5
Neo-Sumerian Field Plan Texts (Ur III)

5.1. MS 1984. A Field Plan Text from Umma with a Summary on the Reverse

MS 1984 (Fig. 5.1 below) is a square clay tablet with rounded corners, inscribed on the obverse with a detailed field plan and on the reverse with a brief summary.

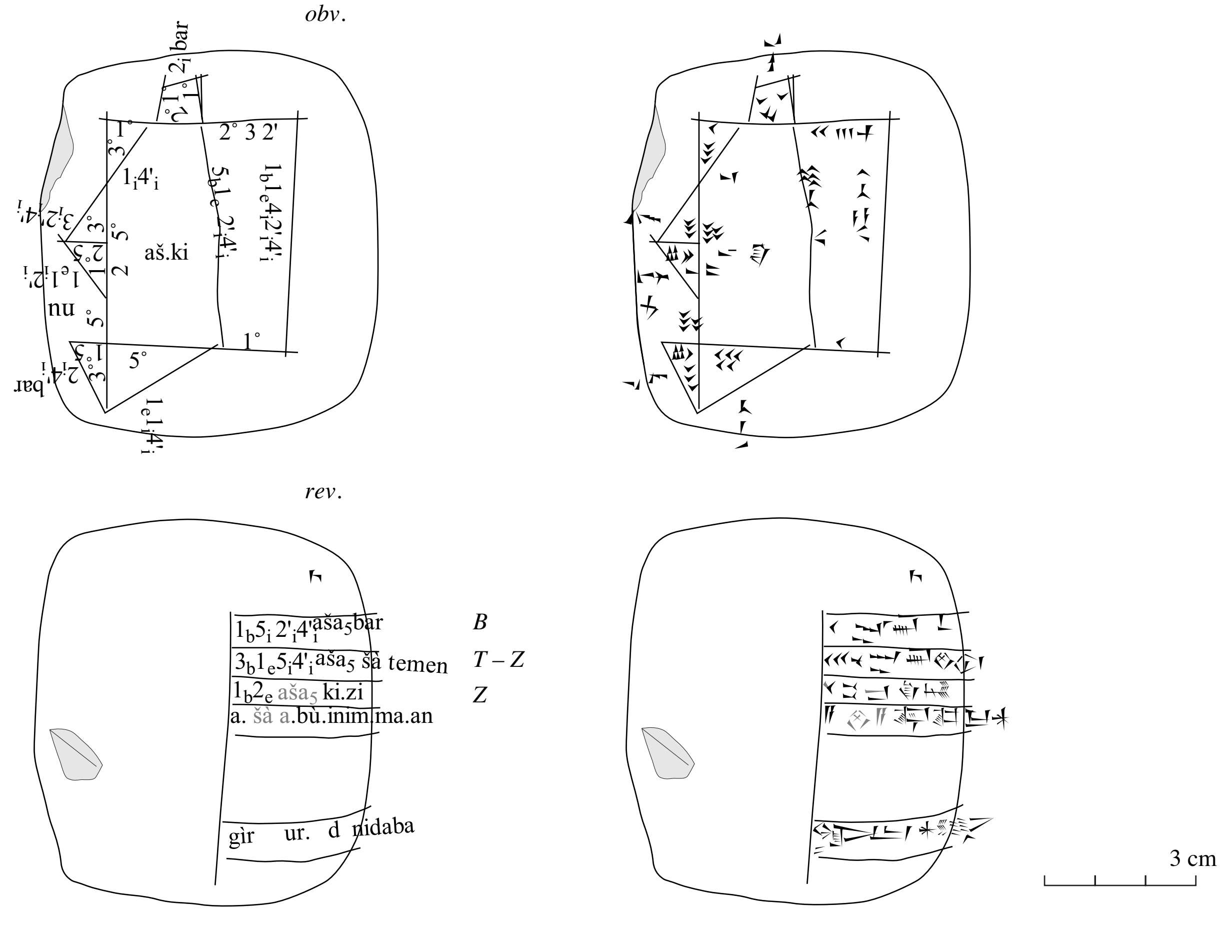


Fig. 5.1. MS 1984. A field plan text, probably from Umma.

The field outlined on the obverse of MS 1984 has an irregular, relatively complicated shape. The computation of its area is carried out in a roundabout way that would have been difficult to reconstruct without the help of the inscription on the reverse. As it turns out, the irregular field, apparently named a.šà a.bù inim.ma.an, is partly overlapping a field of a more regular, trapezoidal shape called the temen (Akk. *temennu*). The standard translation of temen is 'foundation', 'foundation document' (see below for a tentative explanation of the use of this term here). A curving line crossing the temen obliquely cuts off a large part of the temen, a roughly trapezoidal field that is outside the field a.šà a.bù inim.ma.an.

Whatever the meaning is of the term temen, it seems to be clear that the area of the irregular field was computed as the area of the temen, minus the area of the large trapezoidal field and the area of a small triangular field in the upper left corner of the temen, plus the areas of a small rectangular field and four small triangular fields outside the border of the temen.

The author of the text made himself guilty of some minor miscalculations. Immediately below follows an account of the progress of the computation of the area of the irregular field, with the errors in the text corrected. Afterwards, a separate account will be given of the errors, and explanations proposed for where they come from.

