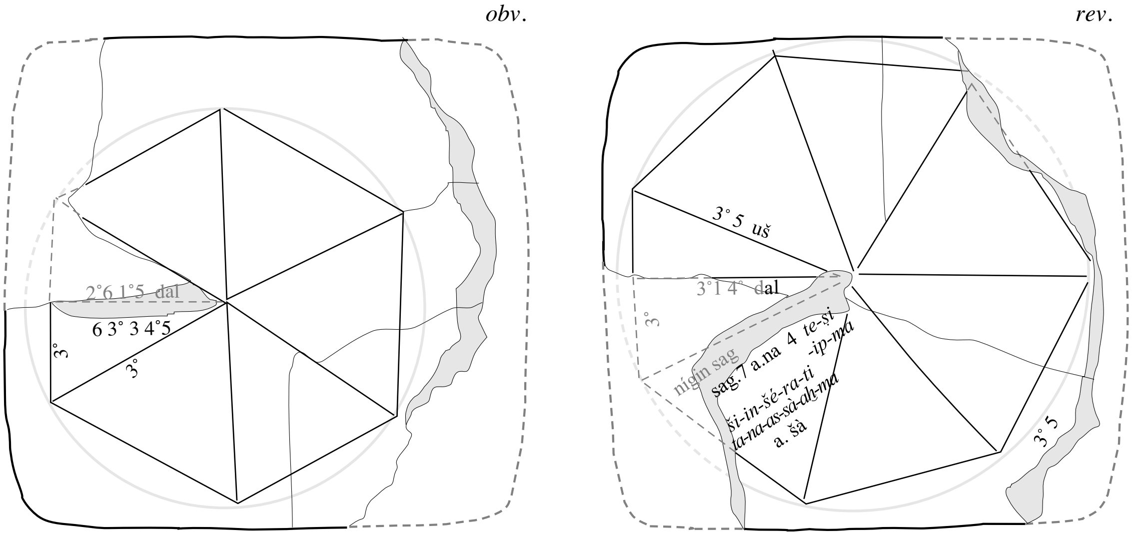


Fig. 8.2.15. *TMS* 2. A '6-front' and a '7-front', with methods for the computation of their areas. (The conform transliterations are based on Bruins' photos of the clay tablet.)

When the area of a *normalized* geometric figure was given as an entry in an Old Babylonian table of constants, it was silently understood that *the areas of similar geometric figures are proportional to the squares of their basic lengths*. In the case of an *n*-front, for instance, with $n = 3$ (an equilateral triangle), 4 (a square) 5, 6, or 7, the area is proportional to the square of the side. On the obverse of *TMS* 2, it is indicated that the length of the upper front (the left-most side) of the figure is 30, while the upper front of a *normalized* 6-front is 1 00, twice as much. Therefore, the area of the 6-front in the drawing is one fourth of the area of a normalized 6-front. Also, the area of each one of the six equilateral triangles in the 6-front is one fourth of the area of a normalized equilateral triangle:

$$A(\text{triangle}) = \text{appr. } 1/4 \cdot 26\ 15 = 6\ 33;45.$$

This is the value recorded inside the upper equilateral triangle in the drawing on *TMS* 2, *obv*.

The length of the upper front of the 7-front on the reverse of *TMS* 3, was also set equal to 30, although the number probably inscribed close to the upper front is no longer present. As a consequence, the length of the circumference of the circumscribed circle was equal to (approximately) $7 \cdot 30 = 3\ 30$, so that the diameter was 1 10, and the radius 35. The area of the 7-front could then be computed as the sum of the areas of 7 symmetric triangles with the front 30 and the length 35. The notation 35 uš '35, the length' is still readable close to one side of the upper triangle.

The next step of the process was to compute the height of the upper triangle as h_7 = appr. 31;40 (= 0 ;30 · 1 02;30). A notation under the central line in the upper triangle in the drawing of the 7-front, 31 40 dal '31;40, the transversal', is almost completely destroyed. Only the last half of the sign dal is preserved.

One would now expect to find the total area of the upper triangle and the area of the whole 7-front recorded in the drawing. This does not happen. Instead one finds a somewhat cryptic inscription, interpreted as follows by Robson in *MMTC* (1999), 49:

[nígin sag] sag.7 a.na *4 te-ṣi- / ip-ma*	*The square of the front* of the 7-front by 4 you repeat, then
ši-in-šé-ra-ti / ta-na-as-sà-aḫ-ma / a.šà	the twelfth you tear out, then the field (= area).

What this means is that the area of a 7-front (regular heptagon) can be computed as

$$A_7 = \text{appr. sq. } s \cdot (4 - ;05 \cdot 4) = \text{sq. } s \cdot (4 - ;20) = \text{sq. } s \cdot 3;40.$$

In other word, you get the area of the 7-front if you first multiply the square of the front by 4, then reduce the result by a twelfth of its value. This computation rule is a handy variant of the more formal computation rule

$$A_7 = \text{sq. } s \cdot 3;40,$$

which can be compared with the entry '3 41 the constant of a 7-front' in BR = *TMS* 3.

8.3. Labyrinths, Mazes, and Decorative Patterns

8.3 a. MS 4515. A Babylonian Square Labyrinth

A labyrinth can be defined as *a convoluted path from the exterior of a square or a circle to its center*. Until now, it has been common knowledge that the idea of a labyrinth can be traced no further back in time than to the Minoan and Mycenaean civilizations on Crete and in mainland Greece, in the latter half of the second millennium BC. Now, however, two clay tablets in the Schøyen Collection suggest that, maybe, the idea of a labyrinth has a Mesopotamian origin. There is no way of dating those clay tablets, but since the overwhelmingly great majority of the mathematical clay tablets in the Schøyen Collection are unmistakably Old Babylonian, it is quite likely that the labyrinth texts, too, are Old Babylonian, hence from the first half of the second millennium BC.

In any case, it is interesting that the Babylonian labyrinths are definitely not of the same type as the "Greek labyrinth" from Crete or Greece (see below). Therefore, even if the presumably Old Babylonian labyrinths are older than the Greek labyrinth, it does not follow that the Greek labyrinth was directly inspired by its Babylonian predecessor. The two can very well have been invented independently (even if this is unlikely, since interesting ideas tend to spread rapidly).

