Tandem Models with Blocking in the Computer Subnetworks Performance Analysis

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Abstract. A new algorithm for computing main measures of effectiveness in a special type of a tandem model with finite capacity of buffer is presented. In such a model, the finite capacity buffer is located between two multi-channel nodes, where tasks are processed. This type of model provides realistic and objective foundation for performance evaluation in the discrete flow systems such as information systems, computer networks, etc. For instant, in the tandem model with finite capacity of buffer, if the buffer is full, the blocking mechanisms restricts the arrival of any new processes and the newly generated tasks are blocked in an input node until the transmission process is resumed.

1 Introduction

In the mathematical models of discrete flow systems, which are effective and realistic tools for performance analysis of wide class systems such as transportation networks, flexible manufacturing systems, or telecommunication subnetworks, tandem models with finite capacity of buffer and blocking mechanisms are often used [1, 6, 13, 14, 22].

Over a period of years, many publications have been written related to the analysis and application of tandem models however, but there is still a great interest to the systems with limitations on the capacity of buffers under different blocking mechanisms [2, 3, 4, 8, 16, 21]. The blocking mechanism restricts the total intensity of input streams by forcing certain limitations on the blocking and synchronization mechanisms [5, 9, 20].

This paper provides a mathematical study of a special type of a tandem model with finite capacity of a buffer. In this type of model, the buffer is located beetween two multi-channel nodes (tandem configuration is shown in Fig. 1). As it is shown in Fig. 1, the tandem has an input node with N parallel service lines and the other, output node consists of c parallel service lines. In the tandem model, all these service lines in the output node have a common waiting buffer with finite capacity which is equal to m. If the buffer is full (blocking), we might experience a storage problem with the newly generated tasks. In this document, blocked tasks are located temporarily on the input node. In this scenario, if the buffer has any free space, the transmission process to the service center is immediately resumed.

In this paper, the all-possible states of the tandem have been defined, that allows the calculation of steady-state probabilities and main measures of the effectiveness. In addition, a number of algorithms are presented allowing calculation of some parameters such as blocking probability, mean response time in the output node, blocking time, the percentage of buffers filling, tandem throughput etc.

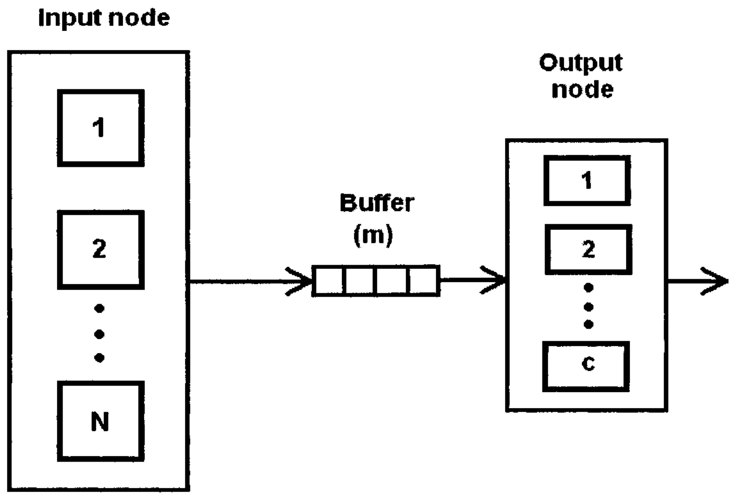

Fig. 1. Tandem configuration with blocking

2 Exact analysis of the mathematical model

Let us consider the tandem model with the finite capacity of buffer as presented in Fig. 1. The general assumptions for this model are:

- the input node has N parallel service lines,
- any input line generates tasks independently and arrival process is exponentially distributed and depicted with parameter $\lambda=1/a$ (where a is the mean inter-arrival time),
- c parallel service lines on the output node are available,
- each service time in the output node is exponentially distributed with the mean value $s=1/\mu$ (where μ is the mean service rate),
- the capacity of buffer is finite, say, of size m.

