10
Three Old Babylonian Mathematical Problem Texts from Uruk

10.1. MS 3971. A Double-Column Mathematical Recombination Text

MS 3971 (Figs. 10.1.1-2 below) is a large fragment of a double-column clay tablet of moderate size. The text on the obverse is only partially well preserved. The text on the reverse is almost completely destroyed, with only the ends of some lines visible on the right edge of the clay tablet.

The nine mathematical exercises inscribed on the obverse can be divided into five different paragraphs, of which the last four deal with 'diagonals' (meaning rectangles). The word used for 'diagonal' is *ṣiliptum*, literally 'that which crosses over', from *ṣalāpum* (Sum. bar) 'to cross over'.

The small vocabulary of Sumerian technical terms used in MS 3971 includes the following words for mathematical operations and structuring of the text:

gar.gar	heap (add)	du$_7$.du$_7$	butt (square)	gaz	break (halve)
zi	tear off (subtract)	nim	lift (multiply)	in.sì	it gives (it is the result)
daḫ	join to (add to)	*n*.e *m* íb.si$_8$	*n* makes *m* equalsided (sqs. *n* = *m*)		
du$_8$	resolve (compute reciprocal)	*aššum* … *amārika*	in order for you to see (as in YBC 4608, *MCT*, 149)		

Consequently, MS 3971 belongs to Goetze's "Group 3" of Old Babylonian mathematical cuneiform texts (see Friberg, *RA* 94 (2000), § 7b). As a member of Group 3, MS 3971 is from Uruk, and middle Old Babylonian, dating to before the year Samsuiluna 11 (1795 BC) (Friberg, *op. cit.*, 174).

10.1 a. MS 3971 § 1. A Broken Reed Problem of a New Type

The upper edge and the uppermost part of MS 3971 are lost. As a consequence of this circumstance, the first seven lines, or so, of § 1, including the question, are missing.

MS 3971 § 1

	… … …	… … …
8	[x x x x x x] x x sa[g gi] /	*x x x x x x* x x the original reed
9	[x x x x x x] x x sag [ki.t]a /	*x x x x x x* x x the *lower* front
10	[x x x x x a].šà *il-li-ik* /	*x x x x x x* the field he went
11	[x x x x x] 30-*šu at-ta-di-ma* /	*x x x x* I have given
12	[x x x x] *at-ta-an-din-nu-ma*$^!$ /	*x x x x that* I have given them, then
13	[x x] x gi.n[a *a*]*t-ta-an-din-ma* /	*x x x* the tru*e x I* have given, then
14	[31] ninda 5 2' kùš sag ki.ta /	*31* ninda 5 1/2 cubits (is) the lower front.
15	6$_{\text{bùr}}$ 1$_{\text{èše}}$ 4$_{\text{iku}}$ 31 šar 15 gín a.šà /	6 bùr 1 è\$e 4 iku 31 \$ar 15 gín (is) the field.
16	sag gi e[n.n]am /	The initial reed i*s w*hat?
17	*ki-ia-a im-ta-x*[x] /	How much x x?
18	[x x x] x [x] ninda$^?$ 1 sag gi.n[a] /	*x x x x x x* x x 1 the true front.

19-20	*ša* [x x 1 *a*]-*na* 2 30 ki.1 daḫ / 2 31 i[n.s]ì	that *x x x 1* to the 1st 2 30 join, 2 31 it gives.
21	2 31 *ù* 2 30 / du$_7$.du$_7$-*m*[*a*] 6 17 30 in.sì /	2 31 and 2 30 (make) butt (each other), 6 17 30 it gives.
22-23	2' 6 17 30 nim *ḫe-ep-pe-e-ma* / 3 08$_v$ 45 in.sì /	1/2 of 6 17 30, the lifted(?), break, then 3 08 45 it gives.
24-25	*mi-na a-na* 3 08$_v$ 45 ḫé.gar$^{!}$ / *ša*	*What* to 3 08 45 shall I set, that
26	31 27 30 sag <ki.ta> in.sì / 10 [šu.si]	31 27 40, the <lower> front, gives. 10, *a finger.*
	2 30 *a-na* 10 nim 25 /	2 30 to 10 lift, 25.
27	25 sag gi in.sì /	25, the initial reed, it gives.
28	25 *a-na* 4 nim 1 40 sag an.na /	25 to 4 lift, 1 40, the upper front.
29	5-*x a-na* 25 daḫ 30 in.sì /	The 5th$^{?}$ to 25 join, 30 it gives.
30	30 *a-na* 6 nim 3 uš gi.na	30 to 6 lift, 3, the true length.

The topic of the imperfectly preserved exercise in MS 3971 § 1 is revealed by a few telltale words:

sag gi	the head of the reed	(the initial length of the measuring stick)	(lines 16, 27)
sag ki.ta	the lower front	(of the trapezoid)	(line 14)
sag an.na	the upper front	(of the trapezoid)	(line 28)
uš gi.na	the true length	(of the trapezoid)	(line 30)
sag gi.na	the true front	(of the trapezoid)	(line 18)

These words make it likely that MS 3971 § 1 is a “broken reed problem”. (See Friberg, *RlA 7* (1990), Secs. 5.4 b, 5.4 e, and 5.7 h.) The lost question, in lines 1-16 of the text, can be reconstructed through a careful analysis of the solution procedure. It appears to have been, essentially, of the following form:

Given a trapezoid and a measuring stick (a ‘reed’) of unknown length.
The reed is applied 6 00 (360) times to measure the length of the trapezoid.
The reed is then shortened by 1/6 of its length.
The shortened reed is applied 4 00(240) times to measure the upper front of the trapezoid.
The shortened reed is finally applied 2 30 (150) times to measure the length of the lower front.
For each application of the reed, a piece (always of the same unknown size) of the length of the reed is lost.
The lower front is 31 ninda 5 1/2 cubits. (line 14)
The area of the trapezoid is 6 bùr 1 èše 4 iku 31 šar 15 gín. (line 15)
What was the initial length of the reed, (and what were the length, the upper front, and the lower front)? (line 16)

The solution procedure is, essentially, the following:

1. ??? (lines 17-18)
2. 2 30 + 2 29 + … 2 + 1 = 2 31 · 2 30 · 1/2 = 3 08 45 (lines 19-23)
3. 3 08 45 · ? = 31;27 30 ninda, the lower front, ? = ;00 10 ninda = 1 finger (lines 24-26)
4. 2 30 · ;00 10 ninda (1 finger) = ;25 ninda (= 5 cubits)= the shortened reed (lines 26-27)
5 ;25 ninda · 4 00 = 1 40 ninda = the upper front (line 28)
6. ;25 ninda · (1 + 1/5) = ;30 ninda = the initial reed (line 29)
7. ;30 ninda · 6 00 = 3 00 ninda= the length (line 30)

The solution procedure starts by making use of the rule for the sum of an arithmetical progression (Friberg, *op. cit.,* Sec. 5.7 h; see also MS 5112 § 5 in Sec. 11.2 h below):

$$n + (n - 1) + \ldots + 2 + 1 = 1/2 \cdot (n + 1) \cdot n.$$

Therefore, the size of the pieces that fell off can be computed as the solution to a division problem:

1/2 · 2 30 · 2 31 = 3 08 45, 3 08 45 · ? = 31 ninda 5 1/2 cubits = 3 08 45 fingers.
Answer: ? = ;00 10 ninda = 1 finger.

Next, the length of the shortened reed is computed as 2 30 times the length of one of the pieces:

The shortened reed = 2 30 · ;00 10 ninda = ;25 ninda (= 5 cubits).
The upper front = ;25 ninda · 4 00 = 1 40 ninda.

The length of the original reed is computed as follows:

Since the shortened reed is 1/6 less than the original reed, the original reed is 1/5 more than the shortened reed.
Hence, the original reed is ;25 ninda + 1/5 · ;25 ninda = ;30 ninda (= 6 cubits = 1 reed).

Finally, when the original length of the reed is known, the length of the trapezoid can be computed:

The length = ;30 ninda · 6 00 = 3 00 ninda.

10.1 b. Other Examples of Broken Reed Problems with Arithmetical Progressions

AO 6770 # 5 (Group 1: Larsa; Thureau-Dangin, *TMB* (1938), 73)

1	gi šu.ba.an.ti	A reed I received.
	1 šu.si *im-ta-qú-ta-an-ni* /	1 finger fell off for me.
2	*a-ša-a*[*r ig*]*-ga-am-ra-an-ni* 4_{v} kùš /	Where it was exhausted: 4 cubits.
3	*re-iš q*[*á*]*-ni-ia mi-nu-um* /	The head of my reed (was) what?
4	*re-iš qá-ni-ia* 2' kùš /	The head of my reed (was) 1/2 cubit.
5	<igi> [ig]i.gub šu.si *a-pa-ṭa-ar*	The <opposite of> the constant for a finger I resolve,
	a-na 4_{v} kùš *a-na-aš-ši* /	to 4 cubits I lift (it),
6	[*a*]*-na ši-na e-ṣí-ip*	to two I repeat.
7	1 *wa-ṣi-ta-am* / [2']-*šu e-ḫe-pe*	1, the extension, its 1/2 I break.
	íb.si_8 *a-na* tab.ba bi.[da]ḫ	(Its) equalside to the doubled I join.

In this text, the stated problem is as follows:

A reed of unknown length is applied an unknown number of times to measure a length of 4 cubits.
For each application of the reed, a piece of length 1 finger falls off. What was the original length of the reed?

The problem in AO 6770 # 5 is complementary to the problem in MS 3971 § 1. In the former, the length of the pieces falling off is known, but not their number, while in the latter the number of pieces is known, but not their length. Exceptionally, the solution procedure in AO 6770 # 5 is *verbal and general* rather than numerical, and in the first person. It is also incomplete, simply for the reason that there was no space left on the tablet for the end of the text. Anyway, if the solution procedure had been explicit and complete, it would have run as follows (in quasi-modern terms, since it is difficult to know how an Old Babylonian writer would have expressed it):

Let n be the number of times that the reed can be applied before it is exhausted.
Then $1/2 \cdot (n + 1) \cdot n$ fingers = 4 cubits = ;20 ninda.
Since 1 finger = ;00 10 ninda, 1 ninda = 6 00 fingers,
and 4 cubits = ;20 ninda = 6 00 · ;20 ninda = 2 00 fingers. (line 5)
Hence, sq. $n + 1 \cdot n = 2 \cdot 2\ 00 = 4\ 00$, where the coefficient 1 is called *wāṣītum* the 'extension'. (line 6)
By completion of the square: sq. $(n + 1/2)$ = sq. 1/2 + 4 00 = 4 00;15 = sq. 15 1/2.
The square side 15 1/2 = n +1/2. Hence, $n = 15$, so that the initial reed was 15 fingers = 1/2 cubit. (line 7)

In both MS 3971 § 1 and AO 6770 # 5 the sum of the arithmetical progression is known but the length of the reed is unknown. In the next example, the length of the reed is given from the start, and the sum of the arithmetical progression has to be computed:

Str. 362 # 5 (Group 3: Uruk; Neugebauer, *MKT 1* (1935), 240; Thureau-Dangin, *TMB* (1938), 83)

1	gi.1.kùš	A 1-cubit-reed.
2	1 šu.si.ta.àm *a-di ig-ga-am-ru* / *im-ta-qú-ta-an-ni*	1 finger each time until it was exhausted fell off for me.
3	uš *ki ma-ṣi* / *al-li-ik*	How much length did I go?
4	1 ninda 3 2' kùš uš / *al-li-ik*	1 ninda 3 1/2 cubits the length I went.

The solution procedure is omitted in this brief text. It would, essentially, have run as follows:

(30 + 29 + … + 2 + 1) fingers = 1/2 · 31 · 30 fingers = 7 45 fingers
7 45 fingers = 7 45 · ;00 10 ninda = 1;17 30 ninda = 1 ninda 3 1/2 cubits

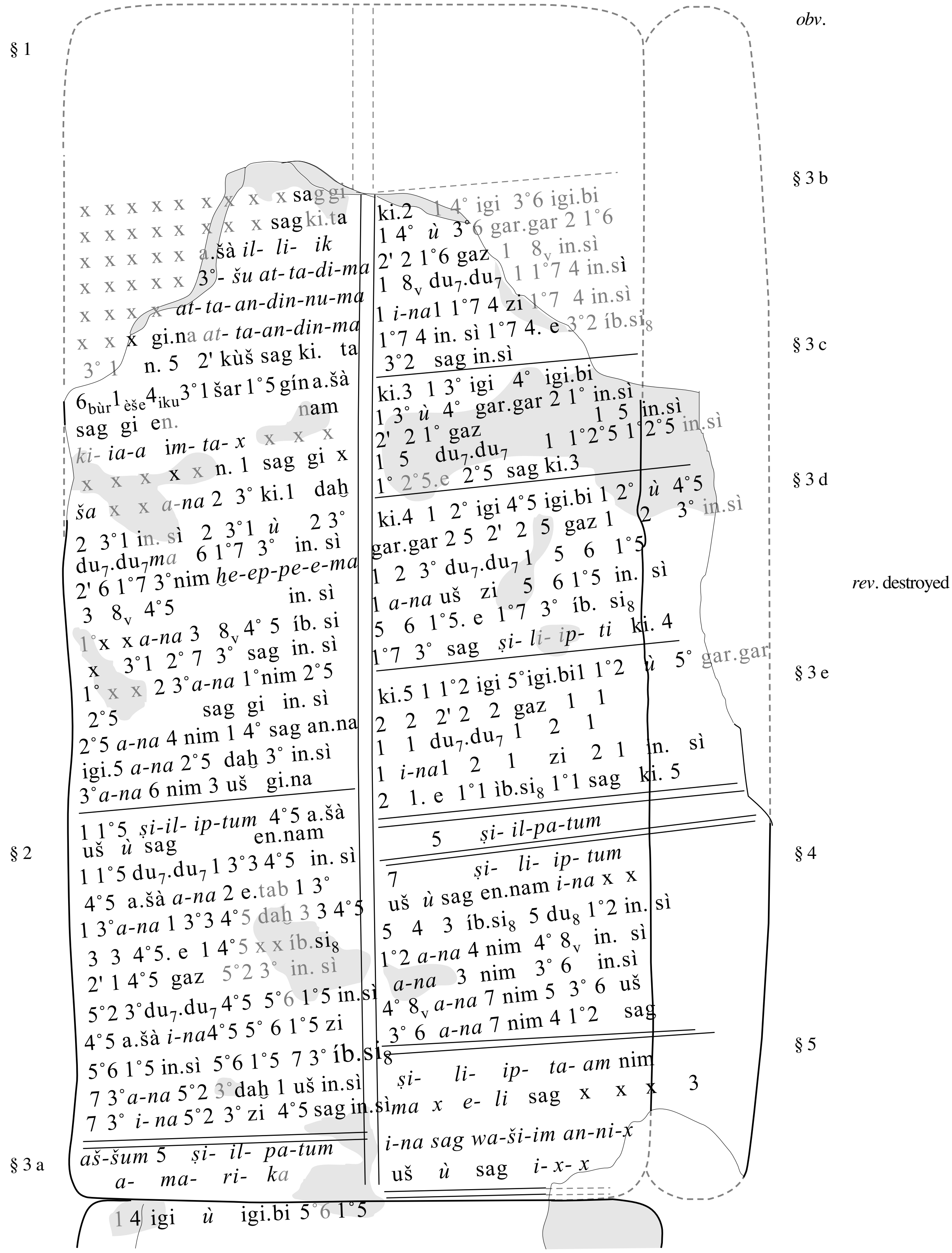
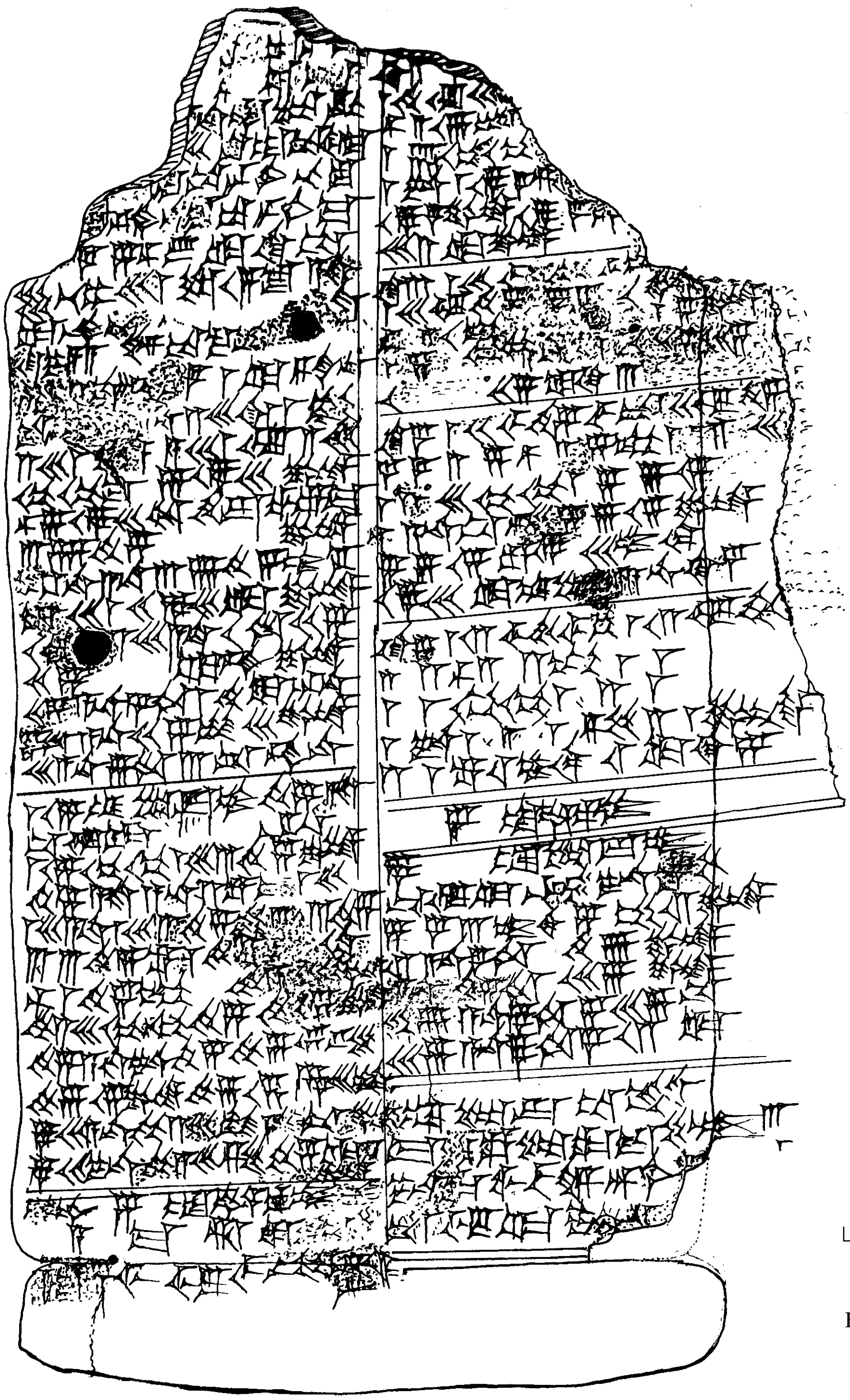


Fig. 10.1.1. MS 3971, *obv*. A large fragment of a mathematical recombination text. Five preserved themes.



Note the three punched holes in the surface of the clay tablet, analogous to those that one finds on clay tablets from the Seleucid period

3 cm

Scale 1 : 1
Hand copy: F. Al-Rawi

Fig. 10.1.2. MS 3971, *obv*. Hand copy of the cuneiform text. (The reverse is completely destroyed.)

10.1 c. Other Examples of Broken Reed Problems for Rectangles or Trapezoids

The simplest example of this kind is concerned with the sides of *a rectangle*:

IM 53965 (Tell Harmal; Baqir, *Sumer* 7 (1951), 39)

1	*šum-ma ki-a-am i-ša-al-*[*ka um-ma šu-ú-ma*] /	If somebody asks you thus:
2	*qa-na-am el-qé-e-ma*	A reed I took,
3	[*mi-in-da-su*] / *ú-ul i-de-e-ma*	its measure I do not know.
4	*šu-ši-da-am al-li-ik* / *am-ma-at aḫ-ṣú-úb-šu-ma*	Sixty-length I walked, a cubit I broke off it, then
5	*ša-la-aš pu-ta-am* / [*a*]*l-li-ik* a.šà 4 10	thirty, the front, I walked. The field (is) 4 10.
6	*ši-di ù pu-ti* / [*ki*] *ma-ṣí* *etc.*	My length and my front how much? *etc.*

The question in this exercise can be reduced to the following system of equations:

u = 1 00 r, s = 30 · (1 r – 1 cubit), $u \cdot s = A$ = 4 10 sq. n., where r is the unknown length of the reed.

This system of equations can be reduced to the following rectangular-linear system of equations:

$u \cdot s^*$ = 2 · 4 10 sq. n. = 8 20 sq. n., $u - s^*$ = 2 · 30 cubits = 5 n., where $s^* = 2 \cdot s$.

The solution to this system of equations, computed in the usual way, is

u = 25 n., s^* = 20 n., so that s = 10 n., and r = ;25 n. = 5 cubits = the original length of the reed.

A more complicated example is concerned with the sides of *a trapezoid*:

VAT 7532 (Group 3: Uruk; Høyrup, *LWS* (2002), 209-213)

1	sag.ki.gud	An ox-head (trapezoid).
	gi kud	A reed I cut.
	gi *e*[*l-qé-ma i-na š*]*u-u*[*l*]*-m*[*i*]*-šu* /	The reed I to*ok in its w*hole (length),
2	1 *šu-ši* uš *al-li-i*[*k*]	1 sixty, the length, I walked.
3	[igi.6.gá]l / *iḫ-ḫa-aṣ-ba-an-ni-ma*	*The 6th-part* he cut off for me, then
	1 12 *a-na* u[š] *ú-r*[*e*]*-ed-di* /	1 12 to the length I added.
4	*a-tu-úr*	I turned back.
	igi.3.gál *ù* 3' kùš *iḫ-*[*ḫa-aṣ-ba-a*]*n-ni-ma* /	The 3rd-part and 1/3 cubit he br*oke off* for me, then
5	3 *šu-ši* sag an.na *al-li-ik* /	3 sixties, the upper front, I walked.
6	*ša iḫ-ḫa-aṣ-ba-an-ni ú-te-er-šum-*[*m*]*a* /	What he broke off for me I returned to it, then
7	36 sag <ki.ta> *al-li-ik*	36, the <lower> front, I walked.
	$1_{\text{bùr}}$ aša$_5$ a.šà	1 bùr (is) the field.
	sag gi en.nam *etc.*	The initial reed is what? *etc.*

The question in this exercise can be expressed as follows:

u = 1 00 r + 1 12 r^*, s_a = 3 00 r^{**}, s_k = 36 r^*,
$u \cdot (s_a + s_k)/2 = A$ = 1 bùr = 30 00 sq. n.,
where $r^* = (1 - 1/6) \cdot r$, and $r^{**} = (1 - 1/3\} \cdot r^* - 1/3$ cubit.

In the solution procedure, this question is reduced to the following quadratic equation:

6 14 24 sq. r^* – 12 00 r^* = 1 00 00 sq. n.

The solution to this equation is computed in the usual way. The resulting values of the unknowns are:

r^* = ;25 n. = 5 cubits,
r = ;30 n. = 6 cubits, r^{**} = ;15 n. = 3 cubits,
u = 1 00 n., s_a = 45 n., s_k = 15 n.

Parallel texts are VAT 7535 ## 1-3 (hand copy: *MKT 2*, pl. 47). Both VAT 7532 and VAT 7535 ##1-3 are illustrated by drawings of trapezoids, with inserted values of various parameters.

10.1 d. MS 3971 § 2. To Find a Rectangle with a Given Diagonal and a Given Area

MS 3971 § 2:

1	1 15 *ṣi-i*[*l-i*]*p-tum* 45 a.šà /	1 15 the cross-over (diagonal), 45 the field.
2	uš *ù* sag en.nam /	The length and the front are what?
3	1 15 du_7.du_7 1 33 45 in.sì /	1 15 (make) butt (itself) 1 33 45 it gives.
4	45 a.šà *a-na* 2 e.ta[b] 1 30 /	45, the field, to 2 you repeat, 1 30.
5	1 30 *a-na* 1 33 45 d[aḫ] 3 03 45 /	1 30 to a 33 45 *join, 3* 03 45.
6	3 03 45.e 1 4[5 íb].si_8 /	3 03 45 makes 1 4*5 equ*alsided.
7	2' 1 45 gaz 5[2 30 in.sì] /	1/2 of 1 45 break, *52 30 it gives*.
8	52 30 du_7.du_7 45 5[6] 15 in.sì /	52 30 (make) butt (itself), 45 *56* 15 it gives.
9-10	45 a.šà *i-na* 45 56 15 zi / 56 15 in.sì	45, the field, from 45 *56 15* tear off, 56 15 it gives.
	56 15 7 30 íb.si_8 /	56 15 <makes> 7 30 equalsided.
11	7 30 *a-na* 52 30 daḫ 1 uš in.sì /	7 30 to 52 30 join, 1, the length, it gives.
12	7 30 *i-na* 52 30 zi 45 sag in.sì /	7 30 from 52 30 tear off, 45, the front, it gives.

In modern language, the question in MS 3971 § 2 can be rephrased as follows:

> The diagonal d of a rectangle is 1 15, and the area A is 45 (00).
> What are the length u and the front s of the rectangle?

The successive steps of the numerical solution procedure are the following:

sq. d = sq. 1 15 = 1 33 45, 2 · A = 2 · 45 (00) = 1 30 (00), 1 33 45 + 1 30 (00) = 3 03 45 (lines 3-5)
sqs. 3 03 45 = 1 45 = p, 1 45 /2 = 52;30 = $p/2$, sq. 52;30 = 45 56;15 (lines 6-8)
45 56;15 – A = 45 56;15 – 45 (00) = 56;15, sqs. 56;15 = 7;30 = $q/2$ (lines 9-10)
52;30 + 7;30 = 1 (00) = u, 52;30 – 7;30 = 45 = s (lines 11-12)

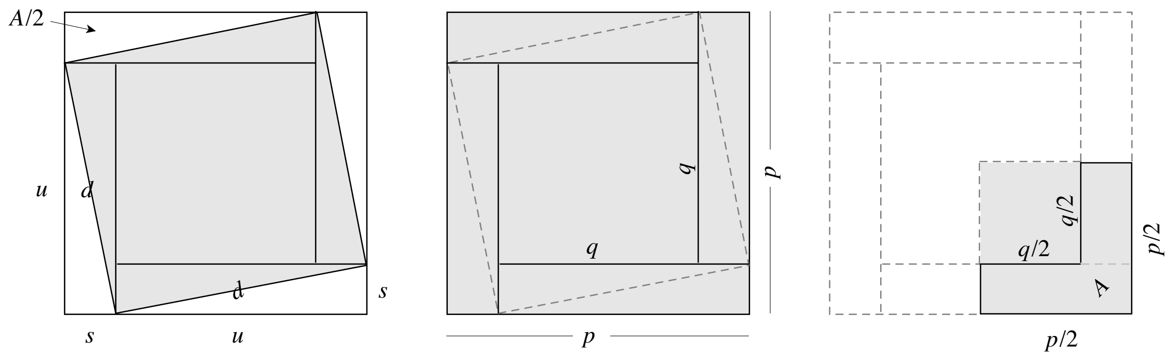
It is possible that the solution procedure in MS 3971 § 2 is best understood as consisting of two steps. In the first step, the given problem is reduced to a rectangular-linear system of equations of the following standard form:

$$u \cdot s = A = 45\ (00), \quad u + s = p = 1\ 45.$$

In the second step, this rectangular-linear system is solved in the usual way.