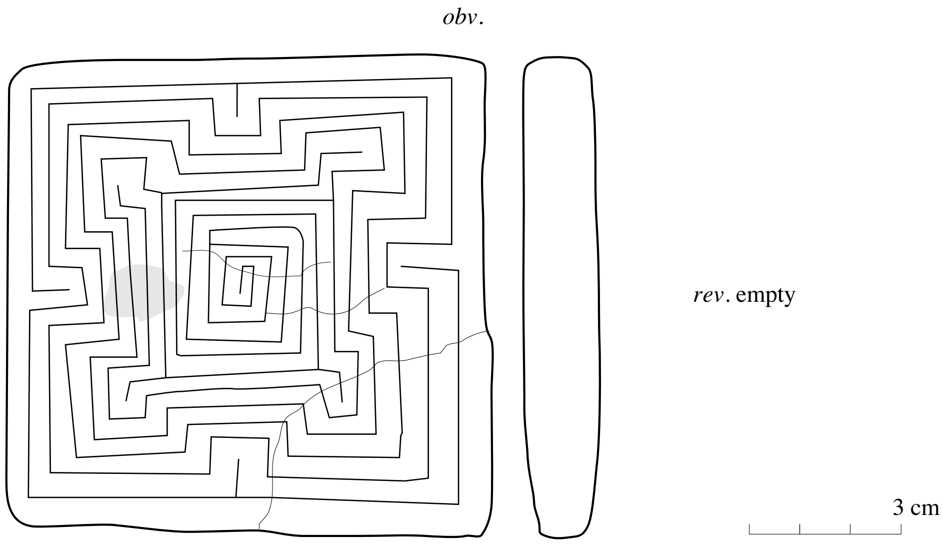


Fig. 8.3.1. MS 4515. The Babylonian square labyrinth with two paths, one good and one bad.

MS 4515 (Fig. 8.3.1 above) is a square clay tablet with a drawing of a labyrinth filling out all available space on the obverse. This labyrinth, in the following called "the Babylonian square labyrinth" appears to be in the form of a fortified city with a city gate in each of the four faces of the city wall. Two of the gates are closed, the other two are open. The path that enters through one of the open gates (the right gate if the clay tablet is oriented as in Fig. 8.3.1), arrives at the center of the square after 2 clockwise loops parallel to the city walls, followed by 4 counter-clockwise loops. The path that enters through the other open gate (the left gate in Fig. 8.3.1) ends in a blind alley after 2 clockwise loops followed by 2 counter-clockwise loops. The five outermost loops of either path are formed as squares with indentations near the four gates. In Fig. 8.3.2 below, the course of the good path is shown in the diagram to the left, while the course of the bad path is shown to the right.

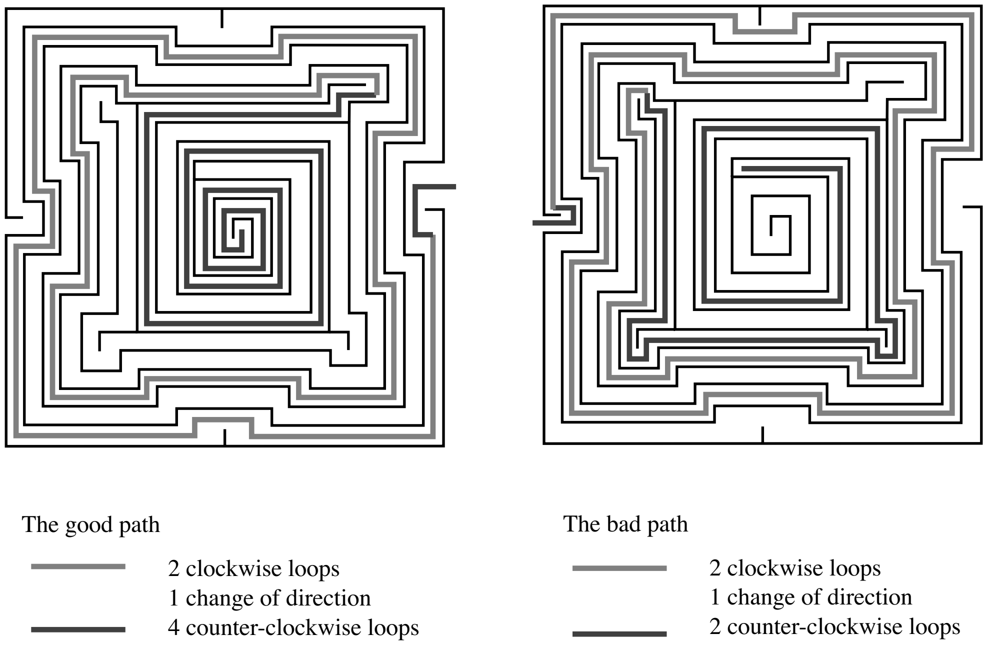


Fig. 8.3.2. The Babylonian square labyrinth. Two paths, only one leading to the center.

It is obvious that it would be difficult to draw the Babylonian labyrinth neatly, in the way it is drawn on MS 4515, without an efficient algorithm for its construction. One such algorithm is demonstrated in a series of six diagrams in Fig. 8.3.3 below. The basic idea is to start with two separate parts of the outer wall of the square labyrinth, on either side of the open gates, and then successively continue the two parts of the wall into the interior of the square, forming segments of inner walls at a constant distance from each other, by use of *a double recursive algorithm*. Two steps at a time of the algorithm are shown in the first five of the diagrams in Fig. 8.3.3. The final result is shown in the sixth diagram.

Here follows a detailed account of how the complicated algorithm operates:

1. Start at the left gate and draw *clockwise, in black,* the upper half of the outer wall.
2. Start at the right gate and draw *clockwise, in grey,* the lower half of the outer wall. Continue to the right gate.
3. Continue the *black* line from the right gate a full loop back to the right gate.
4. Continue the *grey* line from the right gate a full loop back to the right gate.
5. Continue the *black* line from the right gate as part of a loop to the dead end in the upper right corner.
6. Continue the *grey line* from the right gate as part of a loop to the dead end in the upper left corner.
7. Start *a new black line* in the upper left corner and continue *counter-clockwise* to the upper right corner.
8. Start *a new grey* line in the upper right corner and continue *counter-clockwise* back to the upper right corner.
9. Continue the *black* line from the upper right corner a full loop back to the upper right corner.
10. Finish the *grey* line with a short segment and the dead end for the bad path.
11. Finish the *black* line with a spiral in to the center of the square.