The state diagram for the presented tandem has the following structure:

$$H_0 \underset{\mu_1}{\overset{\lambda_0}{\rightleftarrows}} H_1 \ldots H_{c+m} \ldots H_{N+c+m-1} \underset{\mu_{N+c+m}}{\overset{\lambda_{N+c+m-1}}{\rightleftarrows}} H_{N+c+m}$$

For this model, the possible states are:

H_0 – idle tandem (empty buffer and output node),

H_1 – one task in process at any service line in the output node, empty buffer,

. . .

H_c – c tasks in the process, empty buffer,

H_{c+1} – c tasks in the process, one task in the buffer,

. . .

H_{c+m} – c tasks in the process, m tasks in the queue (the buffer is full),

H_{c+m+1} – c tasks in the process, m tasks in the queue (the buffer is full), a newly generated task is blocked in the input node,

H_{c+m+2} - c tasks in the process, m tasks in the queue (the buffer is full), two tasks are blocked in the input node,

...

H_{N+c+m} – c tasks in the output node, m tasks in the buffer, N tasks are blocked in the input node.

Now, we will determine the effective task rates in the tandem:

$$\lambda_0 = \lambda_1 = \ldots = \lambda_k = N \cdot \lambda \qquad \text{for} \quad 0 \le k \le c+m \tag{1}$$

$$\lambda_k = (N+c+m-k) \cdot \lambda \qquad \text{for} \quad c+m+1 \le k \le N+c+m\ .$$

and the service rates in the output node:

$$\mu_1 = \mu\,,\ \ \mu_2 = 2 \cdot \mu\,,\ \ldots,\ \mu_i = i \cdot \mu\,,\ \ldots,\ \mu_c = \ldots = \mu_{N+c+m} = c \cdot \mu \tag{2}$$

Based on queuing theory [3, 12, 14], before we evaluate the main measurements of effectiveness, we must calculate all the probabilities of states p_k $(k = 0, \ldots, N+c+m)$ in the statistical equilibrium.

The steady-state probability p_k can be interpreted as the probability of finding k tasks in the tandem at an arbitrary point of time after the process has reached statistical equilibrium.

The set of equations to get the steady-state solution for p_k [12], may be written as:

$$0 = -(\lambda_k + \mu_k)\, p_k + \mu_{k+1}\, p_{k+1} + \lambda_{k-1}\, p_{k-1} \qquad \text{for} \quad k=1, 2, 3, \ldots, N+m+c-1 \tag{3}$$

$$0 = -\lambda_0\, p_0 + \mu_1\, p_1 \qquad \text{for} \quad k=0$$

$$0 = \lambda_{N+c+m-1}\, p_{N+c+m-1} - \mu_{N+c+m}\, p_{N+c+m} \qquad \text{for} \quad k=N+c+m\,.$$

These equations may be solved recursively, by the first writing of the equivalent equation relating p_1 with p_0 :

$$p_1 = \frac{\lambda_0}{\mu_1}\, p_0\ . \tag{4}$$

similarly for p_k :

$$p_k = \frac{\lambda_0 \cdot \lambda_1 \cdot \lambda_2 \cdots \lambda_{k-1}}{\mu_1 \cdot \mu_2 \cdot \mu_3 \cdots \mu_k}\, p_0\ . \tag{5}$$

or equivalently:

$$p_k = \frac{N\lambda \cdot N\lambda \cdots N\lambda}{\mu \cdot 2\mu \cdots k\mu} \cdot p_0 = \frac{N^k}{k!} \rho^k \cdot p_0 \qquad \text{for} \quad 0 \le k \le c \tag{6}$$

$$p_k = \frac{N\lambda \cdot N\lambda \cdots N\lambda}{\mu \cdot 2\mu \cdots c\mu \cdot c\mu \cdots c\mu} \cdot p_0 = \frac{N^k \cdot c^c}{c! \cdot c^k} \rho^k \cdot p_0 \qquad \text{for} \quad c < k \le c+m$$

$$p_k = \frac{N\lambda \cdot N\lambda \cdots (N+c+m-k)\lambda}{\mu \cdot 2\mu \cdots c\mu \cdot c\mu \cdots c\mu} \cdot p_0 = \frac{N^{c+m} c^c N!}{c^k (N+c+m-k)!\, c!} \rho^k \cdot p_0$$

for $c+m<k\leq N+c+m$ and where $\rho=\dfrac{\lambda}{\mu}$.

