1 Philosophical Accounts of Vagueness, Fuzzy Poverty Measures and Multidimensionality

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1.1 Introduction

There are a number of phenomena studied by economists and other social scientists which involve vague predicates. Yet economists have not developed any explicit methodology to deal with vagueness. Vagueness is, nonetheless, addressed in the growing literature on fuzzy set theoretic poverty measures (or "fuzzy poverty measures", for short). The emergence of this literature is important not only because it has provided insight into the nature of poverty, but also because of its pioneering contribution at the methodological level. In spite of the increasing use of fuzzy poverty measures there has been limited foundational discussion of vagueness and poverty measurement. In this Chapter, I extend earlier work (Qizilbash 2003) and relate the philosophical literature on vagueness to the literature on poverty measurement. I explain why "poor" is regarded as a vague predicate and outline the various philosophical accounts - such as the epistemic view, degree theory and supervaluationism - which attempt to address vagueness. I then discuss various well-known approaches to poverty measurement in the light of these accounts. Amongst a range of issues which arise through relating philosophical accounts to poverty measurement are issues relating to multi-dimensionality. I suggest that these have not as yet been adequately addressed by those involved in applying fuzzy poverty measures.

The Chapter is structured as follows. The chief characteristics of vague predicates are described and it is argued that the predicate "poor" has all these characteristics in Sect. 1.2. In Sect. 1.3, the various philosophical accounts of vague predicates are explained and their strengths and weak-

nesses are evaluated. In Sects. 1.4 and 1.5 these accounts are related to issues in the measurement of poverty, focussing on the epistemic view and fuzzy poverty measures in Sect. 1.4 and supervaluationism and poverty measurement in Sect. 1.5. Sect. 1.6 concludes.

1.2 The Vagueness of "Poor"

A number of predicates[2] are usually classified as vague. Examples include "tall", "bald" and "nice". Furthermore, while the philosophical literature has tended to focus on the predicate vagueness, adverbs (such as "quickly") and quantifiers (such as "many") can also be vague (Keefe and Smith 1996, p 5). I shall focus in this Chapter on the predicate vagueness, since my central claim is that "poor" is a vague predicate. However, "poor" is not the only predicate relevant to poverty measurement. I shall also rely on claims about the vagueness of the predicates "extreme" and "chronic" in addressing issues relating to the measurement of extreme (or "ultra") and chronic poverty. The three well-known and inter-related characteristics of vague predicates are: (1) that they allow for borderline cases, where it is not clear whether the predicate applies or not; (2) that there is no sharp borderline between cases where the predicate does, and does not, apply; and (3) they are susceptible to a Sorites paradox.

First, consider the existence of borderline cases. In the case of the predicate "tall", there are certainly cases where one would unhesitatingly classify people as "tall" and "short". However, there may also be cases where we cannot say that someone is *definitely* tall. One might say, in a case like this, that the relevant person is "borderline" tall. Similarly there are borderline cases of "bald" and "nice". Furthermore, in our ordinary use of "tall", there is no sharp borderline between those who are, and are not, tall. That is, there is no exact height h such that anyone who is shorter than h is not tall, while everyone else is tall. Similarly there is no exact borderline in the case of "bald". There is no precise number of hairs such that if one has less than that number of hairs on top of one's head one qualifies as bald.

Finally, consider the well-known Sorites paradox or "paradox of the heap". Suppose that John is a tall man. It seems plausible that making John a tiny bit shorter will leave him tall. This suggests that whenever we make a tall man a tiny bit shorter he must still be tall. So if we repeatedly make John a tiny bit shorter, he should remain tall. Yet obviously if we make John a tiny bit shorter enough times he will be short. We are thus led to

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[2] A predicate is whatever is affirmed or denied of a subject by means of the copula (i.e. the verb "be") e.g. "mortal" in "all men are mortal".

contradiction - since John will be both tall and short. This is the paradox. Another version of this paradox focuses on a heap. Suppose we are confronted with a heap of pebbles. It seems plausible that taking a single pebble away from the heap will leave a heap of pebbles. One can apply this logic repeatedly each time a pebble is removed, so that each time a pebble is removed we should still be left with a heap of pebbles. Yet if enough pebbles are removed, one by one, all that will be left is a single pebble. One pebble does not make a heap, and so we have a contradiction.

The predicate "poor" has all three characteristics of vague predicates. Consider someone who is poor in terms of income. It seems implausible that giving this person a single penny will make her non-poor. Yet if one repeatedly gives her enough pennies, one by one, she will be rich as regards income. Similarly, there are cases where, in our ordinary usage, we might want to classify someone as "borderline poor". Furthermore, in spite of the use of exact poverty lines to separate out the poor and the non-poor in some official contexts, there does not seem to be a sharp borderline between the poor and the non-poor. So "poor" has all the standard characteristics of vague predicates. It is one among many vague predicates which are used to describe phenomena that social scientists study. It is easy to check that other predicates relevant to poverty measurement - notably the predicates "extreme" and "chronic" - also have the characteristics of vague predicates.