The following notations will be used (cf. Fig. 5.2 below):

A is the area of the irregular field, T the area of the rectangular temen.
B_1, B_2, B_3, B_4, and B_5 are the added areas of the rectangle and the four triangles.
Z_1 and Z_2 are the subtracted areas of the large trapezoid and the small triangle.

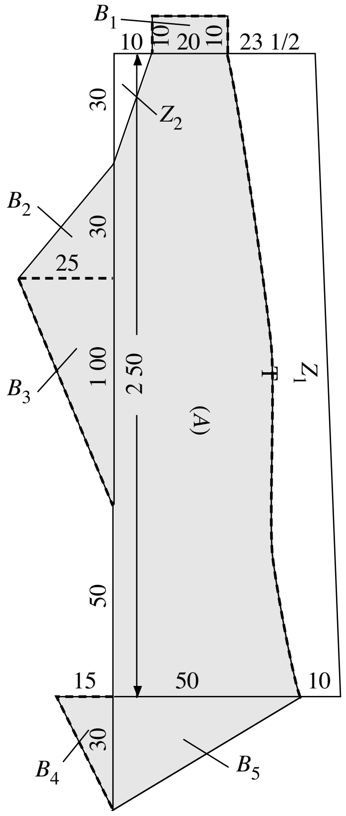
These notations are acronyms alluding to the Sumerian terms a.šà 'field, area', temen 'foundation document', bar 'extra, outside?', and zi 'tear out, subtract'.

Here are the successive steps of the (corrected) computation:

B_1 = 20 n. · 10 n. = 3 20 sq. n. = 2 iku
B_2 = 30 n. · 25 n./2 = 6 15 sq. n. = 3 1/2 1/4 iku
B_3 = 1 00 n. · 25 n./2 = 12 30 sq. n. = 1 èše 1 1/2 iku
B_4 = 30 n. · 15 n./2 = 3 45 sq. n. = 2 1/4 iku
B_5 = 50 n. · 30 n./2 = 12 30 sq. n. = 1 èše 1 1/2 iku
$B_1 + B_2 + B_3 + B_4 + B_5$ = B = 1 bùr 5 iku
Z_1 = 2 50 n. · (23 1/2 + 10) n./2 = 2 50 · 16;45 sq. n. = 47 27;30 sq. n. = 1 bùr 1 èše 4 1/4 iku 22 1/2 šar
Z_2 = 30 n. · 10 n./2 = 2 30 sq. n. = 1 1/2 iku
$Z_1 + Z_2 = Z$ = 1 bùr 1 èše 5 1/2 1/4 iku 22 1/2 šar= appr. 1 bùr 2 èše
T = 2 50 n. · (53 1/2 + 1 00) n./2 = 2 50 · 56;45 sq. n. = 2 40 47;30 sq. n = 5 bùr 1 èše 1/4 iku 22 1/2 šar
$T - Z$ = 3 bùr 2 èše 1/2 iku
$T - Z + B = A$ = 4 bùr 2 èše 5 1/2 iku = 5 bùr – 1/2 iku = appr. 5 bùr

These values can be compared with the ones appearing on the obverse and reverse of MS 1984:

B_1 = 2(iku) bar	*obv.*	correct	
B_2 = 3 1/2 1/4(iku)	*obv.*	correct	
B_3 = 1(èše) 1 1/2(iku)	*obv.*	correct	
B_4 = 2 1/4(iku) bar	*obv.*	correct	
B_5 = 1(èše) 1 1/2(iku)	*obv.*	correct	
B = 1(bùr) 5 1/2 1/4(iku) bar	*rev.*	small error	surplus 1/2 1/4(iku)
Z_1 = 1(bùr) 1(èše) 4 1/2 1/4(iku)	*obv.*	small error	surplus 1/4(iku) 2 1/2 šar
Z_2 = 1 1/2(iku)	*obv.*	correct	
Z = 1(bùr) 2(èše) ki.zi	*rev.*	appr. correct	surplus 2 1/2 šar
T = 5(bùr) 1(èše) 1/2 1/4(iku)	*obv.*	small error	surplus 1/4(iku) 2 1/2 šar
$T - Z$ = 3(bùr) 1(èše) 5 1/4(iku) šà temen	*rev.*	small error	deficit 1/2 1/4(iku)
$T - Z + B = A$ = 4(bùr) 2(èše) 5(iku)	not given in the text	appr. correct	deficit 1/2(iku)



This text was first published and extensively discussed by Allotte de la Fuÿe in *RA* 12 (1915). A new hand copy appeared in Grégoire, *MVN 10* (1981), 214.

The dotted line indicates the boundary of the trapezoidal field called temen.

The field colored gray is the irregularly shaped field called a.gu$_7$.inim.ma.an.

Z_1 and Z_2 are areas to be *subtracted* from the area T of the temen.

B_1, B_2, B_3, B_4, and B_5 are areas to be *added* to the area of the temen.

The area of the field a.bù.inim.ma.an is computed as the area of the temen minus the subtracted areas plus the added areas.

The total area of the field is close to 5 bùr.

Scale: 1 : 12,000 (1 mm representing 2 ninda)

Fig. 5.2. The field plan on MS 1984, *obv.,* drawn to scale.

