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Dynamical Behavior of the Brillouin Precursor in Rocard-Powles-Debye Model Dielectrics

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Abstract: When an ultra-wideband electromagnetic pulse penetrates into a causally dispersive dielectric, the interrelated effects of phase dispersion and attenuation alter the pulse in a fundamental way that results in the appearance of precursor fields. For a Debye-type dielectric, the dynamical field evolution is dominated by the Brillouin precursor as the propagation distance exceeds a penetration depth. Because of its nonexponential peak decay, the Brillouin precursor is of central importance in ultra-wideband electromagnetics. Of equal importance is the frequency structure of the Brillouin precursor which exhibits a complicated dependence on both the material dispersion and the input pulse characteristics. A Brillouin pulse is defined and shown to possess optimal material penetration.

8.1 Introduction

The dynamic evolution of an ultra-wideband electromagnetic pulse, as it propagates through a causally dispersive dielectric, is a classical problem[1–5] in electromagnetic wave theory with considerable current interest.[6–8] For a causally dispersive medium, the frequency dependent phase and attenuation are interrelated through a Hilbert transform pair.[5] Because of this, an ultra-wideband pulse undergoes fundamental structural changes as it propagates through a causally dispersive material. Each spectral component present in the initial pulse travels through the dispersive medium with its own phase velocity so that the phasal relationship between the various spectral components of the pulse changes with the propagation distance. In addition, each monochromatic spectral component is attenuated at its own rate so that the relative amplitudes between the various spectral components of the pulse change with the propagation distance. These two interrelated effects result in a complicated dynamical evolution of the propagated field[6–8] that is accurately described by the asymptotic theory as the propagation distance exceeds a value set by the absorption depth of the material at the input pulse carrier frequency.[9] For an ultra-wideband pulse, these combined effects manifest themselves through the formation of well-defined precursor fields which asymptotically dominate the dynamical field behavior in the mature dispersion regime.[6–10]

The precursor fields are a characteristic of the material dispersion,[10] the input pulse merely providing the requisite spectral energy in the appropriate frequency domain. For the Lorentz model[11] of resonance polarization phenomena, used throughout the classical theory of dispersive pulse propagation in materials that exhibit anomalous dispersion,[1–10] both a high-frequency Sommerfeld precursor and a low-frequency Brillouin precursor are present in the propagated field structure when the input pulse is ultra-wideband. Additional precursor fields may also exist in the passband between each absorption band.[9] For both the Debye model[12] of orientational polarization phenomena and the Rocard-Powles extension[13] of the Debye model, only the Brillouin precursor field is present in the propagated field structure.[10] Because of its unique nonexponential peak decay, the Brillouin precursor has direct application to foliage and ground penetrating radar, remote sensing and wireless communications in adverse environments.

8.2 General Formulation

Plane wave electromagnetic pulse propagation in a temporally dispersive medium may be derived from the *Fourier-Laplace integral representation* of the scalar wave[8]

$$A(z,t) = \frac{1}{2\pi}\int_C \tilde{f}(\omega)\exp\left\{i\left[\tilde{k}(\omega)\Delta z - \omega t\right]\right\}d\omega, \tag{1}$$

for $\Delta z \equiv z - z_0 \geq 0$, where $f(t) = A(z_0, t)$ with temporal frequency spectrum $\tilde{f}(\omega)$. Here $A(z,t)$ represents any transverse component of the electric or magnetic field whose spectrum satisfies the *Helmholtz equation*

$$\left(\nabla^2 + \tilde{k}^2(\omega)\right)\tilde{A}(z,\omega) = 0, \tag{2}$$

with *complex wavenumber* $\tilde{k}(\omega) = (\omega/c)\, n(\omega)$, where $n(\omega) = (\varepsilon(\omega)\mu/\varepsilon_0\mu_0)^{1/2}$ is the *complex index of refraction* of the homogeneous, isotropic, locally linear medium with constant magnetic permeability μ and frequency-dependent dielectric permittivity $\varepsilon(\omega)$. For $f(t) = u(t)\sin(\omega_c t + \psi)$ with fixed carrier angular frequency $\omega_c > 0$, Eq. (1) becomes

$$A(z,t) = \frac{1}{2\pi}\Re\left\{i\exp(-i\psi)\int_C \tilde{u}(\omega - \omega_c)\exp\left\{i\left[\tilde{k}(\omega)\Delta z - \omega t\right]\right\}d\omega\right\}, \tag{3}$$

for all $z \geq z_0$, where ψ is a phase constant and $\tilde{u}(\omega)$ denotes the temporal frequency spectrum of the initial pulse envelope function $u(t)$. If the initial time behavior $A(z_0, t) = f(t)$ of the plane wave field at the plane $z = z_0$ is zero for all time $t < 0$ and if the model of the material dispersion is causal, then[6,8] the propagated field given by either Eq. (1) or (3) identically vanishes for all $t < \Delta z/c$ with $\Delta z > 0$.