The complexity of the construction clearly demonstrates that the drawing of a labyrinth on MS 4515 is not just a meaningless doodle. It is much more likely that this Babylonian labyrinth was the end result of a long series of experiments with various more or less advanced designs with a common objective: to produce one connected path (or two) from the periphery to the center, so convoluted that it is difficult to follow the meanderings. It is conceivable that drawings of that type could draw on experience gathered from executing more conventional and better documented types of drawings on Mesopotamian clay tablets, such as house plans, city maps, serpentine patterns, and so on.

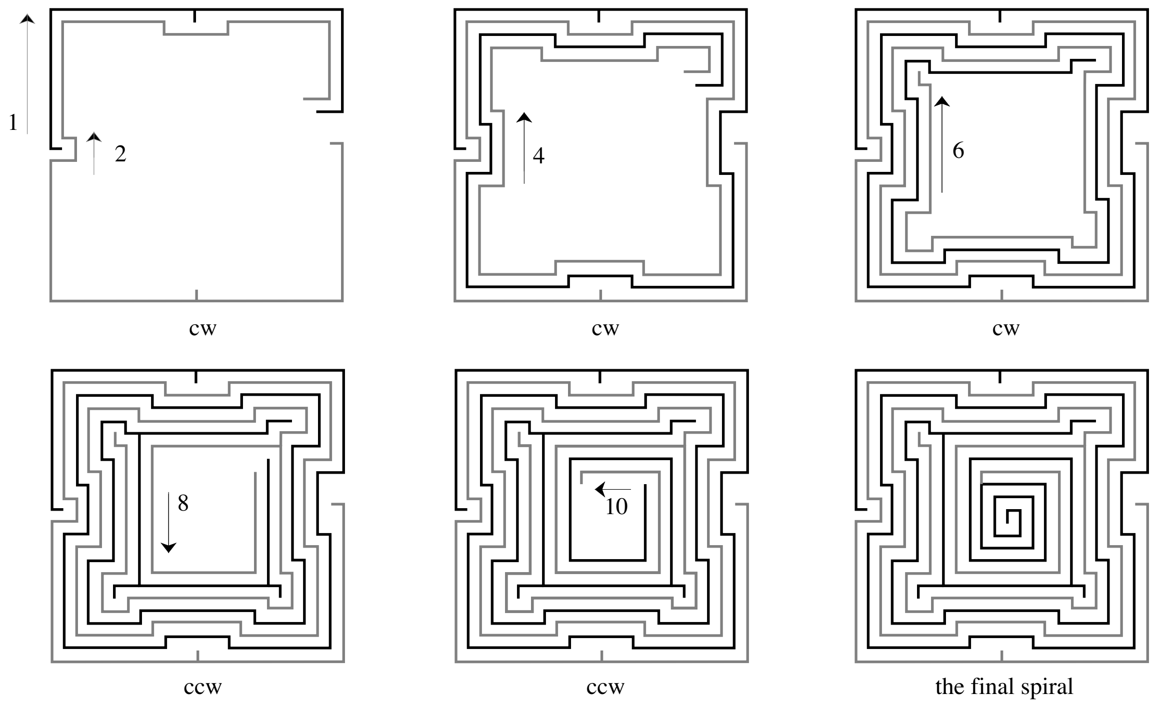


Fig. 8.3.3. The Babylonian square labyrinth. Construction in 11 steps, beginning with the outer walls.

8.3 b. The Greek Labyrinth

The proposed algorithm for the construction of the Babylonian square labyrinth can be compared with a well known algorithm for the construction of the Greek labyrinth. That algorithm starts with a central "core" in the form of a double cross with points in the four corners formed by the outer arms of the double cross:

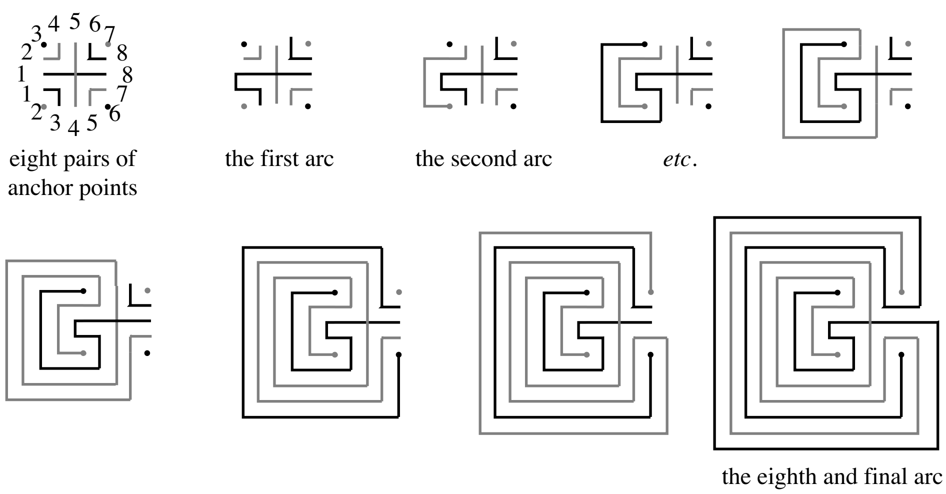


Fig. 8.3.4. The Greek labyrinth. Construction in 9 steps, beginning with the central core.

Eight successive parts of the walls of the labyrinth are then constructed, one at a time, proceeding outwards, by use of the simple idea that the eight numbered points or endpoints of cross arms in the core should be connected to the nearest available point or endpoint to the right by means of a single counter-clockwise arc.

The circumstance that the construction of the Greek labyrinth is simpler than the construction of the Babylonian labyrinth is related to the fact that there is only one path leading into the Greek labyrinth compared to the two paths leading into the Babylonian labyrinth. On the other hand, the one path in the Greek labyrinth is in a certain sense more complicated (see Figs. 8.3.5 and 8.3.12). It proceeds alternatively in four clockwise and three counter-clockwise loops. In addition, the first three loops move from the middle of the labyrinth towards the outer wall, the fourth loop stays in the middle, and the last three begin at the center and move towards the middle.

The difference between the construction of the Babylonian labyrinth on one hand and the Greek labyrinth on the other is accentuated by the drawings in Fig. 8.3.5, which show simplified versions of both labyrinths, with minimal numbers of loops. The simplified Babylonian labyrinth consists, essentially, of two parallel spirals, proceeding inwards from the two gates, but with only one of the spirals actually reaching the center, and with the endpoint of the second spiral resting on a point of the first spiral. The good path through this labyrinth is also a spiral, proceeding from one of the gates all the way to the center of the square.

The simplified Greek labyrinth consists of two spirals crossing each other at one point. There is only one path from the gate to the center of the construction, a spiral that changes direction whenever it comes near the point where the two spirals cross each other.