Here follows a transliteration of the summary and subscript on the reverse of MS 1984:

1(bùr) 5(iku) 2'(iku) a.šà bar /	1 bùr 5 1/2 iku, the field (= area) outside(?).
3(bùr) 1(èše) 5(iku) 4'(iku) a.šà šà temen /	3 bùr 1 èše 5 1/4 iku, the field inside the temen.
1(bùr) 2(èše) a.šà ki.zi /	1 bùr 2 èše the field of the ground torn off.
a.šà a.bù inim.ma.an /	The a.bù field Inim.ma.an.
gìr ur.^{d}nisaba	Inspector: Ur-Nisaba.

Clearly, the author of MS 1984 computed the area of the irregular field in the following way. First he computed *the area of the part of that field inside the temen*, explicitly named a.šà šà temen 'the field (= area) inside the temen' in line 2 of the text on the reverse. Then he *added the area of the small fields outside the temen*, named a.šà bar 'extra? field' in line 1, and subtracted the area of two parts of the temen, called a.šà ki.zi 'the field of the ground torn off' in line 3.

The error made in the computation of the sum *B* of the exterior fields can be explained as a simple addition error. The error made in the computation of the subtracted area Z_1 is more interesting. The scribe's intention was to compute it as

$$Z_1 = 1/2 \cdot 2\ 50 \cdot 33;30 \text{ sq. n.} = 1/2 \cdot 1\ 34\ 55 \text{ sq. n.},$$

but apparently he made the mistake of setting 1/2 · 1 34 55 equal to 47 00 + 55 instead of 47 00 + 27;30. This mistake explains the surplus of 27;30 sq. n. = 1/4(iku) 2 1/2 šar.

The same surplus appears in the incorrect value for the area *T* of the temen. The reason is probably that *T* was computed as the sum of Z_1 and the area of the trapezoid with the height 2 50 and the parallel sides 30 and 50.

It is not clear what caused the small error in the value for $T - Z$.

A possible clue to the meaning of this text is the observation that the area of the irregularly shaped field is *a nearly round number*. Indeed, according to the corrected computation it is 5 bùr minus 1/2 iku, where 1/2 iku is 1/90 of 5 bùr, and according to the slightly incorrect computation on the clay tablet MS 1984 it is 5 bùr minus 1 iku, in both cases a very good approximation to the round number 5 bùr. Hence, the following tentative explanation:

Some high official or wealthy institution originally held the title (the temen!) to a regularly shaped piece of land measuring 5 bùr 1 èše (= 16 èše, about 345,000 sq. meters). Then something catastrophic happened so that a large part of the originally allotted land was lost, the piece to the right of the curved line across the property, and also a small piece in the upper left corner. In compensation, the title holder was allowed to add to what remained of his property several peripheral pieces of land, the rectangle and the triangles together called bar. This was done in a carefully calculated way so that after the change the total holdings came to measure almost exactly 5 bùr (= 15 èše), only slightly less than the area of the original estate.

5.2. *MS 1850. A Field Plan Text without a Summary on the Reverse*

MS 1850 (Fig. 5.3 below) is a badly broken square clay tablet with rounded corners. It is inscribed with a field plan on the obverse and only a few scattered numbers on what remains of the reverse. By sheer luck, almost no important details of the inscription were contained in the damaged parts of the obverse. Therefore, it is possible to reconstruct in its entirety the original drawing of the field plan with all its associated length and area numbers.

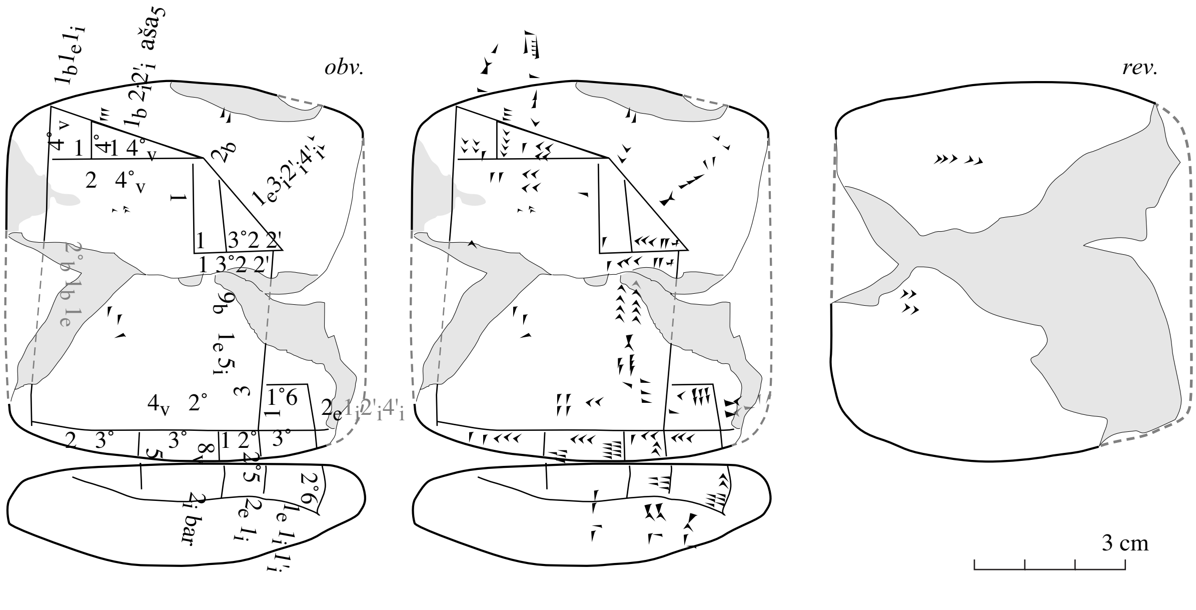
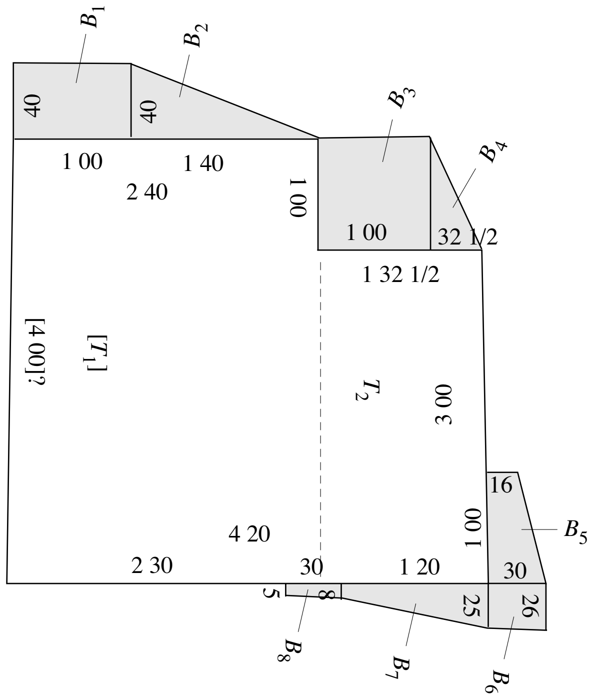