The answer to the stated problem is, of course, that d, u, s = 1 15, 1 (00), 45, the Babylonian standard example of a "diagonal triple". (See MS 3052 § 2 in Sec. 10.2 b below and App. 8.)

A geometric interpretation of the solution procedure is given in Fig. 10.1.3 below:



sq. u + sq. s = sq. d | sq. d + 2 A = sq. p | sq. $p/2$ – A = sq. $q/2$

$u \cdot s = A$ | $u + s = p$, $u - s = q$ | $p/2 + q/2 = u$, $p/2 - q/2 = s$

Fig. 10.1.3. MS 3971 § 2. Given the square on the diagonal of a rectangle and its area.

10.1 e. A Parallel Text: IM 67118 (= Db_2-146)

IM 67118 is from Old Babylonian Eshnunna, dated to the time of Ibal-Piel II (1779-1763).

IM 67118, the first half (Høyrup, *LWS* (2002), 257-261)

1-2	*šum-ma ṣí-li-ip-ta-a-am i-ša-lu-ka* / *um-ma šu-ú-ma*	If (about) a cross-over (rectangle) he asks you thus:
	1 15 *ṣí-li-ip-tum* 45 a.šà /	1 15 the cross-over (diagonal), 45 the field.
3	*ši-di ù* sag.*ki ki ma-a-ṣí*	My length and (my) front, how much?
	at-ta i-na e-pé-ši-ka /	You, in your doing:
4	1 15 *ṣí-li-ip-ta-ka me-ḫe-er-šu i-di-i-ma* /	1 15, your cross-over, its copy lay down.
	šu-ta-ki-il-šu-nu-ti-ma	make them eat each other, then
5	1 33 45 *i-li* / 1 33 45 šu ku.u.zu$^{?}$ /	1 33 45 comes up. 1 33 45 the hand x x.
6	45 a.šà-*ka a-na ši-na e-bi-il-ma* 1 30 *i-li* /	45, your field, bring to two, then 1 30 comes up.
7	*i-na* 1 33 45 *ḫu-ru-uṣ-ma*	From 1 33 45 cut (it) out, then
	{1 3}3 45 *ša-pí-il-tum* /	3 45$^{!}$ the remainder.
8	ib.sí 3 45 *le-qe-e-ma* 15 *i-li*	The equalside of 3 45 take, then 15 comes up.
9	*mu-ta-šu* / 7 30 *i-li*	Its half-part(?) 7 30 comes up.
	a-na 7 30 *i-ši-i-ma* 56 15 *i-li* /	To 7 30 raise (it), then 56 15 comes up.
10	56 15 šu-*ka*	56 15 your hand.
11	45 a.šà-*ka e-li* šu-*ka* / 45 56 15 *i-li*	45, your field, above your hand, 45 56 15 comes up.
12	ib.si 45 56 15 *le-qe-ma* / 52 30 *i-li*	The equalside of 45 56 15 take, then 52 30 comes up.
13	52 30 *me-ḫe-er-šu i-di-i-ma* /	52 30, its copy lay down, then
14	7 30 *ša tu-uš-ta-ki-lu*	7 30 that you made eat
15-16	*a-na iš-te-en* / *ṣí-ib-ma i-na iš-te-en* / *ḫu-ru-uṣ*	to one add, then from one cut out:
	1 uš-*ka* 45 sag.ki	1, your length, 45, the front.

This is the same metric algebra problem as the one in MS 3971 § 2, and the solution procedure is (essentially) the same in both exercises. It is interesting to note, however, that IM 67118 is verbose and written mostly in syllabic Akkadian, while MS 3971 § 2 is concise and written mostly in terms of Sumerian logograms. Another, insignificant, difference between the two texts is that in the solution procedure in IM 67118, the area is *subtracted* from the square of the diagonal, while in MS 3971 § 2 the area is *added* to the square of the diagonal.

10.1 f. MS 3971 § 3. Five Examples of igi-igi.bi Problems

MS 3971 § 3

§ 3 a	1-2	*aš-šum* 5 *ṣi-il-p*[*a-tum*] / *a-ma-ri-k*[*a*] /	In order for you to see five cross-over*s*:
	3	[1 04] igi *ù* igi.bi 5[6] 15$^{!}$ /	*1 04* (is) the igi, and the igi.bi *56* 15.
	5	[… … … … …]	… … … … …
	6	[… … … … …]	… … … … …
	7	[… … … … …]	… … … … …
	8	[… … … … …]	… … … … …
§ 3 b	1	ki.2	The 2nd (example).
		[1 40 igi 36 igi.bi] /	1 40 the igi, 36 the igi.bi.
	2	1 40 *ù* 3[6 gar.gar 2 16] /	1 40 and 36 *heap, 2 16 it gives.*
	3	2' 2 16 gaz [1 08 in.sì] /	1/2 of 2 16 break, *1 08 it gives.*
	4	1 08 du_7.du_7 [1 17 04 in.sì] /	1 08 (make) butt (itself), *1 17 04 it gives.*
	5-6	1 *i-na* 1 17 04 z[i 17 04 in.sì] / 17 04 in.sì	1 from 1 17 04 tear *off, 17 04 it gives.* «17 04 it gives»
		17 04.e [32 íb.si_8] /	17 04 makes *32 equalsided.*
	7	32 sag in.sì	32, the front, it gives

§ 3 c	1	ki.3	The 3rd.
		1 30 igi 40 igi.bi /	1 30 the igi, 40 the igi.bi.
	2	1 30 *ù* 40 gar.gar 2 10 in.sì /	1 30 and 40 heap, 2 10 it gives.
	3	[2'] 2 10 gaz 1 05 in.sì /	*1/2 of* 2 10 break, 1 05 it gives.
	4	1 05 du$_7$.du$_7$ 1 10 25	1 05 (make) butt (itself), 1 10 25.
		<1 *i-na* 1 10 25 zi> 10 25 [in.sì] /	<1 from 1 10 25 tear off> 10 25 *it gives*.
	5	10 [25.e] 25 sag ki.3	10 *25 makes* <25 equalsided>, 25 the third front.
§ 3 d	1	ki.4	The 4th.
		1 20 igi 45 igi.bi	1 20 the igi, 45 the igi.bi.
	2	1 20 *ù* 45 / gar.gar 2 05	1 20 and 45 heap, 2 05.
		2' 2 05 gaz 1 02 30 333[in.sì] /	1/2 of 2 05 break, 1 02 30 *it gives*.
	3	1 02 30 du$_7$.du$_7$ 1 05 06 15 /	1 02 30 (make) butt (itself), 1 05 06 15.
	4	1 *a-na* uš zi 5 06 15 in.sì /	1 to$^!$ the length$^!$ tear off, 5 06 15.
	5	5 06 15.e 17 30 íb.si$_8$ /	5 06 15 makes 17 30 equalsided.
	6	17 30 sag *ṣi-l*[*i-i*]*p-ti* ki.4	17 30, the front of the 4th cross-over.
§ 3 e	1	ki.5	The 5th.
		1 12 igi 50 igi.bi	1 12 the igi, 50 the igi.bi.
	2	1 12 *ù* 50 [gar.gar] / 2 02	1 12 and 50 *heap*, 2 02.
		2' 2 02 gaz 1 01 /	1/2 of 2 02 break, 1 01.
	3	1 01 du$_7$.du$_7$ 1 02 01 /	1 01 (make) butt (itself), 1 02 01.
	4	1 *i-na* 1 02 01 zi 2 01 in.sì /	1 from 1 02 01 tear off, 2 01 it gives.
	5	2 01.e 11 íb.si$_8$ 11 sag ki.5	2 01 makes 11 equalsided. 11, the 5th front.
		5 *ṣi-il-pa-tum*	5 cross-overs.

The five exercises MS 3971 § 3 a-e differ from each other only in the initial choice of parameters, in each case a pair of reciprocals, borrowed from the Old Babylonian standard table of reciprocals (see Sec. 2.5 above). The pair is in each case referred to as igi and igi.bi 'its igi'. (Remember that every regular sexagesimal number is the reciprocal of its own reciprocal.) The five given sets of data are

§ 3 a: igi = 1 04, igi.bi = 56 15
§ 3 b: igi = 1 40, igi.bi = 36
§ 3 c: igi = 1 30, igi.bi = 40
§ 3 d: igi = 1 20, igi.bi = 45
§ 3 e: igi = 1 12 igi.bi = 50

The question may have been formulated in the initial problem, § 3 a, but if so, it is lost, as is also the solution procedure in § 3 a. However, the four other examples are relatively well preserved. Each one of them starts with the setting of the data and continues directly with the solution procedure. The missing question in each case can be reconstructed and seems to have been of the following form (in quasi-modern notations):

Let igi = u and igi.bi = s be a given pair of reciprocals,
and assume that a rectangle has the diagonal $c = (u + s)/2$
and the length b = 1 (00). Find the front a.

This is precisely the same situation as in the single exercise MS 3052 § 2. (See below, Sec. 10. 2 b.) In the case of § 3 b, for instance, the solution procedure is as follows:

u = 1 40, s = 36, $u + s$ = 2 16, $c = (u + s)/2$ = 1 08 (lines 1-3)
sq. c = sq. 1 08 = 1 17 04 (line 4)
sq. c – sq. b = 1 17 04 – 1 (00 00) = 17 04, a = sqs. [sq. c – sq. b] = sqs. 17 04 = 32 (lines 5-7)

The interpretation of the solution procedure is both obvious and simple: With the diagonal c and the length b given, a is computed through an application of the Old Babylonian diagonal rule.

It is less obvious *why* the diagonal should be given as the half-sum of a regular sexagesimal number and its reciprocal in the five exercises MS 3971 § 3 a-e, as well as in the single exercise MS 3052 § 2. The interesting answer to this puzzle is given in App. 8, in connection with a renewed discussion of the famous Old Babylonian table text Plimpton 322.

10.1 g. MS 3971 § 4. A Scaling Problem for a Rectangle with its Diagonal

MS 3971, *obv.*

§ 4	1-2	7 *ṣi-li-ip-tum* /	7, the cross-over (diagonal).
		uš *ù* sag en.nam *i-na* x x /	The length and the front (are) what in the x x?
	3	5 4 3 íb.si$_8$	5, 4, 3, the square sides!.
		5 du$_8$ 12 in.sì /	5 release, 12 it gives.
	4	12 *a-na* 4 nim 48$_v$ in.sì /	12 to 4 lift, 48 it gives.
	5	[12 *a-na* 3] nim 36 in.sì /	*12 to 3* lift, 36 it gives.
	6	48$_v$ *a-na* 7 nim 5 36 uš /	48 to 7 lift, 5 36, the length.
	7	36 *a-na* 7 nim 4 12 sag /	36 to 7 lift, 4 12, the front.

The vaguely formulated question in this exercise can be more precisely reformulated as follows:

Find a rectangle with the diagonal 7, and with the diagonal, the length, and the front proportional to 5, 4, 3.

The solution procedure is simple enough. It begins with the following computations:

$$\text{rec. } 5 = 12, \quad 12 \cdot 4 = 48, \quad 48 \cdot 45 = 36$$

What this means is that first a "normalized" rectangle is constructed with the diagonal $d^* = 1$ and with the diagonal d^*, the length u^*, and the front s^* proportional to 5, 4, 3, so that at least one of the stated conditions is satisfied. This is achieved by setting

$$d^* = ;12 \cdot 5 = 1, \quad u^* = ;12 \cdot 4 = ;48, \quad s^* = ;45 \cdot u^* = ;36 \qquad \text{(lines 3-5)}$$

The wanted rectangle is then obtained simply by setting

$$d = d^* \cdot 7 = 1 \cdot 7 = 7, \quad u = u^* \cdot 7 = ;48 \cdot 7 = 5.36, \quad s = s^* \cdot 7 = ;36 \cdot 7 = 4;12 \qquad \text{(lines 6-7)}$$

The two steps of the solution procedure are illustrated in Fig. 10.1.4 below:

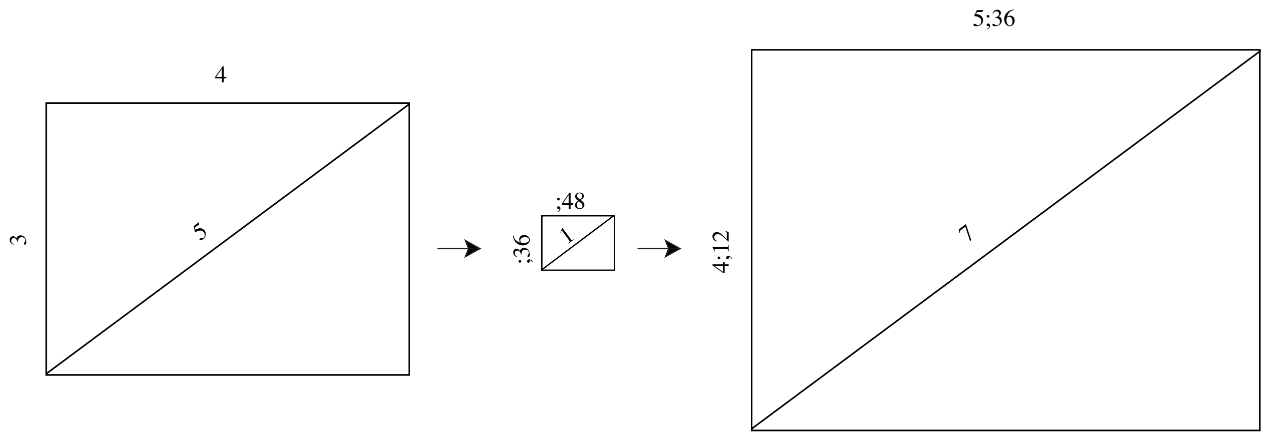


Fig. 10.1.4. Construction of a rectangle with given diagonal and given proportions.

10.2. MS 3052. A Single-Column Mathematical Recombination Text

MS 3052 is a large single-column clay tablet with a well preserved obverse (Figs. 10.2.1-3), but an almost ruined reverse (Figs. 10.2.13-14). It is inscribed with four mathematical exercises on the obverse and four on the reverse. A subscript indicates that five of the exercises deal with partitioned mud walls, while the three remaining exercises deal with, in order, a triangle, an 'excavation', and a square. The small vocabulary of Sumerian technical terms used in the text includes the following words for mathematical operations, *etc*.:

gar.gar	heap (add)	$du_7.du_7$	butt (square)	gaz	break (halve)
zi	tear off (subtract)	nim	lift (multiply)	in.sì	it gives (is the result)
a-na 2 e.tab	to 2 repeat (double)	du_8	resolve (compute reciprocal)		
daḫ	join to (add to)	*n*.e *m* íb.si_8	*n* makes *m* equalsided (sqs. $n = m$)		
aššum … *amārika*	in order for you to see				

Therefore, MS 3052, just like MS 3971, appears to belong to Goetze's "Group 3" of Old Babylonian mathematical cuneiform texts. As probable member of Group 3, MS 3052 should be from Uruk, and middle Old Babylonian, dating to before Samsuiluna 11 (1795 BC). Note, however, in this connection, that no recent removals from Iraq are known to come from Uruk, a site that has remained well guarded by locals after the Kuwait war. The implication of this observation is that MS 3052 and MS 3971 were exported from Iraq and reached the antiquities market *before* the recent events in Iraq.

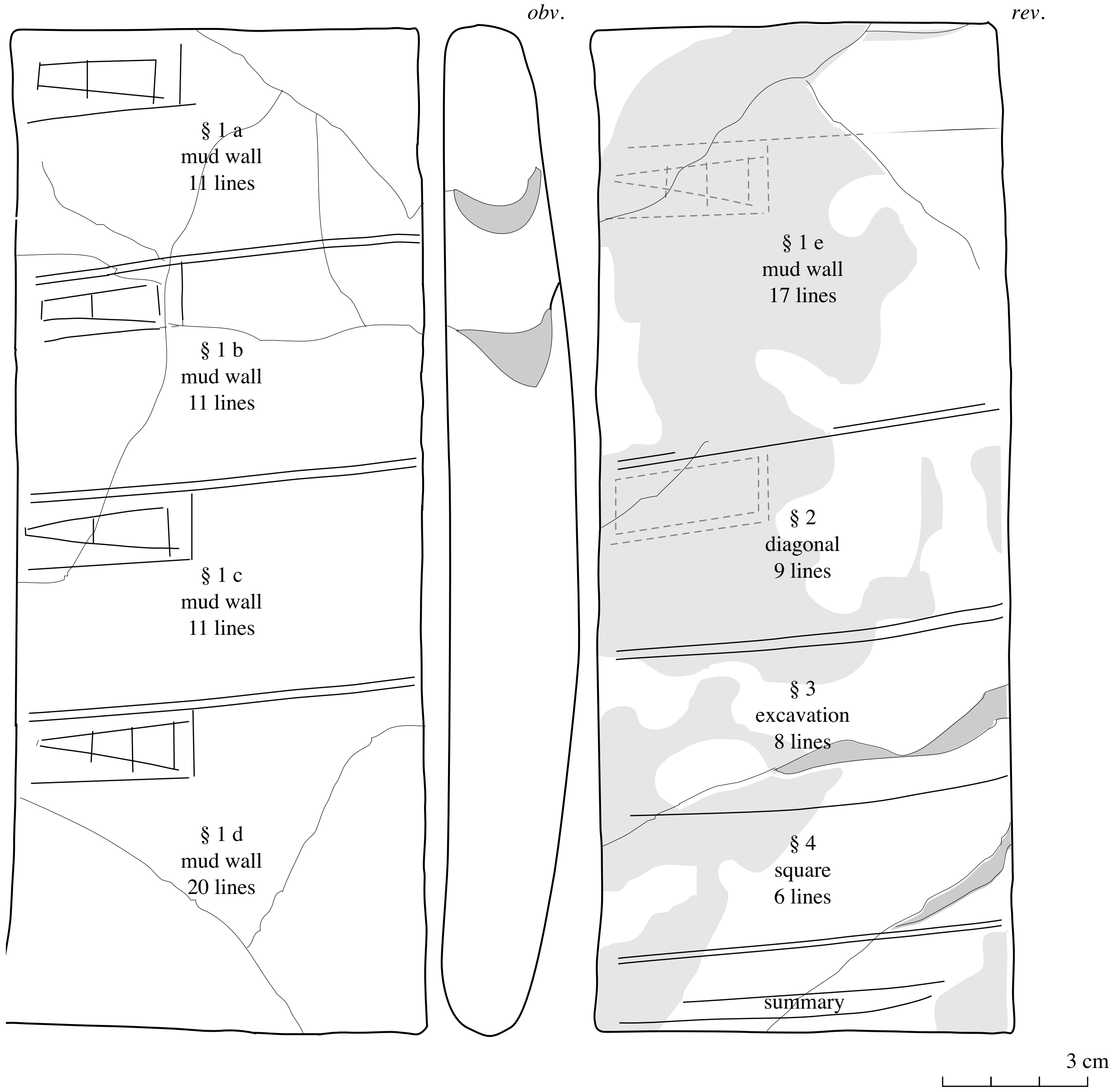


Fig. 10.2.1. MS 3052. Outline with indication of topics.

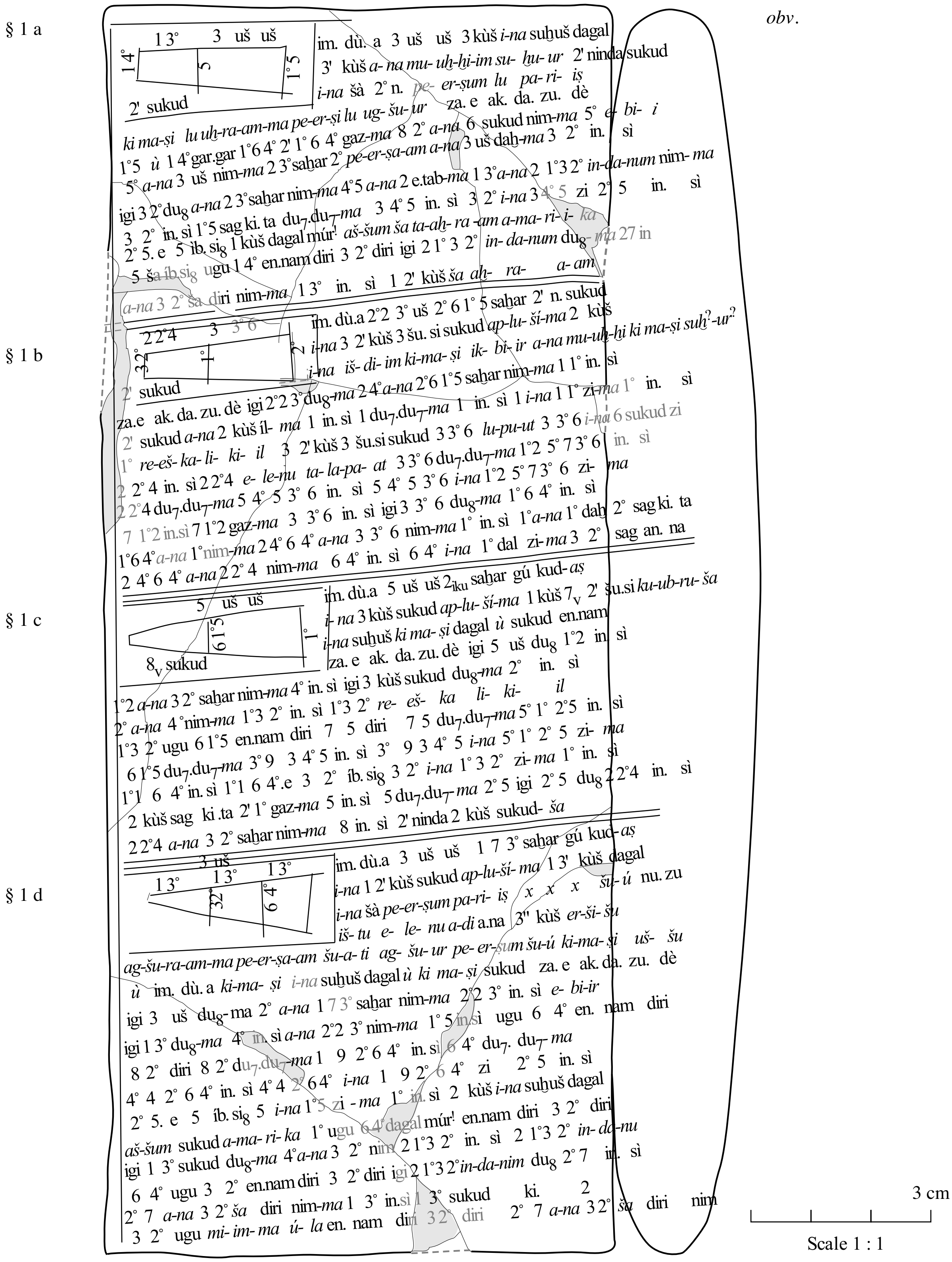
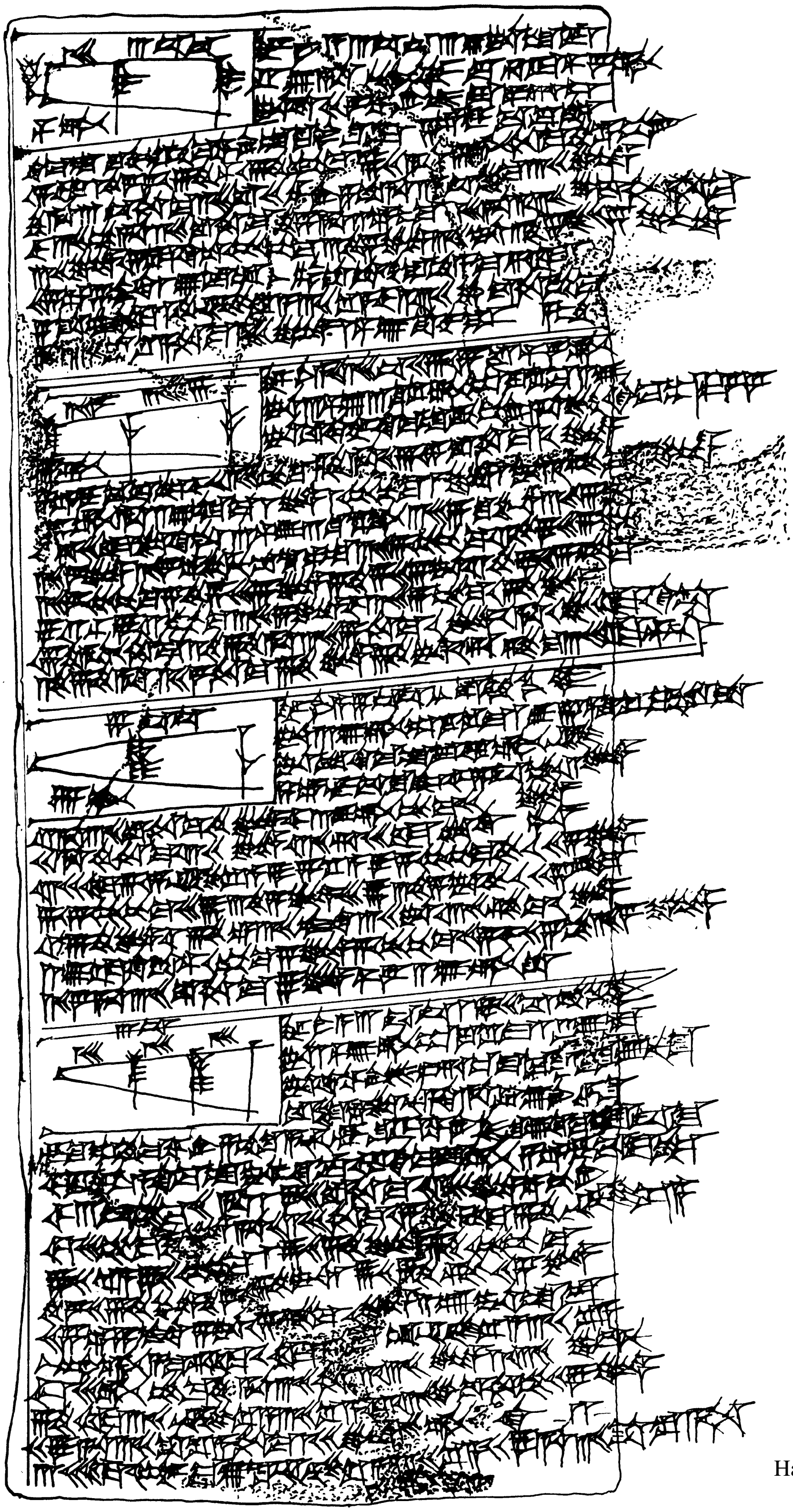


Fig. 10.2.2. MS 3052, *obv*. Conform transliteration.



Hand copy: F. Al-Rawi

Fig. 10.2.3. MS 3052, *obv*. Hand copy of the cuneiform text.

10.2 a. MS 3052 § 1. Mud Walls Partitioned into Two or More Separate Layers

MS 3052 § 1 a. Repairing a Breach in a Wall with Mud from the Top of the Wall

MS 3052 § 1 a

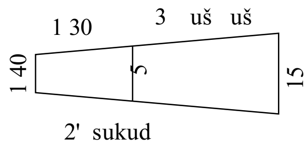


Fig.