8.3 Asymptotic Description in Debye-Type Dielectrics

In the asymptotic approach, the integral representation, Eq. (3), is written as[1-8]

$$A(z,t) = \frac{1}{2\pi}\Re\left\{i\exp(-i\psi)\int_C \tilde{u}(\omega-\omega_c)\exp[(\Delta z/c)\phi(\omega,\theta)]\,d\omega\right\}, \tag{4}$$

with an analogous expression for Eq. (1). Here

$$\phi(\omega,\theta) \equiv i(c/\Delta z)\left[\tilde{k}(\omega)\Delta z - \omega t\right] = i\omega[n(\omega)-\theta] \tag{5}$$

is the *complex phase function* and $\theta \equiv ct/\Delta z$ is a dimensionless *space–time parameter*. The asymptotic description of (4) for large $\Delta z > 0$ proceeds by first determining the set of saddle points of $\phi(\omega,\theta)$ for $\theta > \theta_\infty$, where $\theta_\infty \equiv \lim_{\omega\to\infty}\{n(\omega)\} \geq 1$. The condition that $\phi'(\omega,\theta) = 0$ at a saddle point yields the *saddle point equation* $n(\omega) + \omega n'(\omega) - \theta = 0$.

For a single relaxation time Rocard-Powles-Debye model dielectric,[12,13] the complex index of refraction is given by

$$n(\omega) = \left[\varepsilon_\infty + \frac{a}{(1-i\omega\tau)\left(1-i\omega\tau_f\right)}\right]^{\frac{1}{2}}, \tag{6}$$

where $a \equiv \varepsilon_s - \varepsilon_\infty$ with $\varepsilon_s \equiv \varepsilon(0)$ and $\varepsilon_\infty \equiv \lim_{\omega\to\infty}\{\varepsilon(\omega)\}$. Here τ denotes the *relaxation time* and τ_f the *frictional relaxation time* of the dielectric material, where typically $\tau > \tau_f$. Notice that the Debye model is obtained when $\tau_f = 0$. The branch points of $n(\omega)$ include the singularities $\omega_{p1} = -i/\tau_f$, $\omega_{p2} = -i/\tau$ and zeroes $\omega_{z1} = -i\left(\tau_p - c_z\right)/(2\tau_m^2)$, $\omega_{z2} = -i\left(\tau_p + c_z\right)/(2\tau_m^2)$, with $\tau_p \equiv \tau + \tau_f$, $\tau_m^2 \equiv \tau\tau_f$, and $c_z \equiv \sqrt{\tau_p^2 - 4\tau_m^2(1 + a/\varepsilon_\infty)}$. Appropriate parameter values for triply-distilled water at 25°C are given by $\varepsilon_\infty = 2.1$, $a = 74.1$, $\tau = 8.44\times10^{-12}s$, and $\tau_f = 4.62\times10^{-14}s$.

For a Debye-type dielectric, the saddle point equation yields[10] just a near saddle point solution in the low-frequency domain $|\omega| \leq |\omega_{p2}|$ about the origin. For $|\omega| \ll |\omega_{p2}|$, the complex index of refraction given in Eq. (6) may be approximated as

$$n(\omega) \approx \theta_0 - \frac{a\tau_m^2}{2\theta_0}\left[\frac{\tau_p^2(\varepsilon_\infty + 3\varepsilon_s)}{4\varepsilon_s\tau_m^2} - 1\right]\omega^2 + i\frac{a\tau_p}{2\theta_0}\omega, \tag{7}$$

where $\theta_0 \equiv n(0)$. With this substitution, the saddle point equation yields the approximate near saddle point solution

$$\omega_N(\theta) \approx i\frac{\kappa}{3\varsigma}\left[1 - \sqrt{1 + \frac{3\varsigma}{\kappa^2}(\theta-\theta_0)}\right], \tag{8}$$

for $\theta \geq \theta_\infty \approx \theta_0 - \kappa^2/3\varsigma$ with $\varsigma \equiv \left(a\tau_m^2/2\theta_0\right)\left[\tau_p^2(\varepsilon_\infty + 3\varepsilon_s)/\left(4\varepsilon_s\tau_m^2\right) - 1\right]$ and $\kappa \equiv a\tau_p/2\theta_0$. This saddle point moves down the imaginary axis as θ increases from θ_∞, crosses the origin at $\theta = \theta_0$, and then approaches the branch point $\omega_{p2} = -i/\tau$ in the limit as $\theta \to \infty$.