Fig. 5.3. MS 1850. A field plan with a central region and eight added fields around the border.

The anonymous field depicted on the obverse of MS 1850 consists of a large central region and eight smaller peripheral fields, triangular or trapezoidal. The central region can be divided into two nearly rectangular parts. Fig. 5.4 below shows the field plan drawn to scale.



T_1 and T_2 are the areas of the two nearly rectangular parts of the central region.

$B_1, B_2, B_3, B_4, B_5, B_6, B_7, B_8$ (colored gray) are eight areas *added* to the area of the temen.

The total area of the field is close to 38 bùr.

Scale: 1 : 24,000 (1 mm representing 4 ninda).

Fig. 5.4. The field plan on MS 1850, drawn to scale.

Here are the successive steps of the computation of the areas recorded on MS 1850, *obv*.:

T_1 = 4 00 n. · 2 40 n. =	10 40 00 sq. n. =	21 bùr 1 èše
T_2 = 3 00 n. · (1 32 1/2 + 1 40) n./2 =	4 48 45 sq. n. =	9 bùr 1 èše 5 1/4 iku
$T = T_1 + T_2$ =	30 bùr 2 èše 5 1/4 iku =	31 bùr (– 1/2 1/4 iku)
B_1 = 1 00 n. · 41 1/2 n. =	41 15 sq. n. =	1 bùr 1 èše
B_2 = 1 40 n. · 20 n. =	33 20 sq. n. =	1 bùr 2 iku
B_3 = 1 00 n. · 1 00 n. =	1 00 00 sq. n. =	2 bùr
B_4 = 1 00 n. · 16 1/4 n. =	16 15 sq. n. =	1 èše 3 1/2 1/4 iku
B_5 = 1 00 n. · 23 n. =	23 00 sq. n. =	2 èše 1 1/2 1/4 iku (5 šar)
B_6 = 30 n. · 25 1/2 n. =	12 45 sq. n. =	1 èše 1 1/2 iku (15 šar)
B_7 = 1 20 n. · 16 1/2 n. =	22 00 sq. n. =	2 èše 1 iku (20 šar)
B_8 = 30 n. · 6 1/2 n. =	3 15 sq. n. =	2 iku (– 5 šar)
$B = B_1 + B_2 + \ldots + B_8$ =		7 bùr
$A = T + B$ = 37 bùr 2 èše 5 1/4 iku =		38 bùr (– 1/2 1/4 iku)

Of the values listed above, the area T_1 is lost, being written in one of the destroyed parts on the obverse of the clay tablet. Apparently, the sums T, B, and $A = T + B$ were not recorded, although it is possible that the damaged parts of the obverse contained one or more of these sums.

Both the area T of the central property, and the area A of that central property plus the eight extra pieces of land are *nearly round numbers*. This striking fact supports the proposed interpretation of the field plan MS 1984. Indeed, a similar interpretation of the field plan on MS 1850 is that a wealthy person or institution originally held the title to a very extensive, regularly shaped piece of land, measuring almost exactly 31 bùr (2.4 sq. kilometers). Some higher authority then decided to allow that person or institution to expand this property by adding to it 7 bùr. This was done through expropriation of several adjoining smaller pieces of land owned by less favored individuals or institutions. In line with this interpretation, MS 1850 can be explained as a first draft for an official document recorded in some more elaborate form, perhaps on a stone tablet, to commemorate the occasion.

5.3. Four Ur III Field Plan Texts, Published in 1915, 1898, 1922, and 1962

Although the great majority of the mathematical cuneiform texts in the Schøyen Collection are new additions to the corpus, probably emanating from relatively recent excavations in Iraq, the field plan text MS 1984 has been known for a very long time. It was first published by Allotte de la Fuÿe in ***RA* 12 (1915)**, who presented the text as a clay tablet that had “arrived in France together with a batch (of tablets) from the tell of Djokha, situated not far from Tello”, that is from the site of the ancient city Umma, near Lagaš-Girsu. As a further corroboration of the tablet’s provenience from Umma, Allotte de la Fuye pointed to the circumstance that the name of the inspector mentioned in the inscription on the reverse was Ur-Nisaba, and that there apparently was in Umma a cult of the goddess Nisaba.