Where p_0 is found by the commonly-used way from $\sum_{k=0}^{N+c+m} p_k = 1$, as

$$p_0 = [\sum_{k=0}^{c}\frac{N^k}{k!}\rho^k + \frac{c^c}{c!}\sum_{k=c+1}^{c+m}\frac{N^k}{c^k}\rho^k + \frac{N^{c+m}c^c N!}{c!}\sum_{k=c+m+1}^{N+c+m}\frac{\rho^k}{c^k(N+c+m-k)!}]^{-1} \; . \tag{7}$$

Now, we can derive measurements of effectiveness for the tandem network with finite capacity of buffers using steady-state probabilities given by Equation 6 in the following manner:

1. Idle tandem probability:

$$p_{idle} = [\sum_{k=0}^{c}\frac{N^k}{k!}\rho^k + \frac{c^c}{c!}\sum_{k=c+1}^{c+m}\frac{N^k}{c^k}\rho^k + \frac{N^{c+m}c^c N!}{c!}\sum_{k=c+m+1}^{N+c+m}\frac{\rho^k}{c^k(N+c+m-k)!}]^{-1} \tag{8}$$

2. Blocking probability p_{bl}:

$$p_{bl} = p_{c+m+1} + \ldots + p_{c+m+N} = p_0 \cdot \frac{N^{c+m}c^c N!}{c!}\sum_{k=c+m+1}^{N+c+m}\frac{\rho^k}{c^k(N+c+m-k)!} \; . \tag{9}$$

3. The average number of tasks in the buffer v:

$$v = 1 \cdot p_{c+1} + \ldots + (m-1) \cdot p_{c+m-1} + m \cdot p_{c+m} + m \cdot p_{c+m+1} + \ldots + m \cdot p_{c+m+N} =$$

$$= \sum_{k=c+1}^{c+m}(k-c)\cdot p_k + m\cdot\sum_{k=c+m+1}^{c+m+N}p_k =$$

$$= p_0\cdot\frac{c^c}{c!}\sum_{k=c+1}^{c+m}\frac{(k-c)\cdot N^k}{c^k}\rho^k + m\cdot p_0\cdot\frac{N^{c+m}c^c N!}{c!}\sum_{k=c+m+1}^{N+c+m}\frac{\rho^k}{c^k(N+c+m-k)!} \; . \tag{10}$$

4. The average number of blocked tasks in the input node n_{bl}:

$$n_{bl} = 1\cdot p_{c+m+1} + 2\cdot p_{c+m+2} + \ldots + N\cdot p_{N+c+m} =$$

$$= \sum_{k=c+m+1}^{c+m+N}(k-c-m)\cdot p_k = \frac{N^{c+m}c^c N!}{c!}\sum_{k=c+m+1}^{N+c+m}\frac{(k-c-m)\cdot\rho^k}{c^k(N+c+m-k)!} \; . \tag{11}$$

5. The average number of tasks in the both: the buffer and the output node n:

$$n = 1 \cdot p_1 + 2 \cdot p_2 + \ldots + (c+m-1) \cdot p_{c+m-1} + (c+m) \cdot p_{c+m} + \ldots + (c+m) \cdot p_{c+m+N} =$$

$$= \sum_{k=1}^{c+m} k\cdot p_k + (c+m)\cdot\sum_{k=c+m+1}^{N+c+m}p_k =$$

$$= p_0 \cdot [\sum_{k=1}^{c} \frac{k \cdot N^k}{k!} \rho^k + \frac{c^c}{c!} \sum_{k=c+1}^{c+m} \frac{k \cdot N^k}{c^k} \rho^k + \\ +(c+m) \cdot \frac{N^{c+m} c^c N!}{c!} \sum_{k=c+m+1}^{N+c+m} \frac{\rho^k}{c^k (N+c+m-k)!}] \quad (12)$$