So far, the examples I have focussed on have typically involved only one dimension which is relevant to judging whether or not some predicate applies. In the case of "tall", height was the only relevant dimension. In the case of baldness, the only consideration I invoked was the number of hairs on the top of a person's head. In the case of income poverty, the only dimension was income measured by the number of pennies one had. Yet in the cases of some vague predicates, multiple dimensions are relevant to whether or not the predicate applies. This is the case, for example, with the predicate "nice". Suppose that Jane is extremely polite, sociable and generous, yet is also sometimes bad-tempered. One might say that Jane is not *definitely* nice but only borderline nice. The fact that there are a number of dimensions that are relevant to judging whether Jane is nice is relevant to the fact that she is classified as borderline nice.

This point about multi-dimensionality and vagueness is relevant to the predicate "poor". So far, I have focussed on income poverty. Whether a person was judged to be "poor" was just a matter of the number of pennies, or the amount of income she had. Now consider an alternative example where we allow for multiple dimensions. Suppose that a person can be poor in terms of income, health and educational achievement. Jim, it turns out, has a decent income, but cannot read and write and has a debilitating

disease. He classifies as rich in terms of income, but as poor in terms of health and education, even when one allows for vagueness in these dimensions. Or suppose that Jim has a very low income but enjoys good health and is well educated, so that he counts as being poor with regard to income but is not poor in terms of health and education (again when one allows for the vagueness of "poor" in each of these dimensions). Should we classify Jim as poor, all things considered, in these two scenarios? As long as one adopts a multi-dimensional view of poverty, it may not be obvious whether we should judge him to be poor or not poor in either case. Jim may be classified as "borderline poor" in both cases even though the vagueness in this case does *not* relate to the question of whether or not he would qualify as poor in each of the relevant dimensions. The multi-dimensionality of poverty is thus relevant to its vagueness.

Finally, it is worth noting that where a predicate is vague, it is usually argued that there is not merely vagueness about whether or not the predicate applies. If this was all that mattered, it might be that there are three sharply delineated sorts of cases: those where the predicate applies; those where it does not apply; and "indefinite" cases in between. Yet in the case of vague predicates it seems that there are no precise borderlines between these sorts of cases. For example, in the case of "tall" there is no sharp boundary between those cases which are definitely tall and those which are borderline tall. Vagueness about *this* boundary is vagueness about the limits of a rough borderline. It is a form or "vagueness about vagueness" or "higher-order" vagueness.

1.3 Three Views of Vagueness

Philosophical accounts of vagueness typically attempt to address the characteristics of vague predicates (i.e. the existence of borderline cases, rough borderlines and susceptibility to a Sorites paradox) while also allowing for "higher-order" vagueness. Epistemic views of vagueness are distinct because they suppose that even in the case of vague predicates there is actually a precise borderline between cases where the predicate does, and does not, apply. According to such views it is impossible to know where the exact borderline lies. Williamson (1992, 1994) has championed a version of this view. According to Williamson because it is impossible to know the borderline between cases where a vague predicate such as "poor" does, and does not apply, we must leave a "margin of error" in applying the predicate. Inasmuch as there is any vagueness about where the exact bor-

derline between the poor and the non-poor lies, it is just a matter of *igno-rance*.

There are a number of problems with this view of vagueness, some of which I shall list here. Firstly, this account simply denies one of the key characteristics of vague predicates which a philosophical account of vagueness needs to address: the non-existence of a precise borderline be-tween cases where the predicate does, and does not, apply. Secondly, in many cases of vagueness it simply seems implausible to suppose that where there are borderline cases, ignorance is the root of the problem. Consider the predicates "tall" and "bald", and borderline cases of "tall" and "bald". It seems very implausible that our considering some particular cases to be "borderline tall" or "borderline bald" reflects any sort of igno-rance. There seems to be no knowledge which, if we had access to it, would resolve the question of where the exact borderline is. Furthermore, if someone holding the epistemic view responds by stating that it is impos-sible to know the exact location of the borderline, that does not in itself help much. What one needs is an account of *why* it is impossible to know the exact location of this borderline. The significant advantage of the epis-temic view, for those who hold it, is that it retains classical logic. It retains the "law of excluded middle" according to which, for any predicate such as "tall", either x is, or is not, tall. The vagueness of "tall" does seem to vio-late this law, since in borderline cases of "tall" someone is neither defi-nitely tall nor definitely not tall. From an epistemic view this is not so: if there *appears* to be any vagueness about whether or not someone is tall, this is just a matter of ignorance. Epistemic views also retain two truth values - "true" and "false" - so that the "principle of bivalence" (which only allows for "true" or "false" statements) holds. Finally, the epistemic view can address the Sorites paradoxes that arise in the case of vague predicates. In the case of "tall", for example, the epistemic view would simply reject the idea that it is always true that slightly reducing the height of a tall man will leave him tall.