The integral representation in Eq. (4) may be expressed[6-8] in terms of an integral $I(z,\theta)$ with the same integrand but with a new contour of integration $P(\theta)$. By Cauchy's residue theorem, these two contour integrals are related by

$A(z,t) = I(z,\theta) - \Re\{2\pi i \Lambda(\theta)\}$, where $\Lambda(\theta)$ is the sum of the poles of the integrand in Eq. (4) that were crossed in deforming C to $P(\theta)$. This residue contribution is nonzero only if $\tilde{u}(\omega - \omega_c)$ has poles. For the asymptotic evaluation of the contour integral $I(z,\theta)$, as $\Delta z \to \infty$ in a Rocard-Powles-Debye model dielectric, the path $P(\theta)$ is taken as the Olver-type path[6,8,14] through the near saddle point $\omega_N(\theta)$. Both the nonuniform and uniform asymptotic descriptions of the propagated field may then be expressed either in the form[6–8,10]

$$A(z,t) \sim A_B(z,t) + A_c(z,t) \tag{9}$$

as $\Delta z \to \infty$, or as a superposition of expressions of the form given in Eq. (9). For example, an input rectangular envelope pulse of temporal duration $T > 0$ may be expressed as the difference between two Heaviside unit-step-function signals displaced in time by T, each propagated signal being described by the asymptotic expression given in Eq. (9), the first being referred to as the leading-edge field and the latter as the trailing-edge field. The propagated field component $A_B(z,t)$ appearing in Eq. (9) is due to the asymptotic contribution from the near saddle point $\omega_N(\theta)$ and is referred to as the Brillouin precursor.[10] The propagated field component $A_c(z,t)$ is due to the pole contributions $\Lambda(\theta)$, if there are any, and is referred to as the signal.

The asymptotic description of the Brillouin precursor in a Rocard-Powles-Debye model dielectric is obtained through a direct application of Olver's theorem[6,8,14] with the result

$$\begin{aligned} A_B(z,t) \sim \Re\Bigg\{&\exp(-i\psi)\left[\frac{c}{2\pi\Delta z\phi''(\omega_N(\theta),\theta)}\right]^{1/2} \\ &\times \tilde{u}(\omega_N(\theta)-\omega_c)\exp\left[\frac{\Delta z}{c}\phi(\omega_N(\theta),\theta)\right]\Bigg\} \end{aligned} \tag{10}$$

as $\Delta z \to \infty$ with $\theta > \theta_\infty$. This expression is uniformly valid for all finite $\theta > \theta_\infty$ provided that any pole singularities of the spectral function $\tilde{u}(\omega_N(\theta) - \omega_c)$ are sufficiently well removed from the near saddle point location.

The Brillouin precursor described by Eq. (10), which is characteristic of Debye-type dielectrics,[10] appears as a single positive pulse with peak amplitude occuring at the space–time point $\theta = \theta_0 = n(0)$, so that it propagates with the velocity $v_0 = c/\theta_0 = c/n(0)$ through the dispersive dielectric. Since $n(0) > n_r(\omega)$ for all real $\omega > 0$, the peak amplitude velocity is the minimum phase velocity for a pulse in the given dispersive Debye-type dielectric. Since $\omega_N(\theta_0) = 0$ and $\phi(\omega_N(\theta_0),\theta_0) = \phi(0,\theta_0) = 0$, Eq. (10) then shows that the propagated field amplitude at this space-time point is given by

$$A_B(z,t_0) \sim \Re\left\{\exp(-i\psi)\,\tilde{u}(-\omega_c)\left[\frac{-ic}{4\pi n'(0)\Delta z}\right]^{\frac{1}{2}}\right\} \tag{11}$$

as $\Delta z \to \infty$ with $t_0 \equiv \theta_0 \Delta z/c$, and the peak amplitude point in the Brillouin precursor only decays algebraically as $1/\sqrt{\Delta z}$.