6. The average number of non-blocked tasks on the input node l_1:

$$l_1 = N \cdot p_0 + \ldots + N \cdot p_{c+m} + (N-1) \cdot p_{c+m+1} + \ldots + (N-i) \cdot p_{c+m+i} + \ldots + 0 \cdot p_{c+m+N} =$$

$$= N \cdot \sum_{k=0}^{c+m} p_k + \sum_{k=c+m+1}^{N+c+m} (N+c+m-k) \cdot p_k = \quad (13)$$

$$= p_0 [\sum_{k=0}^{c} \frac{N^{k+1}}{k!} \rho^k + \frac{c^c}{c!} \sum_{k=c+1}^{c+m} \frac{N^{k+1}}{c^k} \rho^k + \frac{N^{c+m} c^c N!}{c!} \sum_{k=c+m+1}^{N+c+m} \frac{(N+c+m-k) \cdot \rho^k}{c^k (N+c+m-k)!}]$$

7. The average number of tasks on the service lines in the output node l_2:

$$l_2 = 0 \cdot p_0 + 1 \cdot p_1 + \ldots + c \cdot p_c + c \cdot p_{c+1} + \ldots + c \cdot p_{c+m+N} =$$

$$= \sum_{k=0}^{c} k \cdot p_k + c \cdot \sum_{k=c+1}^{N+c+m} p_k =$$

$$= p_0 [\sum_{k=0}^{c} \frac{k \cdot N^k}{k!} \rho^k + \frac{c^{c+1}}{c!} \sum_{k=c+1}^{c+m} \frac{N^k}{c^k} \rho^k \\ + \frac{N^{c+m} c^{c+1} N!}{c!} \sum_{k=c+m+1}^{N+c+m} \frac{\rho^k}{c^k (N+c+m-k)!}] \quad (14)$$

8. The mean rate of arrivals into the output node Λ:

$$\Lambda = N \cdot \lambda \cdot p_0 + \ldots + N \cdot \lambda \cdot p_{c+m} + (N-1) \cdot \lambda \cdot p_{c+m+1} + \ldots + (N-k) \cdot \lambda \cdot p_{c+m+k} + \\ \ldots + 0 \cdot \lambda \cdot p_{c+m+N} =$$

$$= N \cdot \lambda \cdot \sum_{k=0}^{c+m} p_k + \lambda \cdot \sum_{k=c+m+1}^{N+c+m} (N+c+m-k) \cdot p_k =$$

$$= p_0 \cdot \lambda \cdot [\sum_{k=0}^{c} \frac{N^{k+1}}{k!} \rho^k + \frac{c^c}{c!} \sum_{k=c+1}^{c+m} \frac{N^{k+1}}{c^k} \rho^k + \\ + \frac{N^{c+m} c^c N!}{c!} \sum_{k=c+m+1}^{N+c+m} \frac{(N+c+m-k) \cdot \rho^k}{c^k (N+c+m-k)!}] \quad (15)$$

9. The mean response time of the output node (waiting + service times) q:

$$q = \frac{n}{\Lambda} . \quad (16)$$

10. The mean waiting time in the buffer w :

$$w = \frac{v}{\Lambda} . \quad (17)$$

11. The mean blocking time of tasks in the input node t_{bl}:

$$t_{bl} = \frac{n_{bl}}{\Lambda} \quad . \tag{18}$$

12. The mean delay time in the tandem t_d:

$$t_d = \frac{1}{\lambda} + t_{bl} + w + \frac{1}{\mu} \quad . \tag{19}$$

13. Tandem throughput σ :

$$\sigma = \frac{N}{t_d} \quad . \tag{20}$$

3 Numerical example

In this section, the results of the tandem examination (configuration as on Fig. 1) are presented. To demonstrate the analysis of tandem with blocking, the following configuration parameters are chosen, for the input node: $N = 28$ with $a=1/\lambda$ that changes within a range from *10.0* to *80.0* time units with the step which is equal to *5* (for studying model with the different coefficient of the utility). The output node contents $c = 6$ parallel service channels, and the service time $s = 4.0$ time units. The buffer has finite capacity (size) $m = 12$.
For the above model with a finite buffer capacity, the following results were obtained, where the most part of them are presented on the Fig. 2, 3 and Tables 1, 2.