1	im.dù.a	A mud wall.
	3 uš uš	3 uš (= 3 00 n.) the length,
	3 kùš *i-na* suḫuš dagal /	3 cubits at the base the width,
2	3' kùš *a-na mu-uḫ-ḫi-im su-ḫu-ur*	1/3 cubit at the top it is turned back (it is narrow),
	2' ninda sukud /	1/2 ninda the height.
3	*i-na* šà 20 ninda *pe-er-ṣum lu pa-ri-iṣ* /	Inside it (for) 20 ninda a breach may be breached (opened).
4	*ki ma-ṣi lu uḫ-ra-am-ma pe-er-ṣi lu ug-šu-ur*	How much may I cut away that I may repair the breach?
	za.e ak.da.zu.dè /	You, with your doing:
5	15 *ù* 1 40 gar.gar 16 40	15 and 1 40 heap, then 16 40.
	2' 16 40 gaz-*ma* 8 20	1/2 of 16 40 break, then 8 20.
	a-na 6 sukud nim-*ma* 50 *e-bi-ir* /	To 6, the height, lift (it), then 50, the stretched-across.
6	50 *a-na* 3 uš nim-*ma* 2 30 saḫar	50 to 3, the length, lift, then 2 30, the mud.
	20 *pe-er-ṣa-am a-na* 3 uš daḫ-*ma* 3 20 in.sì /	20, the breach, to 3, the length, join, then 3 20 it gives.
7	igi 3 20 du$_8$	The opposite of 3 20 release.
	a-na 2 30 saḫar nim-*ma* 45	To 2 30, the mud, lift (it), then 45.
	a-na 2 e.tab-*ma* 1 30	To 2 you repeat, then 1 30.
8	*a-na* 2 13 20 *in-da-nim* nim-*ma* / 3 20 in.sì	To 2 13 20, the *indanum*, lift (it), then 3 20 it gives.
	15 sag ki.ta du$_7$.du$_7$-*ma* 3 45 in.sì	15, the lower front, (make) butt (itself), then 3 45 it gives.
	3 20 *i-na* 3 [45] zi 25 in.sì /	3 20 from the 3 *45* tear away, 25 it gives.
9	25.e 5 íb.sig$_8$ 1 kùš dagal múr$^?$	25 makes 5 equalsided, 1 cubit, the middle$^?$ width
	aš-šum ša ta-aḫ-ra-am a-ma-ri-i-[*ka*] /	In order for you to see that which you cut away:
10	5 *ša* ìb.sig$_8$ ugu 1 40	5 that was made equalsided over 1 40
	en.nam diri 3 20 diri	is what beyond? 3 20 it is beyond.
	igi 2 13 20 *in-da-nim* du$_8$- *ma*	The opposite of 2 13 20, the *indanum*, release, then
	[27 in.sì] /	27 it gives.
11	[*a-na* 3 20 *š*]*a* diri nim-*ma* 1 30 in.sì	*To 3 20 that* is beyond lift (it), then 1 30 it gives.
	1 2' kùš *ša* <*ta-*>*aḫ-ra-a-am-*[*mu*]	1 1/2 cubit that which <you> cut away.

The four exercises on the obverse of MS 3052 are illustrated by four drawings of what, at first sight, looks like trapezoids and triangles. However, the figures in the four drawings are broader at their right ends, while triangles and trapezoids in Old Babylonian mathematical texts normally are shown as tapering off towards the right. See, for instance, the drawing of a triangle on MS 3042 (Fig. 8.1.1), and the drawings of two trapezoids on MS 2107 and MS 3908 (Fig. 8.1.2). The explanation for the deviation from the norm is given in the first lines of the four exercises, where the objects considered are named im.dù.a 'mud wall' (Akk. *pitiqtu*). Clearly, then, what is depicted in the four drawings on the obverse of MS 3052 are pictures of the *cross sections* of four mud walls, in the usual Old Babylonian way *rotated so that the top of each mud wall is shown to the left and the base to the right*.

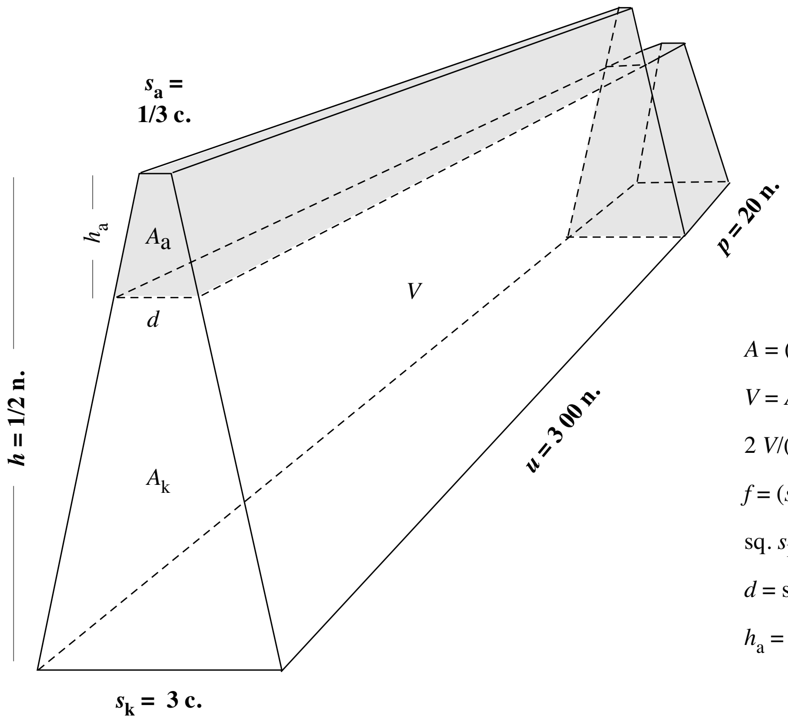
Another peculiarity of drawings associated with Old Babylonian mathematical exercises is that the numerical values indicated in the drawings are not always confined to the parameters given in the statement of the problem. They can include also values computed in the course of the solution procedure. Thus in MS 3052 § 1 a, the *given* values are the following:

uš	the length	3 00 ninda	(c. 1 km)
ina suḫuš dagal	the width at the base	;15 ninda = 3 cubits	(1.5 m)
ana muḫḫim suḫḫur	the 'turned back' at the top[1]	;01 40 ninda = 1/3 cubit	(16 cm)
sukud	the height	1/2 ninda = 6 cubits	(3 m)
perṣum	the breach	20 ninda	(120 m)

In the drawing associated with § 1 a, the four first of these given values are indicated, but also the following two *computed* values

dagal múr?	the middle(?) width	;05 ninda = 1 cubit	(line 9, left)
ša taḫramu	that which you cut away	1;30 cubits	(line 11, right)

The situation is clarified in Fig. 10.2.4 below, with a more accurate "modern" profile of the wall.



$A = (s_k + s_a)/2 \cdot h =$;50 n.c.

$V = A \cdot u =$ 2 30 n.n.c. $= A_k \cdot (u + p)$

$2\ V/(u + p) = 2\ A_k =$ 1;30 n.c.

$f = (s_k - s_a)/h =$;02 13 20 n./c.

sq. s_k – sq. $d = 2\ A_k \cdot f =$;03 20 n.n.

d = sqs. (sq. $s_k - 2\ A_k \cdot f$) = ;05 n.

$h_a = (d - s_a)/f =$ 1;30 c.

Fig. 10.2.4. MS 3052 § 1 a. Repairing a breach in a wall with mud from the top of the wall.

The following standardized notations are used in Fig. 10.2.4:

u (uš)	the length	h	the height	h_a	the upper height
s_a (sag an.ta)	the upper front	s_k (sagki.ta)	the lower front	d (dagal múr)	the middle width
A_a (a.šà an.ta)	the upper area	A_k (a.šà ki.ta)	the lower area	V	the volume
p (*perṣum*)	the breach				

1. The tentative translation 'turned back' is based on the assumption that *su-ḫu-ur* in line 2 stands for *suḫḫur*, the stative of the G-stem of *saḫāru*. It is clear that the term here refers to the upper, narrow end of the trapezoidal cross section of the wall. A related Sumerian word is ba.an.gi$_4$, with the same literal translation as *suḫḫur* and apparently referring to the shorter of the parallel sides in a trapezoid. The term appears in AO 7667, an Ur III account of large numbers of bricks (Friberg, *ChV* (2001), 136-140; Robson, *MMTC* (1999), 149 ff.), and in Clay's *YOS 1* (1915), 24, an Ur III field plan where the sides of a trapezoid are given as '16 ninda uš / 16 ninda sag 14 ninda 4 kùš ba.gi$_4$'. (See Heimpel, *CDLJ* 2004:1.)

Note that the term sag ki.ta 'the lower front' appears in line 8 of § 1 a, although it is more appropriate for the longest parallel of a trapezoid than for the base of a wall.

In Fig. 10.2.4 is also used the notation f for what may be called the "growth rate" or "rate of increase/decrease" for the wall. Apparently this growth rate seems to have been defined as *the difference in thickness of the wall* (measured in ninda) *at two levels differing in height by 1 cubit* (this is $1/12 \cdot 2 = 1/6$ times the inverse of what we would call the *inclination* of the sides of the mud wall). The value of the growth rate in this exercise is ;02 13 20 (= 1/27) n./c. (corresponding to an inclination of 9 : 2). The term used for the growth rate, in lines 7 and 10, is *indanum* (literal meaning unknown). The term is not known from any other cuneiform mathematical text, and it is not listed in the dictionaries. A possible explanation is that it is an Akkadianization of the Sumerian word ninda(n), and that it simply stands for 'ninda (per cubit)'. One would have expected the spelling *nindanum* (listed in the dictionaries) rather than *indanum*, but compare with the following triple of entries in the Old Babylonian lexical series *Proto-Ea* (Civil, *et al.*, *MSL 13* (1979), 208-210):

208	*ni-im*	ninda
209	*gá-ar*	ninda
210	*in-da*	ninda

What this means is, presumably, that the cuneiform sign "ninda" can be read as a) níg (or nì), pronounced more or less like 'nim', b) as gar, and c) as ninda, pronounced more or less like 'inda'.

Another curious term is *ebir* 'stretched across' (stative of *ebēru* 'to cross over, stretch over, lie across'), used here, in line 5, to denote the area of the cross section of the wall.

The question in § 1 a is vaguely formulated, but the form of the solution procedure suggests that the following is an adequate, more understandable reformulation of the question:

> A mud wall with a trapezoidal cross section has the length 3 00 n., the height 1/2 n. = 3 c.,
> and the thickness at the base 3 c. = ;15 n., diminished to 1/3 c. = ; 01 40 n. at the top.
> Originally, the wall was longer, but a breach of length 20 n. has been opened in the wall.
> Mud from the top of the wall is used to return the wall to its original length.
> <What is the new upper width, and> how much lower does the wall become?

The successive steps of the numerical solution procedure are the following:

$A =$		(;15 + ;01 40) n./2 · 6 c. = ;50 n. c.	(line 5)
$V =$	$A \cdot u =$	;50 n. c. · 3 00 n. = 2 30 n. n. c.	(line 6, left)
$u + p =$		3 00 n. + 20 n. = 3 20 n.	(line 6, right)
$A_k =$	$V/(u + p) =$	2 30 n. n. c. / 3 20 n. = ;45 n. c.	(line 7, left)
$f = (s_k - s_a)/h =$	(;15 – ;01 40) n. / 6 c. =	;13 20 n. / 6 c. = ;02 13 20 n./c.	(not mentioned)
$2\ A_k \cdot f =$		1;30 n. c. · ;02 13 20 n./c. = ;03 20 sq. n.	(lines 7 right-8 left)
sq. $s_k - 2\ A_k \cdot f =$		(;03 45 – ;03 20) sq. n. = ;00 25 sq. n.	(line 8 right)
$d =$	sqs. (sq. $s_k - 2\ A_k \cdot f$) =	;05 n. = 1 c.	(line 9 left)
$h_a =$	$(d - s_a)/f =$	(;05 – ;01 40) n. · 27 c./n. = 1;30 n. = 1 1/2 c.	(lines 10-11)

The solution procedure begins in a straightforward way by computing the area A of the cross section of the mud wall and the volume V of the whole mud wall. Then is computed the area of the cross section of the wall after the removal of its top and the mending of the breach. The next step of the solution procedure (cf. Fig. 10.3.4 below) makes use of the fact that, according to *the conjugate rule*,[2]

$$\text{sq. } s_k - \text{sq. } d = (s_k + d) \cdot (s_k - d) = (s_k + d) \cdot (h - h_a) \cdot f = 2\ A_k \cdot f.$$

Since the value of s_k is known, this equation can be used to compute the value of d, as in line 9. The remaining computation of the upper height h_a (called 'that which you cut away') is then easy.

MS 3052 § 1 b. Measuring the Thickness of a Wall by Drilling a Hole through It

2. A metric algebra demonstration of the conjugate rule can be based, for instance, on Fig. 11.2.5, right, in Sec. 11.2 d below. It is shown in that figure that sq. $s + (u + s) \cdot (u - s) =$ sq. u.

MS 3052 § 1 b

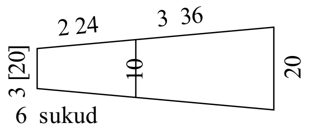


Fig.

1	im.dù.a	A mud wall.
	22 30 uš 26 15 saḫar 2' n. sukud /	22 30, the length, 26 15 the mud, 1/2 ninda the height.
2	*i-na* 3 2' kùš 3 šu.si sukud *ap-lu-ší-ma* 2 kùš /	At 3 1/2 cubits 3 fingers height I drilled through, 2 cubits.
3	*i-na iš-di-im ki-ma-ṣi ik-bi-ir*	In the base, how much is it thick?
	a-na mu-uḫ-ḫi ki-ma-ṣi suḫ$^?$-ur$^?$ /	At the top, how much is it turned back?.
4	za.e ak.da.zu.dè	You, with your doing:
	igi 22 30 du$_8$-*ma* 2 40	The opposite of 22 30 release, then 2 40.
	a-na 26 15 saḫar nim-*ma* 1 10 in.sì /	To 26 15, the mud, lift, then 1 10 it gives.
5	[6] sukud *a-na* 2 kùš íl-*ma* 1 in.sì	*6,* the height, to 2 cubits raise, then 1 it gives.
	1 du$_7$.du$_7$-*ma* 1 in.sì	1 (make) butt (itself), then 1 it gives.
	i-na 1 10 zi-[*ma* 10] in.sì /	From 1 10 tear off, *then 10* it gives.
6	[10] *re-eš-ka li-ki-il*	*10* may hold your head.
	3 2' kùš 3 šu.si sukud 3 36 *lu-pu-ut*	3 1/2 cubits 3 fingers is the height, 3 36 inscribe.
7	3 36 [*i-na* 6 sukud zi] / [2] 24 in.sì	3 36 *from 6, the height, tear off, 2* 24 it gives.
	2 24 *e-le-nu ta-la-pa-at*	2 24 above you inscribe.
	3 36 du$_7$.du$_7$-*ma* 12 57 36 [in.sì] /	3 36 (make) butt (itself), then 12 57 36 *it gives*.
8	[2 2]4 du$_7$.du$_7$-*ma* 5 45 36 in.sì	*2 24* (make) butt (itself), then 5 45 36 it gives.
9	5 45 36 *i-na* 12 57 36 zi-*ma* / 7 12	5 45 36 from the 12 57 36 tear off, then 7 12 it gives.
	2' 7 12 gaz-*ma* 3 36 in.sì	1/2 of 7 12 break, then 3 36 it gives.
	igi 3 36 du$_8$-*ma* 16 40 in.sì /	The opposite of 3 36 resolve, then 16 40 it gives.
10	16 40 [*a-na*] 10 [nim]-*ma* 2 46 40	16 40 *to* 10 *lift,* then 2 46 40.
	a-na 3 36 nim-*ma* 10 in.sì	To 3 36 lift, then 10 it gives.
	10 *a-na* 10 daḫ 20 sag ki.ta /	10 to 10 join, 20, the lower front.
11	2 46 40 *a-na* 2 24 nim-*ma* 6 40 in.sì	2 46 40 to 2 24 lift, then 6 40 it gives.
	6 40 *i-na* 10 dal zi-*ma*	6 40 from 10, the transversal, tear off, then
	3 20 sag an.na	3 20 the upper front.

In MS 3052 § 1 b, the *given* values are the following:

uš	u	the length	22;30 ninda	(135 m)	(line 1)
saḫar	V	the volume	26;15 n. n. c.	(470 m^3)	(line 1)
sukud	h	the height	1/2 ninda = 6 cubits	(3 m)	(line 1)
(sukud ki.ta)	h_k	the lower height	3 1/2 c. 3 f. = 3;36 c.	(1.8 m)	(line 2)
	d	the drilled hole	;10 ninda = 2 cubits	(1 m)	(line 2)

In the drawing illustrating § 1 b, the last three of these five given values are indicated, and also the following three *computed* values

(sukud an.na)	h_a	the upper height	2 1/3 c. 2 f. = 2;24 c.	(1.2 m)	(lines 7-8)
sag ki.ta	s_k	the lower front	;20 n. = 4 c.	(2 m)	(line 10)
sag an.na	s_a	the upper front	;03 20 n. = 2/3 c.	(33 cm)	(line 11)

The situation is clarified in Fig. 10.2.5 below.

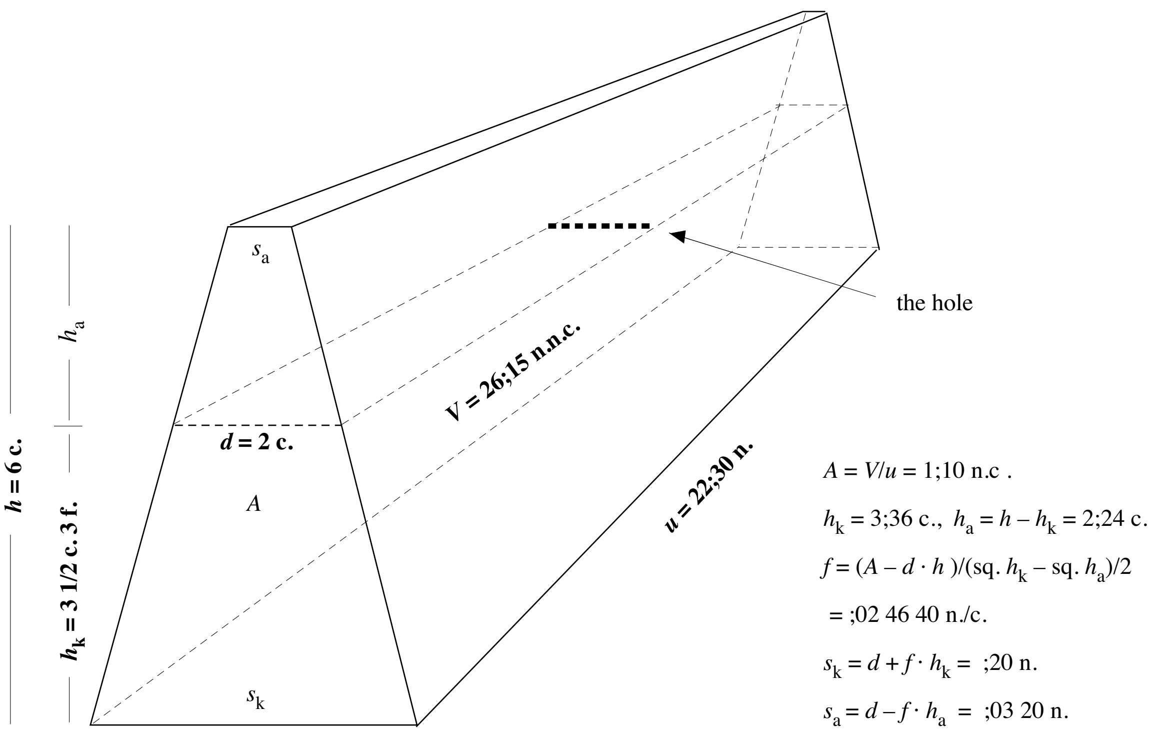


Fig. 10.2.5. MS 3052 § 1 b. A wall with a trapezoidal cross section, and with a hole drilled through it.

Note: It is interesting that in the question (lines 1-3) the transversal d is given as 2 cubits and the lower height as 3 1/2 cubits 3 fingers, while the values indicated in the drawing are 10 (meaning ;10 ninda) and 3 36 (meaning 3;36 cubits). (However, in line 6 it is said directly: 'inscribe 3 36', and in lines 10-11 the length of the drilled hole is referred to simply as '10'.) Also, in the question it is asked how thick the wall is 'in the base' and 'at the top', while in the solution procedure the answer is given in the form '20, the lower front' and '3 20, the upper front', as if the object considered were a trapezoid, rather than the cross section of a mud wall.

In line 4 of § 1 b is computed the area of the cross section of the mud wall:

$$A = V/u = 26;15 \text{ n. n. c.} / 22;30 \text{ n.} = 1;10 \text{ n. c.} \qquad \text{(line 4)}$$

The next step of the computation is totally unexpected and not so easy to understand. It is the computation in line 5 of an unnamed entity with the value

$$A - 1 \cdot d \cdot h = (1;10 - 1 \cdot 6 \cdot ;10) \text{ n. c.} = ;10 \text{ n. c.} \qquad \text{(line 5)}$$

The student is asked to remember this value, until it will be needed again. Then follows, in lines 6-7, the conversion of the given value $h_k = 3\ 1/2$ cubits 3 fingers to the sexagesimal number 3;36, and the computation of the upper height as 6 – 3;36 = 2;24. The student is first asked to 'inscribe' the value 3 36, probably either in his own drawing of the cross section of the mud wall or on a hand tablet. Then he is asked to inscribe the value 2 24 'above' (namely, above 3 36).

The remembered value $A - 1 \cdot d \cdot h = ;10$ n. c. is used in the computation of

$$(A - 1 \cdot d \cdot h) / ((\text{sq.}\ h_k - \text{sq.}\ h_a)/2) = ;10 \text{ n. c.} / \{(12;57\ 36 - 5;45\ 36)/2\} \text{ sq. c.} = ;10 \text{ n.} / 3;36 \text{ c.} = ;02\ 46\ 40 \text{ n./c.}$$

A partial explanation is provided by the final computations, where it becomes clear that ;02 46 40 (= ;25/9) n./c. is the *growth rate f* for the mud wall (corresponding to an inclination of 18 : 5). As a matter of fact, the upper and lower fronts of the trapezoid are computed as

$$s_k = d + f \cdot h_k = ;10 \text{ n.} + ;02\ 46\ 40 \text{ n./c.} \cdot 3;36 \text{ c.} = ;10 \text{ n.} + ;10 \text{ n.} = ;20 \text{ n.} \qquad \text{(line 10)}$$
$$s_a = d - f \cdot h_a = ;10 \text{ n.} - ;02\ 46\ 40 \text{ n./c.} \cdot 2;24 \text{ c.} = ;10 \text{ n.} - ;06\ 40 \text{ n.} = ;03\ 20 \text{ n.} \qquad \text{(line 11)}$$

It remains to explain the following curious equation for the growth rate:

$f = (A - 1 \cdot d \cdot h) / \{(\text{sq. } h_k - \text{sq. } h_a)/2\} = ;02\ 46\ 40$ n./c. (lines 5-10)

Apparently, the idea was to compare the area of a trapezoid divided by a transversal d with the area of the rectangle with base d and with the same height h as the trapezoid. It is clear from Fig. 10.2.6 below that the difference between the two areas is equal to the difference between the areas of two triangles, one with the base $s_k - d$ and the height h_k, the other with the base $d - s_a$ and the height h_a. However, *since the two triangles are similar, the same growth rate f applies to both. This common growth rate is also the growth rate for the trapezoid*. Therefore, $s_k - d = f \cdot h_k$, $d - s_a = f \cdot h_a$, and

$$A - d \cdot h = h_k \cdot (s_k - d)/2 - h_a \cdot (d - s_a)/2 = f \cdot (\text{sq. } h_k - \text{sq. } h_a)/2.$$

Consequently, as in lines 5-10 of the solution procedure in MS 3052 § 1 b,[3]

$$f = (A - d \cdot h) / (\text{sq. } h_k - \text{sq. } h_a)/2.$$

The mentioned equation for $A - d \cdot h$ may be called “the first Old Babylonian transversal rule”.

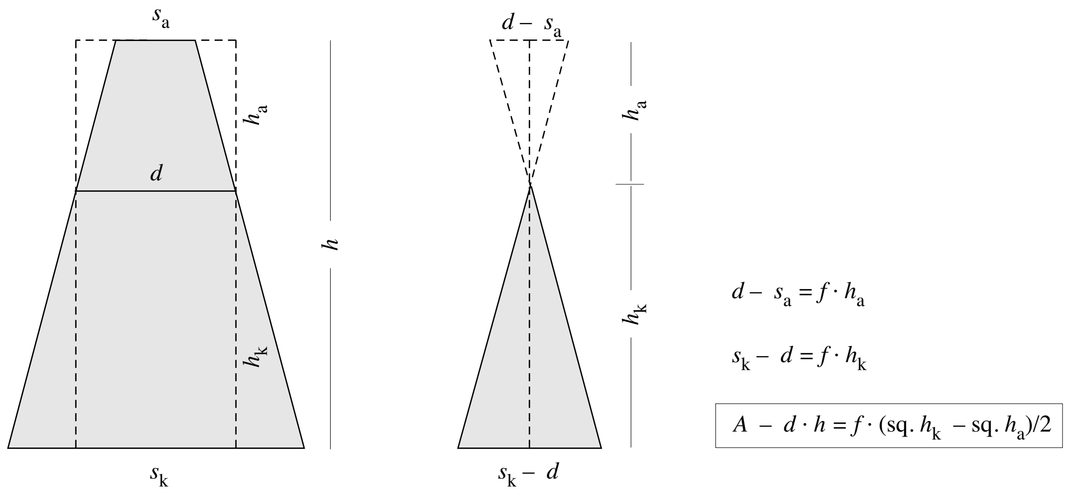


Fig. 10.2.6. MS 3052 § 1 b. The first Old Babylonian transversal rule.

MS 3052 § 1 c. Another Example of a Wall with a Hole Drilled through It

MS 3052 § 1 c

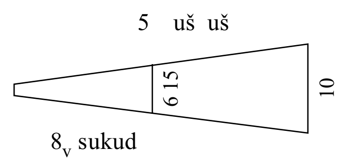


Fig.

1	im.dù.a	A mud wall.
	5 uš uš 2_{iku} saḫar gú kud-*aṣ* (*aḫaraṣ*) /	5 uš (= 5 00 n.) the length, 2 iku the mud, the edge I cut off
2	*i-na* 3 kùš sukud *ap-lu-ši-ma*	At 3 cubits height I drilled through, then
	1 kùš 7_v 2' šu.si *ku-ub-ru-ša* /	1 cubit 7 1/2 fingers its thickness,
3	*i-na* suḫuš *ki ma-ṣi* dagal *ù* sukud en.nam /	In the base, how much is it wide, and the height is what?
4	za.e ak.da.zu.dè	You, with your doing:

3. Note that the squaring of 1 in line 5 is a totally unwarranted step in the solution procedure, but it does not change the result.

	igi 5 uš du_8 12 in.sì /	The opposite of 5, the length, release, 12 it gives.
5	12 *a-na* 3 20 saḫar nim-*ma* 40 in.sì	12 to 3 20, the mud, lift, then 40 it gives.
	igi 3 kùš sukud du_8-*ma* 20 in.sì /	The opposite of 3 cubits, the height, resolve, then 20 it gives.
6	20 *a-na* 40 nim-*ma* 13 20 in.sì	20 to 40 lift, then 13 20 it gives.
	13 20 *re-eš-ka li-ki-il* / 13 20 ugu 6 15 en.nam diri	13 20 may hold your head. 13 20 over 6 15
7	7 05 diri	is what beyond? 7 05 beyond.
	7 05 du_7.du_7-*ma* 50 10 25 in.sì /	7 05 (make) butt (itself), then 50 10 25 it gives.
8	6 15 du_7.du_7-*ma* 39 03 45 in.sì	6 15 (make) butt (itself), then 39 03 45 it gives.
	39 03 45 *i-na* 50 10 25 zi-*ma* /	39 03 45 from 50 10 25 tear off, then
9	11 06 40 in.sì 11 06 40.e 3 20 íb.si_8	11 06 40 it gives. 11 06 40 makes 3 20 equalsided.
	3 20 *i-na* 13 20 zi-*ma* 10 in.sì /	3 20 from 13 20 tear off, then 10 it gives,
10	2 kùš sag ki.ta	2 cubits, the lower front.
	2' 10 gaz-*ma* 5 in.sì	1/2 of 10 break, then 5 it gives.
	5 du_7.du_7-*ma* 25	5 (make) butt (itself), then 25.
	igi 25 du_8 2 24 in.sì /	The opposite of 25 resolve, 2 24 it gives.
11	2 24 *a-na* 3 20 saḫar nim-*ma* 8 in.sì	2 24 to 3 20, the mud, lift, then 8 it gives.
	2' ninda 2 kùš sukud-*ša*	1/2 ninda 2 cubits is its height.

