13 Modelling Fuzzy and Multidimensional Poverty Measures in the United Kingdom with Variance Components Panel Regression

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13.1 Introduction

In this Chapter we propose a methodology to model fuzzy and multidimensional poverty measures in order to study poverty dynamics and influencing socio-demographic factors.

A large amount of literature exists which refers to i) the study of fuzzy and multidimensional poverty in a cross-sectional context, and ii) the study of non-fuzzy poverty dynamics. Not many studies have been carried out on iii) fuzzy and multidimensional measures in a longitudinal perspective.

Concerning category i) above, over the last decades many studies have paid increasing attention to the multidimensional aspects of the phenomenon of poverty and living conditions. These aspects are not taken into account in the so called traditional approach to poverty analysis which only considers monetary indicators (e.g. income or consumption expenditure); in this context the theory of fuzzy sets has been introduced by Cerioli and Zani (1990) and developed by Cheli and Lemmi (1995) in order to overcome some limitations of the traditional approach and in order to define multidimensional fuzzy poverty measures.

A large amount of literature also exists which refers to the study of non-fuzzy poverty dynamics; one of the first contributions, by Lillard and Willis (1978) concentrated on earning dynamics using variance-component models, applied to the Panel Study of Income Dynamics (PSID). More recently, Stevens (1999) has compared duration models with variance component models using an updated set of the PSID. Jenkins (2000) describes a wide range of multivariate models of income and poverty dynamics, including: *i)* longitudinal poverty pattern models, *ii)* transition probability models, *iii)* variance component models, *iv)* structural models, with an ap-

plication to the first 6 waves of the British Household Panel Survey (BHPS). Devicenti (2001) starting from Stevens methodology (1999) studies poverty dynamics in Great Britain from 1991 to 1997.

The first attempt to study poverty dynamics in a longitudinal context was due to Cheli (1995) extending the Totally Fuzzy and Relative (TFR) approach to two periods; this was later developed by Cheli and Betti (1999) and Betti, Cheli and Cambini (2004) introducing fuzzy transition matrices and dynamic indices. Betti and Verma (1999, 2004) have focused more on capturing the multi-dimensional aspects, developing the concepts of "manifest" and "latent" deprivation to reflect the intersection and union of different dimensions; this approach has been further developed and extended to the longitudinal perspective in a more general and consistent way, in Chapter 6 of the present Book. The first attempt to model fuzzy poverty dynamics by means of panel regression is the application of Betti, D'Agostino and Neri (2002) to the first 7 waves of the BHPS. This Chapter is an extension of the latter work: here our original contribution can be summarized as follows:

a) Adoption of a new definition for the membership function to the subset of poor (see Sect. 13.2), in line with the developments reported in the first part of the Book.

b) Extension of the analysis from 7 to 12 waves. Moreover since the BHPS questionnaire has been enriched from wave 6 onwards, two distinguished analyses have been performed: the first based on a small set of common qualitative (supplementary) indicators available for the whole period and the second based on a larger set of indicators present in waves from 6 to 12 only.

c) Particular attention is also paid to the model specification of the time effect; starting from a very flexible parameterisation by means of time indicator variables we look for the best specification of the time effect and then introduce socio-demographic covariates into the model.

The Chapter is organised as follows. In Sect. 13.2 two different measures for the definition of the concept of poverty are presented. The panel regression models are presented in Sect. 13.3. The empirical analysis is reported in Sect. 13.4 (cross-sectional) and in Sect. 13.5 (poverty dynamics); it is based on the data set collected by the BHPS from 1991 to 2002. Finally some concluding remarks are made in Sect. 13.6.

13.2 Fuzzy and multidimensional poverty definitions

As shown in the first part of the present Book, the adoption of a multidimensional approach leads to two main problems: the choice of the indicators and the aggregation process. Although deprivation is widely recognized as a multidimensional phenomenon, we still believe that indicators based on monetary variables have a fundamental role and therefore are worthy of special treatment. For this reason two different fuzzy measures are considered: the first one is based only on a monetary variable and here it is referred to as Fuzzy Monetary (FM); the second measure is based on several indicators relating to housing conditions, durable goods, etc... and here it is referred to as Fuzzy Supplementary (FS).

The monetary variable used for the FM method consists in the net equivalised household income y. Making use of the concepts of the fuzzy set theory, the degree of deprivation of any household *i* at any period *t* is defined as the membership function to the fuzzy set of poor:

$$\mu_{it}=FM_{it}=\left(\sum_{\gamma=i+1}^{n} w_{\gamma t}y_{\gamma t}\Big/\sum_{\gamma=2}^{n} w_{\gamma t}y_{\gamma t}\right)^{\alpha};\quad \mu_{nt}=0 \tag{13.1}$$

As proposed by Cheli and Lemmi (1995) we determine parameters α_t so that the membership function means are not merely equal to 0.5, but are equal to the proportion of poor units according to the traditional approach (the so called head count ratio H).