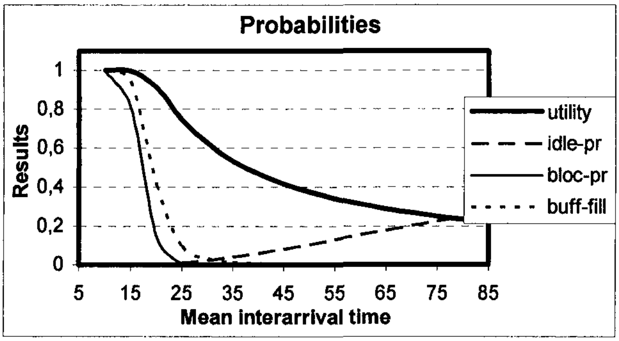

Fig. 2. The probability factors (parameters), where *utility* is the utilization factor, *idle-pr* is the probability of the idle output node, *bloc-pr* is the blocking probability of the input node and *buff-fill* is the filling co-efficient of the tandem buffer

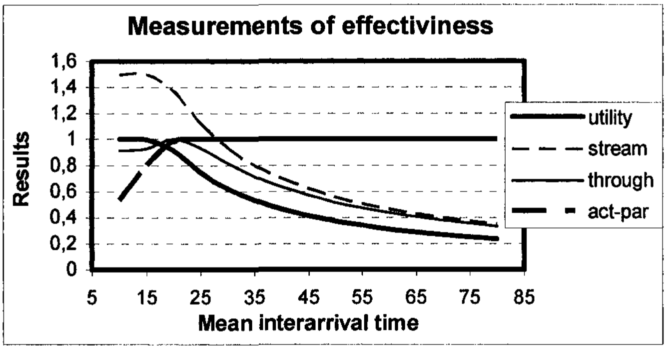

Fig. 3. The parameters related to the mean number of tasks: *utility* is the utilization factor, *stream* is the mean rate of tasks arrival into the output node, *through* is the tandem throughput co-efficient, *act-par* is the relation co-efficient (the average number of non-blocked tasks to N)

Table 1. The main measurements of the effectiveness: a comparison of the mean time parameters

a	*Mean time*				*Utility*
	q	*w*	t_{bl}	t_d	
10	12.00	8.00	8.67	30.66	1.000
15	11.57	7.57	3.70	30.27	0.998
20	7.58	3.58	0.42	28.00	0.914
25	5.04	1.04	0.02	30.06	0.746
30	4.39	0.39	0.00	34.39	0.622
35	4.18	0.18	0.00	39.18	0.533
40	4.09	0.09	0.00	44.09	0.467
45	4.05	0.05	0.00	49.05	0.415
50	4.03	0.03	0.00	54.03	0.373
55	4.02	0.02	0.00	59.02	0.339
60	4.01	0.01	0.00	64.01	0.311
65	4.01	0.01	0.00	69.01	0.287
70	4.01	0.01	0.00	74.01	0.267
75	4.00	0.00	0.00	79.00	0.249
80	4.00	0.00	0.00	84.00	0.233

Table 2. The main measurements of the effectiveness: a comparison of the mean number of tasks

a	*Mean number of tasks*					*Utility*
	n_{bl}	v	l_1	l_2	n	
10	13.00	12.00	15.00	6.00	18.00	1.000
15	5.54	11.34	22.46	5.99	17.33	0.998
20	0.57	4.91	27.43	5.49	10.40	0.914
25	0.02	1.16	27.98	4.48	5.64	0.746
30	0.00	0.37	28.00	3.73	4.10	0.622
35	0.00	0.14	28.00	3.20	3.34	0.533
40	0.00	0.07	28.00	2.80	2.87	0.467
45	0.00	0.03	28.00	2.49	2.52	0.415
50	0.00	0.02	28.00	2.24	2.26	0.373
55	0.00	0.01	28.00	2.04	2.05	0.339
60	0.00	0.01	28.00	1.86	1.87	0.311
65	0.00	0.00	28.00	1.72	1.72	0.287
70	0.00	0.00	28.00	1.60	1.60	0.267
75	0.00	0.00	28.00	1.49	1.49	0.249
80	0.00	0.00	28.00	1.40	1.40	0.233

4 Conclusions

In this paper, a new approach to the exact analysis for a special type of tandem with a finite buffer capacity is proposed. For this kind of the tandem model the exact steady-state solution is provided and studied based on the main performance measurements as functions of the tandem topology and given input parameters.

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