In this exercise (see Fig. 10.2.7 below), the *given* values are the following:

uš	u	the length	5 00 ninda	(1.8 km)
saḫar	V	the volume	2 iku = 3 20 n. n. c.	(3,600 m^3)
(sukud ki.ta)	h_k	the lower height	3 cubits	(1.5 m)
	d	the drilled hole	1 c. 7 1/2 f. = ;06 15 n.	(37.5 cm).

In the drawing illustrating § 1 c, the first and the last of these four given values are indicated, and also two *computed* values, the (width at the) base (in the solution procedure called 'the lower front'), and the height. These computed values are the following:

sukud	h	the height	1/2 n. 2 c. = 8 cubits	(4 m)
sag ki.ta	s_k	the lower front	2 c. = ;10 n.	(2 m)

The solution proceeds in a series of deceptively simple steps:

1. $A/h_k - d =$	;13 20 n. – ;06 15 n. = ;07 05 n.	(lines 4 - 7)
2. sq. $(A/h_k - d)$ – sq. d =	;00 50 10 25 sq. n. – ;00 39 03 45 sq. n. = ;00 11 06 40 sq. n.	(lines 7 - 9)
3. A/h_k – sqs. {sq. $(A/h_k - d)$ – sq. d} =	;13 20 n. – ;03 20 n. = ;10 n. = 2 c. = s	(lines 9 - 10)
4. $V/(u \cdot s/2)$	= 3 20 n. n. c. / (5 00 · ;05) sq. n. = 8 c. = 1/2 n. 2 c. = h	(lines 10 - 11)

One peculiarity of the text is due to the fact that in the Old Babylonian relative place value notation there is no discernible difference in line 10 between u (the length) = 5 00 n. and $s/2$ (half the base) = ;05 n. Both are written simply as '5'. For this reason, the author of the text allowed himself the minor mistake of writing 'square 5', when what he meant was 'multiply 5, the length, with 5, half the base'.

The object considered in MS 3052 § 1 c is a mud wall with a *triangular* cross section. This circumstance is probably what is alluded to by the obscure phrase 'the edge I cut off' in line 1.

The first step of the solution procedure (lines 4-5) is easy to understand. It is the computation of the area of the triangular cross section of the mud wall:

$$A = V/u = 3\ 20 \text{ n. n. c.} / 5\ 00 \text{ n.} = ;40 \text{ n. c.}$$

It remains to compute the *base s* and the *height h* of the cross section of the wall, with departure from the known values for the *area A* of the triangle, the *transversal d*, and the *lower height* h_k. The solution method is based on what may be called "the second Old Babylonian transversal rule", namely that

$$d \cdot h + h_k \cdot s = 2\,A.$$

A geometric proof of the rule is easily obtained through inspection of Fig. 10.2.8 below.

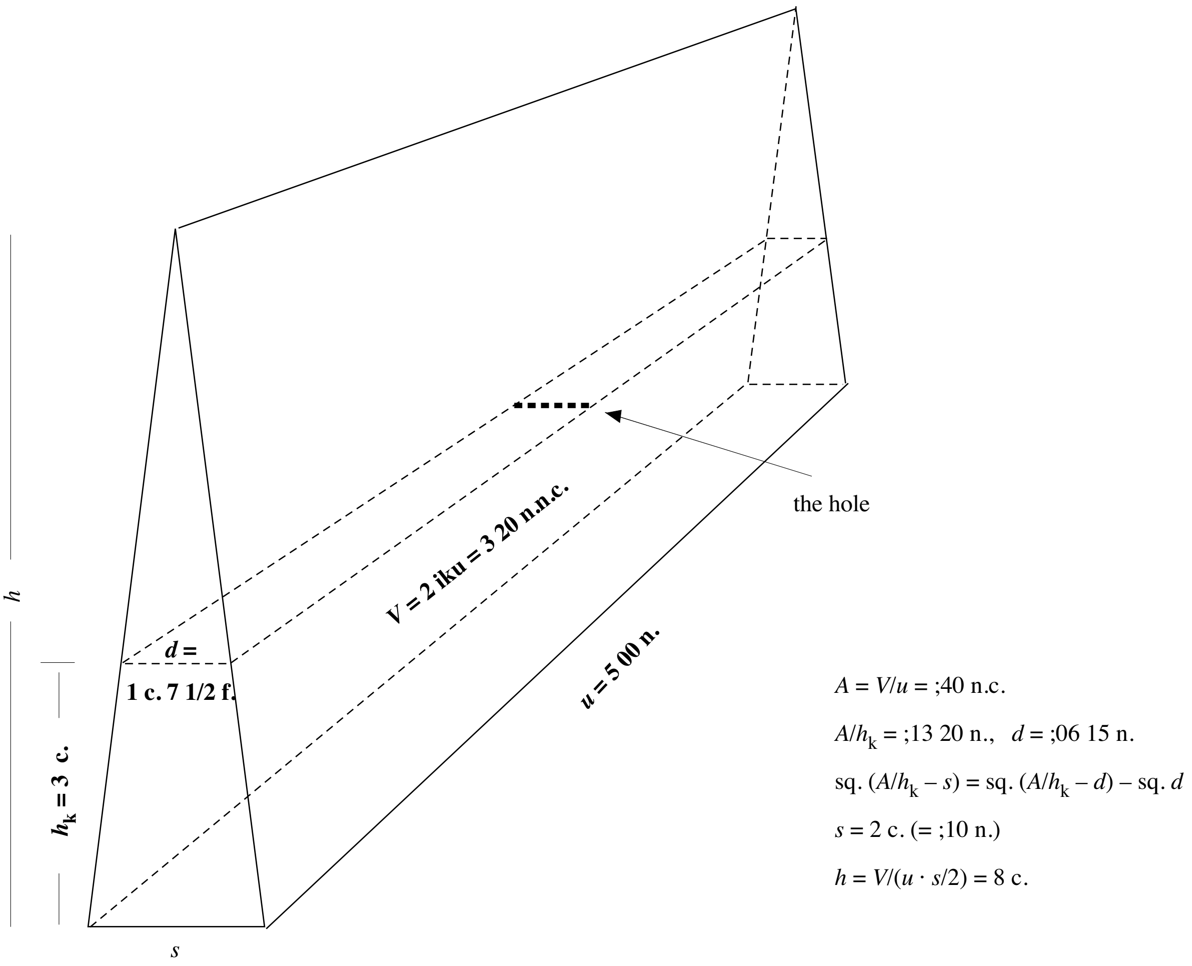


Fig. 10.2.7. MS 3052 § 1 c. A wall with a triangular cross section, and with a hole drilled through it.

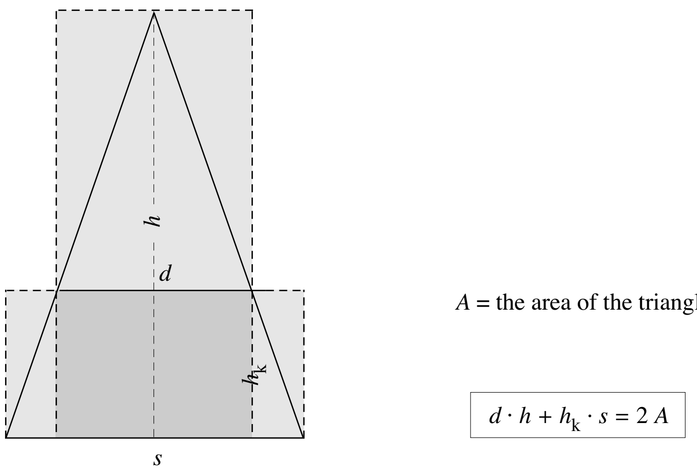


Fig. 10.2.8. MS 3052 § 1 c. The second Old Babylonian transversal rule.

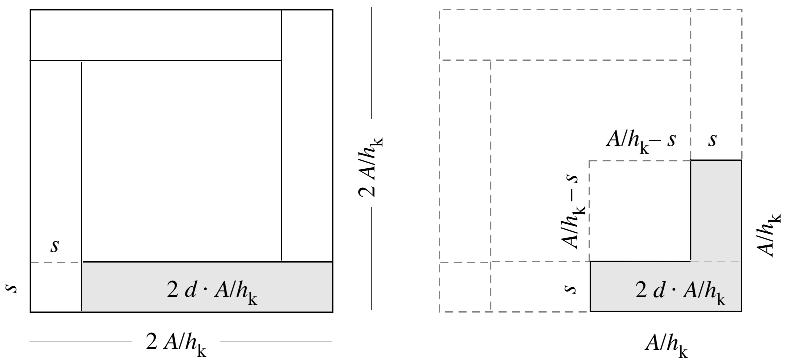
Together with the area equation the second transversal rule leads to the following *rectangular-linear system of equations* for the height h and the base s:

$$h \cdot s = 2\,A, \quad d \cdot h + h_k \cdot s = 2\,A.$$

This system of equations can easily be reduced to one of the Old Babylonian standard forms for such systems, for instance as follows: Change variables by setting $u = d \cdot h/h_\text{k}$. Then the mentioned system of equations can be replaced by the following equivalent system

$$u \cdot s = 2\,d \cdot A/h_\text{k}, \quad u + s = 2\,A/h_\text{k}.$$

This rectangular-linear system of equations, in its turn, can be replaced by a *quadratic equation* for the single variable s. This is done most easily in the appropriate geometric model:



Example:

A = ;40 n. c., h_k = 3 c.

A/h_k = ;13 20 n.

d = ;06 15 n. = 15 · ;00 25

$A/h_\text{k} - d$ = ;07 05 n. = 17 · ;00 25

sq. 17 – sq. 15 = sq. 8

$A/h_\text{k} - s$ = ;03 20 n. = 8 · ;00 25

s = ;13 20 n. – ;03 20 n. = ;10 n. = 2 c.

$2\,A/h_\text{k} \cdot s - \text{sq.}\, s = 2\,d \cdot A/h_\text{k}$

$\text{sq.}\,(A/h_\text{k} - s) = \text{sq.}\,A/h_\text{k} - 2\,d \cdot A/h_\text{k}$
$= \text{sq.}\,(A/h_\text{k} - d) - \text{sq.}\,d$

Fig. 10.2.9. MS 3052 § 1 c. The quadratic equation for the base *s*, and its surprising solution.

The geometric model shows how the mentioned rectangular-linear system of equations for u and s can be transformed into a quadratic equation for s alone (Fig. 10.2.9, left):

$$2\,A/h_\text{k} \cdot s - \text{sq.}\,s = 2 \cdot d \cdot A/h_\text{k}.$$

The model also shows (Fig. 10.2.9, right) how this quadratic equation for s can be reduced to

$$\text{sq.}\,(A/h_\text{k} - s) = \text{sq.}\,A/h_\text{k} - 2 \cdot d \cdot A/h_\text{k}.$$

So far, the solution procedure follows the standard routine, known from a number of earlier published Old Babylonian mathematical texts. According to that routine, the expected next step in the solution procedure would be to compute the value of s as follows (in quasi-modern notations)

$$s = A/h_\text{k} - \text{sqs.}\,(\text{sq.}\,A/h_\text{k} - 2 \cdot d \cdot A/h_\text{k}).$$

Surprisingly, the solution procedure in MS 3052 § 1 c works differently. Apparently, the Old Babylonian author of the exercise had realized that he could refine the standard solution by going one step further. He saw that the expression of which the square side was computed could be expressed as the difference between two squares. Thus, the expression for s in quasi-modern notations which precisely corresponds to steps 1-4 of the solution procedure in MS 3052 § 1 c is the following:

$$s = A/h_\text{k} - \text{sqs.}\,(\text{sq.}\,(A/h_\text{k} - d) - \text{sq.}\,d).$$

This form of the solution is interesting because it shows that the triple

$$c = A/h_\text{k} - s, \quad b = d, \quad a = A/h_\text{k} - d,$$

with A, h_k, d, and s defined as in Fig. 10.2.8, is *always a solution to the diagonal equation*

$$\text{sq.}\,c = \text{sq.}\,a + \text{sq.}\,b.$$

In § 1 c, where A = ;40 n. c., h_k = 3 c., d = ;06 15 n., and s = ;10 n., the triple is

$$A/h_k - d = ;07\ 05 \text{ n.},\quad d = ;06\ 15 \text{ n.},\quad A/h_k - s = ;03\ 20 \text{ n.}$$

It is easy to check that

$$;07\ 05 \text{ n.} = 17 \cdot ;00\ 25 \text{ n.},\quad ;06\ 15 \text{ n.} = 15 \cdot ;00\ 25 \text{ n.},\quad ;03\ 20 \text{ n.} = 8 \cdot ;00\ 25 \text{ n.}$$

In other words, the triple figuring in § 1 c is a multiple of the triple 17, 15, 8, which can easily be shown to be a simple *solution in integers* to the diagonal equation.

It is no coincidence that the data for MS 3052 § 1 c were chosen so that they would lead to an *exact* solution to the problem, without approximate square sides. Indeed, it is likely that the four exercises in MS 3052 § 1 were excerpted from a large *theme text* with a series of exercises of gradually increasing complexity. One of these exercises may have been, for instance, to find the height and the area of the triangular cross section of a mud wall, when the width s at the base and the length d of a drilled hole at the height h_k over the ground are given. A problem of this kind can be reduced to finding the height h and the area A of a triangle with a given front s, divided by a transversal of given length d, a given the distance h_k from the front. The solution to this problem is

$$h = h_k \cdot s/(s-d),\quad A = h_k \cdot s/2 \cdot s/(s-d) \qquad \text{(see Fig. 10.2.8)}$$

If the value for A obtained in this way is used, together with the originally given values for d and h_k, as data for a problem of the type MS 3052 § 1 c, then it is guaranteed that there will exist a solution to that problem with exact (that is, rational) values for s and h.

It is possible to analyze the form of that solution from a modern point of view, as follows:

Set $s = d + e$. Then $A = h_k \cdot s/2 \cdot s/e$, so that A/h_k = sq. $s/(2\,e)$ = sq. $(d+e)/(2\,e)$.
Consequently, $A/h_k - d$, d, $A/h_k - s$ = {(sq. d + sq. e)/2, $d \cdot e$, (sq. d – sq. e)/2}/e.

The result can be reformulated as follows:

Let igi = $d/e = d/(s-d)$ and igi.bi = $e/d = (s-d)/d$.
Then $A/h_k - d$, d, $A/h_k - s = d \cdot$ [(igi + igi.bi)/2, 1, (igi – igi.bi)/2].

When d and s are *regular* sexagesimal numbers, this can be interpreted as an application the Old Babylonian rule for the construction of (rational) solutions to the diagonal equation. (See Fig. A8.4 in App. 8 below.) Note that d = ;06 15 = ;25/4 and s = ;10 *are* regular sexagesimal numbers in MS 3052 § 1 c, with igi = $d/(s-d)$ = 6 15 / 3 45 = 1;40.

MS 3052 § 1 d. A Wall with a Hole Drilled through it and a Breach Repaired

MS 3052 § 1 d

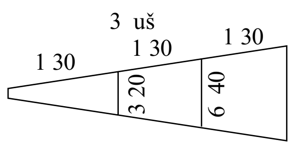


Fig.

1	im.dù.a	A mud wall.
	3 uš uš 1 07 30 saḫar gú kud-*aṣ* (*aḫaraṣ*) /	3 uš (3 00 n.) the length, 1 07 30 the mud, the edge I cut off.
2	*i-na* 1 2' kùš sukud *ap-lu-ší-ma*	At 1 1/2 cubits height I drilled through, then
	1 3' kùš dagal /	1 1/3 cubit is the width.
3	*i-na* šà *pe-er-ṣum pa-ri-iṣ*	Inside it a breach is breached (opened),
	x x x šu-ú nu.zu /	that x x x I do not know.
4	*iš-tu e-le-nu a-di* a.na 3" kùš	From above until as much as 2/3 cubit
5	*er-ši-šu* / *ag-šu-ra-am-ma*	what was asked for I repaired, then
	pe-er-ṣa-am šu-a-ti ag-šu-ur	this breach I repaired

	pe-er-ṣum šu-ú ki-ma-ṣi uš-šu /	This breach how much is its foundation,
6	*ù* im.dù.a *ki ma-ṣi i-na* suḫuš dagal	and the mud wall how much at the base is it wide,
	ù ki ma-ṣi sukud	and how much is the height?
	za.e ak.da.zu.dè /	You, with your doing:
7	igi 3 uš du$_8$-*ma* 20	The opposite of 3, the length, resolve, then 20.
	a-na 1 07 30 saḫar nim-*ma*	To 1 07 30, the mud, lift, then
	22 30 in.sì *e-bi-ir* /	22 30 it gives, the stretched-across.
8	igi 1 30 du$_8$-*ma* 4[0 in].sì	The opposite of 1 30 resolve, then 4*0 it g*ives.
	a-na 22 30 nim-*ma* 15 [in.s]ì	To 22 30 lift, then 15 *it giv*es.
9	ugu 6 40 en.nam diri / 8 20 diri	Over 6 40 what is it beyond? 8 20 beyond.
	8 20 d[u$_7$.du$_7$]-*ma* 1 09 26 40 in.s[ì]	8 20 (make) *butt* (itself), then 1 09 26 40 it gi*ves*.
10	[6] 40 du$_7$.du$_7$-ma / 44 26 40 in.sì	*6* 40 (make) butt (itself), then 44 26 40 it gives.
	44 26 40 *i-na* 1 09 [2]6 40 zi 25 in.sì /	44 26 40 from 1 09 2*6* 40 tear off, then 25 it gives.
11	25.e 5 íb.si$_8$	25 makes 5 equalsided.
	5 *i-na* 1[5 z]i-*ma* 10 [in].sì	5 from 1*5 te*ar off, then 10 it gives,
	2 kùš *i-na* suḫuš dagal /	2 cubits in the base is the width.
12	*aš-šum* sukud *a-ma-ri-ka*	In order for you to see the height(s):
	10 u[gu 6 40 dagal] múr$^?$ en.nam diri	10 *over 6 40,* the middle$^?$ *width*, is what beyond?
	3 20 diri /	3 20 beyond.
13	igi 1 30 sukud du$_8$-*ma* 40	The opposite of 1 30, the height, resolve, then 40.
	a-na 3 20 n[im] 2 13 20 in.sì 2 13 20 *in-da-nu* /	To 3 20 lift, then 2 13 20 it gives, 2 13 20 the ninda.
14	6 40 ugu 3 20 en.nam diri 3 20 diri	6 40 over 3 20 what is it beyond? 3 20 beyond.
	i[gi] 2 13 20 *in-da-nim* du$_8$ 27 in.sì /	The opposite of 2 13 20, the *indanum*, resolve, 27 it gives.
15	27 *a-na* 3 20 *ša* diri nim-*ma* 1 30 in.[sì]	27 to 3 20 that is beyond lift, then 1 30 it g*ives*.
	[1] 30 sukud ki.2 /	*1* 30, the 2nd height.
16	3 20 ugu *mi-im-ma ú-la* en.nam diri	3 20 over nothing is what beyond?
	[3 20 diri]	3 20 it is beyond.
17	27 *a-na* 3 20 *ša* diri nim / 1 30 sukud ki.3	27 to 3 20, that which is beyond, lift, 1 30, the 3rd height.
	[*aš-šum pe-er-ṣum ša pa-ri-iṣ a-ma-ri*]-*ka*	In order for you to see the breach that is breached,
	3' kùš dagal /	1/3 cubit, the width.
18	2' 3 20 gaz-*ma* [1 40]	1/2 of 3 20 *break, then 1 40.*
	[*a-na* 1 30 nim-*ma*] 2 30 in.sì /	*To 1 30 lift, then* 2 30 it gives.
19	2 30 [*a-na*] 3 [nim-*ma* 7 30 in.sì]	2 30 *to* 3 *lift, then 7 30 it gives.*
	[*i-na*] 1 07 [30 z]i-*ma* 1 in.sì /	*From* 1 07 *30 t*ear (it) off, then 1 it gives.
20	1 [*i-na* 1 07 30 zi-*ma* 7 30 in.sì]	1 *from 1 07 30 tear off, then 7 30 it gives.*
	[6 40 *a-na* 3] nim 20 *e-bi-ir* /	*6 40 to 3* lift, 20 the stretched-across.
21	igi [20 du$_8$]-*ma* [3 in.sì]	The opposite *of 20 release*, then *3 it gives.*
	[3 *a-na* 7 30] nim-*ma* 22 30 in.sì /	*3 to 7 30* lift, then 22 30 it gives.
22	22 30 [*pe-er-ṣum*] *ša pa-ri-iṣ*	22 30, *the breach* that is breached.

In MS 3052 § 1 d, the cross section of the mud wall is again triangular, as indicated by the phrase 'the edge I cut off' in line 1. The *given* values are the following:

uš	the length	3 00 ninda	(1.08 km)	(line 1)
saḫar	the volume	1 07;30 n. n. c.	(202.5 m^3)	(line 1)
	the 1st height	1 1/2 c.	(75 cm)	(line 2)
	the drilled hole	1 1/3 c. = ;06 40 n.	(67 cm)	(line 2)
	the new upper width	2/3 c. = ;03 20 n.	(33 cm)	(line 4)

In the drawing associated with § 1 d, all of these five given values are recorded, except that of the volume. In addition, the last two of the following five *computed* values are indicated:

ebir	the area of the cross section	;22 30 n. c.	(1.125 m^2)	(line 7)
ina suḫuš dagal	the width at the base	2 c. = ;10 n.	(1 m)	(line 11)
indanum	the growth rate	;02 13 20 n./c.	(inclination 9 : 2)	(line 13)
sukud ki.2	the 2nd height	1 1/2 c.	(75 cm)	(line 15)
sukud ki.3	the 3rd height	1 1/2 c.	(75 cm)	(line 17)

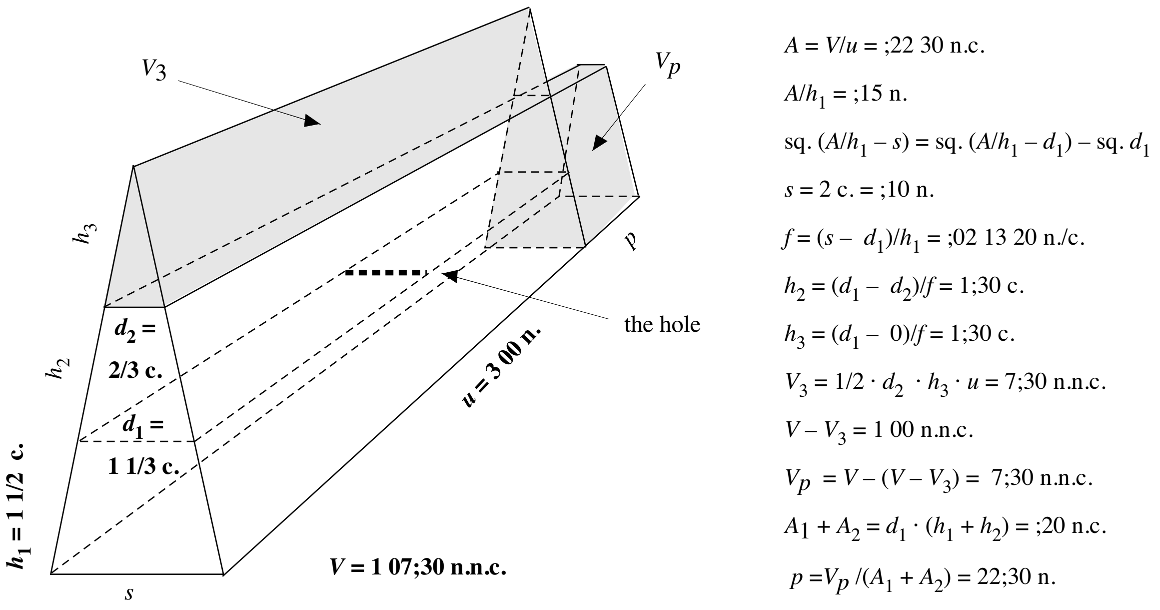


Fig. 10.2.10. MS 3052 § 1 d. Repairing a breach in a wall with a triangular cross section and with a drilled hole.

The first step of the solution procedure is the computation of the 'stretched-across', the area of the cross section of the mud wall:

1. $A = V/u$ = 1 07;30 n. n. c. / 3 00 n. = ;22 30 n. c. (line 7)

Next are computed the *width of the base s*, and the *middle and upper heights* h_2 and h_3, with departure from the known values of the *area A* of the triangular cross section of the wall, the *length of the drilled hole* d_1, the new upper width d_2, and the *lower height* h_1.

The solution procedure in § 1 d is based on the same ideas as the solution procedure in § 1 c. Thus, after the computation of the area of the cross section, the solution continues as follows:

2. A/h_1 = ;22 30 n. c. / 1;30 c. = ;15 n., $A/h_1 - d_1$ = ;15 n. – ;06 40 n. = ;08 20 n. (lines 8 - 9)
3. sq. $(A/h_1 - d_1)$ = ;01 09 26 40 sq. n., sq. d_1 = ;00 44 26 40 sq. n. (lines 9 - 10)
4. sq. $(A/h_1 - d_1)$ – sq. d_1 = ;01 09 26 40 sq. n. – ;00 44 26 40 sq. n. = ;00 25 sq. n. (line 10)
5. A/h_1 – sqs. (sq. $(A/h_1 - d_1)$ – sq. d_1) = ;15 n. – ;05 n. = ;10 n. = 2 c. = s (line 11)
6. $(s - d_1)/h_1$ = (;10 – ;06 40) n. / 1;30 c. = ;02 13 20 n./c. = f (lines 12 - 13)
7. $(d_1 - d_2)/f$ = (;06 40 – ;03 20) n. · 27 c./n. = 1;30 c. = h_2 (lines 14 - 15)
8. $(d_2 - 0)/f$ = ;03 20 n. · 27 c./n. = 1;30 c. = h_3 (lines 16-17)

In steps 2 - 5 of this solution procedure, the base s is computed in precisely the same way as the base was computed in § 1 c, without any notion taken of the "upper" transversal. After that, the middle and upper heights are easily computed, in steps 6 - 8, by use of the growth rate f.

Note the previously unknown term for 'zero' used in line 16 of this text, namely *mimma ula*, the Akkadian phrase for 'nothing', composed of the words *mimma* 'anything' and *ula* 'not'.

