6
An Old Sumerian Metro-Mathematical Table Text (Early Dynastic IIIa)

Shuruppak was one of the Sumerian city states, situated on the Euphrates river in south-central Mesopotamia. Excavations in 1902-03 by the Deutsche Orient-Gesellschaft and in 1931 by the University Museum of the University of Pennsylvania uncovered important remains from the Early Dynastic period, including a wealth of cuneiform documents, both administrative texts and school texts, from the Early Dynastic IIIa period (c. 2600-2500 BC). Among the school texts are some of the earliest known mathematical, or rather “metro-mathematical”, texts (see below, Figs 6.1.1-6.1.2). The term “metro-mathematical” is appropriate, since all numbers appearing in these early mathematical texts are various kinds of measure numbers (length numbers, area numbers, capacity numbers, *etc*.).

6.1. Three Previously Published Metro-Mathematical School Texts from Shuruppak

As an introduction to the discussion of MS 3047 below (Fig. 6.2.1), three previously published metro-mathematical school texts from Shuruppak will be considered, all of them like MS 3047 dealing with areas of quadrilaterals. The texts will be presented here in hand copies with rotated cuneiform signs, like MS 3047 *obv*., although they were probably written with unrotated, upright signs.

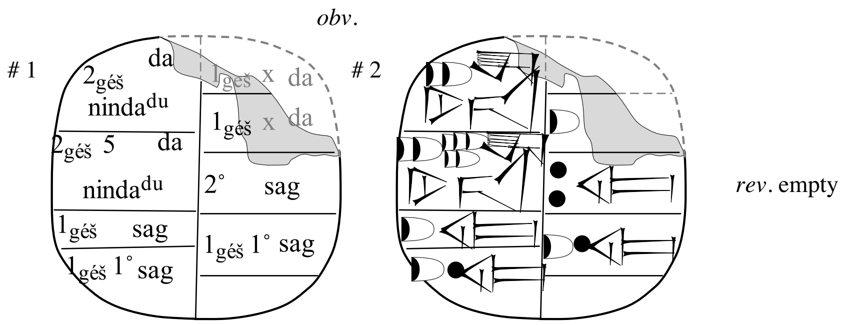


Fig. 6.1.1. *TSS* 926. The four sides of two quadrilaterals.

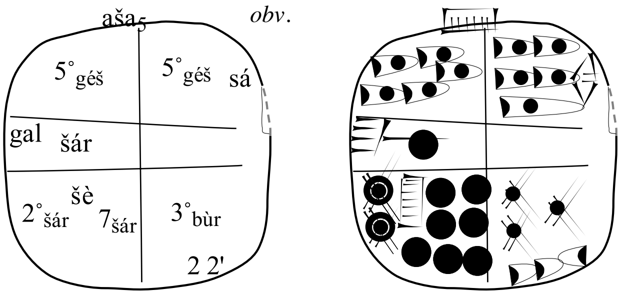
***TSS* 926** (Jestin *TSS* (1937)) is a small tablet inscribed on the obverse with two exercises. In the first exercise, the lengths of the sides of a quadrilateral are given in the following way:

2(géš) ninda^{du}	da	2(géš) ninda the side
2(géš) 5 ninda^{du}	da	2(géš) 5 ninda the side
1(géš)	sag	1(géš) the front
1(géš) 10	sag	1(géš) 10 the front

(An explanation of the “curviform” number signs in cuneiform texts from the third millennium can be found in Figs. A4.1 and A4.10 in App. 4.) It is surprising to find here the Sumerian da ‘side’ as a word for the longer, more or less parallel, sides of a quadrilateral. In later Sumerian and Babylonian texts, the word for the longer sides is normally uš ‘length’. Another difference is that in this text the length measure is written as ninda^{du}, that is ninda plus a determinative du ‘walk’, instead of simply ninda as in later Sumerian and Babylonian texts. (Or, possibly, as ^{ninda}ninda$_x$, with a phonetic determinative.) Anyway, the lengths of the two longer sides of the quadrilateral are, in non-positional sexagesimal numbers, 2(géš) ninda and 2(géš) 5 ninda (corresponding in positional numbers to 2 00 and 2 05 ninda). The two shorter sides are 1(géš) ninda and 1(géš) 10 ninda (corresponding in positional numbers to 1 00 and 1 10 ninda). It is likely that the exercise was an *assignment*, to compute the area of the quadrilateral with the indicated sides. (See Friberg *AfO* 44/45 (1997/98) for a discussion of similar assignments in proto-cuneiform texts from the end of the fourth millennium BC. See, in particular, Figs. 8.1.5-6 below.) The area would have been computed by use of the only approximately correct “quadrilateral area rule”, according to which the area in the given example is the following:

(2(géš) + 2(géš) 5)/2 n. · (1(géš) + 1(géš) 10)/2 n. = 2(géš) 2 1/2 n. · 1(géš) 5 n.
= (12 1/4 · 10 n.) · 6 1/2 · 10 n.) = 1 19 1/2 1/8 sq. (10 n.) = 1 20 iku (– 1/4 1/8 iku).

In the second exercise on *TSS* 926, only the lengths of the shorter sides are perfectly preserved.



rev. empty

the sign transliterated here as šè may be a rotated aša5 sign

Fig. 6.1.2. *TSS* 188. The sides and the area of a very large square.

***TSS* 188** (Jestin *TSS* (1937)) is a small clay tablet from Shuruppak inscribed with both length and area numbers. The area number is so large that it occupies the space of *three* text boxes, instead of only one. The word aša$_5$ (or gán) ‘field, area’ is written on the edge of the clay tablet, so that it can be seen at a glance that this is a text with an area computation. The whole the text can be transliterated and translated as follows, with the help of the factor diagrams in App. 4, Figs. A4.1 and A4.10:

aša$_5$	field (area)
50(géš) 50(géš) sá	50(géš) (n.) 50(géš) (n.) equalsided
1(šár).gal 20(šár) šè 7(šár) 30(bùr)	1(šár) 27(géš) 30(bùr)
2 2'	2 1/2

The meaning of the sign transliterated here as šè is not known. It seems to be part of the number sign for 20(géš) bùr, and may be a rotated aša$_5$ sign. The sign sá (or si$_8$) ‘equal’ probably indicates that the given length numbers are the longer and shorter sides, respectively, of a quadrilateral where the two longer sides are equal, as well as the two shorter sides. Since the same length number is given for both the longer and the shorter sides, the quadrilateral is a square. (Never mind that, from a modern point of view, a quadrilateral with all sides equal can be a parallelogram with equal sides.)

Since 1(šár).gal sq. ninda equals 2(géš) bùr, the area of the square can be computed as follows:

50(géš) n. · 50(géš) n. = 41(šár).gal 40(šár) sq. n. = 2 · 41(géš) 40 bùr = 1(šár) 23(géš) 20 bùr.

This, however, is not the answer given in the text of *TSS* 188. It is clear that the student who wrote the text made an error in his computation. How that error came about can be explained as follows: Near the lower edge of the clay tablet is inscribed a number which probably means ‘2 1/2’. (The sign used here for ‘1/2’ is different from the form of ‘1/2’ in later texts, where an upright cup-shaped sign would have been crossed by a thin wedge

See Fig. A4.1 in App. 4.) The explanation is probably that the area of the square was computed in several small steps, as follows:

sq. (50(géš) n.) = 25 · sq. (10(géš) n.) = 25 · (3(géš) 20) bùr = 2 1/2 · (33(géš) 20) bùr = 1(šár) 23(géš) 20 bùr.

The value of sq. (10(géš) n.) was either held in memory or taken from a table of areas of squares like VAT 12593 (below). Either way, the simple mistake made by the student was to use a slightly incorrect value for sq. (10(géš) n.), 3(géš) 30 bùr instead of 3(géš) 20 bùr. He then counted like this:

sq. (50(géš) n.) = 25 · sq. (10(géš) n.) = 25 · (3(géš) **30**) bùr = 2 1/2 · (35(géš)) bùr = 1(šár) 27(géš) 30 bùr.

VAT 12593 (Deimel, *SF* (1923) 82; Nissen/Damerow/Englund, *ABK* (1993), Fig. 119) is a large clay tablet from Shuruppak, with a "table of areas of squares". It progresses from larger to smaller squares, in contrast to the Old Babylonian (arithmetical) tables of squares of standard type, which always progress from smaller to larger squares.