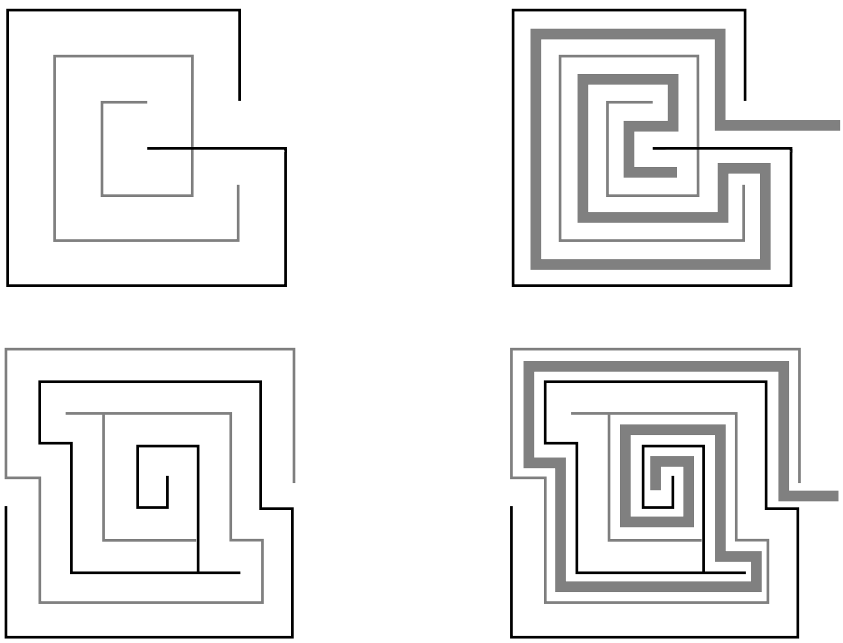


Fig. 8.3.5. The different basic ideas behind the constructions of the Greek and the Babylonian labyrinths.

A Greek labyrinth of a square form appears on the reverse of a clay tablet from king Nestor's palace at Pylos (**Cn 1287**, Athen's National Museum). There is an unrelated Greek inscription in Linear B on the obverse. Two Greek labyrinths of a circular form appear as decorations on two fragments of a clay bowl found at Tell Rifa'at in Syria. Both the clay tablet and the bowl are from the 13th century BC (see Kern (2000 (1982)), nos. 102-104) and consequently (probably) younger than the Babylonian labyrinth on MS 4515.

(The conform transliteration of Cn 1287 in Fig. 8.3.6 below is based on Kern's photo of the clay tablet.)

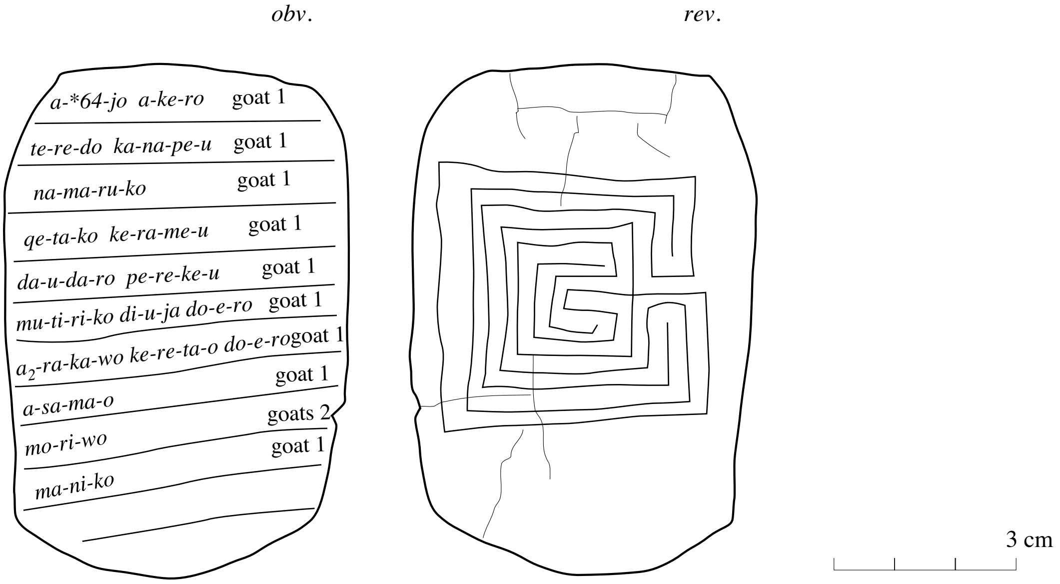


Fig. 8.3.6. Cn 1287. A Mycenaean clay tablet with the oldest known representation of the Greek labyrinth.

A drawing of a kind of double spiral on the round hand tablet **VAT 744** may be a precursor of the Babylonian labyrinth. However, according to Kern, *Labyrinthe* (2000 (1982)), the clay tablet may be Late Babylonian, and the drawing is not a labyrinth but a model of the intestines of a sacrificial animal.

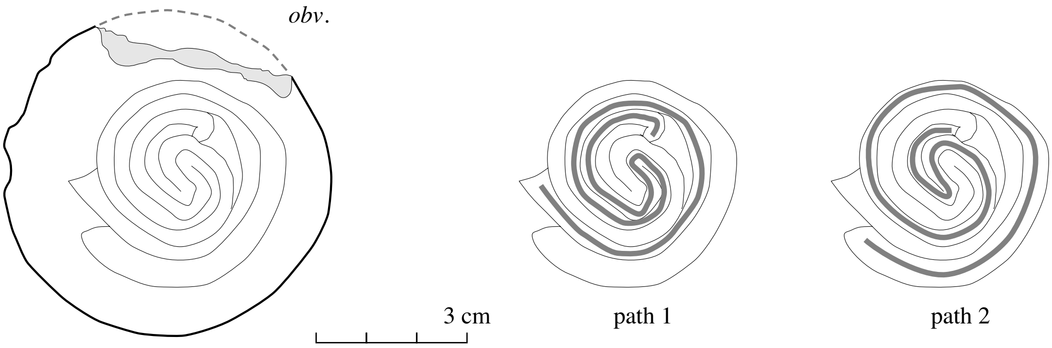


Fig. 8.3.7. VAT 744. A possible precursor of the Babylonian labyrinth.