The two other well-known accounts of vagueness - degree theory and supervaluationism - drop classical logic. Degree theories do so by suppos-ing that there are more than two truth values. Truth, on these views, comes in degrees. Thus, in borderline cases of "tall", it is true to some degree that people are tall. There are many different forms of degree theory. One sort just adds another value - such as "indefinite" - which holds for cases which fall between those which are definitely true and those that are definitely false. Fuzzy set logic goes further and quantifies the degree of truth in bor-derline cases. Views of this sort have been illustrated by Zadeh (1965, 1975), Goguen (1969) and Machina (1976). They typically measure a de-gree of truth on the [0,1] interval, with 0 signifying definite falsehood and

1 signifying definite truth. Unlike epistemic views, degree theories allow explicitly for rough borderlines as well as for the existence of borderline cases. They can also address Sorites paradoxes. If, for example, it is taken to be *nearly* true, or true to a high degree, that each time one takes a penny from a rich person, she remains rich, then repeatedly taking a penny away from a rich person enough times may make it definitely *false* that she is rich. These are clearly strengths of degree theory.

However, degree theory also faces potential criticism. One worry relates to the very idea of a degree of truth. How is one to make sense of this? One way of doing so goes like this. Suppose we are concerned with whether or not x is F-er than y, where F is a vague predicate. Then it might be argued that it is *more true* that x is F than that y is F. Is this plausible? There are cases where it clearly is not at all plausible. Suppose that we are concerned with "tall", and that both John and Jim are very tall. As it happens John is a little taller than Jim. If the account of a degree of truth just given is correct, then it follows that it is more true that John is tall than that Jim is tall. However, since both John and Jim are very tall, this is surely not so: it is simply true that John and Jim are tall. In responding to this point, a degree theorist might argue that the intuition about degrees of truth only applies to borderline cases. Yet there seems to be no reason for which the intuition which underlies this account of degrees of truth should apply only to borderline cases. This seems to be a weakness in degree theories which make sense of degrees of truth in this way.

Some criticisms of degree theory apply specifically to those variations of it - such as fuzzy set theory - which attempt to put a numerical value on the degree of truth. Some worry that it is inappropriate to put numerical values on degrees of truth, because of considerations relating to higher-order vagueness. Assigning a precise numerical value to the degree of truth of vague statements seems inconsistent with allowing for vagueness about the degree of truth. Another related worry concerning these forms of degree theory is that they assume that there is a precise cut-off between those cases that are definitely true or false (i.e. true to degree 0 or degree 1) and those that are not. Proponents of these forms of degree theory can respond in a number of ways. They may respond by suggesting that degrees of truth of statements involving vague predicates are also true to some degree. There can be, on this response, a degree of truth about the degree of truth of a statement. Yet this response may fail to convince many because of the precision involved in assigning numerical values to degrees of truth. Alternatively, a degree theorist may suggest that there *is* a precise cut-off between cases which are definitely true and those which are not, and that higher-order vagueness is just ignorance about the exact degree of truth assigned to a statement. Here degree theorists end up taking a line which is

similar to that taken in epistemic views. Again this response might not adequately address the worry about higher-order vagueness if the nature and source of ignorance is not clarified.

Finally, Keefe (1998, 2000) has recently discussed a number of potential problems with degree theories which use numerical values to capture degrees of truth. For measurement of degrees of truth to be possible, a basic requirement is that such degrees can be ordered. For this requirement to hold, it must be the case that all sentences are comparable as regards degrees of truth. Writing "$\exists_T$" for "true to a greater or the same degree", then for all sentences p and q, it must be true that $p \exists_T q$ or $q \exists_T p$. Yet is it obvious that this is so? In the case of predicates involving more than one dimension it is not at all clear that it is. If there are borderline cases of "nice" involving people who are pleasant and unpleasant in quite different ways, then it is not clear that we can compare the degree to which it is true that they are nice in different dimensions. Similarly, it may be difficult to compare the degree of truth of "John is nice" and "the chair is red". Keefe argues that the confusion in attempts to measure degrees of truth might arise from the fact that some vague predicates - such as "tall" - allow for measurement. Yet it is a mistake to jump from the plausible thought that height can be measured to the view that the degree of truth of "Jack is tall" can also be measured, when Jack is borderline tall[3]. The jump might be plausible if the degree of truth of "x is F" is closely related to how Fx is relative to others - for example whether x is more or less F than y. Yet as we saw earlier, one might resist the claim that "x is F-er than y" implies that "it is more true that x is F than that y is F". Finally, it is also worth mentioning that further complications might arise because it may not be entirely clear which dimensions are relevant to judging whether or not some predicate (such as "nice") actually applies. These various worries about some versions of degree theory in the context of multi-dimensionality are clearly also relevant to the case of poverty. In spite of these potential problems with degree theory, fuzzy set theory is the most widely applied account of vagueness. The fact that it involves numerical values no doubt makes it attractive to economists and social scientists.