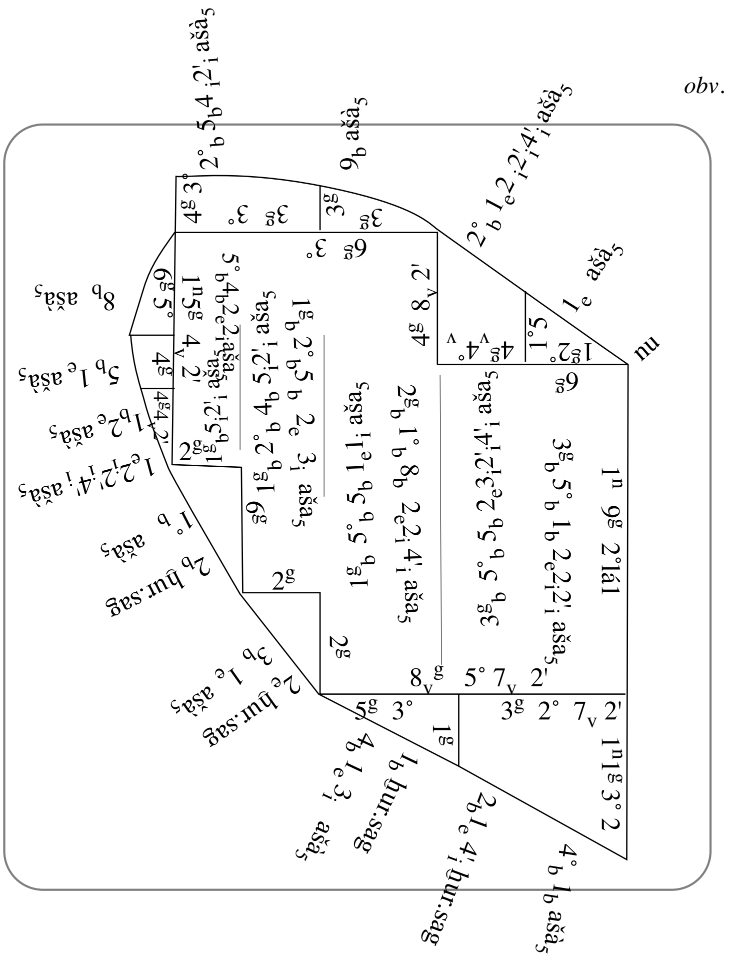


Fig. 5.5. Ist. O (MIO) 1107. A field plan with a central region and eleven added fields around the border.

A similar text is **Ist. O (MIO) 1107** (see Figs. 5.5-5.7), an Ur III text from the region around Lagaš-Girsu, published by F. Thureau-Dangin first in *RA* 4 (1897), then in *RTC* (1903) # 416. It has a field plan on the obverse, and a summary and subscript on the reverse.

The field plan on Ist. O 1107 resembles the one on MS1850 but has a couple of extra features that make it particularly interesting. The most conspicuous extra feature is that the central region, in the summary called the temen, has been divided into four nearly rectangular sub-regions, and that for each of the four sub-regions its area has been computed twice, with different results. Thus, in each of the four nearly rectangular parts of the temen, two area numbers are recorded, one facing left, the other facing right. A convincing explanation for this strange feature was lacking for a long time but has been presented quite recently by Quillien in *RHM* 9 (2003).

Apparently, the area of the temen was computed twice. The first computation was based on the assumption that *the first three of the four sub-regions of the temen, counted from the left, are rectangular, while the fourth is trapezoidal*. The second computation was based on an identical assumption, after the field plan had been rotated to an *upside-down* position. The situation is made clear in Fig. 5.6 below, where the temen in its normal position is shown to the left, while the temen in its upside-down position is shown to the right.

The length numbers that are explicitly given in the drawing on the obverse of Ist. O 1107 are written in their correct positions along the sides of the temen in the two copies of the temen in Fig. 5.6. Computed length numbers for parts of the sides and for transversal lines are written within brackets. Thus, in the copy of the temen to the left, since the leftmost rectangular sub-region has one of its short sides given as 2 00, its other short side must be equal to 2 00, too. Similarly in the case of the second rectangular sub-region. It follows that the two short sides of the third rectangular sub-region must both be equal to 6 30 (given) minus twice 2 00 (computed). That is, they are both equal to 2 30. The fourth sub-region has then one of its parallel sides equal to 6 00 (given), while its second parallel side is 8 57 1/2 (given) minus 2 30 (computed). That is, it is equal to 6 27 1/2, as indicated.