The FS measure is based on some supplementary variables x_{ijt} (j = 1, 2, . . ., k), such as amenities in the household, ability to afford durable goods, accommodation problems, and any other variables relevant for the multidimensional definition of deprivation. The construction process of this measure is fully described in Betti and Verma (1999). When supplementary variables are ordinal with two or more categories, for each variable j, with ordered categories from 1 (least deprived) to M (most deprived), the single poverty indicator for all households in category m is defined as follows:

$$s_{ijt}=\frac{m-1}{M-1} \tag{13.2}$$

When supplementary variables are quantitative poverty indicators they can be calculated in a way similar to formula (13.1). The aggregation process of the single indicators into the multidimensional measure is constructed as a weighted mean:

$$s_{it}=FS_{it}=\frac{\sum_{j=1} w_j s_{ijt}}{\sum_{j=1} w_j} \tag{13.3}$$

The weights w_j are determined by two statistical considerations: *i)* firstly, the weight is determined by the power of the variable to "discriminate" among individuals in the population, that is, by its dispersion, measured by its coefficient of variation; *ii)* in order to avoid redundancy, it is necessary to limit the influence of those characteristics that are highly correlated to the others. For a detailed description of the weighting procedure see Betti and Verma (1999).

13.3 Panel regression models with variance components

In most applications, data sets present multiple measurements over time on the same statistical unit. Regression model is assuming independent errors are not appropriate in this case because repeated measures can be correlated. Hence modelling an appropriate covariance structure is essential so that inferences about means are valid. Therefore, the parameters of the mean model are referred to as *fixed-effect parameters* (associated with known explanatory variables) and as *random-effect parameters* (associated to the chosen covariance structure).

In the present framework, dependent variables are FM_{it} and FS_{it} respectively the poverty indicator based on income (FM measure) and the poverty indicator based on qualitative variables (FS measure) at time t (t = 1,...,T) for each statistical unit i (i = 1,...n). The following logit transformation:

$$y_{it}^{(FM)}=\text{logit}\left(FM_{it}\right);\quad y_{it}^{(FS)}=\text{logit}\left(FS_{it}\right) \tag{13.4}$$

is performed in order to create two variables ranging between -∞ and +∞. The statistical model for both indicators is specified as:

$$y^{(.)}_{it}=\boldsymbol{\beta}'\mathbf{x}_{it}+\phi_t+u_{it}. \tag{13.5}$$

Here $\boldsymbol{\beta}$ is an unknown vector of fixed-effect parameters associated to k time-varying exogenous variables $\mathbf{x}_{it}$ on individual i, ϕ_t is a parametric or non parametric specification for the time effect and u_{it} is the error structure that takes into account correlation among repeated measures. We assume that:

$$u_{it}=\delta_i+\xi_{it}, \tag{13.6}$$

where $\delta_i \approx N\left(0,\sigma_\eta^2\right)$ and ξ_{it} follows a AR(1) structure, e.g.:

$$\xi_{it}=\rho\xi_{it-1}+\eta_{it}. \tag{13.7}$$

Here η_{it} is a purely random component assumed to be *i.i.d.* as $N(0,\sigma_\eta^2)$ and ρ is the serial correlation coefficient common to all statistical units. We also assume that δ_i and η_{it} are independent of each other and of $\mathbf{x}_{it}$ and ϕ_t (Lillard and Willis, 1978).

Note that this is a particular specification of covariance error structure that combines autoregression with variance component so as to obtain a model allowing for both heterogeneity and autocorrelation (Anderson and Hsiao, 1981; Mansour et al. 1985; Goldstein et al. 1994).