Although the instantaneous oscillation frequency of the Brillouin precursor at the peak amplitude point is identically zero, this does not mean that the Brillouin precursor is a static field. A physically meaningful frequency measure is determined by the temporal width of the Brillouin precursor, which is determined by the e^{-1} points of the exponential function $\exp[(\Delta z/c)\phi(\omega_N(\theta),\theta)]$, given by $(\Delta z/c)\phi(\omega_N(\theta), \theta) = -1$. Since these two points occur about the peak amplitude point $\theta = \theta_0 = n(0)$ when the near saddle point crosses the origin, it is appropriate to approximate $\phi(\omega, \theta)$ about the origin by the first few terms of it's Maclaurin's series expansion as $\phi(\omega, \theta) \cong \phi(0, \theta) + \phi'(0, \theta)\omega + (1/2)\phi''(0, \theta)\omega^2$, where $\phi(0, \theta) = 0$, $\phi'(0, \theta) = i(\theta_0 - \theta)$, and $\phi''(0, \theta) = 2in'(0)$. As a first approximation, take $\phi(\omega_N(\theta), \theta) \approx i(\theta_0 - \theta)\omega_N(\theta)$. With the approximate near saddle point location given by Eq. (8), the space-time locations of the e^{-1} points are found to be given by

$$\theta_\pm \approx \theta_0 \pm \left(\frac{a(\tau + \tau_f)c}{\theta_0 \Delta z}\right)^{\frac{1}{2}}. \tag{12}$$

The *temporal width* of the Brillouin precursor is then given by

$$\Delta T_B \equiv \frac{c}{\Delta z}(\theta_+ - \theta_-) \approx 2\left(\frac{a(\tau + \tau_f)}{\theta_0 c}\Delta z\right)^{\frac{1}{2}} \tag{13}$$

as $\Delta z \to \infty$. This then corresponds to the *oscillation frequency*

$$f_B \equiv \frac{1}{2\Delta T_B} \approx \frac{1}{4}\left(\frac{\theta_0 c}{a(\tau + \tau_f)\Delta z}\right)^{\frac{1}{2}} \tag{14}$$

of the Brillouin precursor as $\Delta z \to \infty$. These two results then show that the temporal width and oscillation frequency of the Brillouin precursor are set by the material parameters independent of the input pulse for sufficiently large propagation distances. Notice that the oscillation frequency of the Brillouin precursor approaches zero as the propagation distance increases to infinity, in which limit the Brillouin precursor becomes a static field (with zero amplitude). Numerical results presented in Figure 8.1 show that for an input single cycle rectangular envelope pulse, the effective oscillation frequency f_{eff} of the propagated pulse decreases monotonically from the initial pulse carrier frequency f_c at $\Delta z = 0$ and asymptotically approaches the curve described by Eq. (14) as $\Delta z \to \infty$, the transition to the asymptotic behavior occuring when $\Delta z / z_d \sim 1$, where $z_d \equiv \alpha^{-1}(\omega_c)$ denotes the e^{-1} penetration depth at the carrier angular frequency ω_c, where $\alpha(\omega) \equiv \Im\{\tilde{k}(\omega)\}$ is the attenuation coefficient of the dispersive dielectric.

8.4 Optimal Pulse Penetration

Because the peak amplitude of the Brillouin precursor decays only as $(\Delta z)^{-1/2}$, it is then seen that an input pulse that is comprised of a pair of Brillouin precursor structures with the second precursor delayed in time and π phase shifted from the first will possess optimal penetration into the dispersive dielectric. This so-called *Brillouin pulse* is obtained from Eq. (10) with Δz set equal to $z_d \equiv \alpha^{-1}(\omega_c)$ in the exponential and the other factors not appearing in the exponential set equal to unity. The input Brillouin pulse is then given by

$$f_{BP}(t) = \exp\left[\frac{\phi(\omega_N(\theta), \theta)}{\omega_c n_i(\omega_c)}\right] - \exp\left[\frac{\phi(\omega_N(\theta_T), \theta_T)}{\omega_c n_i(\omega_c)}\right], \tag{15}$$

where $\theta_T \equiv \theta - cT/z_d$ with $T > 0$ describing the fixed time delay between