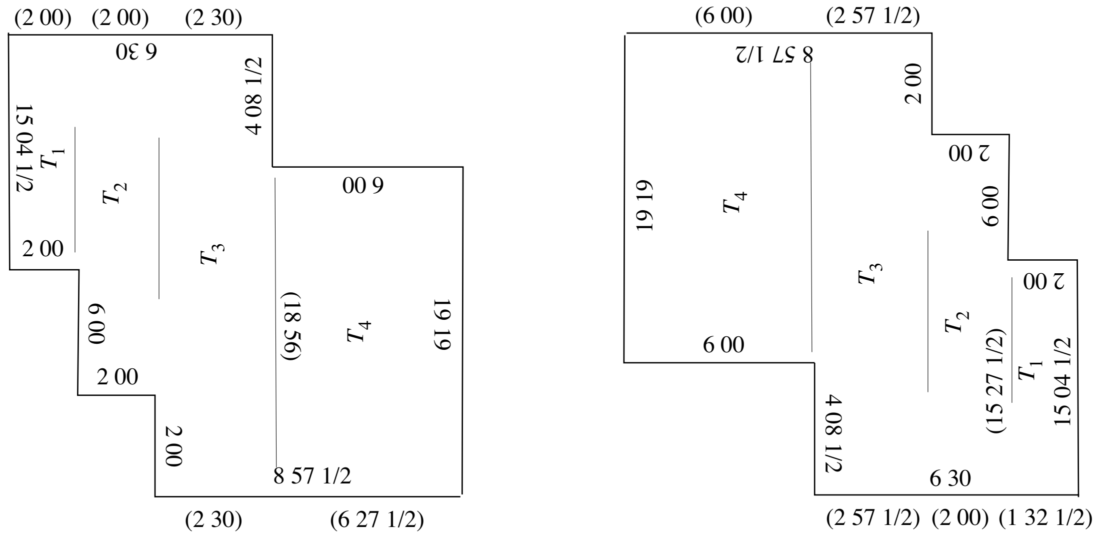


Fig. 5.6. Ist. O (MIO) 1107. The temen in its normal and upside-down positions.

The long sides of the three rectangular sub-regions to the left are clearly

15 04 1/2 (given), 15 04 1/2 + 6 00 = 21 04 1/2, and 21 04 1/2 + 2 00 = 23 04 1/2.

The crucial observation made by Quillien is that in this first round of computations, the height of the trapezoidal sub-region to the right is assumed to have the *computed* length 23 04 1/2 – 4 08 1/2 = 18 56 rather than the *given* length 19 19! Now, with all the side lengths of the four sub-regions of the temen either given or computed, the four areas and their sum can be computed as follows:

T_1 = 15 04 1/2 n. · 2 00 n. = 30 09 00 sq. n. = 1 00 bùr 5 1/2 iku (– 10 šar)
T_2 = 21 04 1/2 n. · 2 00 n. = 42 09 00 sq. n. = 1 24 bùr 5 1/2 iku (– 10 šar)
T_3 = 23 04 1/2 n. · 2 30 n. = 57 41 15 sq. n. = 1 55 bùr 1 èše 1/2 1/4 iku
T_4 = 18 56 n. · 6 13;45 n. = 1 57 56 20 sq. n. = 3 55 bùr 2 èše 3 1/2 1/4 iku (+ 5 šar)
$T_1 + T_2 + T_3 + T_4 = T$ = 8 15 bùr 2 èše 3 1/2 iku (– 15 šar).

In the second round of computations, with the temen turned upside-down, the leftmost rectangle has the sides 19 19 and 6 00. The sides of the second rectangle can then be computed as

19 19 + 4 08 1/2 = 23 27 1/2 and 8 57 1/2 – 6 00 = 2 57 1/2.

The third rectangle has the sides

23 27 1/2 – 2 00 = 21 27 1/2 and 2 00.

Finally, the height and the two parallel sides of the fourth sub-region, the trapezoid, are

21 27 1/2 – 6 00 = 15 27 1/2, 2 00, and 6 30 – (2 57 1/2 + 2 00) = 1 32 1/2.

With these given or computed side lengths for the four sub-regions of the upside-down temen, the four areas and their sum are computed as follows, beginning with the one at the left:

T_4 = 19 19 n. · 6 00 n. = 1 55 54 00 sq. n. = 3 51 bùr 2 èše 2 1/2 iku (– 10 šar)
T_3 = 23 27 1/2 n. · 2 57 1/2 n. = 1 09 23 51 1/4 sq. n. = 2 18 bùr 2 èše 2 1/4 iku (+ 6 šar)
T_2 = 21 27 1/2 n. · 2 00 n. = 42 55 00 sq. n. = 1 25 bùr 2 èše 3 iku
T_1 = 15 27 1/2 n. · 1 46 1/4 n. = 27 22 26;52 30 sq. n. = 54 bùr 2 èše 1 1/2 iku (– 3 šar)
$T_1 + T_2 + T_3 + T_4 = T$ = 8 31 bùr 3 1/4 iku (– 7 šar).