The model assumes that there are two random effects: one (δ_i) is assumed to persist through the period of observation (called, for this reason, permanent variation); and the other one (ξ_{it}) has the desired property of correlations, being larger for nearby times than far-apart times. In order to explain the amount of variation due to permanent component δ_i, the intra-class correlation coefficient can be computed as:

$$\hat{\gamma} = \hat{\sigma}_\delta^2 \Big/ \hat{\sigma}_u^2 . \tag{13.8}$$

However, in this specification the assumption concerning the initial observation plays a crucial role in interpreting the model and in devising consistent estimates. Therefore for the first response on each unit it is assumed that $y_{i0} \sim N\left(\boldsymbol{\beta}'\mathbf{x}_{i0}, \frac{\sigma_\eta^2}{1-\rho^2}\right)$. For t >1 the residual covariance structure is (Anderson and Hsiao 1982):

$$E\left(u_{it}u_{\gamma t'}\right) = \begin{cases} \sigma_\delta^2 + \dfrac{\sigma_\eta^2}{1-\rho^2} & \mathrm{i} = \gamma \quad \mathrm{t} = \mathrm{t}' \\ \sigma_\delta^2 + \rho^s \dfrac{\sigma_\eta^2}{1-\rho^2} & \mathrm{i} = \gamma \,, \left|\mathrm{t}\text{-}\mathrm{t}'\right| = \mathrm{S} > 0 \\ 0 & \mathrm{i} \neq \gamma \end{cases} . \tag{13.9}$$

13.4 Cross-sectional empirical analysis

The empirical analysis has been conducted using twelve waves of data of the British Household Panel Survey (BHPS) for the years 1991 – 2002[1].

The BHPS was designed as an annual survey of each adult (16+) member of a nationally representative sample of more than 5,000 households, making a total of approximately 10,000 individual interviews annually. The same individuals are re-interviewed in successive waves and, if they split-off from original households, all adult members of their new households are also interviewed. Children are interviewed once they reach the age of 16. Thus the sample is expected to remain broadly representative of the population of Britain as it changes through the reference period, except for the effect of immigration and panel attrition.

The derived BHPS data set we work with is the one used by Bardasi et al. (2004); this data set reports incomes defeated to January 2003 prices.

The sample used to construct the household poverty indicators (see formulas (13.1) and (13.3)) consists of those households in which all eligible adults gave a full interview in each wave. In this data set the net equivalised household income[2] is present for all individuals; missing values in the supplementary variables have been imputed using the approach adopted by Raghunathan et al. (2001). For the reference year 1991, the poverty line has been calculated as half of the mean net equivalent household income; the line results as being equal to £ 153.17 per week among the 4826 households. Table 13.1 reports the percentages of poor households in waves 1-12 according to the traditional approach (the head count ratios H_t) and the values of parameters α_t of formula (13.1) so that:

$$E[FM_{it}] = H_t \tag{13.10}$$

Therefore the head count ratios coincide with the household membership function means calculated year-by-year. These show a declining behaviour pattern from 1991 to 1998, and from 1999 to 2002, while there is a slight increase between 1998 and 1999.

Note that in order to identify the year-by-year household head count ratios H_t, the poverty line in the analysis presented here has been calculated for the first period only and is kept fixed (in real terms) for the following years.

[1] Wave number 13 is currently available but it was not so during the data analysis.

[2] This is the sum of all individual net incomes deflated by the McClements (1977) equivalence scale.

Table 13.1. Cross-sectional membership functions

Wave	1991	1992	1993	1994	1995		
$E[FM_{it}]=H_t$	0.2075	0.1690	0.1659	0.1588	0.1460		
α_t	1.9515	2.2065	2.2272	2.2580	2.3825		
$E[FS_{it}^{1-12}]$	0.2536	0.2415	0.2286	0.2136	0.2039		
N	4826	4556	4354	4378	4259		
Wave	1996	1997	1998	1999	2000	2001	2002
$E[FM_{it}]=H_t$	0.1323	0.1284	0.1190	0.1230	0.1113	0.0974	0.0921
α_t	2.4981	2.6155	2.5357	2.6182	2.7943	2.8602	2.8578
$E[FS_{it}^{1-12}]$	0.1864	0.1751	0.1636	0.15895	0.1424	0.1312	0.1215
$E[FS_{it}^{6-12}]$	0.0793	0.0712	0.0657	0.0660	0.0623	0.0575	0.0555
N	4372	4384	4328	4273	4194	4104	3969

In order to evaluate the household membership functions according to the FS measure (formula (13.3)) two sets of supplementary variables are considered. The first set ($FS_{1\text{-}12}$) consists of those variables collected in the entire time period (waves 1 to 12) and are denoted with an asterisk in Table 13.2: they refer to housing conditions and to the presence of durable goods. The exhaustive list of poverty symptoms is: house which is not owned; and lack of central heating, colour TV, video recorder, washing machine, dishwasher, home computer, CD player, microwave, car or van. The second set ($FS_{6\text{-}12}$) consists of a larger group of variables collected from wave 6 onward; the complete list is reported in Table 13.2.