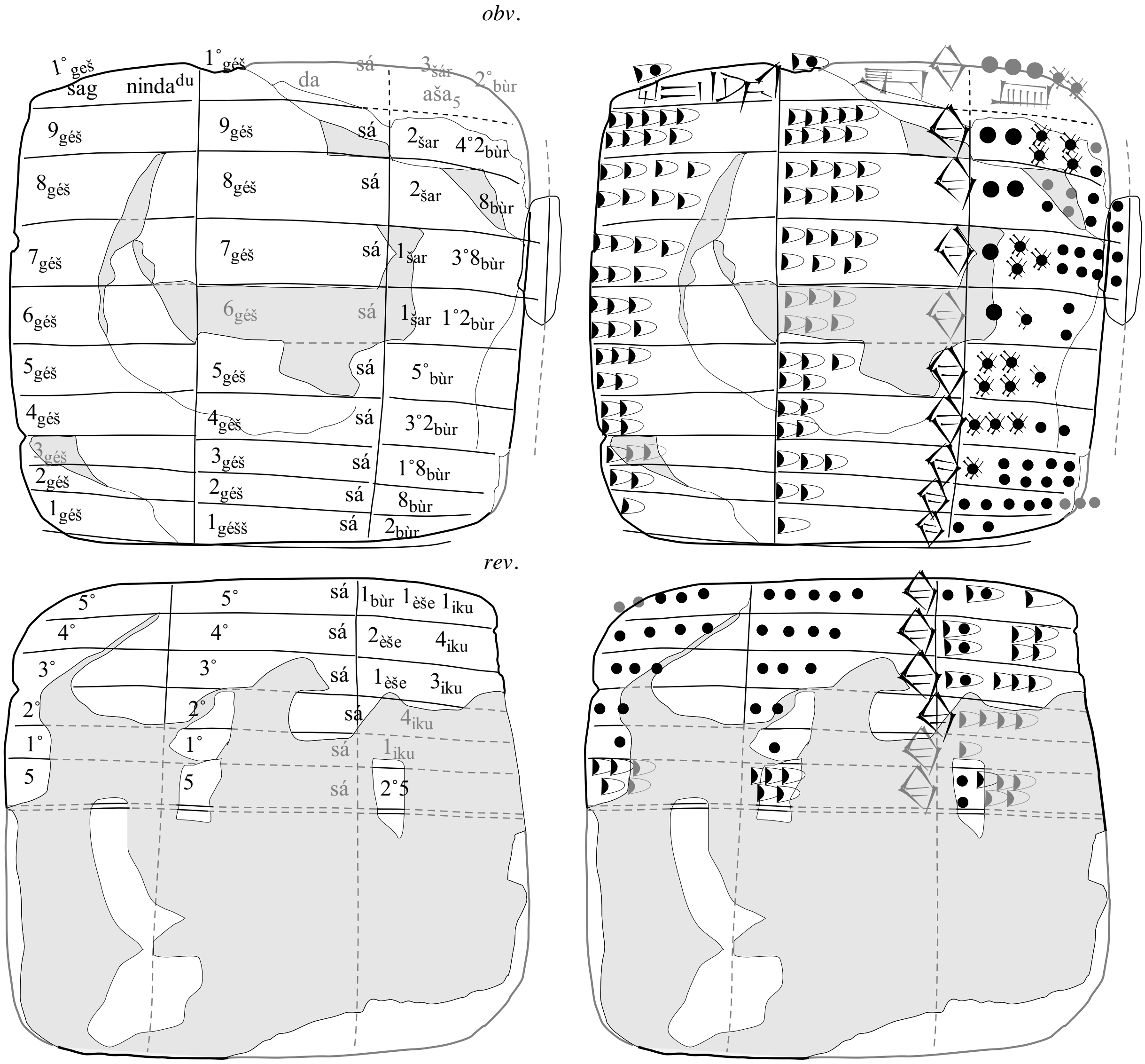


Fig. 6.1.3. VAT 12593. A descending table of areas of large squares, from sq. (10 · 60 ninda) to sq. (5 ninda).

The table begins with the area of the large square

sq. (10(géš) n.) = 3(géš) 20 bùr.

(This is the value that was remembered incorrectly by the author of *TSS* 188!) This initial line of the table, which is now partly lost, was originally inscribed on the edge of the clay tablet, in the same way as the word aša$_5$ is inscribed on the edge of *TSS* 188, as a quick indication of the content of the text.

The two smallest values recorded in this table of areas of squares are

sq. (10 n.) = 1(géš) 40 sq. n. = 1(iku) and
sq. (5 n.) = 25 sq. n., which may have been written as 2[5 šar], although this is not certain.