An account of vagueness which has not been widely explored by economists and social scientists is supervaluationism. Supervaluationism develops the thought that statements involving vague predicates might, or might not, be true depending on the manner in which they are made more

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[3] Smith (2003) argues that Keefe's argument here only applies to some (confused) versions of fuzzy set theory. However, as Keefe (2003) writes in response to Smith, this point does not undermine the claims she makes against degree theories, like fuzzy set theory, which use numbers to capture degrees of truth.

precise. If a statement involving a vague predicate is true in all acceptable ways in which it can be made more precise, one might say that it is "super-true". This is the central intuition running through the best known version of supervaluationism, which has been developed by Fine (1975). On Fine's account, for any vague predicate, there are a number of "admissible" ways of making statements involving the predicate more precise or "precisify-ing" it. Fine "maps" the various ways of making statements involving a vague predicate more precise in terms of a "specification space". Points in this space include "base points" where the statement is initially specified. These points are "extended" by making the statement more precise. If a statement has not been completely precisified a "partial" specification point has been reached. Once a statement has been made as precise as pos-sible a "complete" specification point has been reached. A vague statement is then "super- true" in Fine's formal sense if and only if it is true in all admissible ways of making it more precise or, equivalently, in all admissi-ble "precisifications".

This account has some attractive features. It clearly allows for border-line cases. From a supervaluationist view these are statements which are true in some, but not all, admissible precisifications. Supervaluationism also allows for rough borderlines since there are a number of admissible ways of drawing borderlines in the case of vague predicates such as "tall" and no single borderline is privileged. Furthermore, this account seems to get round the Sorites paradox. To see why, let's consider "tall". For each admissible way of making a statement such as "John is tall" completely precise there is an exact height h such that making John a tiny bit shorter than h means that he is not tall. Nonetheless, since no such exact height h is privileged, there is no h such that (it is super-true that) someone at or above this height is tall, while anyone shorter than h is not tall. So, unlike the epistemic view, supervaluationism addresses the Sorites paradox with-out giving up on the existence of rough borderlines. Finally, Fine's super-valuationism attempts to allow for higher-order vagueness by suggesting that the predicate "admissible" is vague, so that the set of admissible pre-cisifications of a statement is also vague.

Fine has been criticised because his account clearly makes a great deal of use of the notion of precision. To this degree, his approach can be seen as an attempt to address vagueness by insisting on precision. This can be seen as an inappropriate response to vagueness[4]. Finally, it is sometimes argued in defence of supervaluationism that it comes close to preserving classical logic. Firstly, supervaluationism retains the law of excluded mid-

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[4] There are also other more technical objections to supervaluationism. See Wil-liamson (1994).

dle in virtue of the fact that for any vague predicate F either x is or is not F for *all* ways of making the borderline between those objects which are or are not F as precise as possible[5]. Finally, unlike degree theory, Fine's version of supervaluationism does not require degrees of truth. This might be an attraction for some. However, some versions of supervaluationism do involve degrees of truth (Lewis 1970; Kamp 1975). Intuitively one statement might be truer than another if it is true on a larger number of admissible precisifications than the other. This point serves to remind us that the three-fold distinction between epistemic views, degree theories and supervaluationism itself has rough borderlines.

1.4 Epistemic and Fuzzy Set Theoretic Views and the Measurement of Poverty

It is important to recognise that while it is only recently that poverty researchers have explicitly begun to take on board the implications of vagueness, the issue has implicitly been addressed in some literature. Most notably in the "mainstream" literature on poverty - which does not explicitly address vagueness - it has been recognised that even if there is an exact cut-off between the poor and the non-poor there may be difficulties about establishing where this cut-off lies. In some of the literature, the problem with establishing an exact cut-off is seen as deriving from the "noisiness" of data on living standards (Ravallion 1994). In the light of such noisy data, there is an advantage in allowing for a range of poverty lines, to allow for a margin of error in making poverty judgements. Clearly, the implicit view of vagueness adopted here is an epistemic one. Furthermore, this approach has the standard problems of an epistemic approach. It seems implausible that even if we had perfect, "noiseless" data we could establish an exact, non-arbitrary, cut-off. It is important to distinguish the issue of noisy data, or ignorance which derives from other sources, from evaluative disagreement. Sometimes it is argued that people differ about where they might set the poverty line because of evaluative disagreement. Given the variety of evaluative judgements, some advocate allowing for a range of poverty lines in making poverty judgements (most notably Atkinson 1987; Foster and Shorrocks 1988). This well-known approach does not address vagueness, though it is quite possible that evaluative judgements can also

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[5] On the other hand, supervaluationism violates the principle of bivalence because according to supervaluationism it is *not* the case that all statements are true or false. In cases where statements are true on some, but not all, precisifications, they are neither true or false (Keefe and Smith 1996, p 7).

be imprecise, especially where there are multiple dimensions involved in making such judgements[6]. So vagueness may also be relevant here.