It should be noted that the indicators reported in Table 13.2 are not proper poverty symptoms: sometimes, it could merely be a matter of choice whether to own a car or not (especially if someone lives in Central London); therefore it would be more informative to know whether or not someone can afford a particular good *if they wanted it*. Unfortunately, this information is not collected by the BHPS, at least in the first waves.

Let us now analyse household means of the two FS indicators reported in Table 13.1: in this case we can observe a regular decrease of the indicator $FS_{1\text{-}12}$ over twelve years, while the indicator $FS_{6\text{-}12}$ shows a slight increase between 1998 and 1999.

Table 13.2. Supplementary indicators

Variable name	Variable label	Variable name	Variable label
wHSWND	House owned	wHSPRBO	Accommodation has rot in windows, floor
wHSCANA	Keep home adequately warm	wHSPRBP	Pollution and environmental problems
wHSCANB	Pay for annual holiday	wHSPRBQ	Vandalism or crime
wHSCANC	Replace furniture	wHSGDN	Accommodation has a terrace or garden
wHSCANE	Buy new clothes	wFNCARS*	Car or van for private use
wHSCAND	Eat meat on alternate days	wHEATCH*	House has central heating
wHSCANF	Feed visitors once a month	wHCD1USE*	Colour TV
wHSKCH	Accommodation has a separate kitchen	wHCD2USE*	VCR Video recorder
wHSBTH	Accommodation has a separate bathroom	wHCD3USE*	Washing machine
wHSTLT	Accommodation has an indoor toilet	wHCD6USE*	Dish washer
wHSPRBG	Accommodation has shortage of space	wHCD7USE*	Microwave oven
wHSPRBH	Accommodation has noise from neighbours	wHCD8USE*	Home computer
wHSPRBI	Accommodation has street noise	wHCD9USE*	CD player
wHSPRBJ	Accommodation has not enough light	wHCD10USE	Satellite dish
wHSPRBK	Accommodation has lack of adequate heating	wHCD11USE	Cable TV
wHSPRBL	Accommodation has condensation problem	wHCD12USE	Telephone in accommodation
wHSPRBM	Accommodation has leaky roof	wXPHSD1	Housing payment required borrowing
wHSPRBN	Accommodation has damp walls, floors		

- The analysis conducted on waves 1-12 is based on those variables denoted by the asterisk.

13.5 Longitudinal empirical analysis

BHPS identifies the individual person, as the "unit of analysis" and establishes such a rule independently of the phenomenon being studied. However, this Chapter deals with the multidimensional aspects of poverty dynamics, and in this context there is no unanimity in the choice of the longitudinal unit; the controversy is about choosing individuals or house-

holds. Considering the unit identification problem, it seems to be reasonable to consider the individual person as a unit of the analysis: persons remain identifiable over periods, but the identification of families is complicated by marriages, divorces, births and deaths, and movement of individual family members (Trivellato, 1998).

On the other hand, it is reasonable to associate poverty dynamics to household variables rather than to individual characteristics. Particularly in the FS approach which is based on supplementary variables such as housing conditions, the presence of some durable goods, etc.; moreover in the FM approach the poverty indicator is computed considering the net equivalised household income which also depends on the size and composition of the whole household. Furthermore, the choice of the individual as the longitudinal unit with household variables generates some complicated econometric problems concerning the specification of the models, already introduced in Sect. 13.3, specifically the effect of: *i)* presence of correlation among members sharing the same household over time; *ii)* introduction of different individual effects for units having exactly the same values for the dependent variable and covariates.

For these reasons, the present analysis is based on the *longitudinal household.* In order to follow a complex unit such as the household, the definition of a set of follow-up or tracing rules becomes more and more important. These rules can be simple for individuals sharing the same household across the reference period; in the other cases, it becomes more complicated to construct rules because of longitudinal changes. There are different ways for defining the longitudinal household. In the present analysis we have chosen to follow adult individuals in the current household for each wave, even if they split-off from the original household; for a detailed discussion about the longitudinal unit of analysis see Betti, D'Agostino and Neri (2002).

The analysis refers to the unbalanced panel of longitudinal households considering different reference time periods separately: i) the whole panel, waves 1-12, in this case the definition of the FS indicator concerns only the subset of indicators common to the twelve waves; here the total sample size consists of 51936 repeated measurements. ii) For waves 6-12, the FS indicator concerns a richer set of indicators common to these final waves; here the total sample size consists of 29624 repeated measurements. For details on the variables involved in the two analyses see Table 13.2.

The models specified in (13.4) have been estimated and in each model the dependent variable consists, alternatively, of one of the two poverty indicators. In order to compare results of the parameter estimation they have been standardized.