The reason why the table stops there, quite abruptly, is probably that 25 square ninda equals 1/4 iku, and that smaller fractions of the iku normally do not appear in cuneiform texts.

Note: CUNES 50-08-001, a previously unpublished Early Dynastic clay tablet from the collections of the department for Near Eastern Studies at Cornell University, is a metro-mathematical combined table text with a sub-table that is closely related to the table on VAT 12593. See App. 7 below.

6.2. MS 3047. An Old Sumerian Metro-Mathematical Table Text

MS 3047 (Fig. 6.2.1) is another example of an Old Sumerian metro-mathematical text. It is likely that it, too, comes from Early Dynastic Shuruppak, in view of both its shape (a roundish square tablet) and the form of the cuneiform signs it is inscribed with. Its content is new and interesting.

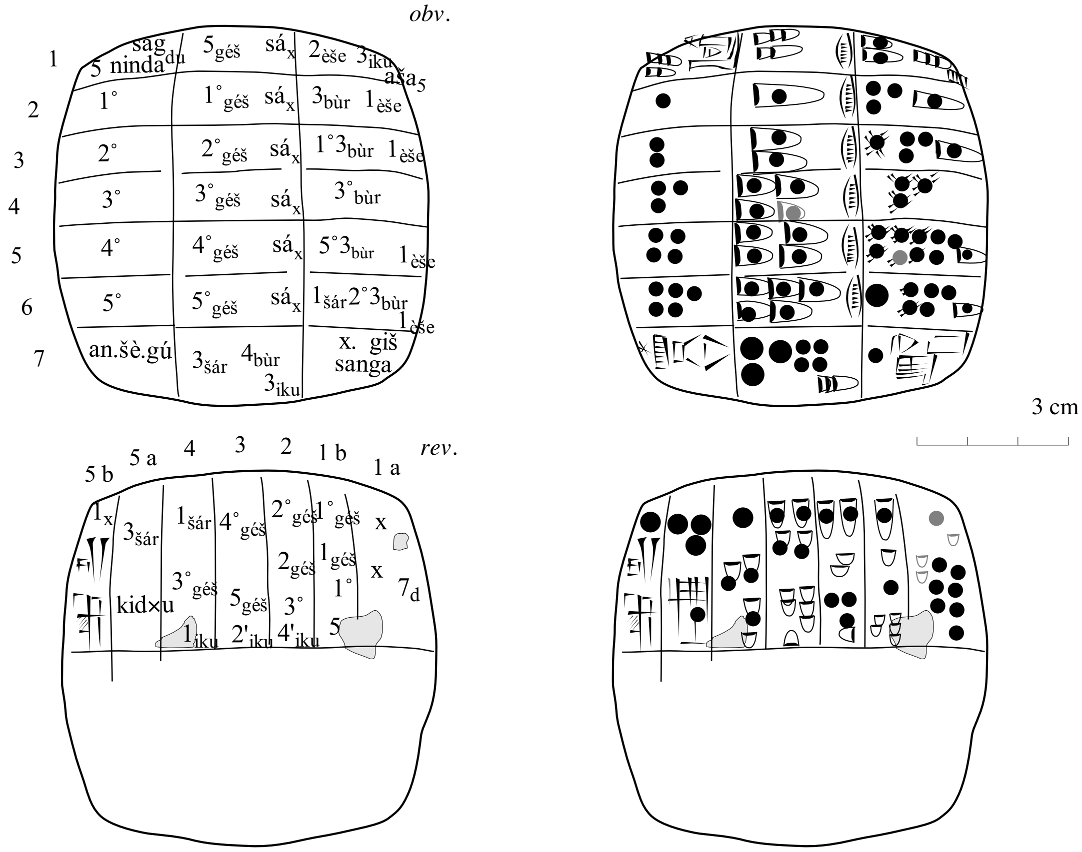


Fig. 6.2.1. MS 3047. A metro-mathematical table text, probably from Shuruppak, Early Dynastic IIIa.

Note: When this clay tablet first entered the Schøyen collection, about half its surface was covered with salt incrustations, covering the text. When the tablet had been cleaned, the appearance of a total in the last entry on the obverse came as a complete surprise. See the photos of MS 3047 in App. 10.