The computation of the upper height h_3, ends in line 1 of the badly damaged reverse of MS 3052. After that, the last six lines of § 1 d are devoted to the computation of the 'foundation' (meaning the length) of the breach. It is computed as follows. First is computed the volume V_3 of the mud removed from the top of the clay wall, and the volume $V - V_3$ of what then remains of the wall:

9. $V_3 = 1/2 \cdot d_2 \cdot h_3 \cdot u$ = 1/2 · ;03 20 n. · 1;30 c. · 3 00 n. = 7;30 n. n.c. (lines 18-19)
10. $V - V_3$ = (1 07;30 – 7;30) n. n.c. = 1 00 n. n.c. (line 19)

(See Fig. 10.2.10.) Next is computed the volume of the mud used to repair the breach:

11. $V_p = V - (V - V_3)$ = (1 07;30 – 1 00) n. n.c. = 7;30 n. n. c. (line 20)

(It would have been simpler to say directly that the volume of the mud used to repair the breach was the same as the volume of the mud removed from the top of the wall.) Then the volume is computed of the cross section of the wall with the top removed:

12. $A_1 + A_2 = d_1 \cdot (h_1 + h_2)$ = ;06 40 n. · 3 c. = ;20 n.c. (line 20)

In the final step of the computation, the length of the breach is computed as follows:

13. $p = V_p/(A_1 + A_2)$ = 7;30 n. n.c./;20 n.c. = 22;30 n. (lines 21-22)

Earlier Published Parallel Texts: YBC 4673 § 12, Str. 364, and VAT 8512

Only one previously published Old Babylonian mathematical text is concerned with a partitioned mud wall. It is **YBC 4673 § 12**, an isolated exercise in the middle of a large mathematical recombination text, without solution procedures (Muroi, *SBM* 2 (1992), Robson, *MMTC* (1999), 89: Friberg, *ChV* (2000), 106). The transliteration and translation of that text is quite problematic.

YBC 4673 §12 (Group 2, hence from a southern site, possibly Ur)

1-2	im.dù.[a] / 5 uš uš.bi /	A clay wall. 5 uš (5 00 n.) its length,
3-4	2 kùš dagal / 2' ninda sukud.bi /	2 c. the width, 1/2 n. its height.
5	*i-na* 1 kùš 3' kùš gu$_7$ ì.gu$_7$-*ma* /	In 1 c., 1/3 c. fodder it eats.
6	guruš$^?$ gul.gul-*ma* /	A worker$^?$ demolishes.
7	1 2' kùš sukud *ur-dám*$^?$ /	1 1/2 c. it has come down$^?$.
8	saḫar$^?$ en.nam ḫé.kur.ru$^?$	Of mud$^?$, what shall he accomplish$^?$?

The wall in this exercise resembles the wall in MS 3052 § 1 d. Both walls have a triangular cross section, both have the same width at the base, and both have 1 1/2 cubit torn off at the top. Only the heights are different. Note also that different expressions are used in MS 3052 § 1 and in YBC 4673 for the inclination of the wall. In YBC 4673 § 12, the curious phrase *ina* 1 kùš 3' kùš gu$_7$ ì.gu$_7$, here tentatively translated 'in 1 cubit it eats 1/3 cubit of fodder', seems to be a graphic description of the circumstance that as the wall goes upward by 1 cubit its width decreases by 1/3 cubit.

Even if there are no more known direct parallels to MS 3052 § 1, there is one text whose theme is closely, although indirectly, related to the theme of MS 3052 § 1. After all, the theme of MS 3052 § 1 is only superficially "mud walls". The real theme is "striped triangles". (See Friberg, *RlA 7* (1990) Sec. 5.4 i.) In this respect, **Str. 364** (*MKT 1*, 248-256; Fig. 10.2.12 below) is a parallel to MS 3052 § 1.

Str. 364 is a fairly well preserved medium size clay tablet, probably belonging to Group 3, hence a text from Uruk. The first and last exercises are lost, as well as important parts of the second exercise. Unfortunately, the second (and presumably also the first) exercise is where a contrived explanation of the striped triangles was given, in terms of canals, dams, and dikes. The other exercises are formulated entirely in terms of metric algebra, that is as numerical problems for geometric figures.

All the exercises in Str. 364 have the form of a question, without solution procedure and answer, but illustrated by a drawing of a triangle with one or several transversals parallel to the front. The triangles are oriented in the Old Babylonian standard way, with the front facing left.

Str. 364 is a well organized topic text with eight related themes. It is likely that it is a condensed version of much larger theme text, complete with solution procedures and answers. (Cf. the way in which YBC 4657 (*MCT,* G) is a condensed copy, with only questions and answers, of the complete theme texts YBC 4663 (MCT, H) and YBC 4662 (MCT, J). Those three texts have the same combined theme: "digging", "work norms", and "expenses in terms of man-power or silver".)

The text of **Str. 364 § 3** is perfectly preserved (see below). The situation is essentially the same as in MS 3052 § 1 c. Given are the area of the whole triangle, A = 1 bùr 2 èše = 50 00 sq. n., the transversal, d = 40 n., and the upper length u_a = 33;20 n. Therefore, the second Old Babylonian transversal rule (Fig. 10.2.8) is applicable, so that the front s and the length u (actually the height) of the triangle can be computed as the solutions to the rectangular-linear system of equations

$$u \cdot s = 2\,A = 2 \cdot 50\;00 \text{ sq. n.}$$
$$d \cdot u + u_a \cdot s = 2\,A, \quad \text{that is} \quad 40 \text{ n.} \cdot u + 33;20 \text{ n.} \cdot s = 1\;40\;00 \text{ sq. n.}$$

As shown in the discussion of MS 3052 § 1 c, this system of equations can be solved as follows:

$$A/u_a = 50\;00 \text{ sq. n.} / 33;20 \text{ n.} = 1\;30 \text{ n.}$$
$$s = 1\;30 \text{ n.} - \text{sqs. (sq. } (1\;30 \text{ n.} - 40 \text{ n.}) - \text{sq. } 40 \text{ n.}) = 1\;30 \text{ n.} - 30 \text{ n.} = 1\;00 \text{ n.}$$
$$u = 1\;40\;00 \text{ sq. n.} / 1\;00 \text{ n.} = 1\;40 \text{ n.}$$
$$u_{\text{k}} = 1\;40 \text{ n.} - 33;20 \text{ n.} = 1\;06;40 \text{ n.}$$

Thus, in this exercise, the triple

$$A/u_a - d = 50 \text{ n.}, \quad d = 40 \text{ n.}, \quad A/u_a - s = 30 \text{ n.}$$

is equal to the simple diagonal triple 5, 4, 3 multiplied by 10 ninda. The data for the exercise may have been constructed with departure either from this simple diagonal triple, or from the simple condition that the transversal divides the length of the triangle in two parts in the ratio 1 : 2.

Str. 364 § 3

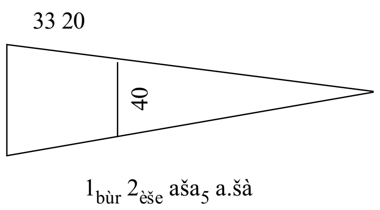


Fig.

1	sag.kak	A peg-head (= triangle).
	uš *ù* sag an.<na> nu.zu /	The length and the up<per> front I do not know.
2	1$_{\text{bùr}}$ 2$_{\text{èše}}$ aša$_5$ a.šà	1 bùr 2 èše the field.
3	*iš-tu* sag an.<na> / 3 20 *ur-dam-ma*	From the upper front 33 20 I went down, then
	40 dal /	40, the transversal.
4	uš *ù* sag en.nam	The length and the front are what?

Str. 364 § 4 a is concerned with another example of a striped triangle:

Str. 364 § 4 a

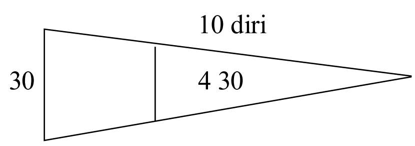
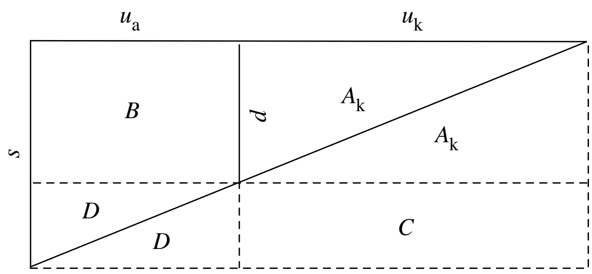


Fig.

1	sag.kak	A peg-head.
	i-na šà 2 íd	Inside it 2 canals.
	30 sag an.<na> 4 30 a.šà ki.<ta> /	30 the up<per> front, 4 30 the lo<wer> field.
2	uš ki.<ta> ugu uš an.<na> 10 diri	The lo<wer> length over the up<per> length is 10 beyond.

Here, the given parameters for the striped triangle are the lower area $A_{\text{k}} = 4\;30$ sq. n., the front $s = 30$ n., and the difference between the lower and upper lengths (actually heights), $u_{\text{k}} - u_{\text{a}} = 10$ n. The parameters to be computed are the transversal d, the partial lengths u_{a} and u_{k}, and the upper area A_{a}. If two of these unknown parameters can be found, for instance and u_{k} and d, then it will be easy to find also the values of the remaining two parameters. Now, the product of u_{k} and d is known, since it is equal to twice the lower area. Therefore, what is needed is to find a linear relation between u_{k} and d, so that u_{k} and d can be found as the solutions to a rectangular-linear system of equations.

A geometric derivation of such a linear relation is shown in Fig. 10.2.11 below. (In order to avoid unnecessary complications of the arguments, in the figure the triangle is assumed to be a right triangle.) The rule demonstrated in Fig. 10.2.11 may be called “the third Old Babylonian transversal rule”. So far, it is only a conjecture that Old Babylonian mathematicians were familiar with this rule, but it seems to be a fairly reasonable conjecture.



$A_k + B + D = A_k + C + D \Rightarrow \boxed{B = C}$

$s \cdot u_k = 2\,A_k + C$, and

$(u_k - u_a) \cdot d = 2\,A_k - B \;\Rightarrow$

$\boxed{s/2 \cdot u_k + (u_k - u_a)/2 \cdot d = 2\,A_k}$

Fig. 10.2.11. Str. 364 § 4 a. The (alleged) third Old Babylonian transversal rule.

The linear relation $s/2 \cdot u_k + (u_k - u_a)/2 \cdot d = 2\,A_k$ between the unknown lower length u_k and the unknown transversal d can be obtained through a simple geometric argument as shown in Fig. 10.2.11 above. It follows that u_k and d can be computed as the solutions to the following rectangular-linear system of equations

$$u_k \cdot d = 2\,A_k, \quad s/2 \cdot u_k + (u_k - u_a)/2 \cdot d = 2 \cdot A_k.$$

This is a system of equations of the same type as the system $u \cdot s = 2\,A,\ d \cdot u + u_a \cdot s = 2\,A$ for the unknowns u and s in Str. 363 § 3. Clearly, then, the unknowns u_k and d play the same role in Str. 364 § 4 a as the unknowns u and s in § 3, and the coefficients $2\,A_k$, $s/2$ and $(u_k - u_a)/2$ play the same role in Str. 364 § 4 a as the coefficients $2\,A$, d and u_a in § 3. Therefore, in Str. 364 § 4 a

$$d = A_k\,/(u_k - u_a)/2 - \text{sqs. } \{\text{sq. } (A_k\,/(u_k - u_a)/2 - s/2) - \text{sq. } s/2\}, \quad u_k = 2\,A_k/d.$$

With the given values $A_k = 4\ 30$ sq. n., $s = 30$ n., and $(u_k - u_a) = 10$ n., it follows that

$A_k\,/(u_k - u_a)/2 = 4\ 30$ sq. n. / 5 n. = 54 n.
$d = 54$ n. – sqs. (sq. 39 n. – sq. 15 n.) = 54 n. – 36 n. = 18 n.
$u_k = 9\ 00$ sq. n. / 18 n. = 30 n., hence $u_a = 30$ n. – 10 n. = 20 n.

This solution to the problem in Str. 364 § 4 a is associated with the diagonal triple

$$A_k/(u_k - u_a)/2 - s/2 = 39 \text{ n.}, \quad A_k/(u_k - u_a)/2 - d = 36 \text{ n.}, \quad s/2 = 15 \text{ n.}$$

This, of course, is a multiple of the well known diagonal triple 13, 12, 5.

The problem in **Str. 364 § 5 a** is concerned with a similar problem for the same triangle, but with the transversal in a different position, so that $u_a - u_k = 10$ instead of $u_k - u_a = 10$ as in Str. 364 § 4 a. There is also a new value for the lower area, $A_k = 2$ (00 sq. n.). The problem leads to the following rectangular-linear system for the unknowns u_k and d, resembling the one in the case of Str. 364 § 4 a:

$u_k \cdot d = 2\,A_k$
$s/2 \cdot u_k - (u_a - u_k)/2 \cdot d = 2 \cdot A_k$ (see again Fig. 10.2.11 above).

With the given values $A_k = 2$ (00 sq. n.), $s = 30$ (n.), and $u_a - u_k = 10$ (n.) inserted, the solution is

$d = \text{sqs. } \{\text{sq. } (A_k/(u_a - u_k)/2 + s/2) - \text{sq. } s/2\} - A_k/(u_a - u_k)/2 = \text{sqs. (sq. } (24 + 15) - \text{sq. } 15) - 24 = 36 - 24 = 12$ n.
$u_k = 2\,A_k\,/d = 20$ n.

This solution to the problem in Str. 364 § 5 a is associated with the diagonal triple

$$A_k/(u_k - u_a)/2 + s/2 = 39 \text{ n.}, \quad A_k/(u_k - u_a)/ + d = 36 \text{ n.}, \quad s/2 = 15 \text{ n.}$$

This is, again, a multiple of the well known diagonal triple 13, 12, 5.

The remaining exercises on the obverse of Str. 364 are routine variations on the theme. The exercises on the reverse with five-striped triangles look formidable but each one of them can be reduced to a pair of simpler exercises, one for a two-striped trapezoid, and another for a two-striped triangle.

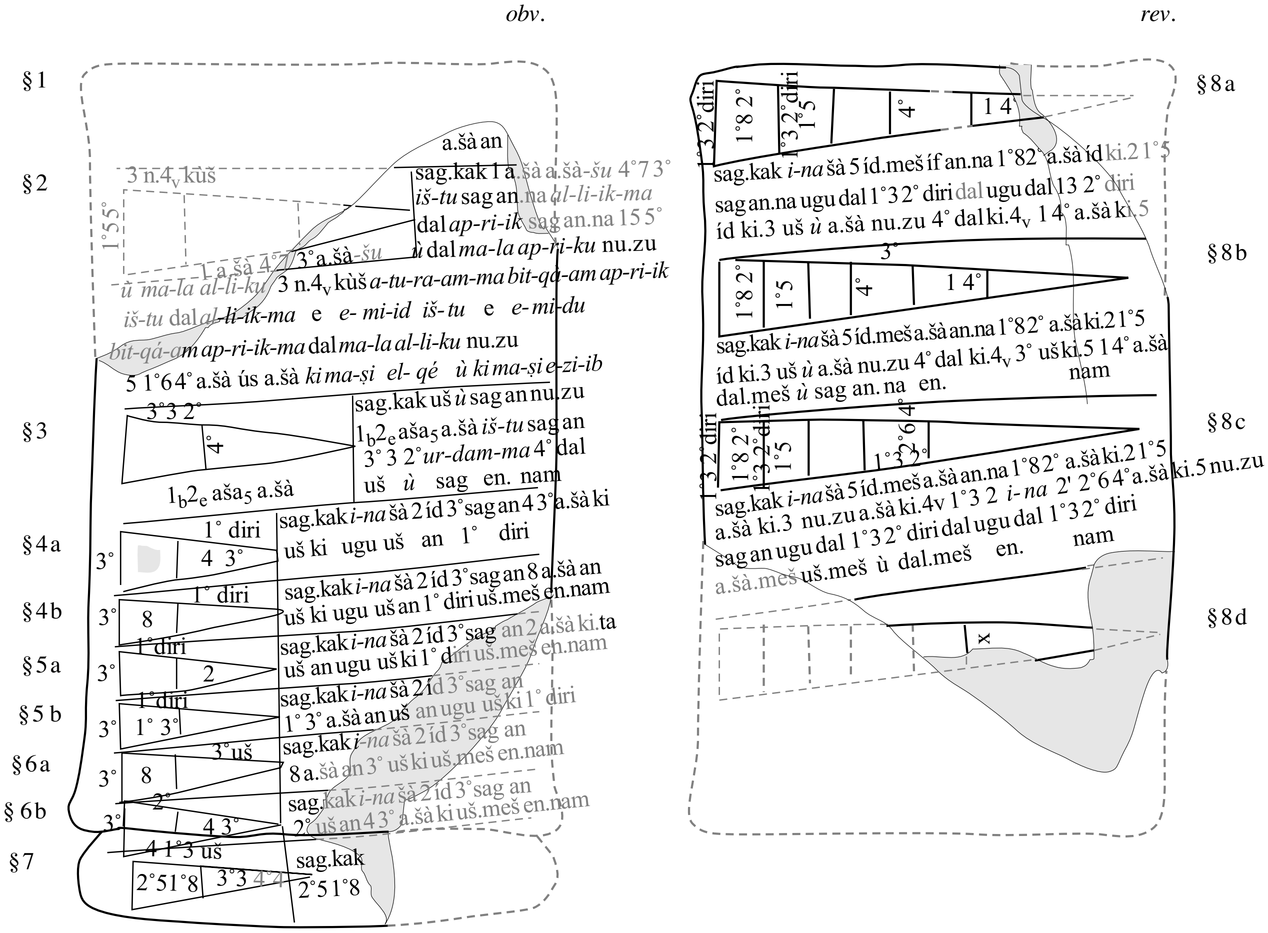


Fig. 10.2.12. Str. 364. A theme text with metric algebra problems for striped triangles.
(The conform transliteration here is based on Neugebauer's hand copy of the text in *MKT*.)

VAT 8512 (Høyrup, *LWS* (2002), 234-238; Group 4 a, hence a southern text; cf. Robson, *HM* 28 (2001), 183, fn. 21) is a problem text with a single exercise that is closely related to the theme of Str. 364 § 4 a, hence also to the theme of MS 3052 §§ 1 c-d. In VAT 8512 is considered a striped triangle with a single transversal. With the usual notations, the given parameters are

$$s = 30 \text{ n.}, \quad u_\mathrm{k} - u_\mathrm{a} = 20 \text{ n}, \quad A_\mathrm{a} - A_\mathrm{k} = 7\ (00) \text{ sq. n.}$$

The solution is given explicitly by an elegant solution procedure. In modern notations, the first step of the solution procedure is the computation of the transversal, called *pirkum* 'cross-line', as

$$d = \text{sqs. } \{(\text{sq. } (s + p) + \text{sq. } p)/2\} - p, \quad \text{where} \quad p = (A_\mathrm{a} - A_\mathrm{k})/(u_\mathrm{k} - u_\mathrm{a}) = 7\ 00\ /\ 20 = 21, \quad \text{so that} \quad d = 18.$$

This equation may have been found as follows: If f is the growth rate for the triangle, then

$$s - d = f \cdot u_\mathrm{a} \quad \text{and} \quad d = f \cdot u_\mathrm{k}, \quad \text{hence} \quad f = (2\,d - s)/(u_\mathrm{k} - u_\mathrm{a}).$$

In addition,

$$\text{sq. } s - \text{sq. } d = f \cdot 2\,A_\mathrm{a} \quad \text{and} \quad \text{sq. } d = f \cdot 2\,A_\mathrm{k}, \quad \text{so that}$$

$$\text{sq. } s - 2 \text{ sq. } d = f \cdot 2\,(A_\mathrm{a} - A_\mathrm{k}) = 2\,(d - s)/((u_\mathrm{k} - u_\mathrm{a}) \cdot 2\,(A_\mathrm{a} - A_\mathrm{k}).$$

It follows that

$$\text{sq. } s - 2 \text{ sq. } d = 2\, p \cdot (2\, d - s), \quad p = (A_a - A_k)/(u_k - u_a).$$

Therefore, the value of *d* can be found as a solution to the quadratic equation

$$\text{sq. } d + 2\, p \cdot d = (\text{sq. } s + 2\, p \cdot s)/2.$$

It is, of course, tempting to complete the squares on both sides of this equation. The result is the reformulated quadratic equation

$$\text{sq. } (d + p) - \text{sq. } p = \{\text{sq. } (s + p) - \text{sq. } p\}/2,$$

or, finally,

$$\text{sq. } (d + p) = \{\text{sq. } (s + p) + \text{sq. } p\}/2.$$

The solution given explicitly in the text of VAT 8512 follows immediately from this equation for *d*.
Note: An ingenious *geometric* derivation of the equation was found by Gandz (1948) and Huber (1955). See the references to these authors in Høyrup (*op. cit.*) and, in particular, Fig. 59 on p. 235.

Note the following interesting similarity between MS 3052 § 1 c and VAT 8512: The explicit solution in MS 3052 § 1 c, was given in a form that made it clear that the triple

$$c = A/h_k - d, \quad b = d, \quad a = A/h_k - s \quad \text{is a solution to the } \textit{diagonal equation} \; \text{sq. } c = \text{sq. } a + \text{sq. } b.$$

Similarly, the explicit solution in VAT 8512 is given in a form that makes it clear that the triple

$$c = d + p, \quad b = s + p, \quad a = p \quad \text{is a solution to the } \textit{equipartitioned trapezoid equation} \quad \text{sq. } c = (\text{sq. } a + \text{sq. } b)/2.$$

With the given values in VAT 8521, the triple, $s + p$, $d + p$, p = 51, 39, 21 = 3 · (17, 13, 7). Equipartioned trapezoids was a popular topic in Old Babylonian mathematics. (See Friberg, *RlA 7* (1990) Sec. 5.4 k.) The idea is the following: If *a*, *b*, *c* is an equipartioned trapezoid triple, then any trapezoid with the lower front *a*, and the upper front *b* is divided into *two sub-trapezoids of equal area* by a parallel transversal of length *c*. Common Old Babylonian examples of equipartitioned trapezoid triples are 7, 5, 1 or 17, 13, 7 and multiples of these triples.

MS 3052 § 1 e. A Badly Preserved Exercise Dealing with a Mud Wall with a Breach

The text of **MS 3052 § 1 e** is inscribed on the badly damaged reverse of MS 3052. Too little is preserved of the text of this exercise to allow a meaningful reconstruction. See Fig. 10.2.13 below. Anyway, since a subscript on the reverse of MS 3052 mentions '8 hand tablets (= exercises)', and, more specifically, '[x] mud walls, 1 diagonal (= rectangle), 1 excavation, and 1 square', it is clear that there were, originally, five exercises for mud walls on MS 3052. It follows that § 1 e must be the fifth exercise for a mud wall, probably one with a trapezoidal cross section, a breach, and a hole.

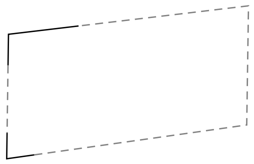
The word *ag-šu-ur* 'I repaired' in line 2 of § 1 e (cf. line 5 of § 1 d) shows that also § 1 e is concerned with the reparation of a breach in a wall. The fact that the numbers 3 4[5], 5, 6 15, 25, 2 36 15, 12 30, and 7 30 are mentioned in lines 6-12 of § 1 e suggests that the solution procedure in § 1 e in some way makes use of the two diagonal triples

6 15, 5, 3 45, with sq. 6 15 – sq. 5 = 39 03 45 – 25 = 14 03 45 = sq. 3 45. and
12 30, 7 30, 10 with sq. 12 30 – sq. 10 = 2 36 15 – 56 15 = 1 40 = sq. 10.

10.2 b. MS 3052 § 2. A 'Diagonal'. The Basic igi-igi.bi Problem

The text of this exercise is badly preserved, but the fact that MS 3971, § 3 a-e are closely parallel exercises (see above, Sec. 10.1 c) makes it easy to reconstruct most of the text.

MS 3052 § 2



	[igi] 2 uš sag *ù ṣi-li-ip-tum* /	*The igi* (is) 2.
1	en.nam /	Length, front, and cross-over, what?
2	[za].e ak.da.zu.dè /	Fig.***You* with your doing:**
3	[2] igi *ù igi-bé-e* en$^?$.nam$^?$ /	[2] the igi and the igi.bi what?
4	2 igi *ù* 30 *ig*[*i-b*]*é-e* gar /	2, the igi, and 30, the ig*i.b*i, set.
5	2 igi *ù* 30 [igi.bi gar]$^?$.gar-*ma* /	2, the igi, and 30, *the igi.bi, he*ap, then
6	2 30 in.[sì 2' 2 30 gaz 1 15 in.s]ì	2 30 it gives. *1/2 of 2 30 break, 1 15 it giv*es.
	1 15 [*ṣi*]*-li-ip-tum* /	1 15 (is) the cross-over.
7	1 [15 du$_7$.du$_7$-*ma* 1 33 45 in.sì]	1 *15 (let) butt (itself), then 1 33 45 it gives.*
	1 uš du$_7$.du$_7$-*ma* 1 in. sì /	1, the length, (let) butt (itself), then 1 it gives.
8	1 [*i-na* 1 33 45 zi-*ma*] 33 45 in. sì /	1 *from 1 33 45 tear off, then* 33 45 it gives.
9	33 [45.e 15 íb.si$_8$]	33 45 makes 15 equalsided.
	[15 sa]g in. sì	15, the front, it gives.

The drawing accompanying this text is badly preserved. However, the few traces of the drawing that remain seem to indicate that what was depicted was a rectangle *without* any diagonals. On the other hand, the question is almost completely preserved. It asks for the sizes of *the length, the front, and the diagonal* of the rectangle, given that a number called the igi is equal to 2.

The first step of the solution procedure is the computation of the igi.bi, the reciprocal of the igi, in the present case obviously equal to 1/2 = ;30. The diagonal c, the length b, and the front a of the rectangle are then computed as follows:

$$c, b, a = (\text{igi} + \text{igi.bi})/2,\ 1,\ \text{sqs. (sq. } d - \text{sq. } u) = 2;30/2,\ 1,\ \text{sqs. (sq. } 2;30/2 - \text{sq. } 1) = 1;15,\ 1,\ ;45.$$

Another possibility is that the length called ‘1’ was interpreted as 1 (00). In that case, the computed triple has to be understood as

$$c, b, a = 1\ 15,\ 1\ (00),\ 45.$$

Whichever the case may be, it is easy to see that the rectangle computed in this way is *similar* to a rectangle in which the diagonal, the length, and the front form the well known “Pythagorean” triple 5, 4, 3. (Compare with the exercise MS 3971 § 4, where it is shown that 7, 5;36, 4;12 are the diagonal, the length, and the front of another rectangle similar to the 5, 4, 3 rectangle.)

Actually, the triple 1 45, 1 (00), 45 is one of the triples listed in the famous Old Babylonian table text Plimpton 322. See App. 8 for a further discussion of this fascinating topic.

10.2 c. MS 3052 § 3. An ‘Excavation’ of the igi-igi.bi Type

The text of § 3 of MS 3052 is so damaged that no reconstruction can be attempted. The statement of the problem and the answer are both lost, except the phrase igi uš ù igi.bi sag in line 1, which probably means that the floor of the excavation is a rectangle with the sides u and s, where $u \cdot s = 1$. (square ninda). Cf. **BM 85200+, ## 15-18** (Høyrup, *LWS* (2002), 158; Sec. A8 b in App. 8 below). Of the solution procedure all that remains are the computations $5 \cdot 18 = 1\ 30$ in line 3 and sq. 45 + 1 = sq. 1 15 in lines 4-5.

10.2 d. MS 3052 § 4. A ‘Square’. Another Badly Damaged Exercise

The text of § 4 is not as extensively damaged as the text of § 3. A transliteration of the text is given below.