Another possible precursor of the Babylonian labyrinth is the small clay cone **MS 3195**, inscribed with spirals on its bottom face which appear to be continued into spirals along the sides of the cone:

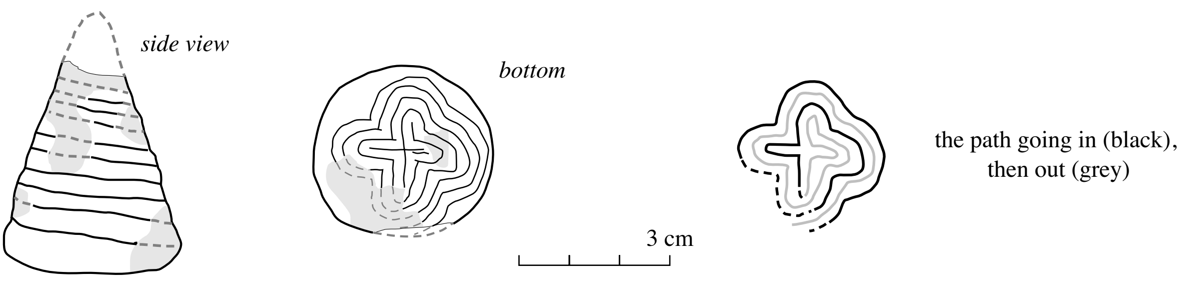


Fig. 8.3.8. MS 3195. Another possible precursor of the Babylonian labyrinth.

8.3 c. MS 3194. A Babylonian Rectangular Labyrinth

MS 3194 (Fig. 8.3.9) is a relatively well preserved clay tablet with a drawing on the obverse of a complicated labyrinth, in the following called "the Babylonian rectangular labyrinth". Some parts of the design close to one edge of the clay tablet are missing, and the surface of the obverse is damaged in some places, all of which makes it difficult to reconstruct with certainty the original form of the labyrinth. Fortunately, the apparent symmetry of the design makes the task somewhat easier.

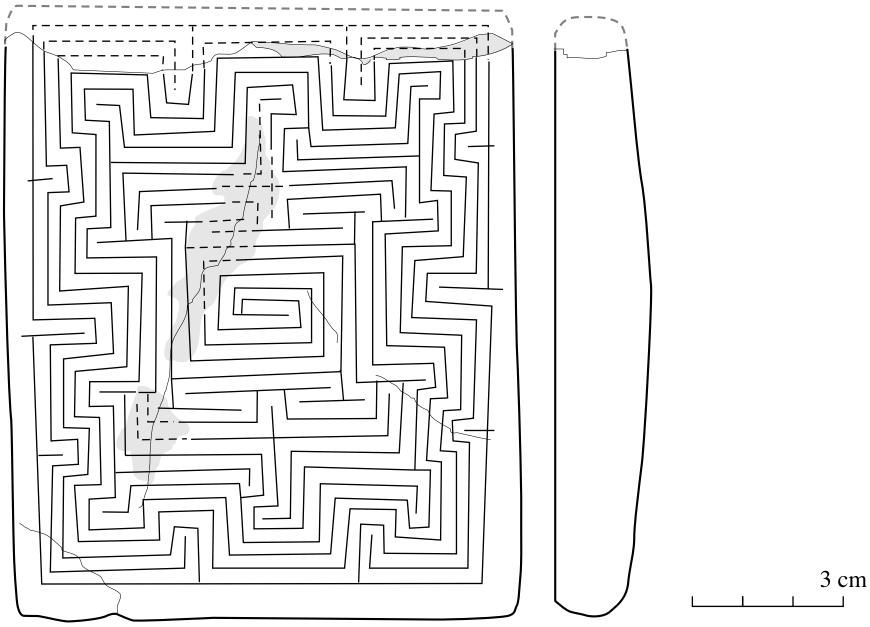


Fig. 8.3.9. MS 3194. The Babylonian rectangular labyrinth.

This Babylonian labyrinth appears to be a more elaborate variant of the Babylonian square labyrinth of MS 4515 (Fig. 8.3.1). Thus, the basic design is the same: to the left and to the right two open gates, with two paths spiralling inwards from the gates, with only one reaching the center of the labyrinth. On the other hand, there are also some interesting differences between the two designs. While the square labyrinth has the form of a *square* city with *four* gates, two open and two closed, with one gate on each side of the square, the rectangular labyrinth has the form of a *rectangular* city with *ten* gates, 2 open and eight closed, with three gates on each one of the long sides of the rectangle, and two gates on each one of the short sides. As a result of the greater complexity of the design, the "good" path leading from the outside to the center is even more convoluted in the case of the rectangular labyrinth than it was in the case of the square labyrinth. See Fig. 8.3.12 below.

The construction of the Babylonian rectangular labyrinth is also much more complicated than the construction of the Babylonian square labyrinth. Thus, while 11 steps are needed for the construction of the square labyrinth (Fig. 8.3.3 above), 38 steps are required for the construction of the Babylonian rectangular labyrinth (Figs. 8.3.10-11).

Yet, the basic idea of the construction is the same in both cases. One has to start from the outer wall of the city, with the positions of the gates indicated (steps 1-2). The outer wall is divided into two separate halves by the open gates. In the initial design, in the upper left corner of Fig. 8.3.10, the two parts of the outer wall are drawn with grey and black lines, respectively. The construction proceeds from this initial design in a number of simple steps. In each step, a black wall section is built parallel to and a constant distance away from the currently innermost grey wall section.