The time indicator is specified in two different ways: a linear specification, with the reference variable TREND; and alternatively with a non parametric specification based on dummy variables (DT1 – DT12).

13.5.1 Trend estimation

Particular attention is given to the specification of time effect. For this reason we estimate the models specified in formula (13.4) without covariates. The time effect is introduced in a non parametric form in order to obtain a more flexible model, using dummy variables as time indicators in each wave. Four different models have been estimated taking into account the two data sets described in Sect. 13.5 and the two poverty indicators FM and FS. The estimated trends are plotted in Figures 13.1 and 13.2, and as expected, both measures show a decreasing level of poverty, whichever data set is used[3].

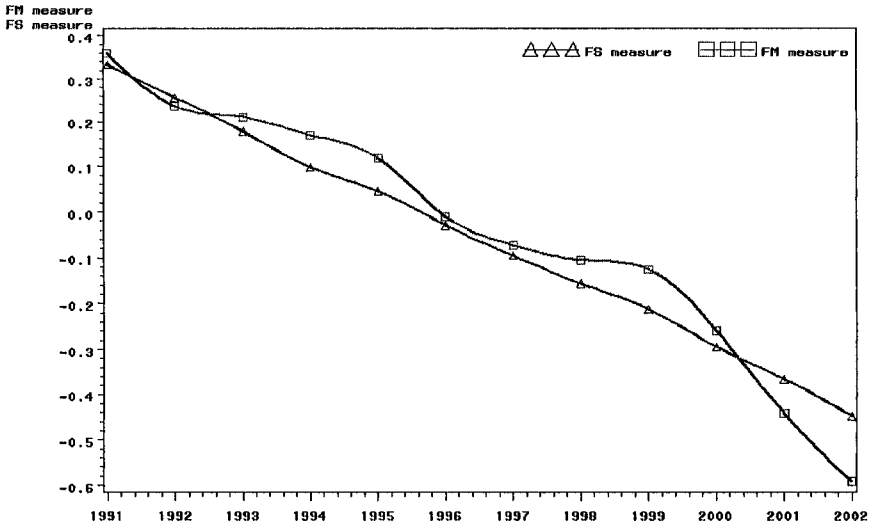

Fig. 13.1. Estimated trends in waves 1 to 12

[3] Note that this decline does not necessary imply a decreasing level of relative poverty, defined in relation to poverty lines determined independently at each wave. In this analysis, the poverty line is anchored at year 1991, though income amounts have been adjusted for price inflation. Also note that since the FM measure is defined in the same way for both the periods, obviously the shape drawn, using dataset waves 6-12 and the dataset waves 1-12 is the same between 1996 and 2002.

This generally means a higher improvement of living conditions over time. Moreover the shapes are very different and depend on the two measures and on the two data sets used. Results are very interesting as follows.

Using waves 1-12, the FS measure substantially shows a linear trend; on the contrary, the FM measure needs time indicator dummies to properly capture its modulation over time. In fact, it is quite plausible that income fluctuations make a smoother slope over time than the poverty measure based on supplementary variables that takes into account housing conditions and possession of durable goods. A test based on Akaike's Information Criterion (AIC) suggests reducing parameters estimate for the FS measure by introducing a time dependence specified as a first degree polynomial (AIC_{FM}=107747, AIC_{FS}=87986 with time indicator dummies and AIC_{FM}=108072, AIC_{FS}=87981 with linear trend specification).

Using waves 6-12, both poverty measures show a non-linear trend; the AIC criterion confirms this hypothesis (AIC_{FM}=62917, AIC_{FS}=65514 with time indicator dummies and AIC_{FM}=63079, AIC_{FS}=65576 with linear trend specification).

Differences in the shape of the trend suggest that the choice of supplementary variables can affect the estimated time dependence. In fact, variables included in waves 6-12 represent aspects of life-style and of environmental problems, which are expected to be more variable over time.