The suggestion that fuzzy set theory might be applied to the economics of poverty and inequality can be traced to Sen's writings. In his writings on economic inequality Sen (1971, p 5) recognised that the notion of inequality "that we carry in our mind is, in fact, much less precise" than that involved in most inequality measures. Sen thought that imprecision implied that inequality rankings are "incomplete" - so that there are cases where, of two states of affairs, it is neither true that one is more unequal than another, nor true that they are equally unequal. He made similar observations in the context of poverty. For example, in his *Poverty and Famines* he wrote that "while the concept of a nutritional requirement is a rather loose one, there is no reason to suppose that the concept of poverty is clear cut or sharp ... a certain amount of vagueness is implicit in both concepts" (Sen 1981, p 13).

In discussing inequality measurement, Basu (1987) argued that Sen had taken an "all or nothing" view in suggesting that we should deal with imprecision by adopting incompleteness, which allows for cases where one cannot make any judgement at all. He argued there are cases that fall between those where one can make a precise judgement and those where one could make no judgement at all: cases where one can only make an *imprecise* judgement. On this basis, Basu developed his axiomatic fuzzy set theoretic measure of inequality. Sen's writings are also supportive of the use of fuzzy set theory and measures based on it. In fact the precision of such measures is clearly an attraction for Sen. If the relevant concept is ambiguous, Sen suggests that "the demands of precise measurement call for *capturing* that ambiguity rather than replacing it with some different idea - precise in form but imprecise in representing what is to be represented" (Sen 1989, p 317). In this context, Sen suggests that fuzzy set theoretic measures and incomplete orderings have quite a bit to offer economics. It is worth noting that it is just this precision with which fuzzy set theoretic accounts of vagueness capture imprecision or ambiguity that worries those who are concerned about higher-order vagueness.

The use of fuzzy set theory unsurprisingly spread to the measurement of poverty with important early contributions from Cerioli and Zani (1990) and Cheli and Lemmi (1995). In the application of fuzzy set theory to poverty measurement, there is typically taken to be a degree to which someone

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[6] Literature on fuzzy preferences has emerged in economics to address such evaluative vagueness. See Barrett and Pattanaik (1989) for an introductory survey. There is also literature on vagueness and topics in welfare economics. On this see Broome (2004) and Qizilbash (2005a, 2005b).

is a member of the set of the poor. In the terms used above, the degree of membership captures the degree to which it is true that someone (or some household) belongs to the set of the poor. The membership function to the set of the poor is typically taken to lie on the [0,1] interval, with 0 meaning definite non-membership, 1 definite membership and numbers in between capturing the degree of membership. One key element in this context is the "membership function" which maps an individual's (or household's) performance in terms of an indicator, or in terms of a set of indicators, on to a degree of membership of the set of the poor. The first attempt at fuzzy poverty measurement - advanced by Cerioli and Zani - involved a linear membership function. The simplest version of their measure was income based, though Cerioli and Zani also developed variations on their measure which allow for the multi-dimensionality of poverty. In the simplest case, they took a level of income at or below which a person (or household) is judged to be definitely (income) poor, and one at or above which she (or it) is taken to be definitely not (income) poor. In between these levels, the degree of membership of the set of the poor increases in a linear way as income falls. In the multi-dimensional context, Cerioli and Zani suggested alternative measures, based on the same approach. In one variation, they suggested an ordinal ranking of levels of disadvantage for each dimension. In each dimension there is some level at or below which a person (or household) counts as definitely poor, and one at or above which she (or it) classifies as definitely not poor. In between these levels, the degree of membership of the set of the poor (for each dimension) depends on the person's (household's) position in the ordinal ranking. Cerioli and Zani explored various ways of weighting the dimensions of poverty in judging whether or not a person (or household) is definitely poor taking into account all the dimensions of poverty. It is worth noting that the various approaches they discuss imply that - as long as each dimension has positive weight - a person (or household) must qualify as definitely poor on all dimensions - i.e. get a score of 1 on all dimensions - to gain a score of 1 overall and to count as definitely poor overall.

In their important contribution, Cheli and Lemmi (1995) criticised the arbitrary use of two critical levels which define the range of levels of income or other indicators where there is fuzziness in Cerioli and Zani's methodology. They suggested an alternative "Totally Fuzzy and Relative" (TFR) approach. The approach works so that the cut-offs used to establish the relevant range of levels is driven by the distribution itself. The TFR approach can be applied to both income and multi-dimensional contexts. Only those who are most (least) deprived in terms of the distribution of the relevant indicator (which may be income or some indicator used in a multi-dimensional application) are definitely poor (not poor) in terms of

that indicator. Between these levels, the degree of membership of the set of the poor in terms of the relevant indicator depends on, and "mirrors" the distribution of the relevant indicator. Like Cerioli and Zani, Cheli and Lemmi suggest a multidimensional variation of their measure which involves weighting. The TFR approach has now been applied in a number of contexts, and Chiappero-Martinetti (1994, 1996, 2000) has made a number of influential applications in the Italian context.