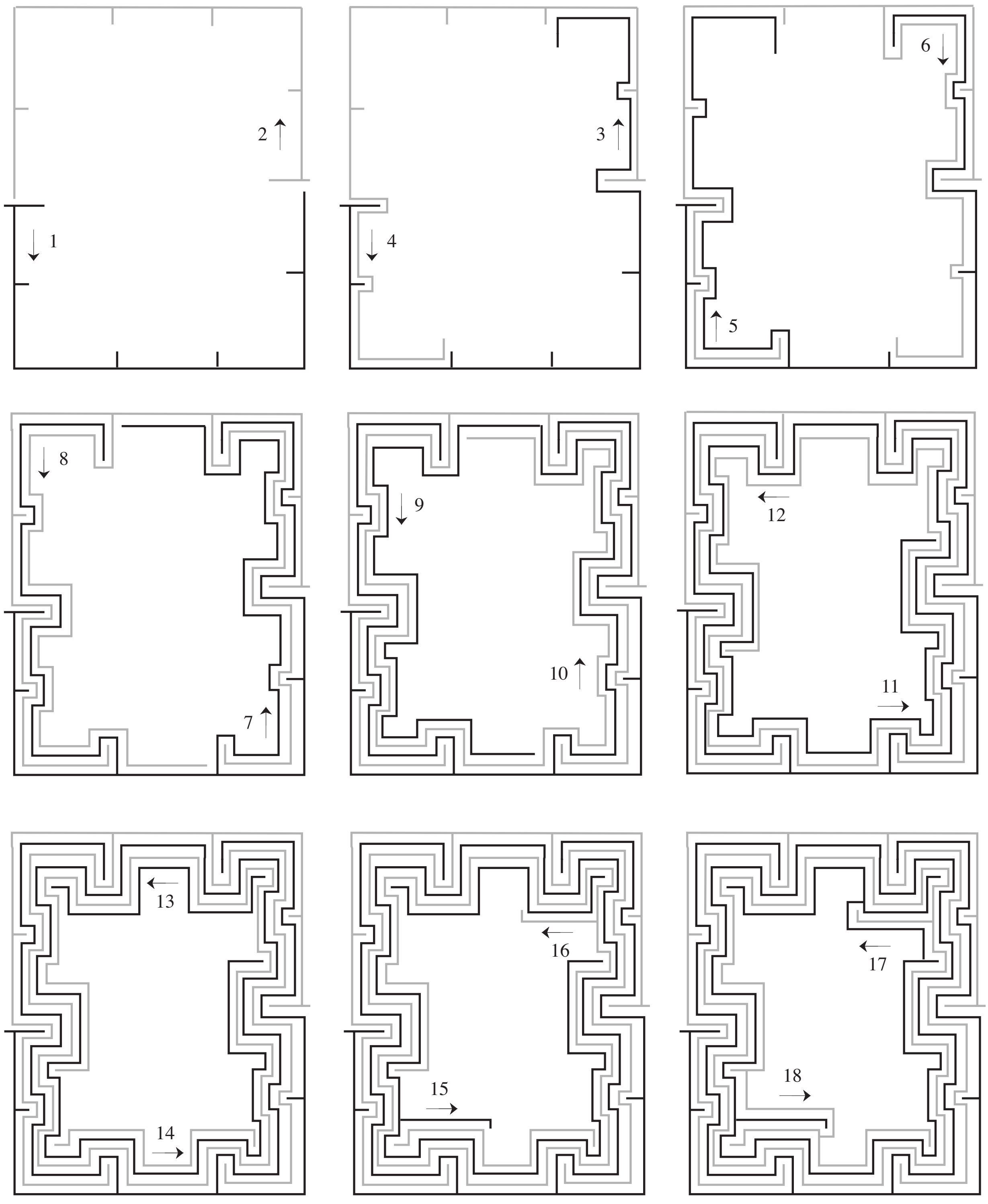
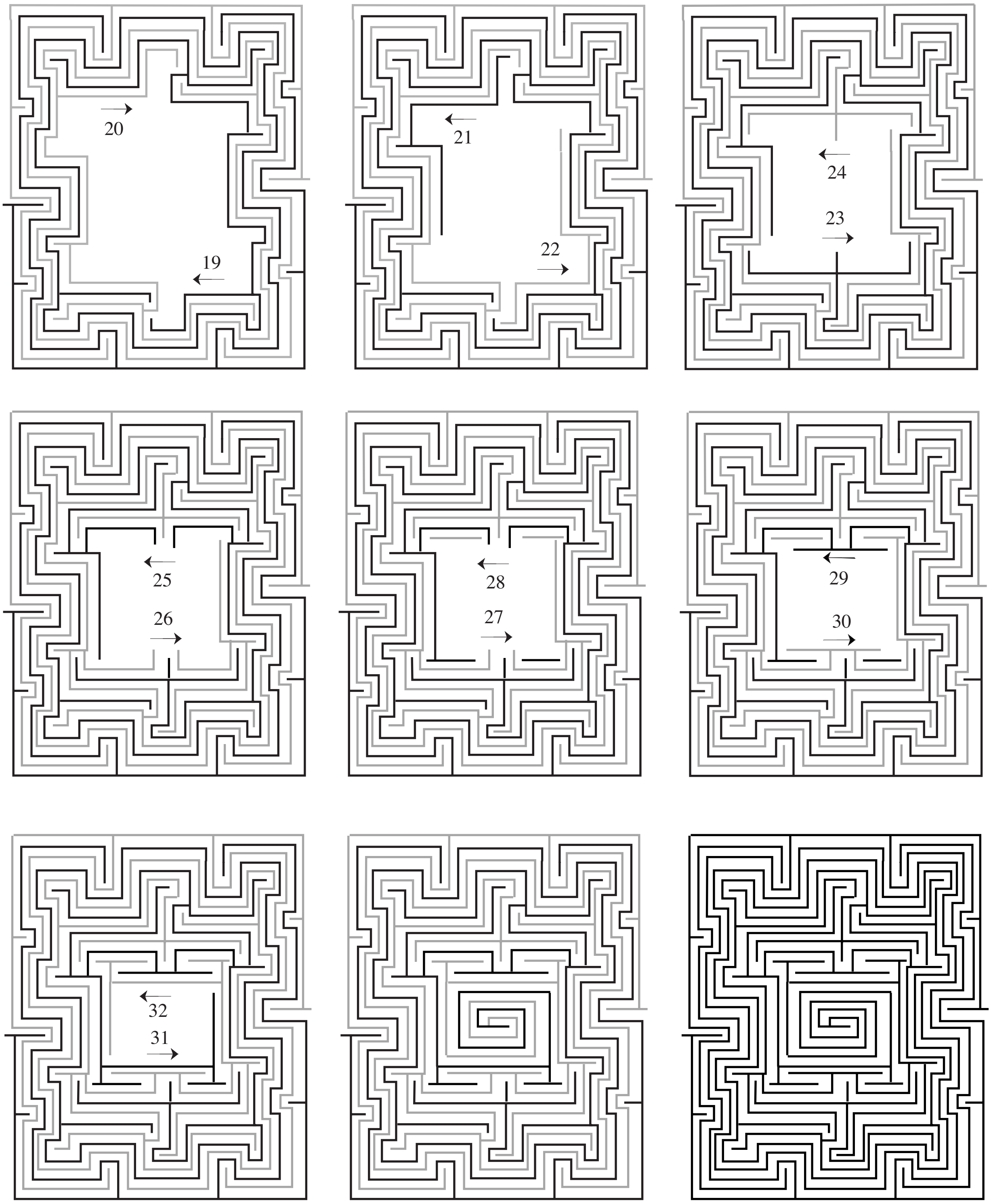


Fig. 8.3.10. The first 18 steps of the construction of the Babylonian rectangular labyrinth.



the last 6 steps the final view

Fig. 8.3.11. The 16 final steps of the construction of the Babylonian rectangular labyrinth.