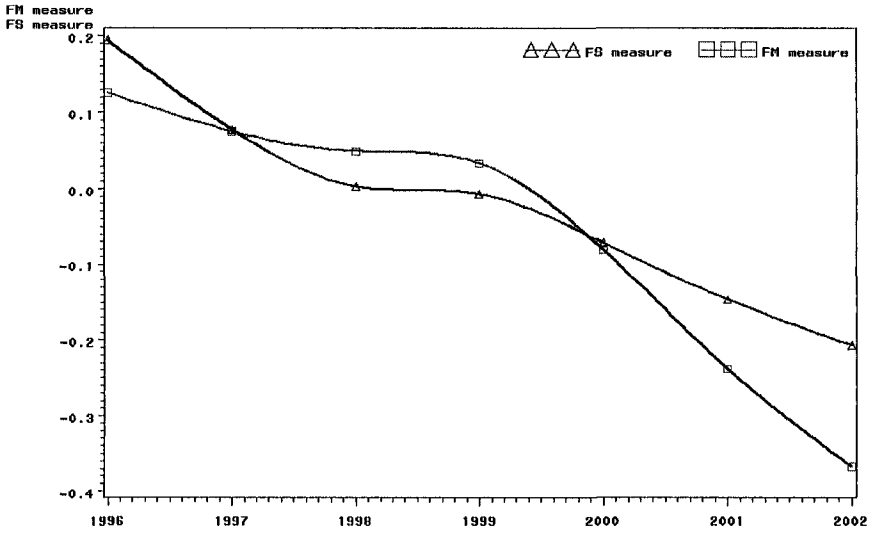

Fig. 13.2. Estimated trends in waves 6 to 12

Let us now consider the random-effect parameters and the autoregressive component. All estimates reported in Table 13.3 are significantly different from zero at least at 1% level. Results for waves 1-12 show the $\hat{\rho}$ estimate for the FS measure to be larger than the one for the FM measure; the opposite result is observed for waves 6-12. The former evidence can be explained by considering that housing conditions and possession of durable goods (which are the main items in the FS measure for waves 1-12) are much less volatile than the monetary variable. On the other hand, the latter evidence leads to reflection on the nature of supplementary variables used for computing the FS measure for waves 6-12: many of these variables (such as environmental and life style indicators) can be much more volatile than income over time, and, on the basis of statistical criteria noted in Sect. 13.2, they receive larger weights than housing conditions and possession of durable goods.

Let us consider the random-effects. We first refer to waves 1-12. The total within-year variance is 0.903 for the FM measure; the amount of variation due to the permanent component (δ_i) is about 43% of total variation (see the column referring to $\hat{\gamma}$ in Table 13.3). The remaining 57% is due to purely stochastic variation, from period to period, and to serial correlation contained in the component ξ_{it}. In the case of the FS measure, the total within-year variance is 0.846, and about 59% of it is explained by permanent variation. This last value is higher than the one computed for the FM measure and it confirms the evidence already highlighted in the case of the trend model. The effect of the purely stochastic variation ($\hat{\sigma}^2_\eta$) for the FM measure is stronger than the one for the FS measure; it seems to be reasonable given that monetary variables can be more affected by measurement errors.

Table 13.3. Random-effects estimates and autoregressive component.

		$\hat{\sigma}^2_u$	$\hat{\sigma}^2_\delta$	$\hat{\sigma}^2_\xi$	$\hat{\sigma}^2_\eta$	$\hat{\gamma}$	$\hat{\rho}$
FM	waves 1-12	0.903	0.385	0.518	0.378	0.427	0.519
	waves 6-12	0.968	0.501	0.467	0.363	0.518	0.471
FS	waves 1-12 (linear trend)	0.846	0.494	0.351	0.242	0.585	0.557
	waves 6-12	0.943	0.505	0.437	0.392	0.536	0.322

All estimates are significantly different from zero at least at 1% level; for this reason we do not report p-values.

Let us now consider the random-effects for waves 6-12. In this case differences among parameter estimates are less evident except for the autoregressive parameter, which substantially seems to capture differences between the two measures.

13.5.2 The effect of covariates

The covariates considered in the model refer to household characteristics, and all of them are time dependent. The variables referring to the household head are: the age and the age square, AGE and AGE2; a dummy variable for the gender, SEX (1 if male); two dummies for the employment status, JOBSTATUS1 (1 if self or in paid employment) and JOBSTATUS2 (1 if unemployed); four dummies for educational level, EDUC1 (1 if first degree or more), EDUC2 (1 if HND, HNC2 or Teaching qualification), EDUC3 (1 if A level), EDUC4 (1 if O level); a dummy variable for the marital status, MARSTATUS (1 if married or in common law status); household size, HSIZE and HSIZE2 (size square); finally two dummies for macro regions, REGION3 and REGION4. REGION3 refers to Regions of the South West, West Midlands, Manchester and Merseyside; REGION4 refers to a large set of regions: region of the North West, Yorkshire, region of York & Humber, Tyne & Wear, region of the North, Wales and Scotland. The remaining areas are London (inner and outer, REGION1) and the South-East, East Anglia and East Midlands (REGION2).

All models have been estimated by maximum likelihood estimation using SAS PROC MIXED (Littell et al, 1996).