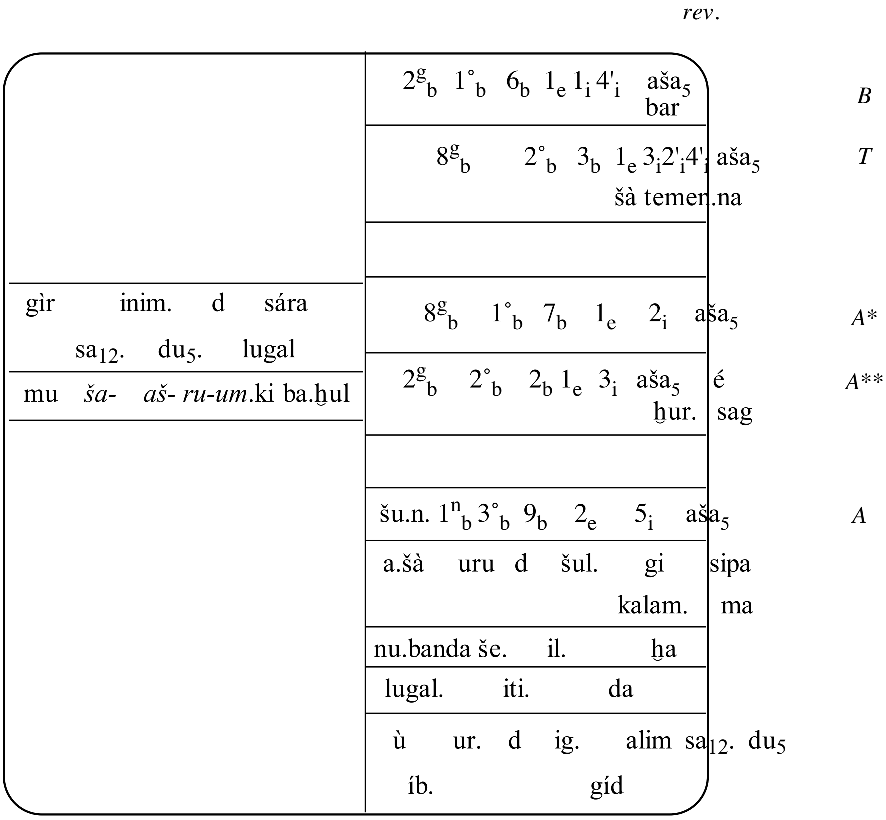


Fig. 5.7. Ist. O (MIO) 1107. The reverse with the summary and a subscript.

The reverse of Ist. O 1107 with its summary and subscript is shown in Fig. 5.7 above. The summary can be explained as follows:

B = 2 16 bùr 1 èše 1 1/4 iku is the sum of the areas of the eleven added fields (Sum. bar) around the temen
T = 8 23 bùr 1 èše 3 1/2 1/4 iku is the average of the two computed values for the area of the temen
A = *T* + *B* = 10 39 bùr 2 èše 5 iku is the total (Sum. šu.nígin) area of the field.

As an afterthought, the total area is split in two parts, *A** = 8 17 bùr 1 èše 2 iku, called aša$_5$ ‘field’, and *A*** = 2 22 bùr 1 èše 3 iku, called aša$_5$ e ‘fields with houses’ and ḫur.sag ‘hilly terrain’. On the obverse, five of the added areas are split in the same way, but the figures don’t add up. Conceivably, the explanation for the discrepancy is that Ist. O 1107 is an incomplete copy of a more complete text, where *all* the added areas were divided into their cultivated and non-cultivated constituents.

Just as in MS 1984 and MS 1850, the total area in the case of Ist. O 1107 is *very close to a round number*. Indeed,

A = 10 39 bùr 2 èše 5 iku = 10 40 bùr – 1 iku = appr. 10 40 bùr (= 41.5 sq. kilometers).

The subscript states that the field plan depicts a town or village (Sum. uru) called Šulgi.sipa.kalam.ma '(king) Šulgi is the shepherd of the country', further mentions the names of the responsible overseer (Sum. nu.bànda), of two 'surveyors' ($sa_{12}.du_5$) who measured (íb.gíd) the fields, and of the 'inspector' (gìr) with the title ($sa_{12}.du_5$.lugal) 'royal surveyor'. The subscript ends with the year name 'the year when the city Šašrū was destroyed'.

A third Ur III field plan is **Wengler 36**, first published by Deimel in *Or* 5 ed. 2 (1930). It has a very elaborate field plan on the obverse, a summary and subscript on the reverse. According to the subscript, it is a text from Umma. A beautiful hand copy, by Maul, in Nissen/Damerow/Englund, *ABK* (1993), Figs. 58-59, is reproduced in Quillien (*op.cit.*), Fig. 8. As in the other Ur III field plans discussed above, the field plan on Wengler 36 shows a field composed of a central temen and a large number of peripheral fields (actually 48). The temen, in its turn, can be composed into six or seven rectangular or trapezoidal fields, for each of which the area is computed in two ways, in the same way as in the case of the field plan on Ist. O 1107. See the detailed analysis in Quillien (*op. cit.*). Unfortunately, although the text is only slightly damaged, it is not clear what the total area $A = T + B$ amounts to. (It seems to be an integral multiple of 1 bùr.) The text was written in 'the year when king Amar-Sîn destroyed Urbilum'.

A fourth Ur III field plan is **HSM 1659** (Dunham, *RA* 80 (1986), 34). It has a relatively simple field plan on the obverse, a summary and subscript on the reverse. According to the subscript, it is a text from Lagash. In this field plan, too, there is a temen. It is composed of a rectangle and a trapezoid. There are also 4 small trapezoidal fields and 2 small triangular fields added outside the temen. The total area $A = T + B$ = 3 bùr 2 èše 1/2 iku. The text was written in "the year when king Shu-Sîn built a big ceremonial boat for the god En-Lil".