A puzzling detail is that the direction of writing on the reverse of MS 3047 does not agree with the direction of writing on the obverse. (Normally a clay tablet is first inscribed on the obverse, and then turned over along a horizontal or vertical axis so that it can be inscribed also on the reverse. In the case of MS 3047, the tablet has also been rotated around its center.) An effort has been made to show this anomaly in the hand copy above. Note how the longish "cup-shaped" number signs seem to be written horizontally on the obverse, but vertically on the reverse.

It is probable that the conflicting orientations are the result of a mistake. The writing on the reverse is weakly inscribed, as if the tablet after being inscribed on the obverse had been lying around for a while and become somewhat dry, before it was picked up again and inscribed on the reverse. It could then easily happen that the author of the text on the reverse did not remember to orient the reverse correctly with respect to the obverse.

In this connection it must be pointed out that it is not clear which the correct orientation of a text from Early Dynastic IIIa really is. What is known is that at some time during the course of the third millennium BC (probably close to the end of the millennium) the direction of writing in cuneiform texts was changed. Indeed, on clay tablets with proto-cuneiform script from the end of the fourth millennium, the text is written *from top to bottom in vertical text boxes*, arranged from right to left in horizontal registers, while on Old Babylonian clay tablets from the early part of the second millennium, the text has come to be written *from left to right in horizontal rows*, arranged from top to bottom in vertical columns. (See Picchioni, *OrNS* 49 (1980).) When the direction of writing changed, most of the cuneiform signs were rotated with it, from an upright to a lying down position. (However, according to an established convention in assyriological publications, hand copies of cuneiform texts are normally showing the text in the rotated position, no matter from which chronological period the text happens to be.)

6.2 a. ***MS 3047, obv.*** *The Sum of the Areas of a Series of Similar Large Rectangles*

The survey above of earlier published metro-mathematical texts from Shuruppak concerned with the sides and areas of squares and other quadrilaterals was a necessary preparation for the following discussion of the text of MS 3047 (Fig. 6.2.1). On the obverse of MS 3047, there is the following metro-mathematical table:

sag 5 nindadu	5(géš) $sá_x$	2(èše) 3(iku) $aša_5$
10	10(géš) $sá_x$	3(bùr) 1(èše)
20	20(géš) $sá_x$	13(bùr) 1(èše)
30	30(géš) $sá_x$	30(bùr)
40	40(géš) $sá_x$	53(bùr) 1(èše)
50	50(géš) $sá_x$	1(šár) 23(bùr) 1(èše)
an.šè.gú	3(šár) 4(bùr) 3(iku)	x.giš.sanga

(The phrase an.šè.gú is a commonly occurring notation for 'sum, total' in Early Dynastic administrative texts. The phrase x.giš.sanga is probably a signature. The frequently occurring word sanga stands for 'priest/accountant'. The sign sanga itself may be the picture of a box for number tokens.)

It is clear that the first six lines of MS 2047, *obv.* is a table of sides and areas of *rectangles*. All the rectangles have the long side 60 times longer than the short side. The table is in several ways similar to the table of areas of squares on VAT 12593. In particular, the sequence 5, 10, 20, 30, 40, 50, in the first column of MS 3047, *obv.* is the same as the last six entries in the first column of VAT 12593, except for the ordering of the entries. The table on MS 3047, *obv.* is an *ascending* table of areas of rectangles, in contrast to the table on VAT 12593, which is a *descending* table of areas of squares. Note also that the unrealistic proportions of the rectangles indicate that MS 3047, *obv.* is a series of metrological exercises for use in the school rather than a practically useful reference table.

Another obvious difference between MS 3047, *obv.* and VAT 12593 is that in the second column of the former text each entry ends with the sign ki 'place, ground' while in the latter text each entry ends with the sign sá (or si_8) 'equal'. It is possible that ki functions here as a substitute for $aša_5$ 'field, area', and is inserted as a reminder that the product of the length numbers in the first two columns is the area number in the third column. Clearly, the two words ki and $aša_5$ are semantically related. Moreover, in proto-cuneiform texts ki sometimes has the same function as $aša_5$. (See Friberg, *AfO* 44/45 (1997/98), Figs 5.1 and 7.2.) An alternative, relatively plausible, explanation is that the ki signs are badly written versions of the normal sá signs, which is why they here are called $sá_x$.