(It is easy to check that if the constant distance between the gray and black wall sections is, say, 1 ninda, then the lengths of the long and short outer walls of the rectangular labyrinth are 40 and 33 ninda, respectively, if no account is taken of the thickness of the walls, while in the case of the square labyrinth, the lengths of the four outer walls are all equal to 22 ninda.)

The sections of black and gray walls have to be drawn alternately. It is important to keep in mind that because of the requirement that there must be a constant distance of 1 ninda between the black and gray wall sections, those sections are not allowed to touch each other until the last step of the construction. Consecutive black wall sections, on the other hand (like consecutive gray wall sections), are required to touch each other. See steps 5-8, for instance.

The construction is essentially symmetrical, in the sense that the black and the gray wall sections are constructed in pairs, with each gray wall section being more or less the mirror image, with respect to the center of the labyrinth, of the corresponding black wall section.

There is one exception this rule. In steps 33-38 of the construction, the black wall is allowed to spiral inwards, and the gray wall ends by touching the black spiral. In this way, it is made sure that one of the two paths leading inwards from the open gates will end at the center of the labyrinth, while the other path will stop short of the center, at the point where the gray wall touches the black wall. (The situation is the same in the case of the square labyrinth, as shown in Fig. 8.3.3.)

Note that there are no fork points, where a path is allowed to continue through the labyrinth in more than one way. Thus, there is only one choice to be made, namely between the two open gates, and only one possible dead end, namely if one makes the wrong choice of gate to enter through.

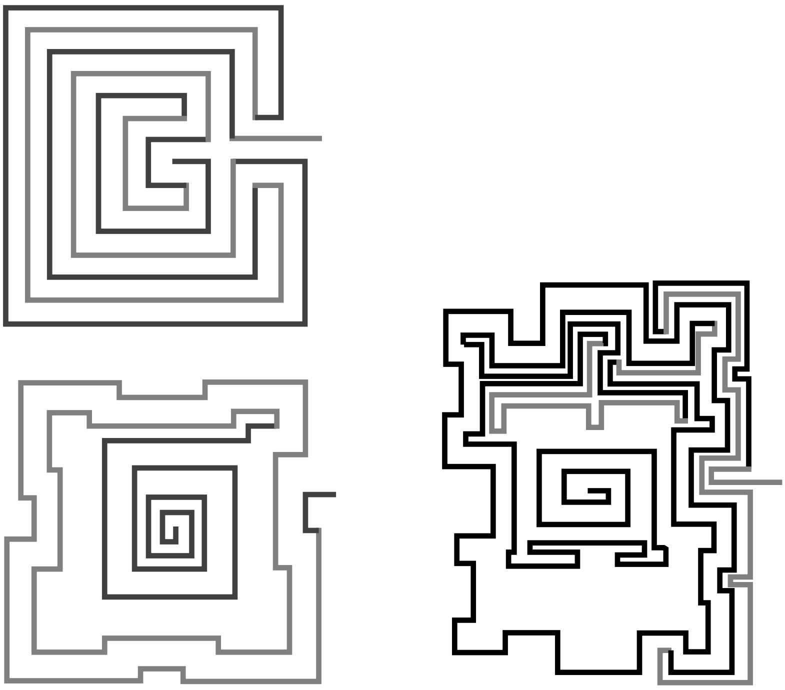


Fig. 8.3.12. The only thread in the Greek labyrinth and the good threads in the two Babylonian labyrinths.

The forms of the good threads in the Greek and the two Babylonian labyrinths are compared in Fig. 8.3.12 above. Clearly, the Greek labyrinth is at the same time simpler and more sophisticated than the two Babylonian labyrinths. A comparison of the algorithms in Figs. 8.3.3, 8.3.4, and 8.3.10-11 for the construction of the three types of labyrinth shows that the Greek labyrinth is also easier to construct. This may be why the Greek labyrinth is the only one of the three that never lost its popularity.

8.3 d. MS 4516. A Geometric Theme text with 8 Assorted Mazes

MS 4516 (Fig. 8.3.13 below) is a rectangular hand tablet inscribed on the obverse with four rows of drawings in two columns. There is no accompanying text. One of the eight drawings is lost, and one is so damaged that it is not possible to reconstruct all its details. The other six drawings can be reconstructed, even the ones that are quite extensively damaged.

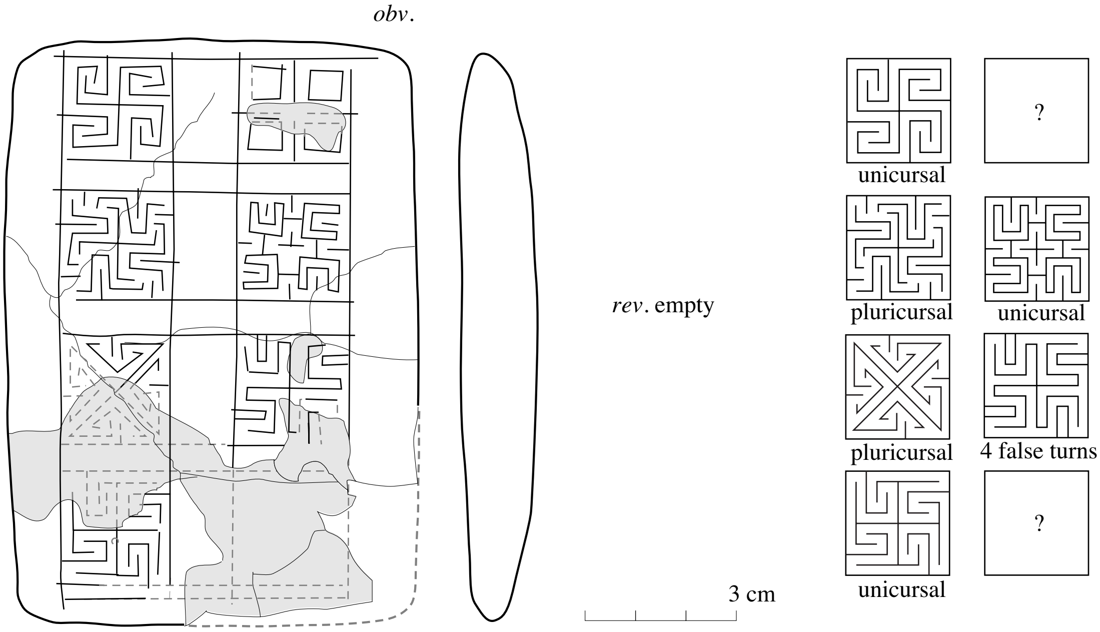


Fig. 8.3.13. MS 4516. A geometric theme text with 8 assorted mazes.