For each reference period (waves 1-12 and waves 1-6) two models have been estimated; one for the FM and one for the FS measure. Maximum likelihood estimates of the parameters are reported in Tables 13.4 and 13.6. In each model the dependent variable is the poverty indicator $y_{it}^{(\cdot)}$: thus a positive sign for a significant parameter corresponds to a higher deprivation risk, or more precisely to a higher membership function to the set of poor.

Let us first consider the longer reference period (waves 1-12). With regard to the time effect, considering the empirical evidence shown in Sect. 13.5.1, we specify a linear trend for the model based on the FS measure (variable TREND in Table 13.4), and use dummy variables for the time-effect in the model for the FM measure (variables DT2 – DT12 in Table 13.4). As expected, a decreasing level for both measures may be observed: this suggests a decreasing poverty risk from 1991 to 2002.

Let us now consider the effect of covariates. Observing Table 13.4 we note that for a subset of covariates included in the analysis there are more

or less no differences between the FS and FM measures and the effects are as expected. In fact, the household head age has a quadratic effect on the degree of deprivation, with a minimum at about fifty years for the FM measure (this result is coherent with the lifecycle theory, see Modigliani, 1966), and at 55 years for the FS approach.

When compared to the reference category ("not to be in the labour force"), the poverty indicator is lower if the head of the household is employed or self-employed. According to the FM measure, the effect of the variable JOBSTATUS2 is, as expected, positive and therefore the poverty risk due to being unemployed is higher than the one for the reference category.

Table 13.4. Maximum likelihood estimates: waves 1-12

	FM measure		FS measure	
Fixed effects	Estimates	S.E.	Estimates	S.E.
Intercept	1.3284	0.043	2.9245	0.035
DT2	-0.1391	0.011		
DT3	-0.1656	0.013		
DT4	-0.2066	0.014		
DT5	-0.2516	0.014		
DT6	-0.3707	0.014		
DT7	-0.4276	0.014		
DT8	-0.4510	0.014		
DT9	-0.4715	0.015		
DT10	-0.5988	0.015		
DT11	-0.7805	0.015		
DT12	-0.9311	0.015		
TREND			-0.0724	0.001
AGE	-0.0303	0.002	-0.0769	0.001
AGE2	0.0003	0.000	0.0007	0.000
SEX	-0.0840	0.012	-0.0215	0.010
HHSIZE	0.0716	0.016	-0.4205	0.013
HHSIZE2	0.0065	0.023	0.0423	0.002
REGION3	0.1470	0.017	0.0839	0.015
REGION4	0.1463	0.016	0.0415	0.014
MARSTATUS	-0.2709	0.014	-0.1812	0.011
JOBSTATUS1	-0.3866	0.012	-0.1468	0.009
JOBSTATUS2	0.1028	0.018	-0.0287	0.013
EDUC1	-0.6498	0.020	-0.2497	0.017
EDUC2	-0.3106	0.023	-0.2791	0.019
EDUC3	-0.1891	0.017	-0.1742	0.014
EDUC4	-0.1205	0.015	-0.1403	0.012
-2log L	102515.1		76467.6	

By contrast, the effect of JOBSTATUS2 is not significantly different from zero in the case of the FS measure. The difference in the effects of JOBSTATUS2 in the two models is likely to be related to the volatility of the income in comparison with the possession of durable goods (we remind the reader that in this analysis the FS indicator is computed essentially by variables regarding housing conditions and possession of durable goods, see Table 13.2). The effect of the educational level of the household head is the same for the two measures: the degree of deprivation tends to decrease as the educational level increases. Married heads of household or in common law status make the membership function smaller than other marital statuses; such an effect is likely to be associated with the age of the head of household and/or with there being more than one wage earner in the household. According to both FM and FS measures a quadratic specification of the household size is significant. With regard to the FS measure, the membership function decreases with the increase of the household size up to five members (the minimum of the parabola). Where there are more than five members, it seems that there are not sufficient economic resources to meet the needs of the household members in terms of durable goods. On the contrary, monetary deprivation generally increases as the size of the household increases; it is reasonable to think that the increasing trend of the membership function is associated with the increasing number of children. The SEX variable is always significantly different from zero and its effect is negative; that is, households with a male household head are at an advantage. The degree of poverty is higher in Northern and Western regions than in the reference regions (Eastern regions and London).