Some issues that arise in the context of applications of these measures relate directly to points raised in the context of the degree theories of vagueness discussed earlier. Firstly, just as there were problems with providing an account of degrees of truth, there are problems with giving an intuitive interpretation of a measure of the degree of membership of the set of the poor. Secondly, the issue of comparability of degrees of truth in different dimensions also arises for multi-dimensional poverty measures. Are we right to assume that we can compare degrees of membership in diverse dimensions such as health, education and housing? If we cannot, it is perhaps best to use fuzzy measures in specific dimensions without attempting to form judgements across dimensions. Finally, again relating to the issue of dimensions, the Cerioli and Zani and TFR approaches take the dimensions of poverty as given. As we saw in the discussion of vague predicates, this may not be sensible: we may not be able to pin down precisely the range of dimensions which are relevant to poverty measurement.

1.5 Supervaluationism and the Measurement of Poverty

In an earlier paper (Qizilbash 2003) I attempted to address some of the problems that arise for fuzzy poverty measures by developing a framework which is inspired by supervaluationism. An intuitive interpretation of fuzzy poverty measures emerges in this framework. I only sketch the framework in broad terms here so as to show how it attempts to address some of the problems just noted. First of all, I follow Kit Fine in allowing for a set of admissible specifications of "poor". The set of such specifications can, of course, be vague (because of the vagueness of "admissible"). Each admissible specification involves a set of dimensions of poverty and a range of critical levels relating to each dimension. Any dimension of poverty which appears on *all* admissible specifications is termed a *core* dimension. In each dimension, someone (or some household) who falls at or below the lowest admissible critical level is judged to be *definitely* poor in that dimension. If she (it) is definitely poor on a core dimension, she (it) is *core poor*. Someone (or some household) who falls at or above the high-

est critical level is definitely not poor in that dimension. Anyone (or any household) who (that) is definitely not poor on all admissible dimensions is *non-poor*. Those who are neither core poor nor non-poor fall at the *margins of poverty*. If someone is core poor, I have suggested that it is "super-true" that he/she is poor - in Fine's terms - since he/she falls at or below the lowest critical level in a dimension which is admissible on *all* specifications of "poor". There is no ambiguity about whether or not such a person is poor, taking account of all the dimensions of poverty. So, for example, if nutrition is a core dimension and someone falls at or below the lowest admissible critical level one would classify that person as core poor without worrying about how he/she is doing on other dimensions.

In this framework, fuzzy poverty measures can be interpreted as measures of "vulnerability" in each dimension. In each dimension, there will be some who falls between the highest and lowest critical levels, and so are neither definitely poor nor definitely not poor in that dimension. These people (or households) can be seen as "vulnerable" in as much as they are poor in terms of *some* admissible critical level in the relevant dimension, and would be defined as poor if that critical level was used. Fuzzy poverty measures capture how "close" these individuals (or households) come to being definitely poor in the relevant dimension. This, intuitively, is the sense of vulnerability which is relevant to the interpretation of fuzzy poverty measures. On this interpretation, the Cerioli and Zani measure is a linear measure of vulnerability while Cheli and Lemmi provide a relative measure. However, it is worth being clear about what is meant by "vulnberability" here, given the way in which Cheli and Lemmi express the intuition behind their measure. They write that the "membership function will express the exposure of risk to poverty" (Cheli and Lemmi 1995, p 129). There is scope for confusion here because much of the discussion of "vulnerability" in economics and development studies has to do with the risk of *becoming* poor as a consequence of some event. That sense of vulnerability also clearly relates to the "exposure of risk to poverty" and focuses on the *probability* of some person (or household) falling below some (possibly exact) borderline (see, for example, Morduch 1994).

The notion of vulnerability which underlies the interpretation of fuzzy poverty measures in my framework is different. Fuzzy measures are conceived as measures on the "specification space" (in Fine's terms) in a particular dimension. So they relate to the range of precisifications of "poor" on which someone is judged to be poor in a particular dimension. As the range or proportion of precisifications on which someone classifies as poor in a particular dimension increases that person classifies as more vulner-