What makes the reconstruction possible in the cases when the drawings are damaged is the following observation. Each drawing consists of a square with a complex pattern of segments of straight lines inside it. All those interior patterns exhibit the same kind of *fourfold rotational symmetry*: If the pattern in, say, the upper left corner of one of the squares is rotated around the center of the square for one quarter of a full revolution, then it comes to coincide with the pattern in the lower left corner of the square. After another quarter revolution, it coincides with the pattern in the lower right corner, and after a third quarter revolution, it coincides with the pattern in the upper right corner.

The squares with their interior patterns are no labyrinths, because there is no entrance into the squares. Somewhat arbitrarily, they will here be called "mazes", which seems to be a fitting name for a pattern forming a passage way for a convoluted path with no well defined start or finish.

The first of the eight mazes can be called "unicursal" because there is only one possible path through it. The fourth and seventh mazes are also unicursal. The third maze is "pluricursal" in the sense that there is more than one path leading through it. The sixth maze differs from all the others in that there are four false turns in it leading to dead ends for the path.

There are several possible explanations for a text like MS 4516. It may be just a catalog of esthetically pleasing patterns, maybe intended to be used for decorative purposes (layouts for mosaics, tilings of floors, gardens, *etc*.). It may also have been a school text, used to teach students the basic principles of maze construction. (Cf. the well known geometric theme text BM 15 285 with its 41 illustrated exercises of the kind shown in Fig. 8.2.13 above.) Or, it may have been a preliminary exercise, a preamble to the construction of a true labyrinth.

8.3 e. MS 3940. A Pattern Superimposed on a Dense Grid of Guide Lines

MS 3940 (Fig. 8.3.14,left) is a fragment, the upper or lower half of a clay tablet, on which is drawn a dense grid of horizontal and vertical guidelines. These guidelines are used for the accurate construction of a pattern, presumably for a tiled floor or a woven fabric. The basic design in the pattern is 4 series of concentric oblique serrated squares flanked on all sides by series of oblique serrated rectangles. This basic design is repeated until all available space is filled.

This clay tablet is interesting in the present context for a couple of reasons. First, it is an excellent demonstration of the rather obvious close relation between decorative art and geometry. Recall, in particular, that concentric squares was a popular topic in Old Babylonian geometry. (Cf. Figs. 8.2.3 and 8.2.10 above.) Secondly, anyone who tries to draw for the first time a complex design like the labyrinths on MS 4515 and MS 3194 or one of the mazes on MS 4516 will soon realize that this is a difficult task unless it is done with the help of an appropriate grid of guide lines.

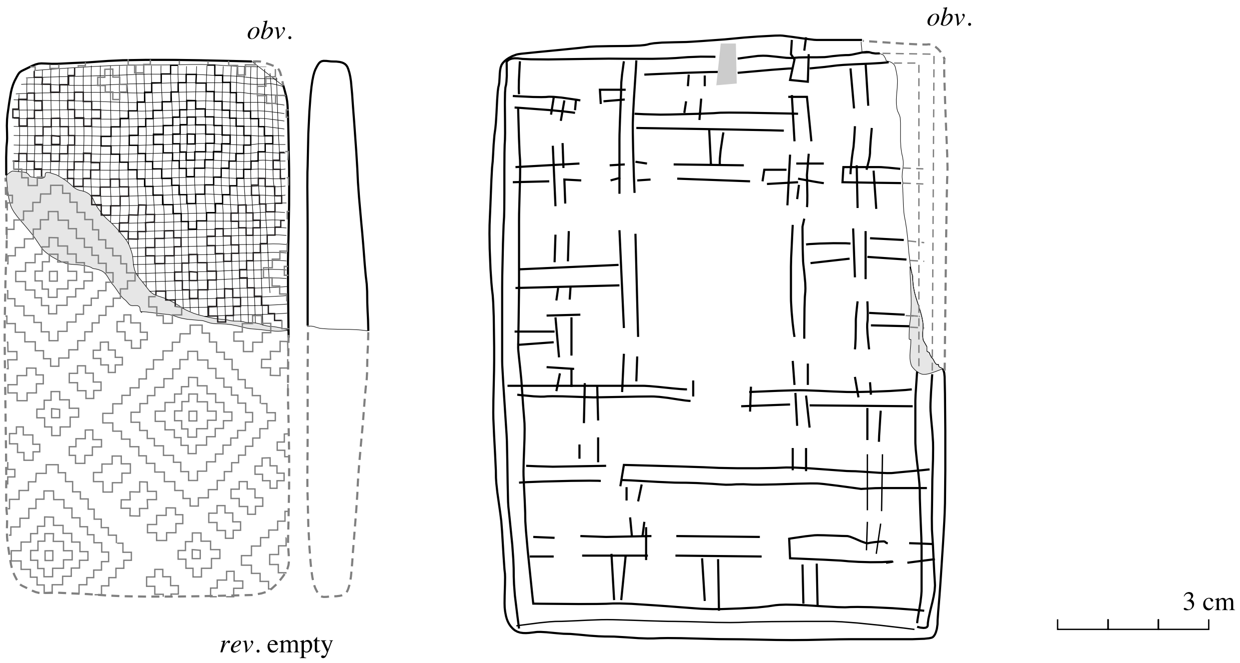


Fig. 8.3.14. MS 3940. A pattern superimposed on a dense grid of guide lines.
MS 3031. The ground plan of the palace of Nur Addad at Larsa.

8.3 f. MS 3031. The Ground Plan of a Palace

It is interesting to compare the layout of the Old Babylonian square and rectangle labyrinths with an actual labyrinthine ground plan of an Old Babylonian palace. **MS 3031** (Fig. 8.3.14, right) is a detailed drawing of the various halls, rooms, and antechambers around the central courtyard of a palace. Although there is no inscription on the clay tablet one can be sure of what the drawing depicts. The design of the palace and the proportions correspond nicely with the remains of the palace of Nur Adad at Larsa unearthed by French excavations since 1903. Besides, the clay tablet is made of a purplish clay that is said to be characteristic for clay tablets from Larsa.

So far this is the only example of a clay tablet with an identifiable house plan of a known building.