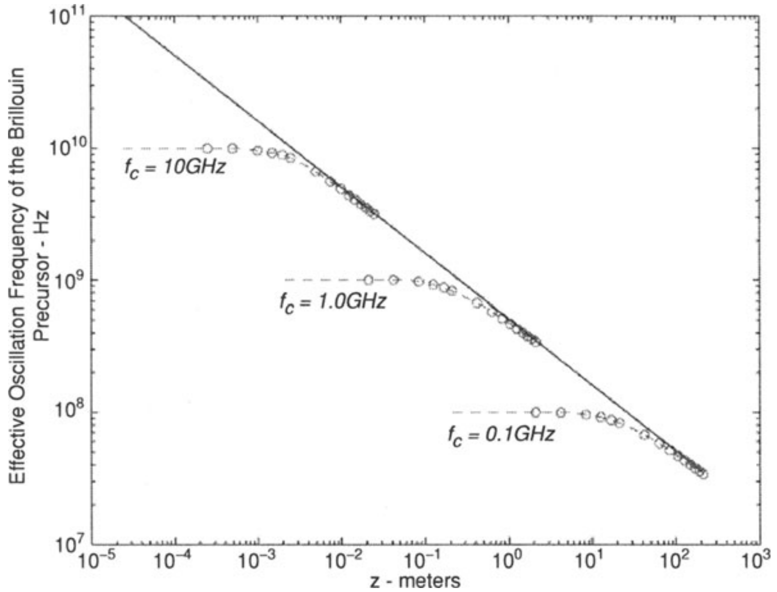

Figure 8.1 Effective oscillation frequency (in Hz) of a single cycle rectangular envelope pulse as a function of the propagation distance (in meters) in triply-distilled water for 0.1 GHz, 1.0 GHz, and 10 GHz input pulse frequencies. The solid curve describes the asymptotic behavior given by Eq. (14).

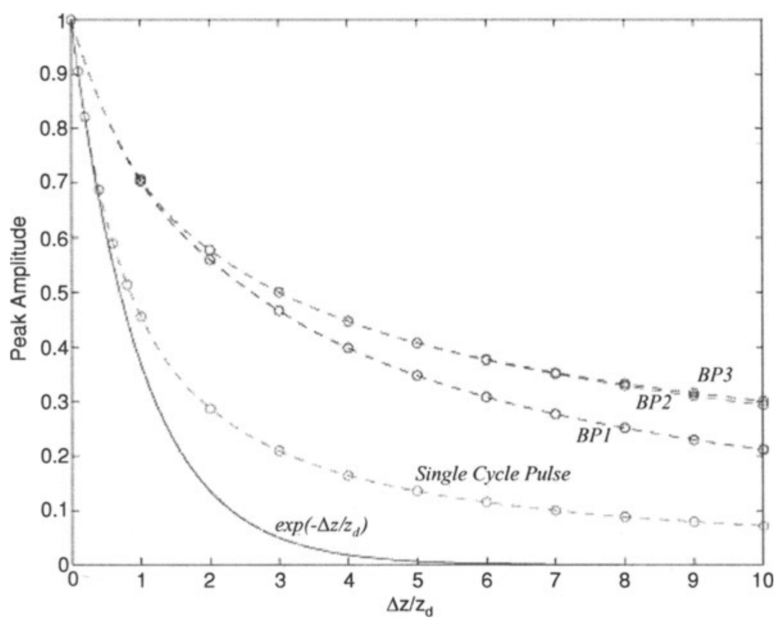

Figure 8.2 Peak amplitude as a function of the relative propagation distance in triply-distilled water for the input unit amplitude single-cycle rectangular envelope pulse and the Brillouin pulses *BP1*, *BP2*, and *BP3* with *1 GHz* carrier frequency. The solid curve describes pure exponential decay with propagation distance.

the leading and trailing edge Brillouin precursor components. If T is chosen too small, then there will be significant destructive interference between the leading and trailing edge precursors and the pulse will be rapidly extinguished. For practical reasons, $2T$ should be chosen near to the inverse of the operating frequency f_c of the antenna used to radiate this pulse.

The numerically determined peak amplitude decay with relative propagation distance $\Delta z/z_d$ is presented in Figure 8.2. The lower solid curve describes exponential attenuation as given by $\exp(-\Delta z/z_d)$, and the lower dashed curve describes the peak amplitude decay for a single cycle pulse with $f_c = 1$ GHz. Notice that the departure from pure exponential attenuation occurs when $\Delta z/z_d \approx 0.5$, as the leading and trailing edge precursors emerge from the pulse. The dashed curve *BP1* describes the peak amplitude decay for the Brillouin pulse in Eq. (15) with $T = 1/(2f_c)$, *BP2* describes that for the Brillouin pulse with $T = 1/f_c$, and *BP3* describes that for $T = 3/(2f_c)$. If the initial field is perturbed from that given in Eq. (15), the peak amplitude is decreased. Hence, by adjusting the time delay between the leading and trailing edge Brillouin precursors, optimal pulse penetration can be obtained over a given finite propagation distance.

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