Table 13.5. Components of variance; autocorrelated individual component models

		$\hat{\sigma}_u^2$	$\hat{\sigma}_\delta^2$	$\hat{\sigma}_\xi^2$	$\hat{\sigma}_\eta^2$	$\hat{\gamma}$	$\hat{\rho}$
FM	waves 1-12	0.687	0.232	0.455	0.355	0.337	0.470
	waves 6-12	0.714	0.298	0.416	0.344	0.417	0.416
FS	waves 1-12 (linear trend)	0.529	0.255	0.275	0.203	0.481	0.510
	waves 6-12	0.766	0.369	0.397	0.365	0.482	0.281

Let us now consider the parameters of the variance components for the analysis referring to waves 1-12, reported in Table 13.5. All the parameters are significantly different from zero. This result suggests that the effect of unobserved heterogeneity, interpreted as the effect of permanent differences among longitudinal units, plays an important role in the analysis of poverty dynamics. It is evident that parameter estimates in Table 13.5 are

smaller than the ones in Table 13.3 due to the significant effect of missing covariates. Regarding the interpretation of these parameters we can refer to the remarks discussed in Sect. 13.5.1.

Finally, considering the parameter estimates using waves 6-12, reported in Table 13.6, the effect of the covariates does not change substantially with respect to the 1-12 waves analysis; the exceptions being the time effect, already observed in Sect. 13.5.1, and the regional effect. Obviously these differences concern only the model based on the FS measure, since the model is computed in the same way for both the analyses (waves 1-12 and waves 6-12). It is reasonable that in the Western Region the membership function to the set of poor decreases since the FS measure consists of a set of supplementary variables including environmental problems as well as life style factors.

Referring to the variance components (Table 13.5) the figures are similar to the ones already commented on in Sect. 13.5.1.

Table 13.6. Maximum likelihood estimates: waves 6-12

	FM measure		FS measure	
Fixed effects	Estimates	S.E.	Estimates	S.E.
Intercept	1.2035	0.056	2.5628	0.058
DT7	-0.0467	0.011	-0.1141	0.012
DT8	-0.0638	0.013	-0.1836	0.013
DT9	-0.0804	0.014	-0.1985	0.014
DT10	-0.1890	0.014	-0.2608	0.014
DT11	-0.3437	0.014	-0.3340	0.014
DT12	-0.4729	0.015	-0.3902	0.014
AGE	-0.0321	0.002	-0.0562	0.002
AGE2	0.0003	0.000	0.0005	0.000
SEX	-0.1016	0.015	-0.1714	0.016
HHSIZE	0.0668	0.021	-0.2618	0.022
HHSIZE2	0.0085	0.003	0.0361	0.003
REGION3	0.1513	0.021	-0.0686	0.022
REGION4	0.1650	0.019	-0.0165	0.020
MARSTATUS	-0.2614	0.018	-0.1870	0.019
JOBSTATUS1	-0.4132	0.015	-0.2036	0.016
JOBSTATUS2	0.1259	0.026	0.1018	0.028
EDUC1	-0.7075	0.025	-0.2431	0.026
EDUC2	-0.3527	0.029	-0.3096	0.030
EDUC3	-0.2140	0.021	-0.2154	0.022
EDUC4	-0.1537	0.019	-0.2069	0.020
-2log L	59475.7		62533.4	

13.6 Concluding remarks

In this Chapter we have proposed a methodology to model fuzzy and multidimensional poverty measures in order to study poverty dynamics; moreover we have analysed and discussed socio-demographic factors influencing those dynamics. We have taken into account two fuzzy measures, one based on a monetary variable only (FM) and the other based on supplementary variables (FS).

As far as the comparison between the two measures (FM and FS) is concerned, interesting results suggest that the FS measure can be used to complement the picture of poverty dynamics based on income, and that the simultaneous use of the two measures can help to understand the phenomenon of deprivation better.

We have also illustrated how fuzzy measures can overcome a further limitation of the traditional approach: overestimation of the mobility of the units near the poverty line.

From a methodological point of view, we are conscious that the specified model has the restrictions such as stationarity and interdependence between unobserved heterogeneity and covariates, which in principle could be relaxed. However, the relaxation of these restrictions would greatly increase the complexity of the estimation procedure. We also suspect that the improvements in the model specification do not necessarily improve the final results.

It is also important to point out that it would be interesting to consider dummy variables for household changes as covariate in the model, since these changes could influence the poverty process.

From an empirical point of view, we have observed that the number and mainly the nature of the supplementary indicators used for constructing the FS measure can greatly influence the model estimation results. This is particularly true for the model estimated for waves 6-12 which is based on a large number of heterogeneous items. For this reason, a further development of the analysis could consist in modelling several multidimensional poverty measures based on homogenous groups of indicators (dimensions) as proposed in Chapter 6, and in Sect. 7.3 of Chapter 7.

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