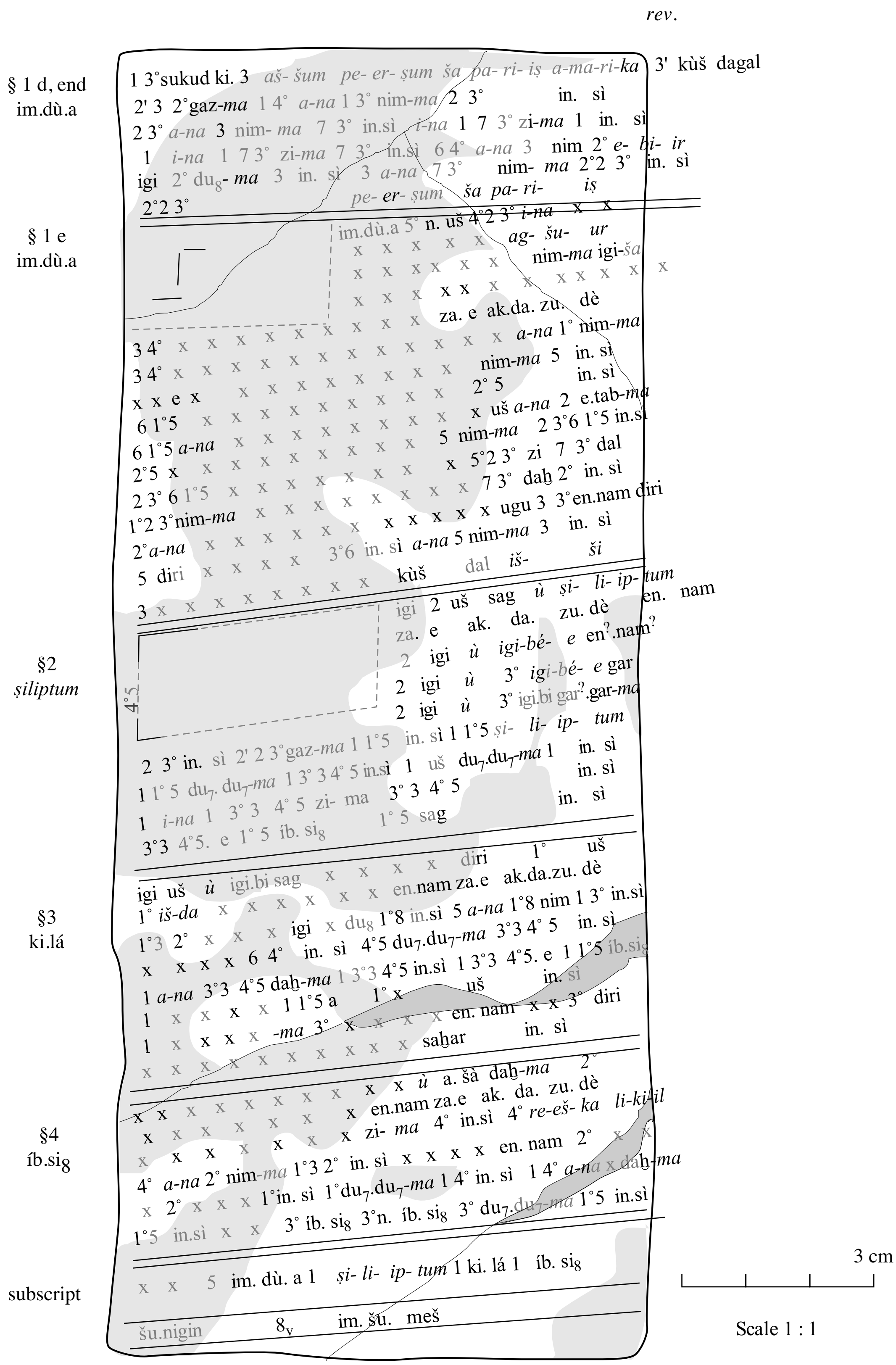
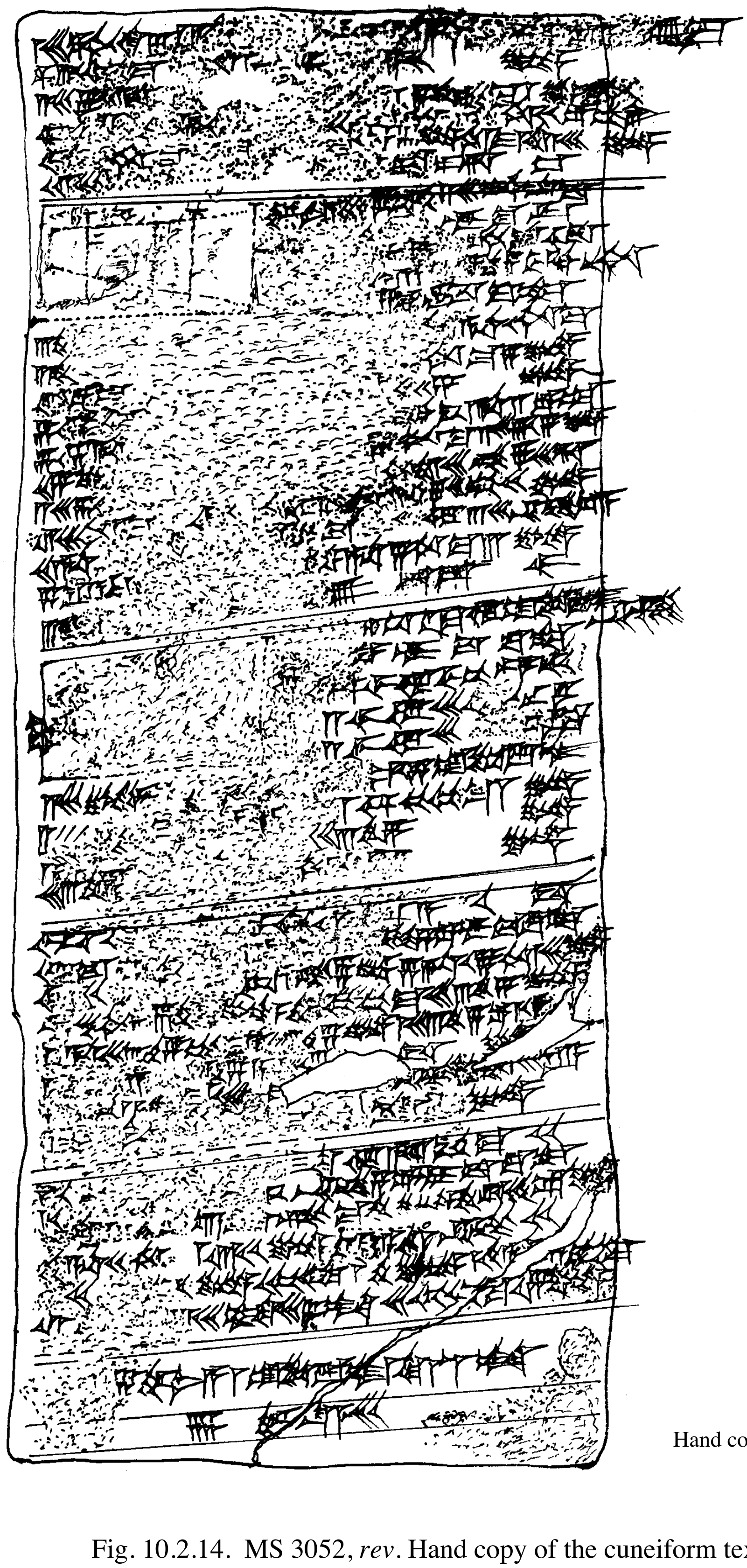


Fig. 10.2.13. MS 3052, *rev*. Conform transliteration.



Hand copy: F. Al-Rawi

Fig. 10.2.14. MS 3052, *rev*. Hand copy of the cuneiform text.

MS 3052 § 4

1	x x [x x x x x x x x x x]	x x x x x x x x x x x x
	x x *ù* a.šà daḫ-*ma* 20 /	x x and the field join, then 20.
2	x [x x x x x x x x x] en.nam	x x x x x x x x x x (are) what?
	za.e ak.da.zu.dè /	You, with your doing:
3	[x] x x [x] x [x] x zi-*ma* 40 in.sì	x x x x x x x tear off, then 40 it gives.
	40 *re-eš-ka li-ki-il* /	40 may hold your head.
	40 *a-na* 20 nim-[*ma*] 13 20 in.sì	40 to 20 lift, *then* 13 20 it gives.
4	x x x x en.nam 20 [x x] /	x x x x (is) what? 20 the *x x*.
	[x] 20 [x x x] 10 in.sì	*x* 20 *x x x*, 10 it gives.
5	10 du$_7$.du$_7$-*ma* 1 40 in.sì	10 (let) butt (itself), 1 40 it gives.
	1 40 *a-n*[*a* 13 20 da]ḫ-*ma* /	1 40 to *13 20* join, then
	1[5 in.sì x x] 30 íb.si$_8$	1*5 it gives x x*, 30 the equalside.
6	30 ninda íb.si$_8$	30 ninda (is) the equalside.
	30 du$_7$.du$_7$-*ma* 15 aša$_5$	30 (let) butt (itself), 15 the field.

Most of the question is lost in this exercise. What remains is in many places not clearly legible. Apparently, something added to the area (of a square?) makes 20. The solution procedure begins with a subtraction, with the remainder 40, which is to be remembered. This number 40 is multiplied with 20, which gives 40 · 20 = 13 20. Then some other operations yields the result 10, and the solution procedure seems to end with the computation sq. 10 + 13 20 = 15 = sq. 30, where 30 = 30 ninda is said to be the side of the square. It is not at all clear what all this means.

10.2 e. MS 3052, Subscript. A List of the Separate Topics in the Text

MS 3052, summary

1	[x x 5] im.dù.a 1 *ṣi-li-ip-tum* 1 ki.lá 1 íb.si$_8$ /	*x x 5* mud walls, 1 cross-over, 1 excavation, 1 equalside
2	[šu.nígin] 8$_v$ im. šu.meš	*Together* 8 hand tablets (assignments).

This kind of subscript with a detailed summary of the topics in the text is an almost unique feature of MS 3052. The only known parallel is the subscript of MS 3049. (See Fig. 11.1.4 below.)

10.3. MS 2792. Two Exercises Dealing with a Divided Ramp

10.3 a. MS 2792 # 1. A Layer on Top of a Ramp Divided Equally along the Length

MS 2792 (Figs. 10.3.1-2 and 10.3.5-6 below) is a large, fairly well preserved single-column clay tablet, inscribed with one long mathematical exercise on the obverse and another on the reverse. The common topic of the two exercises is an *arammu* 'ramp'. Both exercises are accompanied by drawings showing a rectangle or trapezoid divided by three transversals into four sub-rectangles or sub-trapezoids.

The small vocabulary of Sumerian technical terms used in MS 2792 includes the following terms for mathematical operations:

gar.gar	heap (add)	du$_7$.du$_7$	butt (square)	gaz	break (halve)
zi	tear off (subtract)	nim	lift (multiply)	in.sì	it gives (the result)
a-na 2 e.tab	to 2 you repeat (double)	du$_8$	resolve (compute the reciprocal)		
n.e *m* íb.si$_8$	*n* makes *m* equalsided (sqs. $n = m$)			gar.ra	set (make a note of)

There is also, as in MS 3971 (Sec. 10.1) and MS 3052 (Sec. 10.2), the following Akkadian phrase, marking a new section of the solution procedure:

aššum … amārika in order for you to see.

Consequently, MS 2792 belongs to “Group 3” of Old Babylonian mathematical cuneiform texts, just like MS 3971 and MS 3052. As a member of Group 3, MS 2792, too, can be classified as a text from the ancient southern Mesopotamian city Uruk. Note, by the way, that in MS 2792 and MS 3052, but not in MS 3971, the solution procedures begin with the phrase

za.e ak.da.zu.dè You, with your doing:

MS 2792 # 1 (the obverse):

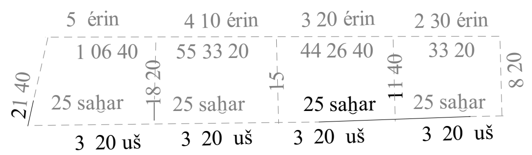


Fig.

1	*a-ra-mu* 13 20 uš	A ramp, 13 20 the length,
	a-ra-mu a-na ki.là 8 20 kùš	The ramp at the ki.là 8 20 cubits.
	10.bi *i-za-*[*bi-lu*] /	10 of it they shall ca*rry.*
2	1 30 sag 5 kùš sukud	130 the front, 5 cubits the height.
	4$_v$ šagina.meš	4 officers.
3	*iš-t*[*e-e*]*n* 5 *šu-ši* [érin-*šu*] / *ša-nu-ú* 4 10 érin-*šu*	On*e*, 5 sixties *his soldiers*, the second 4 10 his soldiers,
	ša-al-šu 3 20 érin-*šu re-bu-ú* 2 30 [érin-šu] /	the third 3 20 his soldiers, the fourth 2 30 *his soldiers*.
4	uš *p*[*u-lu-u*]*k-šu-nu-ši-im*	The lengths ma*rk out* for them.
	za.e ak.da.zu.dè /	You, with your doing:
5	13 2[0 uš 1 30] sag gar.ra *ù*	13 *20, the length (and) 1 30* the front set, and
	8 20 ki.lá [*i-na*] 21 40 z[i 1]3 20 in.sì /	8 20, the ki.lá, *from* 21 40 tear *off, 1*3 20 it gives.
6	21 40 *ù* 8 20 gar.gar-*ma* 30	21 40 and 8 20 heap, then 30,
	2' 30 gaz-*ma* 15	1/2 of 30 break, then 15.
	a-na 13 20 nim 3 20 in.sì /	To 13 20 lift, *3* 20 it gives.
7	érin.ḫá gar.gar-*ma* 15	The soldiers heap, then 15.
	igi 15 du$_8$-*ma* 4	The opposite of 15 resolve, then 4.
	a-na 3 20 nim 13 20 in.[sì] /	To 3 20 lift, 13 20 it gi*ves*.
8	13 20 *a-na* 5 érin nim 1 06 40 in.sì	13 20 to 5 soldiers lift, 1 06 40 it gives.
	13 20 *a-na* 4 10 érin nim 55 [33] 20 in.sì /	13 20 to 4 10 soldiers lift, 55 *33* 20 it gives.
9	13 20 *a-na* 3 20 érin nim 44 26 40 in.sì	13 20 to 3 20 soldiers lift 44 26 40 it gives.
	13 20 *a-na* <2> 30 érin nim 33 20 in.sì /	13 20 to <2> 30 soldiers lift, 33 20 it gives.
10	21 40 ugu 8 20 en.nam diri 13 20 diri	21 40 over 8 20 is what beyond? 13 20 beyond.
	igi 13 20 du$_8$-*ma* 4 30	The opposite of 13 20 resolve, then 4 30.
11	*a-*[*na* 13] 20 *ša* diri nim 1 / 1 *in-da-nu*	T*o 13* 20, that is beyond lift, 1. 1 is the *indanum*.
	1 *a-na* 2 e.tab-*ma* 2	1 to 2 you repeat, then 2.
	a-na 1 06 40 nim 2 13 20 in.sì	To 1 06 40 lift, 2 13 20 it gives.
12	21 40 du$_7$.du$_7$-*ma* / 7 49 26 40 in.sì	21 40 (make) butt (itself), then 7 49 26 40 it gives.
	2 13 20 zi-*ma* 5 36 06 40 in.sì /	2 13 20 tear off, then 5 36 06 40 it gives.
13	5 36 06 40.e 18 20 íb.si$_8$	5 36 06 40 makes 18 20 equalsided.
	dal ki.1 in.sì	The 1st transversal it gives.
	1 *a-na* 2 e.tab-*ma* 2 in.sì /	1 to 2 you repeat, then 2 it gives.
14	*a-na* 55 33 20 nim 1 51 06 40 in.sì	To 55 33 20 lift, 1 51 06 40 it gives.
15	18 20 du$_7$.du$_7$-*ma* / 5 36 06 40 in.sì	18 20 (make) butt (itself), then 5 36 06 40 it gives.
	1 51 06 40 zi-*ma* 3 45.e 15 íb.si$_8$ /	1 51 06 40 tear off, 3 45, makes 15 equalsided.
16	dal ki.2 in.sì	The 2nd transversal it gives.
	1 *a-na* 2 e.tab-*ma* 2	1 to 2 you repeat, then 2.
	a-na 44 26 40 nim 1 28 53 20 in.sì /	To 44 26 40 lift, 1 28 53 20 it gi*ves*.
17	15 du$_7$.du$_7$-*ma* 3 45 in.sì	15 (make) butt (itself), then 3 45 it gives.
	1 28 53 20 *i-na* 3 45 zi /	1 28 53 20 from 3 45 tear off,

18	2 16 06 40.e 11 40 íb.si$_8$	2 16 06 40 makes 11 40 equalsided,
	dal ki.3 in.sì /	the 3rd transversal it gives.
19	*aš-šum* uš *a-ma-ri-ka*	In order for you to see the length:
	21 40 ugu 18 20 en.nam diri 3 20 diri /	21 40 over 18 20 is what beyond? 3 20 beyond.
20	igi 1 *in-da-nim* du$_8$	The opposite of 1, the *indanum*, release, 1.
	1 *a-na* 3 20 *ša* diri nim 3 20 uš ki.1 /	To 3 20 that is beyond lift, 3 20 the 1st length.
21	18 20 ugu 15 en.nam diri 3 20 diri	18 20 over 15 is what beyond? 3 20 beyond.
	igi 1 du$_8$-*ma* 1	The opposite of 1 release, then 1.
22	*a-na* 3 20 nim-*ma* / 3 20 uš ki.2	To 3 20 lift, then 3 20, the 2nd length.
	15 ugu 11 40 en.nam diri 3 20 diri	15 over 11 40 is what beyond? 3 20 beyond.
	igi 1 du$_8$-*ma* 1 /	The opposite of 1 release, then 1.
23	*a-na* 3 20 nim-*ma* 3 20 uš ki.3	To 3 20 lift, then 3 20, the 3rd length.
24	11 40 ugu 8 20 en.nam diri / 3 20 diri	11 40 over 8 20 is what beyond? 3 20 beyond.
	igi 1 du$_8$-*ma* 1	The opposite of 1 release, then 1.
	a-na 3 20 nim 3 20 uš ki.4$_v$ /	To 3 20 lift, 3 20 the 4th length.
25	*aš-šum* saḫar *a-ma-ri-ka*	In order for you to see the mud (volumes):
	1 30 sag *a-na* 3 20 uš nim 5 in.sì /	1 30, the front, to 3 20, the length, lift, 5 it gives.
26	5 *a-na* 5 sukud nim-*ma*	5 to 5, the height, lift, then
	25 saḫar ki.1 in.sì	25, the 1st mud it gives.
	1 30 sag *a-na* 3 20 uš ki.2 nim /	1 30, the front, to 3 20 the 2nd length lift, 5.
27	5 *a-na* 5 sukud nim-[*ma*]	To 5, the height, lift, then
	25 saḫar.ki 2 in.sì	25, the 2nd mud it gives.
	1 30 sag *a-na* 3 20 uš ki.3 nim / [5]	1 30, the front, to 3 20 the third length lift, *5*.
28	[*a-na* 5] sukud nim-[*ma*]	*To 5,* the height, lift, then
	25 saḫar.ki 3 in.sì	25, the 3rd mud it gives.
	1 30 sag *a-na* 3 20 uš ki.4 nim / [5]	1 30, the front, to 3 20 the 4th length lift, *5*.
29	[*a-n*]*a* 5 sukud nim-*ma*	*To* 5, the height, lift, then
	[2]5 saḫar ki.4$_v$ in.sì	25, the 4th mud it gives.

The question in this exercise is vaguely stated, and some pieces of it are lost. The question mentions a 'ramp' (*arammu*), of length 13;20 (ninda), a height? (probably at the end of the ramp), equal to 8;20 cubits, a distance? of 10 (ninda?), a 'front' equal to 1;30 (ninda), and another 'height' equal to 5 cubits. It also mentions 4 šagina (written gìr.níta) (Akk. *šakkanakkum*) 'officers, military governors', and the sizes of their respective troops of men. With departure from these data, the 'lengths' for the four troops are to be computed.

The ensuing computations give some clues to the meaning of this vaguely stated question. Thus, a likely interpretation seems to be that a ramp is already present, built by the four troops of men. Its height has to be increased by an additional 5 cubits, and the question is what the lengths (and volumes) are of the added top layers on the four sections of the ramp. The situation is clarified in Fig. 10.3.3 below, with a drawing of the ramp in perspective. The notations used in Fig. 10.3.3 are the following:

E	(érin)	soldiers	d	(dal)	transversal
A	(a.šà)	area	u	(uš)	length
s	(sag)	front	h		height

The drawing associated with the exercise MS 2792 # 1 is badly damaged. Only the line of numbers below the drawing remains intact. However, since the related drawing illustrating the exercise MS 2792 # 2, on the reverse of the clay tablet, is fairly well preserved, it is easy to reconstruct the drawing on the obverse in its original form, with all the numbers inscribed in and around it.

The values recorded in the (reconstructed) drawing on the obverse can be interpreted as:

man-days for section 1	$E_1 =$ 5 00 'soldiers'	(300)
man-days for section 2	$E_2 =$ 4 10 'soldiers'	(250)
man-days for section 3	$E_3 =$ 3 20 'soldiers'	(200)
man-days for section 4	$E_4 =$ 2 30 'soldiers'	(150)

the height of the ramp at its upper end	h_a =	21 2/3 cubits		(10.83 m)
the first transversal	d_1 =	18 1/3 cubits		(9.17 m)
the second transversal	d_2=	15 cubits		(7.5 m)
the third transversal	d_3 =	11 2/3 cubits		(5.83 m)
the height of the ramp at its lower end	h_k =	8 1/3 cubits		(4.17 m)
the area of the side of section 1	A_1 =	1 06;40	(ninda · cubits)	
the area of the side of section 2	A_2 =	55;33 20	(ninda · cubits)	
the area of the side of section 3	A_3 =	44;26 40	(ninda · cubits)	
the area of the side of section 4	A_4 =	33;20	(ninda · cubits)	
the length of section 1	u_1 =	3;20	(ninda)	(20 m)
the length of section 2	u_2 =	3;20	(ninda)	
the length of section 3	u_3 =	3;20	(ninda)	
the length of section 4	u_4 =	3;20	(ninda)	
the volume of the top layer of section 1	V_1 =	25	(sq. ninda · cubits)	
the volume of the top layer of section 2	V_2 =	25	(sq. ninda · cubits)	
the volume of the top layer of section 3	V_3 =	25	(sq. ninda · cubits)	
the volume of the top layer of section 4	V_4 =	25	(sq. ninda · cubits)	

In line 1 of the text, the number '8 20' is specified by the phrase *arammu ana* ki.lá 'the ramp at the ki.lá'. It is not clear what that means. Normally, the meaning of ki.lá (Akk. *kalakkum*) is 'excavation, pit, trench'. Here, it may refer to the lower, abruptly ending, part of the ramp. The related number '21 40' is given without any specification in the text. Cf. **BM 85194 # 17** (Neugebauer, *MKT 1*, 143]), where the lower end of a ramp is qualified by the phrase *i-na* suḫuš saḫar.ḫá 'at the base of the mud', while the upper end is said to be

i-na pa-ni a-bu-li-im 'in front of the gate'.

Not recorded in the drawing on the obverse of MS 2792 are three more values given in the text:

the total length of the ramp	u =	13;20	(ninda)	(80 m)	(line 1)
the front of the ramp	s =	1;30	(ninda)	(9 m)	(line 2)
the height of the added top layer	h =	5	cubits	(2.5 m)	(line 2)

Now the vaguely stated question in MS 2792 # 1 can be rephrased in the following more precise way:

> Given a ramp with the length 13;20 n., the front 1;30 n., the lower height 8;20 c., and the upper height 21;40 c., divided in four sections that it took 5 00, 4 10, 3 20, and 2 30 'soldiers', respectively, to build, and with an added top layer of height 5 c., find the lengths of the four sections and the volumes of the added layers on top of them.

The solution procedure begins with the computation of the area of the vertical side of the ramp, which has the form of a trapezoid:

A = 13;20 n. · (21;40 + 8;20)/2 c. = 13;20 n. · 15 c. = 3 20 n. c. (line 6)

Next, the areas of the vertical sides of the four sections of the ramp are computed, by use of the obvious circumstance that these four areas are proportional to the four given numbers of 'soldiers':

E = (5 00 + 4 10 + 3 20 + 2 30) 'soldiers' = 15 00 'soldiers' (meaning *man-days*) (line 7)
A/E = 3 20 n. c. / 15 00 md = ;13 20 n. c./md. (line 7)
$A_1 = A/E \cdot E_1$ = ;13 20 n. c./md. · 5 00 md. = 1 06;40 n. c. (line 8)
$A_2 = A/E \cdot E_2$ = ;13 20 n. c./md. · 4 10 md. = 55;33 20 n. c. (line 8)
$A_3 = A/E \cdot E_3$ = ;13 20 n. c./md. · 3 20 md. = 44;26 40 n. c. (line 9)
$A_4 = A/E \cdot E_4$ = ;13 20 n. c./md. · 2 30 md. = 33;20 n. c. (line 9)

The inclination of the ramp is then computed as

f = (21;40 – 8;20) c. / 13;20 n. = 13;20 c. / 13;20 n. = 1 c./n. (inclination 1 : 12) (lines 10-11)

In this text, as in MS 3052 (Sec. 10.2 above), the Akkadian word *indanum* is used for 'growth rate'. The value f of the growth rate is needed in the next step of the solution procedure, the computation of the three transversals in the trapezoid formed by the vertical cross section of the ramp.

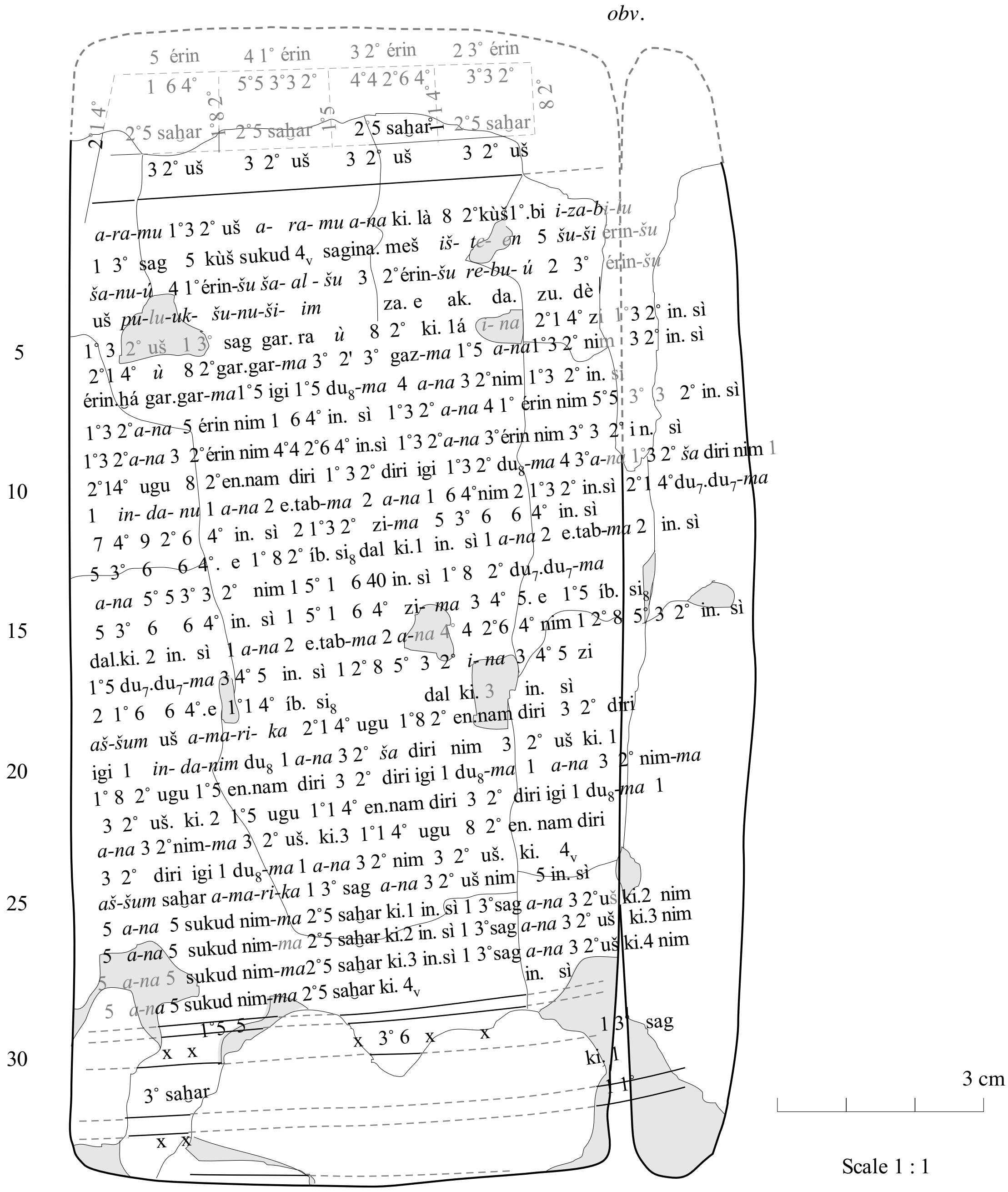


Fig. 10.3.1. MS 2792, *obv*. Conform transliteration. An equally divided ramp with an added layer on top of it.