able[7]. In this context, anyone who is defined as poor on all but one critical level (or a very small proportion of critical levels) in some dimension might classify as "extremely vulnerable", in the sense that a tiny relaxation of the standards used for judging whether or not someone is definitely poor in that dimension will lead to that person classifying as definitely poor[8]. So if one uses Cheli and Lemmi's notion of "exposure to risk of poverty" in my framework, this must be interpreted in terms of the notion of vulnerability described here. Obviously whether or not someone counts as vulnerable in the framework *might* be related to whether or not she is vulnerable in the "standard" sense. However, the two senses of vulnerability are quite distinct. In using the term "vulnerability" to capture the intuition underlying fuzzy measures when they are interpreted within this framework, there is obviously a danger of confusion for those who use alternative notions of vulnerability. Nonetheless, the notion of vulnerability involved in the interpretation of fuzzy measures enriches the analysis of vulnerability by introducing a new conception of it. There is no reason why the most commonly used interpretation of vulnerability in economics should be the only one that is permitted. Finally, it is worth noting that the interpretation of fuzzy poverty measures within this framework is related to the interpretation of degrees of truth on those versions of supervaluationism which overlap with degree theory (Lewis 1970; Kamp 1975). In those versions, the possibility of some form of measurement on the specification space is the underlying intuition for degrees of truth. Yet one need not accept degrees of truth to accept the interpretation of fuzzy measures as measures of vulnerability.

One advantage of the framework sketched here is that it allows for two kinds of vagueness. It allows for vagueness about the critical level at or below which a person (household) classifies as poor. This is "vertical vagueness". It is the focus in the literature on fuzzy poverty measures. However, my framework also allows for vagueness about the dimensions of poverty. As we saw, Keefe (1998) raised this issue in the context of her critique of accounts of vagueness which use numerical values. In the case of a predicate like "nice", the set of dimensions which is relevant to applying the predicate is not sharply defined. This is also true of the predicate "poor".

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[7] One difference between the Cerioli and Zani and Cheli and Lemmi measures, when they are interpreted in this way, has to do with the way in which the specification space is defined. In the Cerioli and Zani measure it merely has to do with the range of critical levels, while in the TFR methodology it is driven by the distribution. On this see Qizilbash (2003).

[8] This sense of "extremely vulnerable" is used in Qizilbash (2002). It is worth noting that the vagueness of "extremely" would be relevant if one were to develop this idea much further within this framework.

In my framework such vagueness about the dimensions of poverty is "horizontal vagueness". The distinction between dimensions which are core and other admissible dimensions of poverty reflects such vagueness. Chiappero-Martinetti (2005) has cast some doubt on the notion of horizontal vagueness, suggesting that the underlying issue here may be the "complexity" of poverty, which is, in part, constituted by its multidimensionality. Chiappero-Martinetti here tries to distinguish issues relating to multi-dimensionality and vagueness in a sharp way. Yet we saw earlier that multi-dimensionality is often invoked in the context of the vagueness of some predicates. Furthermore, in some accounts of poverty, horizontal vagueness can be motivated by the use of the predicate "basic", when poverty is seen in terms of falling short of some "basic" standard. In versions of the "basic needs" approach (Streeten et al. 1981) and in Sen's capability approach - which involves the notion of "basic capability failure" (Sen 1992, 1999) - researchers need to decide on those dimensions of well-being, or those capabilities, that count as "basic". Yet it is highly plausible that "basic" is a vague predicate. Certainly there seems to be no sharp borderline between those needs or capabilities which are, and are not, "basic"[9]. So in these accounts, horizontal vagueness might relate to what is, and is not, judged to be "basic". Neither the Cerioli and Zani measure, nor the TFR methodology (nor any other poverty measure I know of) accommodates such vagueness.

There is a number of further issues about the use of fuzzy poverty measures and the framework sketched here which are worth noting. First, vertical vagueness is often confused with the depth of poverty. Indeed fuzzy measures of poverty are sometimes confused with measures of the depth of poverty. It should be clear that this is a mistake. For any measure of the depth of poverty, we need to establish some critical level relative to which one might measure how far someone who is judged to be poor falls. Measures of the depth of poverty thus usually begin with some precise poverty cut-off and "resolve" vertical vagueness in some arbitrary way. In this context, the vagueness of "extreme" is also relevant. Since whether or not a poor person's (household's) condition is judged to be extreme is the key to whether or not that person (household) is treated as "extremely poor" or "ultra poor", the adverb "extremely" in "extremely poor" is also no doubt vague. Certainly, there appears to be no exact borderline between those who are, and are not, extremely poor. So over and above any vagueness about whether or not someone (or some household) classifies as poor, there is further vagueness about whether that person (or household) quali-

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[9] Indeed, this would be true even in the absence of evaluative disagreements about what counts as a "basic" need or capability.

fies as extremely poor. The framework sketched above can easily allow for this further level of vagueness. It would do so by adding a set of admissible critical levels for someone to qualify as "extremely" poor in each dimension. If someone (some household) fell at or below the lowest of these, she (it) would be definitely extremely poor in that dimension. If she (it) was definitely extremely poor in a core dimension, one might say that she is "extremely core poor". However, the use of this term would be misleading if it were taken to imply that core poverty comes in degrees. It does not.