Without recourse to sexagesimal numbers in Babylonian place value notation, the author of the table on the obverse of MS 3047 may have counted in the following way:

5 n. · 5(géš) n. =	25(géš) sq. n. =			2(èše)	3(iku)
10 n. · 10(géš) n. =	4 · 2(èše) 3(iku) =		3 (bùr)	1(èše)	
20 n. · 20(géš) n. =	4 · 3(bùr) 1(èše) =		13(bùr)	1(èše)	
30 n. · 30(géš) n. =	9 · 3(bùr) 1(èše) =		30(bùr)		
40 n. · 40(géš) n. =	16 · 3(bùr) 1(èše) =		53(bùr)	1(èše)	
50 n. · 50(géš) n. =	25 · 3(bùr) 1(èše) =	1(šár)	23(bùr)	1(èše)	
	sum:	3(šár)	4(bùr)		3(iku)

*6.2 b. **MS 3047, rev.** A Geometric Progression of Areas(?)*

On the reverse of MS 3047, there is a table of a completely different type, with no known parallel. The interpretation of the table is quite difficult, not only because the table is of a new type, but also because its layout is confusing, and because some of the numbers in the first text case of the table are so weakly imprinted that they are hard to read. In addition, the meaning of the non-numerical signs in the last couple of text cases is not clear.

In order to begin to understand the table on the reverse, it is necessary to realize that there are (probably) only five or six, not seven, lines in the table, and that the entries in each line can be separated into two measure numbers. The first number in each pair of measure numbers appears to be a sexagesimal number, while the second number in each pair may (or may not) be a small area number. The following seems to be a relatively adequate transliteration of the table:

11(géš) 15	[x] 7(d) [x]	2 · 11(géš) 15	= 22(géš) 30
22(géš) 30	4'(iku?)	2 · 22(géš) 30	= 45(géš)
45(géš)	2'(iku?)	2 · 45(géš)	= 1(šár) 30(géš)
1(šár) 30(géš)	1(iku?)	2 · 1(šár) 30(géš)	= 3(šár)
3(šár) kid×u	1(šár?) x x		

In spite of this partial success, there are several remaining obscure points in this interpretation. First of all, it is only a conjecture that the second number in each entry is an area number. The second numbers in entries 2 and 3 do look like the cuneiform signs for 1/2 iku and 1/4 iku (App. 4, Fig. A4.9). Unfortunately, this conjecture cannot be supported by the form of the corresponding numbers in entries 1 a and 5 b. In addition, the number which according to this interpretation should have the meaning '1/8 iku' in entry 1 is written in the form [x] 7(d) [x] (where 7(d) stands for 7 disk-shaped number signs). If 7(d) is a sexagesimal number, it has the value 70, if 7(d) is an area number, it has the value 7(bùr), but 7(d) can also be some other kind of number, previously not attested.

It is regrettable that it seems impossible at the present time to give a complete explanation of the metro-mathematical table on the reverse of MS 3047, but the fact remains that MS 3047 is a very important text for the history of mathematics. Already the table of areas of rectangles on its obverse, with the sides of the six rectangles in the ratio 1 : 60, demonstrates that the mathematics taught in the scribe schools in Mesopotamia in the middle of the third millennium was unexpectedly sophisticated. Such a table cannot have been of any practical use; it is an example of mathematics for its own sake. The computation of the sum of the areas of the six rectangles seems to have been added as an afterthought, to make the text look more like an ordinary administrative text.

The geometric progression of sexagesimal numbers in the table on the reverse is also an unexpectedly sophisticated feature of the text. *It is the only pre-Babylonian example of a geometric progression.*

It is interesting that the combination of the number sign šár with the sign kid×u (of unknown meaning), which appears in text case 5 a on the reverse of MS 3047, also appears as an entry in Jestin's ***TSS* 190** (1937), a small lexical text on the theme ŠÁR from Early Dynastic Shuruppak (see Fig. 6.2.2 below). There is also a parallel text from Ebla (Pettinato, *MEE 3*, 72 (1981); see Sec. A6 b below, footnote 8), in which the entry šár kid×u is replaced by the simpler šár kid (that is, without the punched circular hole in the kid sign).

šár šè.gín?	šár nì.búr?.gu$_7$
šár giš	šár u$_4$.ni.gi.gu$_7$
[šár] kid×u	
[š]ár sá	šár u$_4$.u$_4$.u$_4$
šar dirig	šár an.[x]
šár šu nu.gi	

3 cm

Scale 1 : 1

Fig. 6.2.2. *TSS* 190. A small Early Dynastic lexical text on the theme ŠÁR.

This photo of *TSS* 190 is published here with the assistance and gracious permission of Veysel Donbaz.