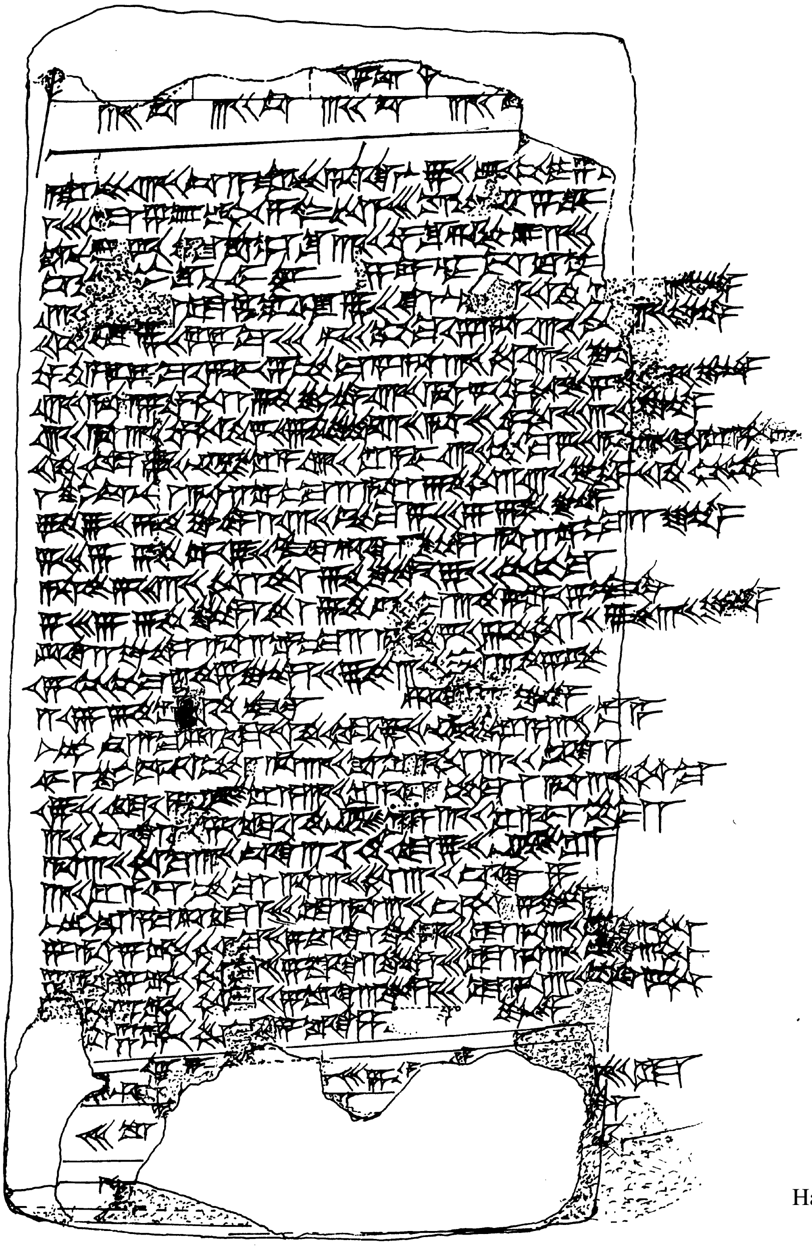


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Fig. 10.3.2. MS 2792, *obv*. Hand copy of the cuneiform text.

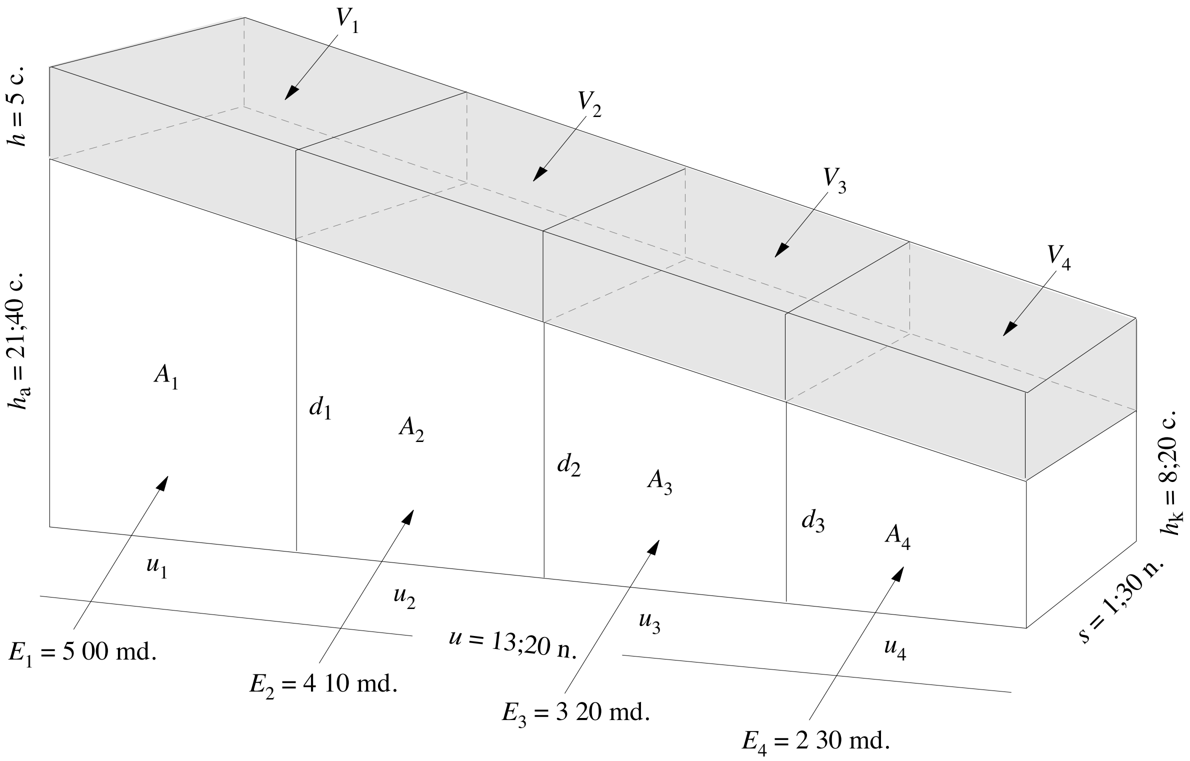

Fig. 10.3.3. MS 2792 # 1. A layer on top of a ramp, built by four troops of soldiers in sections of equal length.

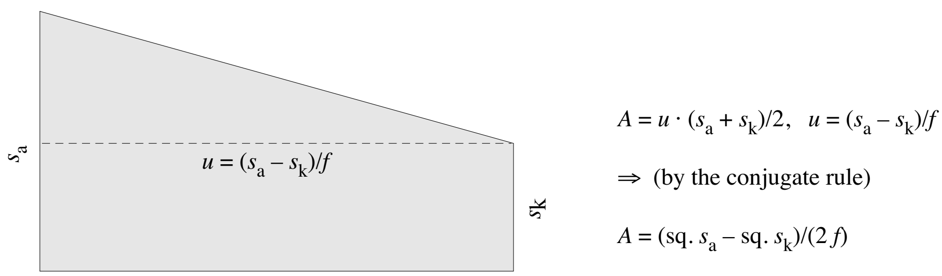

Fig. 10.3.4. MS 2792 # 1. The Old Babylonian alternative trapezoid area rule.

The Old Babylonian "alternative trapezoid area rule", derived as in Fig. 10.3.4, states that

$$A = (\text{sq. } s_a - \text{sq. } s_k)/(2f).$$

This is the explanation for the following computations of the three trapezoid transversals (see Fig. 10.3.3) in the solution procedure of MS 2792 # 1:

sq. s_a – sq. d_1 =	$(2f) \cdot A_1 = (2 \cdot 1$ c./n.$) \cdot 1\ 06;40$ n. c. $= 2\ 13;20$ sq. c.	(lines 11-13)
sq. s_a =	sq. (21;40 n.) = 7 49;26 40 sq. n.	
sq. d_1 =	sq. s_a – (sq. s_a – sq. d_1) = sq. $s_a - 2f \cdot A_1$ = (7 49;26 40 – 2 13;20) sq. n. = 5 36;06 40 sq. n.	
d_1 =	sqs. (5 36;06 40 sq. n.) = 18;20 c.	
sq. d_1 – sq. d_2 =	$(2f) \cdot A_2 = (2 \cdot 1$ c./n.$) \cdot 55;33\ 20$ n. c. $= 1\ 51;06\ 40$ sq. c.	(lines 13-16)
sq. d_1 =	sq. (18;20 n.) = 5 36;06 40 sq. n.	
sq. d_2 =	sq. d_1 – (sq. d_1 – sq. d_2) = sq. $d_1 - 2f \cdot A_2$ = (5 36;06 40– 1 51;06 40) sq. n. = 3 45 sq. c.	
d_2 =	sqs. (3 45 sq. n.) = 15 c.	

sq. d_2 – sq. d_3 = $(2f) \cdot A_3 = (2 \cdot 1$ c./n.$) \cdot 44;26\ 40$ n. c. $= 1\ 28;53\ 20$ sq. c.(lines 16-18)

sq. d_2 = sq. (15 n.) = 3 45 sq. n.
sq. d_3 = sq. d_2 – (sq. d_2 – sq. d_3) = sq. d_2 – 2 $f \cdot A_3$ = (3 45 – 1 28;53 20) sq. n. = 2 16;06 40 sq. c.
d_3 = sqs. (2 16;06 40 sq. n.) = 11;40 c.

The solution procedure continues with the phrase

aššum uš *amārika* in order for you to see the length(s),

meaning something like 'since you also want to know the lengths'.[4] The four partial lengths (see Fig. 10.3.3) are computed as follows:

u_1 =	$(s_a - d_1)/f$ =	(21;40 – 18;20) c. / (1 c./n.) =	3;20 n.	(20 m)	(lines 19-24)
u_2 =	$(d_1 - d_2)/f$ =	(18;20 – 15) c. / (1 c./n.) =	3;20 n.	(20 m)	
u_3 =	$(d_2 - d_3)/f$ =	(15 – 11;40) c. / (1 c./n.) =	3;20 n.	(20 m)	
u_4 =	$(d_3 - d_4)/f$ =	(11;40 – 8;20) c. / (1 c./n.) =	3;20 n.	(20 m)	

Thus, in this problem the partial lengths are equal.

The final part of the solution procedure is preceded by the phrase

aššum saḫar *amārika* in order for you to see the mud (the volumes),

clearly meaning that it remains to compute the volumes of the added layers on top of the four sections of the ramp. The simple computations are as follows:

V_1 =	$s \cdot u_1 \cdot h$ =	1;30 n. · 3;20 n. · 5 c. =	25 n. n. c.	(lines 25-29)
V_2 =	$s \cdot u_2 \cdot h$ =	1;30 n. · 3;20 n. · 5 c. =	25 n. n. c.	
V_3 =	$s \cdot u_3 \cdot h$ =	1;30 n. · 3;20 n. · 5 c. =	25 n. n. c.	
V_4 =	$s \cdot u_4 \cdot h$ =	1;30 n. · 3;20 n. · 5 c. =	25 n. n. c.	

10.3 b. MS 2792 # 2. A Layer on Top of a Ramp Divided Unequally along the Length

The problem considered in problem # 2 on the *reverse* of MS 2792 (Fig. 10.3.5)is of the same type as problem # 1 on the obverse, only with partly different data. Thus, in this new problem, a ramp is again divided into four parts, the length of the ramp is 13;20 n. and the front is 1;30 c. as before. See Fig. 10.3.7 below.

However, the height of the added layer is now 8 c. instead of 5 c., the upper height of the ramp is 23;20 c. instead of 21;40 c., and the lower height of the ramp is 10 c. instead of 8;20 c. Nevertheless, the difference between the upper and the lower height is unchanged, so that also the growth rate *f* remains the same.

In the exercise MS 2792 # 1, the numbers of 'soldiers' needed for the construction of the four sections of the ramp form an arithmetical progression, but in the exercise on the reverse, the corresponding numbers of 'soldiers' are chosen more at random:

man-days for section 1	E_1 =	4 30 'soldiers'	(270)
man-days for section 2	E_2 =	8 00 'soldiers'	(480)
man-days for section 3	E_3 =	6 40 'soldiers'	(400)
man-days for section 4	E_4 =	7 30 'soldiers'	(450)

In spite of the differences between problem # 1 on the obverse and problem # 2 on the reverse, there is no visible difference between the drawing on the obverse and the one on the reverse.

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4. Similar phrases are used also in MS 3052 §§ 1 a and 1 d (Sec. 10.2 a above), and in MS 3971 § 3 a (Sec. 10.1 c above). Cf. Neugebauer and Sachs, *MCT*, p. 50, text D = **YBC 4608** (Group 3; Uruk): *aš-šum* sag an.na *ù* sag ki.ta *a-ma-ri-i-ka* (lines 21-22), and *aš-šu* a.šà *a-ma-ri-i-ka* (line 28). Cf.also in **AO 6555**, a text from the first millennium BC (Friberg, *BagM* 28 (1997), 298) § 4, 1-2: *mín-da-a-ti ki-gal-li* / é.te.me.en.an.ki / uš *ù* sag *a-na* igi.du$_8$.[a-*ka*] / 3(1.šu) uš 3(1.šu) sag / *ina*1 kùš *as$_4$-lum* 'Measures of the base of Etemenanki (the ziggurrat of Babylon). In order for you to see the length and the front: 3 sixties the length, 3 sixties the front, in the *aslu* cubit.' and in § 5, 1-2: *ša-ni-iš* ka+x.meš ki.gal é.te.me.en.an.ki uš *ù* sag *a-na a-ma-ri-*[*ka*] / [10 ninda u]š 10 ninda sag *i-na* 1 kùš a.rá-*e* 'Secondly. Measures(?) of the base of Etemenanki. In order for you to see the length and the front: *10 n. the length*, 10 n. the front, in the *arû* cubit.'

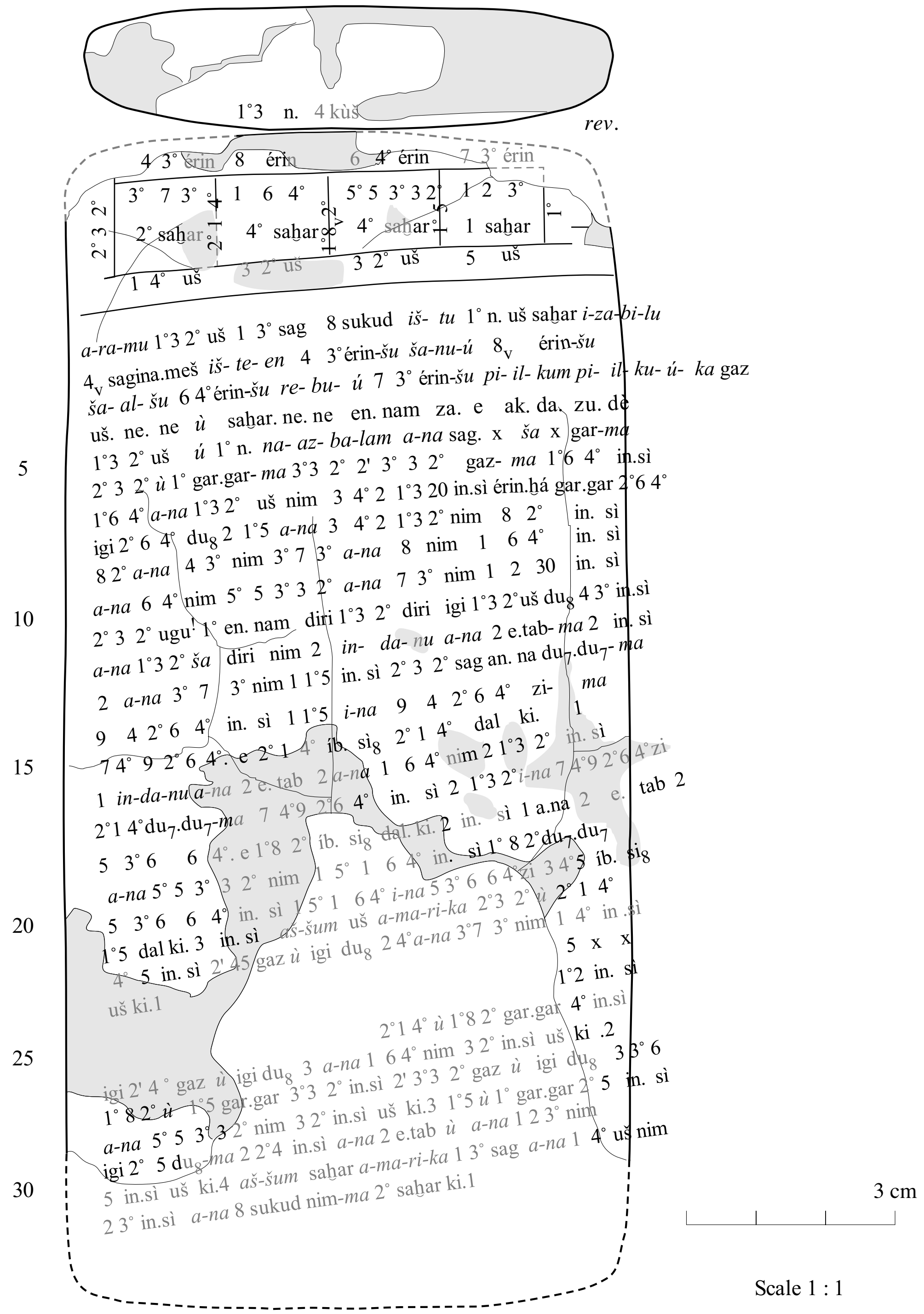
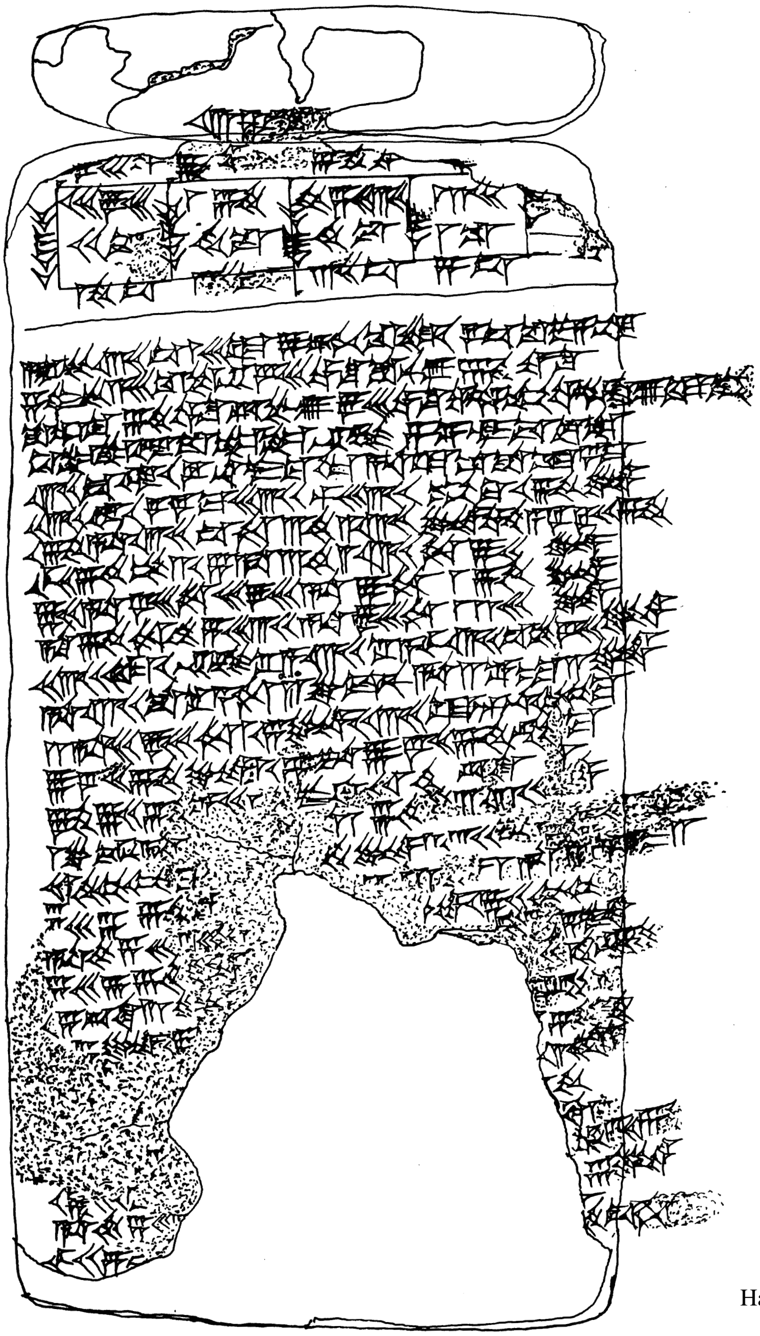


Fig. 10.3.5. MS 2792, *rev*. Conform transliteration. An unequally divided ramp with an added layer on top of it.



Hand copy: F. Al-Rawi

Fig. 10.3.6. MS 2792, *rev*. Hand copy of the cuneiform text.

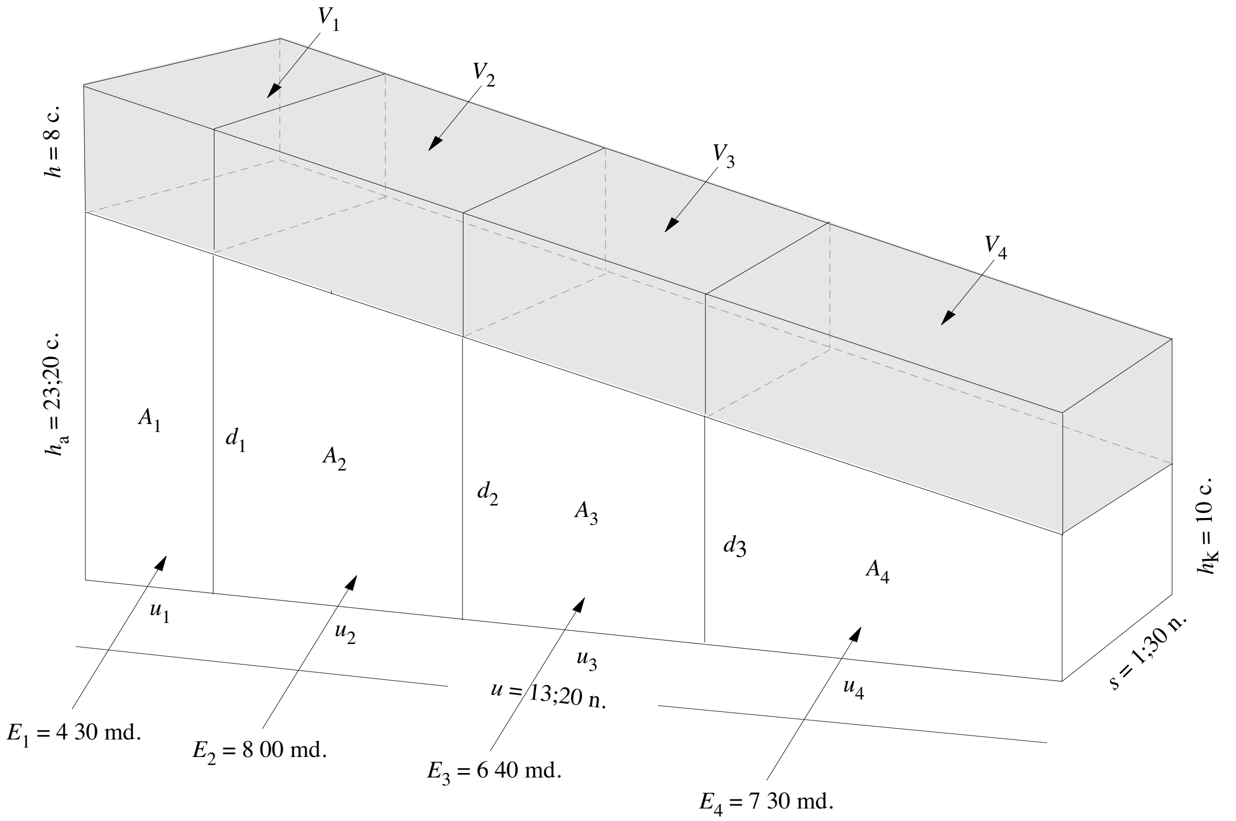


Fig. 10.3.7. MS 2792 # 2. A layer on top of a ramp, built by four troops of soldiers in sections of unequal length.

MS 2792 # 2 (the reverse)

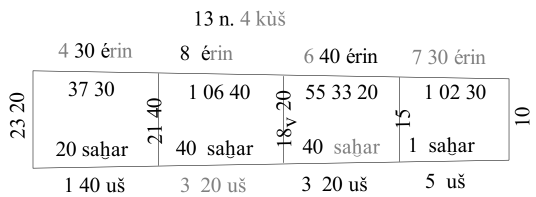


Fig.

1	*a-ra-mu* 13 20 uš 1 30 sag 8 sukud	A ramp. 13 20 the length, 1 30 the front, 8 the height.
	iš-tu 10 ninda uš saḫar *i-za-bi-lu* /	From 10 ninda's distance, mud they shall carry.
2	4_v šagina.meš	4 officers.
	iš-te-en 4 30 érin-*šu ša-nu-ú* 8_vérin-*šu* /	One, 4 30 his soldiers, the second, 8 (00) his soldiers,
3	*ša-al-šu* 6 40 érin-*šu re-bu-ú* 7 30 érin-*šu*	the third, 6 40 his soldiers, the fourth, 7 30 his soldiers.
	pi-il-kum pi-il-ku[!]*-ú-ka* ga[z] /	The marked off, your marked off (section), bre*ak*.
4	uš.ne.ne *ù* saḫar.ne.ne en.nam	Their lengths and their muds (volumes) are what?
	za.e ak.da.zu.dè /	You, with your doing:
5	13 20 uš *ù* 10 ninda *na-az-ba-lam*	13 20, the length and 10 ninda, the carrying (of mud),
	a-na sag x *ša* x gar-*ma* /	to the front x that x set, then
6	23 20 *ù* 10 gar.gar-*ma* 33 20	23 20 and 10 heap, then 33 20.
	2' 33 20 gaz-*ma* 16 40 in.sì /	1/2 of 33 20 break, then 16 40 it gives.
	16 40 *a-na* 13 20 uš nim 3 42 13 20 in.sì	16 40 to 13 20, the length, lift, 3 42 13 20 it gives.