The same basic point holds in the case of the width of poverty. The number of dimensions on which someone (or some household) is poor is the central focus when measuring the width of poverty. Yet measuring width is quite different from capturing horizontal vagueness. Only when issues of horizontal vagueness are resolved - so that the dimensions of poverty are clearly defined - is it possible to measure the width of poverty. Again if one wanted to examine whether or not a person (household) is extremely poor - as regards the width of poverty - one would need to allow for the vagueness of "extreme".

In this context, it is also worth mentioning the amount of time someone (or some household) has been poor. In the literature on poverty measurement, the distinction is sometimes made between those who are "temporarily" (or "transitory") poor and those who are "chronically" poor. Here again there is more than one level of vagueness. On the one hand, there is vagueness about whether or not someone (some household) is poor at a point in time. This is addressed by the framework described above. Once this issue is settled, there is the further issue of whether that person's (household's) condition is "chronic". Since "chronic" is a vague predicate, fixing on any precise number of years (months or other time units) one must be poor to be counted as chronically poor in some dimension is arbitrary, and the vagueness of the predicate "chronic" needs to be taken into account. The framework outlined above can be easily extended to allow for this further level of vagueness, by allowing for a range of admissible periods of time for which a person (household) has been poor in some dimension for that person's (household's) condition to be classified as chronic in that dimension. If a person (household) qualifies as poor for all the relevant admissible time periods for some dimension and critical level, that person's (household's) condition would classify as definitely chronic in terms of the relevant dimension and critical level. If her (its) condition is definitely chronic in a core dimension for the lowest admissible critical

level, one might say that she is "chronically core poor"[10]. Again, this term would be misleading if it suggested that core poverty comes in degrees.

Finally, it is worth noting another way in which the framework described here differs from standard approaches which use fuzzy poverty measures in the context of multi-dimensionality. As we saw in the previous section, in most fuzzy set theoretic measures one has to qualify as definitely poor on all dimensions to qualify as definitely poor overall, as long as all dimensions have positive weight in arriving at the overall judgement. By contrast, in the supervaluationist approach outlined here one only needs to be definitely poor on a core dimension to be defined as core poor, so that one is poor on all admissible specifications of "poor". I think this is intuitively forceful, since one might want to classify someone who is starving as unambiguously poor irrespective of how she is doing in terms of other dimensions. Nonetheless, it is worth noting that while this holds on the framework I have developed, it is possible to develop supervaluationism differently. One might, for example, develop it so that it is only "super-true" that someone (some household) is "poor" if she (it) is poor for all admissible dimensions and critical levels. If one developed supervaluationism in this way it would be compatible with the standard form that multi-dimensional fuzzy poverty measures take. However, I would resist this version of supervaluationism. To see why, consider a case where there are just three dimensions of poverty, involving education, health and housing. If one pursued this variation of supervaluationism it would *not* be super-true that a person is poor, even if she is starving and illiterate as long as she happens to live in a high quality house. If find this both implausible and unattractive.

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[10] My articulation of this idea emerged through discussion with Clark, who was, at the time, working on extending or modifying the framework to allow for time. Clark first used the term in work in progress co-authored with Hulme (Clark and Hulme 2005). In parts of their text Clark and Hulme use it in the same sense that I am using it here. However, their analysis is distinct, and they propose a notion of "temporal" vagueness, alongside "horizontal" and "vertical" vagueness. One difference between my view and that adopted by Clark and Hulme is that they would not take someone to be unambiguously poor if she were core poor at a point in time. They only classify the chronically core poor as unambiguously poor. They would, thus, not be able to judge that a famine victim who is very seriously malnourished at a point in time is unambiguously poor. I find this highly implausible. By contrast, my view is that one must separately establish whether some person (or household) is core poor at a point in time - this would imply that there is no ambiguity about whether or not the person is poor at that moment - and whether that person's (household's) condition is definitely chronic.

1.6 Conclusions

Vagueness must be addressed by those who attempt to measure poverty because "poor" is a vague predicate. Philosophers have developed a range of different accounts of vagueness, of which degree theory is one. Fuzzy set theory is one particular form of degree theory. Some problems with fuzzy set theory - as a theory of vague predicates - arise from the precision with which it attempts to capture vagueness. Others arise from its attempt to measure degrees of truth when multiple dimensions are involved in the application of a predicate. While the precision with which fuzzy set theory attempts to capture vagueness appears to be a problem when it comes to higher-order vagueness, it is this very precision and the use of numerical values to capture degrees of truth which makes it attractive to some economists. Problems regarding multi-dimensionality arise for fuzzy set theory both as an account of vagueness and as a methodology for measuring poverty. An alternative framework which is inspired by supervaluationism can allow for vagueness about the dimensions of poverty, while also providing a more intuitive interpretation of fuzzy poverty measures. This framework can also be extended to allow for the vagueness of predicates such as "extreme" and "chronic". However, this framework addresses the multidimensionality of poverty in a way which is quite different to that implicit in some fuzzy poverty measures. This multidimensionality will need further attention in future attempts to develop fuzzy poverty measures.

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