A late addition to the manuscript:

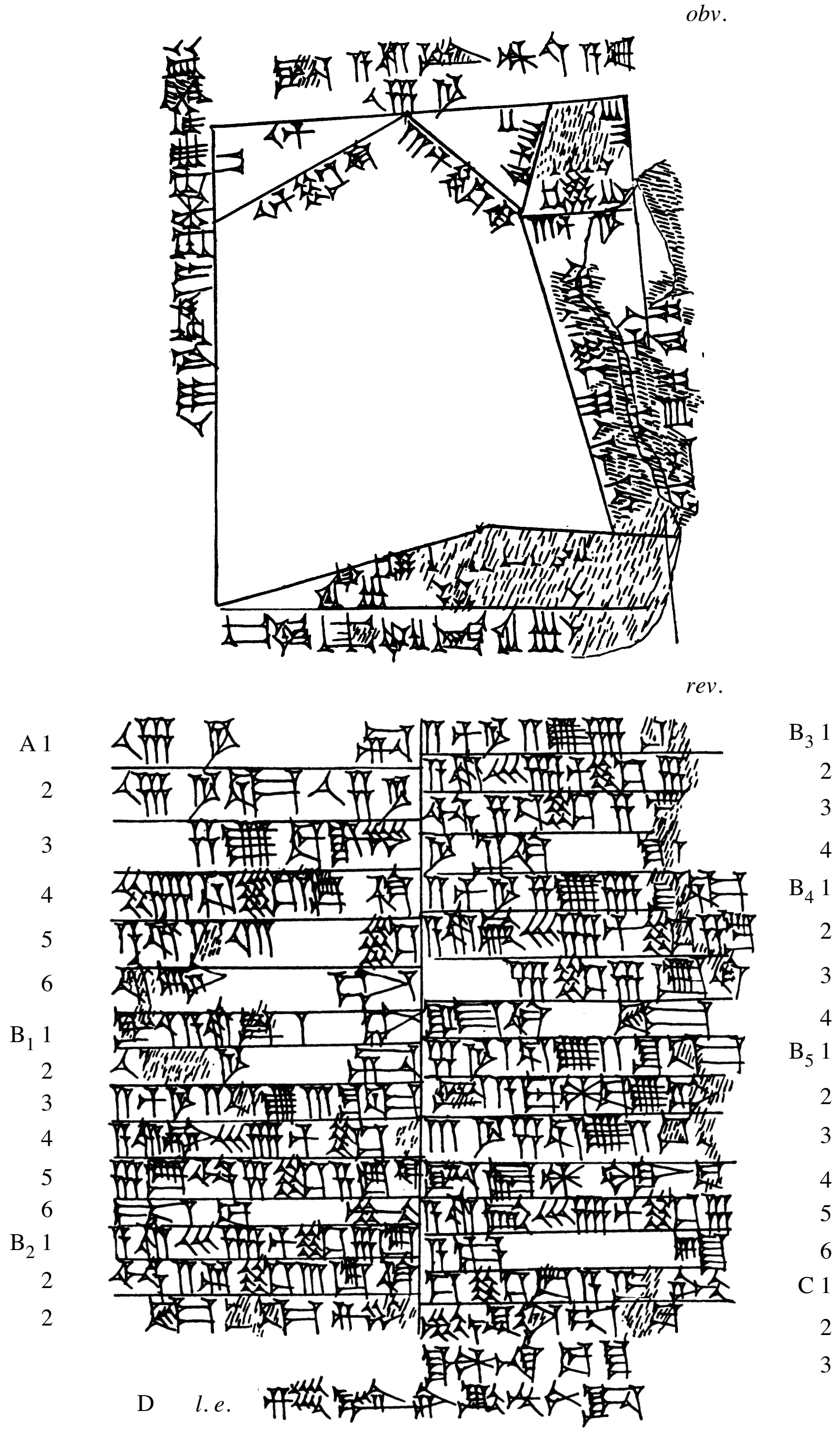
Hand copies of two further field plan texts involving a temen (**VAT 7029** and **VAT 7030**) were published by Schneider in *O*r 47-49 (1930). The hand copies are so sketchy that it is difficult to see what the precise layout actually may have been of the two field plans in these texts.

YBC 3879 (Fig. 5.8 below) is yet another field plan text from Ur III Umma. It was published by A. T. Clay as *Yale Oriental Series 1* (1915) text 24. The field plan on the obverse of the clay tablet shows a trapezoidal temen with six subtracted fields, all denoted by the term ki. The reverse contains a summary of standard type, followed by the data for *a division* into *five parallel stripes of equal area of the 'good' land*, denoted by the term sig_5 and meaning the temen minus the six subtracted fields.

It can be shown that the computation of the widths of the five parallel stripes required, among other things, *the calculation of the 'feed' or growth rate of the subtracted trapezoid* along the right part of the temen, as well as *the solution of a series of quadratic equations*. This is quite surprising since there are no other known examples of *pre-Old Babylonian* texts demonstrating a knowledge of the concept of the 'feed' of a trapezoid or a familiarity with quadratic equations. There are also, by the way, no previously known examples of the use of quadratic equations in a *non-mathematical* cuneiform text!

Moreover, the series of calculations on the reverse of YBC 3879 of the widths of the five parallel stripes of equal area is the earliest known example of a "geometric algorithm". (An astonishing new example of an *Old Babylonian* geometric algorithm, a "chain of trapezoids with fixed diagonals", is discussed in Friberg, *Amazing Traces* (2007), App. 1.)

A detailed discussion of YBC 3879 = *YOS 1*, 24 will appear in a separate publication with the title "A Geometric Algorithm Making Use of Quadratic Equations in a Neo-Sumerian Field Plan Text".



Hand copy: A. T. Clay

Fig. 5.8. *YOS 1*, 24 = YBC 3879. A field plan text from Ur III Umma with a summary and a field division.