7	érin.ḫá gar.gar 26 40 /	The soldiers heap, 26 40.
	igi 26 40 du$_8$ 2 15	The opposite of 26 40 resolve, 2 15.
8	*a-na* 3 42 13 20 nim 8 20 in.sì /	To 3 42 13 20 lift, 8 20 it gives.
	8 20 *a-na* 4 30 nim 37 30	8 20 to 4 30 lift, 37 30,
9	*a-na* 8 nim 1 06 40 in.sì /	to 8 lift, 1 06 40 it gives.
	a-na 6 40 nim 55 33 20	To 6 40 lift, 55 33 20,
10	*a-na* 7 30 nim 1 02 30 in.sì /	to 7 30 lift, 1 02 30 it gives.
	23 20 ugu 10 en.nam diri 13 20 diri	23 20 over 10 is what beyond? 13 20 beyond.
11	igi 13 20 uš du$_8$ 4 30 in.sì /	The opposite of 13 20, the length, resolve, 4 30 it gives.
	a-na 13 20 *ša* diri nim 2 *in-da-*[*n*]*u*	To 13 20, what is beyond, lift, $1^!$ the *indanum*.
12	*a-na* 2 e.tab-*ma* 2 in.sì /	To 2 you repeat, then 2 it gives.
	2 *a-na* 37 30 nim 1 15 in.sì	2 to 37 30 lift, 1 15 it gives.
13	23 20 sag.an.na du$_7$.du$_7$-*ma* /	23 20 the upper front (make) butt (itself), then
	9 04 26 40 in.sì	9 04 26 40 it gives.
14	1 15 *i-na* 9 04 26 40 zi-*ma* /	1 15 from 9 04 26 40 tear off, then
	7 49 26 40.e 21 [40] íb.si$_8$	7 49 26 40, makes 21 *40* equalsided.
15	21 40 dal ki.1 /	21 40 the 1st transv*ersal*.
	1 *in-da-nu a-*[*na* 2.e.tab 2]	1, the *indanum*, t*o 2 repeat, 2*.
16	[*a-n*]*a* 1 06 40 [ni]m 2 13 20 [in.s]ì /	*To* 1 06 40 *lift*, 2 *13* 20 *it gi*ves.
	21 40 du$_7$.du$_7$-*m*[*a* 7 49 26] 40 in.sì	21 40 (make) butt (itself), the*n 7 49 26* 40 it gives.
17	2 13 20 [*i-na* 7 49 26 40 zi] /	2 13 20 *from 7 49 26 40 tear off,*
	5 36 06 [40.e 18 20 íb.si$_8$]	5 36 06 *40, makes 18 20 equalsided,*
18	[dal ki.]2 in.sì	the 2*nd transversal* it gives.
	1 *a-na* [2 e].tab 2 /	1 to *2 you* repeat, 2.
	a-na 55 3[3 20 nim 1 51 06 40 in].sì	To 55 3*3 20 lift, 1 51 06 40 it* gives.
19	18 20 du$_7$.du$_7$ / 5 36 06 40 [in.sì]	18 20 (make) butt (itself), 5 36 06 40 *it gives*.
20	[1 51 06 40 *i-na* 5 36 06 40 zi 3 4]5	*1 51 06 40 from 5 36 06 40 tear off, 3 4*5.
	íb.si$_8$ / 15 dal ki.3 i[n.sì]	Equalside 15, the 3rd transversal i*t gives*.
21	[*aš-šum* uš *a-ma-ri-ka*]	In order for you to see the length:
	[23 20 *ù* 2]1 40 [gar.gar / 45 in.sì]	*23 20 and* 21 40 *heap, 45 it gives*.
22	[2' 45 gaz *ù* igi du$_8$ 2 40]	1/2 of 45 break and the opposite release, 2 40.
	[*a-na* 37 30 nim 1 40 in.sì / uš ki.1]	To 37 30 lift, 1 40 it gives, the 1st length.
23	[… … … … …] 5 x x /	… … … … … 5 x x.
	[… … … … … … … …] 12 in.sì /	… … … … … … … …, 12 it gives.
24	[… … … … …]	… … … … …
25	[21 40 *ù* 18 20 gar.gar] 40 [in.sì] /	*21 40 and 18 20 heap,* 40 *it gives*.
	[2' 40 gaz *ù* igi du$_8$ 3]	1/2 of 40 break and the opposite release, 3.
26	[*a-na* 1 06 40 nim 3 20 in.sì] uš ki.2 /	To 1 06 40 lift, 3 20 it gives, the 2nd length.
	18 20 *ù* [15 gar.gar 33 20 in.sì]	18 20 and 15 heap, 33 20 it gives.
27	[2' 33 20 gaz *ù* igi du$_8$] 3 36 /	1/2 of 33 20 break and the opposite release, 3 36.
	a-na 55 33 [20 nim 3 20 in.sì uš ki.3]	To 55 33 20 lift, 3 20 it gives, the 3rd length.
28	[15 *ù* 10 gar.gar 2]5 in.sì /	15 and 10 heap, 25 it gives.
	igi 25 d[u$_8$-*ma* 2 24 in.sì]	The opposite of 25 release, then 2 24 it gives.To 2 repeat and
29	[*a-na* 2 e.tab *ù a-na* 1 02 30 nim] /	to 1 02 30 lift,
	[5 in.sì uš ki.4]	5 it gives, the 4th length.
30	[*aš-šum* saḫar *a-ma-ri-ka*]	In order for you to see the mud:
	[1 30 sag *a-na* 1] 40 uš nim / [2 30 in.sì]	1 30 the front to 1 40 the length lift, 2 30 it gives.
31	[*a-na* 8 sukud nim-*ma* 20 saḫar ki.1]	to 8 the height lift, then 20, the 1st mud.
	etc.	*etc.*

The solution procedure for the problem on the reverse of MS 2792 (# 2) is essentially identical with the solution procedure for the problem on the obverse. The computed values are the following:

A = 3 42;13 20 n. c.				(line 7)
E = 26 40 ‘soldiers’				(line 8)
A/E = ;08 20 n. c./md.				(line 9)
A = 37;30 n. c.,	A_2 = 1 06;40 n. c.,	A_3 = 55;33 20 n. c.,	A_4 = 1 02;30 n. c	(lines 10-11)
f = 1 c./n.				(lines 12-13)
d_1 = 21;40 c.,	d_2 = 18;20 c.,	d_3 = 15 c.		(lines 14-22)
[u_1 = 1;40 n.,	u_2 = 3;20 n.,	u_3 = 3;20 n.,	u_4 = 5 n.]	(lines 22-30)
[V_1 = 20 n. n. c.,	V_2 = 40 n. n. c.,	V_3 = 40 n. n. c.,	V_4 = 1 00 n. n. c.]	(lines 30 ff)

While the solution procedure in this exercise is relatively clear, the statement of the question in lines 1-4 is obscure and incomplete. It begins by mentioning ‘the length 13 20’, ‘the front 1 30’ and ‘the height 8’. These are apparently the dimensions of the added layer on top of the ramp. The question continues by mentioning that something is carried ‘here’, presumably to the construction site, from a distance of 10 ninda. (Something similar is mentioned, even more cryptically, in the question at the beginning of the exercise on the obverse.) There is nothing in the solution procedure that explains what this is about. Next, the question gives the four numbers of ‘soldiers’, and it ends by asking for the ‘lengths’ and ‘volumes’, presumably of the added layers on top of the four sections of the ramp.

10.3 c. The Work Norm for Building a Ramp

It might be a good idea to investigate what the ratio is in MS 2792 ## 1-2 between the number of ‘soldiers’ needed to build the ramps, and the volumes of the ramps. In # 1, the volume of the ramp can be computed as

$$V = 13;20 \text{ n.} \cdot 1;30 \text{ n.} \cdot (21;40 + 8;20)/2 \text{ c.} = 20 \text{ sq. n.} \cdot 15 \text{ c.} = 5\ 00 \text{ sq. n.} \cdot \text{c. (volume-šar).}$$

Hence, the number of volume-units per ‘soldier’ in this case is

$$V/E = 5\ 00 \text{ volume-šar} / 15\ 00 \text{ ‘soldiers’} = ;20 \text{ volume-šar/‘soldier’}.$$

In view of what work-norms look like in other Old Babylonian mathematical texts (see Friberg, *RlA 7* (1990) Sec. 5.6 h), this almost certainly means that the word ‘soldier’ must be interpreted as a word for ‘man-day’, a unit of labor corresponding to 1 man working for a full day. In other words, in this text, *the work norm for building a ramp is 20 volume-shekels per man-day*, a volume-shekel being a sixtieth of a volume-šar.

In # 2, the work norm ought to be the same. However, in that case

$$V = 13;20 \text{ n.} \cdot 1;30 \text{ n.} \cdot (23;20 + 10)/2 \text{ c.} = 20 \text{ sq. n.} \cdot 16;40 \text{ c.} = 5\ 33;20 \text{ sq. n.} \cdot \text{c. (volume-šar).}$$
$$V/E = 5\ 33;20 \text{ volume-šar/26 40 ‘soldiers’} = ;12\ 30 \text{ volume-šar/‘soldier’}.$$

The conflict between the two implicitly given values for the work norm is probably due to a simple mistake on the part of the author of the text. In exercise # 1, the work norm is ‘20’, the man-days (or ‘soldiers’) ‘15’, and the volume ‘13 20’ (= 20 · 15). In # 2, on the other hand, the man-days are ‘26 40’ and the volume ‘5 33 20’, where 5 33 20 can be factorized as 20 · 16 40. Therefore, it is likely that a mistake made by the author of the text was to let the number of ‘soldiers’ be 26 40 instead of 16 40!

10.3 d. The Construction of the Data for the Two Problems

It is now obvious how the data for exercise # 2 were constructed with departure from the data for exercise # 1. The author of the text wanted the problem in # 2 to be a more complicated version of the problem in # 1. For that purpose, he added the same amount, 1;40 c., to both the upper and the lower height of the ramp, keeping the rate of decrease of the ramp the same as before. In doing so he changed the volume from 5 00 = 20 · 15 in # 1 to 5 33 20 = 20 · 16 40 in # 2. Assuming that he had not made the mentioned mistake, he would then have let the number of ‘soldiers’ be 16 40.

Another change made in the construction of the problem on the reverse of MS 2792 is that in this second problem the length of the ramp is divided into four *unequal* parts, whereas in the problem on the obverse the length was divided into four equal parts. In spite of the differences, however, both problems are solved in essentially the same way.

Actually, it is hard to know precisely how the partial lengths and the added volumes were computed in MS 2792 #2, since more than a third of the text is lost on the reverse of the tablet, perhaps as much as the fifteen last lines of the text. (See Figs. 10.3.5-6.) Repeated attempts to reconstruct the lost text have been only partly successful. (In particular, it is not at all clear what is going on in lines 23-25.) One of the difficulties is that the author of the text apparently chose not to use the same solution procedure in # 2 as in # 1 for the computation of the lengths of the four sections of the ramp. Thus, in # 1, lines 19-24, the four partial lengths are computed as follows:

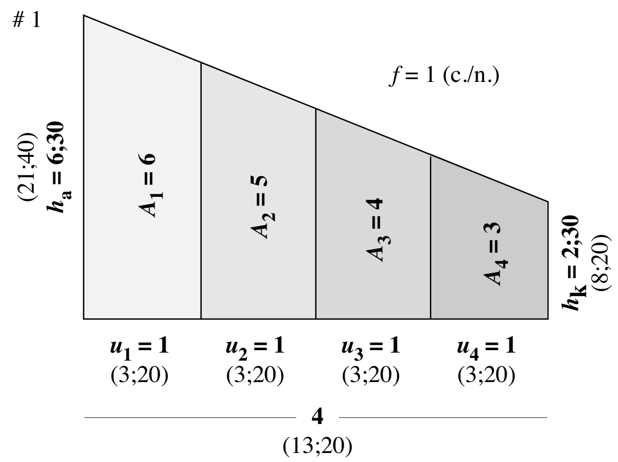
$$u_1 = (s_a - d_1)/f, \; etc.$$

In # 2, lines 22-23 and 25-30, on the other hand, the four partial lengths are computed as

$$u_1 = 2\,A_1 \,/\, (s_a + d_1), \; etc.$$

There are clearly also other changes to the solution procedure, but what they are is not clear.

In order for the mentioned modified solution procedure in # 2 to work, it is imperative that the sums $s_a + d_1$, $d_1 + d_2$, *etc*., are *regular* sexagesimal numbers. It is not a trivial task to choose the data for a problem like the one in MS 2792 # 2 so that this requirement is satisfied. For that reason, it is interesting to try to explain how the author of MS 2792 actually constructed the data appearing in the text.



A = 18 (n. c.) and s = 1 (n.)
$\Rightarrow$
V = 18 (n. n. c.)

E = 18 (n. n. c.) / ;20 (n. n. c./man-day)
= 54 man-days

scale factors: 3;20 (10/3), 3;20 (10/3), 1;30 (3/2):
54 man-days · 10/3 · 10/3 · 3/2 = 15 00 man-days

Fig. 10.3.8. A possible construction of the data for MS 2792 # 1. (The figure is not drawn to scale.)

It is likely that the construction of the data for MS 2792 #1 started with a “basic” trapezoid like the one depicted in Fig. 10.3.8 above. In this trapezoid, the length is divided into four equal pieces, all four of length 1. The growth rate for the trapezoidal cross section of the ramp is assumed to be 1, and the upper and lower fronts are set equal to 6;30 and 2;30, respectively. The lengths of the three transversals are then, of course, equal to 5,30, 4;30, and 3;30, and the areas of the four sub-trapezoids are 6, 5, 4, and 3. Therefore, the total area of the cross section is A = 6 + 5 + 4 + 3 = 18 (ninda · cubits). If, in addition, the width of the ramp in this basic construction is 1 (ninda), it follows that the volume is V = 18 (square ninda · cubits). Thus, finally, if the work norm for building the ramp is 20 volume-shekels per man-day, the needed man power for the whole ramp is

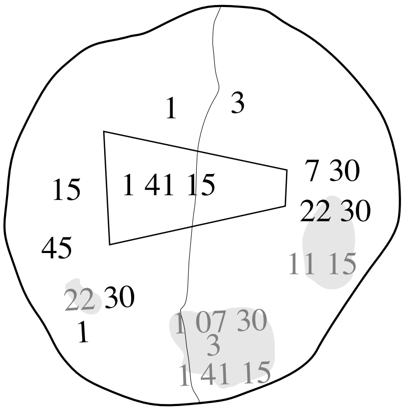
$$E = 18 \text{ n. n. c.} \,/\, ;20 \text{ n. n. c./man-day} = 54 \text{ man-days}.$$

In order to get more realistic numbers for the ramp, the author of the problem now decided to scale up the horizontal and vertical dimensions of the cross section by the factor 3;20 (10/3), and the width of the ramp by the factor 1;30 (3/2). The result was a ramp with the length u = 3;20 · 4 = 13;20 (n.), the heights at the two ends equal to h_a = 3;20 · 6;30 = 21;40 (c.) and h_k = 3;20 · 2;30 = 8;20 (c.), and the width s = 1;30 · 1 = 1;30 (c.). As a result of the change of scale, the needed man power then became

$$E = 10/3 \cdot 10/3 \cdot 3/2 \cdot 54 \text{ man-days} = 15\ 00 \text{ man-days}.$$

In Fig. 10.3.8, the scaled up numbers are within brackets. Note that f = 1 (c./n.) remains the same.

Compare the way in which the sides of a trapezoid are scaled up by a factor 3 in the drawing on the Old Babylonian hand tablet **YBC 11126** (Neugebauer, *MCT* (1945), 44; Fig. 10.3.9 below:



Question (reconstructed):
The given area of a trapezoid is 1 41 15.
The upper front is 1/4 of the length,
and the lower front is half the upper front.
What are the length and the fronts?
Solution by use of the rule of false value:
If the (false) length is 1(00), then the fronts
are 15 and 7;30, and the area 11 15.
The desired "true" area, 1 41 15, is 9 times larger.
Hence the correction factor is sqs. 9 = 3,
the true length is 3 (00), and the true
fronts are 3 · 15 = 45 and 3 · 7;30 = 22;30.

Fig. 10.3.9. YBC 11126. A hand tablet with scaled up numbers for the sides of a trapezoid.

The construction of the data for MS 2792 # 2 was probably similar to the construction of the data for # 1. Presumably, it started with a "basic" divided trapezoid like the one in Fig. 10.3.10 below, an 8-striped trapezoid with the length 8 and the fronts 14 and 6 and, consequently, with the growth rate f = 1 (c./n.). The lengths of the 7 transversals are then, of course, equal to 14 – 1 = 13, 13 – 1 = 12, *etc*. Hence, the area of the first sub-trapezoid is A_1 = (14 + 13)/2 · 1 = 13;30., and the area of each sub-trapezoid is less by 1 than the area of the preceding sub-trapezoid. This means that the 8 sub-areas are

13;30 = 27/2, 12;30 = 25/2, 11;30 = 23/2, 10;30 = 21/2, 9;30 = 19/2, 8;30 = 17/2, 7;30 = 15/2, and 6;30 =13/2.

Here 27, 25, and 15 are regular sexagesimal numbers, but 23, 21, 19, 17, 14, and 13 are non-regular. Therefore, not all the sub-areas in the basic trapezoid are equal to regular sexagesimal numbers, and neither are all the eight sums

$$s_a + d_1 = 2\,A_1/1, \quad d_1 + d_2 = 2\,A_2/1, \textit{ etc.}$$

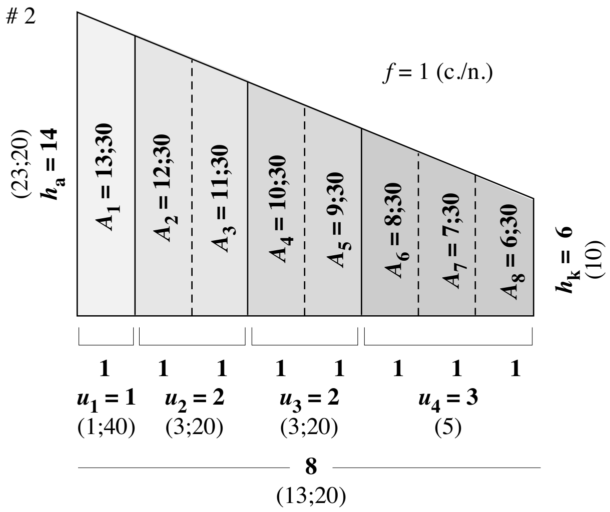
The author of the text cleverly circumvented this difficulty by bunching together some of the sub-rectangles, so that the basic trapezoid became divided in four parts, with the areas equal to

A_1 = 13;30 = 27/2, $A_2 + A_3$ = 12;30 + 11;30 = 24, $A_4 + A_5$ = 10;30 + 9;30 = 20, and
$A_6 + A_7 + A_8$ = 8;30 + 7;30 + 6;30 = 22;30 = 45/2.

In this way he could construct a basic trapezoid divided in four parts, with all the areas of the sub-trapezoids equal to regular sexagesimal numbers. The total area of this basic trapezoid was 1 20 (ninda · cubits). With the width assumed to be 1 (ninda), the volume of the ramp in this basic form would then be 1 20 (square ninda · cubits), corresponding to 1 20 / ;20 = 4 00 man-days of labor. After application of the scale factors 1;40, 1;40, 1;30, the scaled up ramp had the same length (13;20 n.) and width (1;30 n.) as the scaled up ramp in # 1, and the needed man power had increased to

1;40 · 1;40 · 1;30 · 4 00 man-days = 16 40.

The corresponding numbers of man-days for the four parts of the divided ramp could then be computed by use of proportionality. In this final step of the construction, the author of the problem by mistake counted with proportional parts of 26 40 instead of 16 40.



A_1 = 13;30 (27/2)
A_2 = 12;30 (25/2)
A_3 = 11;30 (23/2) non-regular
A_4 = 10;30 (21/2) non-regular
A_5 = 9;30 (19/2) non-regular
A_6 = 8;30 (17/2) non-regular
A_7 = 7;30 (15/2)
A_8 = 6;30 (13/2) non-regular

A_1 = 13;30
$A_2 + A_3$ = 24
$A_4 + A_5$ = 20
$A_6 + A_7 + A_8$ = 22;30
all regular

A = 1 20 (n. c.) and s = 1 (n.) ⇒ V = 1 20 (n. n. c.),
E = 1 20 (n. n. c.) / ;20 (n. n. c./man-day)
= 4 00 man-days
scale factors: 1;40 (5/3), 1;40 (5/3), 1;30 (3/2):
4 00 man-days · 5/3 · 5/3 · 3/2 = 16 40 man-days

Fig. 10.3.10. A proposed construction of the data for MS 2792 # 2. (The figure is not drawn to scale.)

A Related Text: Str. 362 # 6. A Combined Work Norm for Carrying and Building

Str. 362 (*MKT 1,* 239) is a small Old Babylonian clay tablet, probably from Uruk, with six miscellaneous mathematical exercises. The last exercise is a problem for a ramp built in three sections:

Str. 362 # 6

1	*a-ra-mu*	A ramp.
	10 ninda uš 1 2' ninda sag /	10 n. the length, 1 1/2 n. the front.
2	3 šagina.meš	3 officers.
	3 ninda 4 kùš uš *iṣ-ba-tu* /	3 n. 4 c. they took.
3	*iš-te-en* 1 *šu-ši* érin	One, 1 sixty soldiers,
	ki.2 1 20 érin /	the 2nd, 1 20 soldiers,
4	ki.3 1 40 érin	the 3rd, 1 40 soldiers.
	iš-tu 5 [*ṣu-up-pa-am*] / saḫar.ḫá *iz-za-bi-*[*lu-nim*] /	From 5 (n.), a *ṣuppān*, mud *they* shall carry *here*.
5	*a-ra-mi šu-up-li* /	The depth of the ramp
6	*ù* saḫar.ḫá *ki ma-*[*ṣi iz-za-bi-lu-nim*]	and mud how m*uch shall they carry here?*

In this problem, a ramp is built by 3 troops of 'soldiers' under the command of three officers. Just as in MS 2792 # 1, the front is 1 1/2 n. (= 9 m), and each troop builds 3;20 n.(= 20 m) along the length of the ramp. The total length of the ramp is then 3 · 3;20 n. = 10 n. The soldiers in the first troop yield 1 00 (60) man-days of work, those in the second troop 1 20 (80) man-days, and those in the third troop 1 40 (100) man-days. The mud to build the ramp is brought from a specified distance, 5 ninda. The purpose of the exercise is to compute the height of the ramp, apparently the same everywhere, and the volume of the ramp. The solution procedure, not given in the text (nor in the commentary in Neugebauer, *MKT 1* (1935)), would probably have been as follows:

Assume that in this text, as in MS 2792 # 1 (and # 2!), the work norm for building a ramp out of mud is ;20 (= 1/3) volume-šar/man-day. However, since the mud is to be carried to the ramp from some place 5 n. away, the work norm has to be correspondingly reduced. The standard work norm for carrying mud in Old Babylonian mathematical texts is 1;40 volume-šar · ninda/man-day. (See § 7.3 a in connection with the discussion there of the table of constants on MS 2221, *obv.*) That work norm is the same as ;20 volume-šar · 5 ninda per man-day. Consequently, what is required in order to carry mud to the construction site over a distance of 5 n. and then build the ramp is 6 man-days per volume-šar, namely *3 days for the carrying* and *3 days for the building*. Now, in Str. 362 # 6, the given number of man-days is 1 00 + 1 20 + 1 40 = 4 00. Hence, the volume of the ramp must be 4 00 man-days · ;10 volume-šar/man-day = 40 volume-šar (sq. n. · c.). Since the length of the ramp is 10 n. (60 m) and the front 1 1/2 n. (9 m), the constant height of the ramp must be 40 sq. n. · c. / 15 sq. n. = 2;40 c. = (1 1/3 m).

The obvious similarities between Str. 362 # 6 and MS 2792, make it fairly reasonable to conjecture that both texts are (copies of) extracts from an original large theme text dealing with divided ramps and single or combined work norms. This would help to explain some peculiar features of MS 2792 ## 1-2, namely that the statements of the questions are so incomplete, and that they mention the carrying (of mud) from some distance, although this information is never used in the solution procedures.

Another, somewhat more distant, relative to MS 2792 is **BM 85194 # 1** (*MKT 1*, 143; from Sippar), an isolated exercise in a large recombination text. In that exercise, an *a-ra-am-mu-um* 'ramp' of a rather complicated form has the length 10 ninda and different trapezoidal cross sections of given dimensions at the lower end ('at the base of the mud') and at the upper end ('in front of the gate'). The work norm is ;10 volume-šar/man-day, and is explicitly referred to as 10 éš.kàr. The stated question is the following (cf. MS 2792 # 1, l. 4):

saḫar.ḫá en.nam *a-na* 1 lú uš *pu-lu-uk*	the mud is what? to 1 man, (his) length mark off.

In the solution procedure, it is shown that the volume of the ramp is 50 volume-šar, and that the needed amount of labor is 5 00 man-days (expressed in the form 5 érin '5 soldiers'). The final answer is computed as 10 n. / 5 00 érin = ;02 n. (= 20 cm) / érin and is expressed in the following form:

2 1 lú *i-ṣa-ba-at*	2 is (what) 1 man takes.

This is, of course, nonsense, and also mathematically incorrect. (Cf. Thureau-Dangin, *TMB* (1938), 22, fn. 1: "Ce n'est qu'une moyenne, car, dans le calcul, il n'a pas éte tenu compte de la difference de hauteur d'une extrémité à l'autre de l'ouvrage.")

The small work norm ;10 volume-šar in BM 85194 # 1, can be explained as follows: This exercise is an excerpt from a large theme text. It is possible that it was stated in some of the preceding exercises in the theme text that the mud for the construction of the ramp had to be carried there from a distance of 5 ninda, precisely as in Str. 362 # 6. In that case, 6 man-days would be needed for the construction of 1 volume-šar, and the *combined* work norm would be ;10 volume-šar/man-day.

The work norm ;20 volume-šar/man-day for some kind of construction work, possibly the building of a ramp, is behind the following curious series of entries in an Old Babylonian table of constants (G = **IM 52916**; Goetze (1951); cf. Friberg, *ChV* (2001), 6.5):

na-az-ba-al saḫar	1 40 *i-gi-gu-bu*	carrying of mud,	1 40 the constant	G *rev*. 23'
a.na 40 ninda *a-za-bi-il*	2 13 20 *al-lu-um*	for 40 ninda I shall carry,	2 13 20 the *allum*	G *rev*. 25'
a.na 20 ninda *a-za-bi-il*	4 *al-lu-um*	for 20 ninda I shall carry,	4 the *allum*	G *rev*. 26'
a.na 15 ninda [*a-za-bi-il*]	[5] *al-lu-um*	for 15 ninda *I shall carry,*	5 the *allum*	G *rev*. 27'
[a.na] 10 ninda *a-za-*[*bi-il*]	6 40 *al-lu-um*	*for* 10 ninda I shall car*ry*,	6 40 the *allum*	G *rev*. 28'
[a.na 5] ninda *a-za-*[*bi-il*]	10 *al-lu-um*	*for 5* ninda I shall car*ry*,	10 the *allum*	G *rev*. 29'

The last of these entries, for instance, must be interpreted as stating that the combined work norm for carrying from a distance of 5 ninda and building, say, a ramp, is ;10 volume-šar/man-day. This is the case in Str. 362 # 6, and also, possibly, in BM 85194 # 1. In MS 2792, on the other hand, where the distance is 10 ninda, hence, according to this table, the combined work norm would be ;06 40 volume-šar/man-day.