Introduction

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One of the results that nowadays seems to be well-established and shared by scholars in international literature is the definition of poverty and deprivation or, more generally, of the hardships facing individuals and families in multidimensional terms. In fact it is not only theoretical socio-economic research which focuses on this aspect, following up and refining the arguments and formulations put forward by precursors of this approach (Townsend 1979; Sen 1985; Desai and Shah 1988; Bourguignon and Chakravarty 2003), but also several important public statistical agencies which welcome the above-mentioned theoretical arguments and transform them into empirical measures and analyses. The most relevant application of such a method has been conducted by Eurostat (2002) in the second Social Report in Poverty and Social Exclusion in the European Union.

Very recently (August 2005) at an International Conference organized in Brasilia[1] by the UNDP International Poverty Centre on "Many Dimensions of Poverty", the multidimensional nature of poverty was examined from a socio-economic point of view, and also analyzed in an anthropological, psychological, juridical and institutional framework.

Space and attention have also been dedicated to multidimensional poverty measurement in both developing and developed countries, where it has not only been studied theoretically, but also applied empirically. Such quantitative aspects which seem to be strongly representative of the most recent and widespread contributions in international literature all have the same problem of how to deal with the basic information which is very vast and dispersed and therefore have the need for aggregation in synthetic measures.

The two most important types of approach are as follows: i) one which is first of all represented by theoretical construction based on suitable and coherent logical reference models; ii) a second which builds on multivariate statistical methodology (discriminant analysis, factor analysis, cluster analysis, correspondence analysis) and attempts to aggregate, within rea-

[1] For draft versions of presented papers go to web site
http://www.undp-povertycentre.org/md-poverty/index.htm

sonable dimensions, the basic information dispersed in considerably extensive indicator vectors.

An approach that belongs to category i) can be derived from the mathematical theory of the fuzzy sets, proposed by Zadeh (1965) and developed by Dubois and Prade (1980). Following this approach the so called Totally Fuzzy and Relative (TFR) method has been proposed by Cheli and Lemmi (1995) starting from an idea of Cerioli and Zani (1990). Further contributions (either for implementing the robustness and for considering the dynamic of poverty) have strongly improved the initial proposal and many applications have been undertaken in several economic and political realities at international, national and regional level, for developed, developing and less developed countries.

An important characteristic of the multidimensional poverty measures derived from the fuzzy approach consists of its particular suitability for small area poverty estimation; in such a sense the European Commission (2005) has adopted the fuzzy approach in the report on "Regional Indicators to reflect social exclusion and poverty".

Moreover the Fuzzy poverty approach shows an intrinsic strong capacity when dynamic statistical information is involved; it is a matter of fact that longitudinal fuzzy poverty estimates allow us to analyze the duration and the intensity of poverty together. Finally, several Authors (Clark and Qizilbash 2002; Chiappero-Martinetti 2000; Lelli 2001) have shown the strict consistency of the fuzzy approach with the theory of the Nobel Prize Amartya Sen on the Capability approach.

It is therefore evident that the international literature on poverty analysis shows an increasing interest on the method mentioned above, but the contributions are wide spread and often not well linked to each other.

The aim of the Book is to provide the interested reader with an organic, consistent and fully comprehensive overview of the fuzzy approach. For reaching such an aim, the Book is divided into two Sections; the first devoted to the theoretical, philosophical, mathematical and statistical aspects, and the second containing further developments based on empirical analyses conducted on actual cross sectional and longitudinal data sets.

Since the fuzzy approach is very innovative with respect to the traditional socio-economic literature, an introduction to its philosophical fundaments and to its mathematical background seems to be appropriate for allowing the reader to fully comprehend the basic content of the Book.

In Chapter 1 Mozaffar Qizilbash discusses the "Philosophical Accounts of vagueness, Fuzzy Poverty Measures and Multidimensionality". The justification for using fuzzy set theory to study poverty is derived from the vague predicate nature of the phenomenon itself.

The Author considers vagueness within the framework of the most diffused philosophical accounts (epistemic view, degree theory and supervaluationism) for defining in correct and detailed terms the foundation of vagueness itself.

At first sight the degree theory appears to be the most appropriate philosophical background since fuzzy set theory is one particular form of such theory. Anyway, an alternative framework inspired by supervaluationism also provides intuitive interpretation of fuzzy poverty estimates, also in the multidimensional case. Further extensions to the longitudinal dimension of poverty allow for the vagueness of predicates such as "extreme" and "chronic" poverty.

The second Chapter of the Book deals with "The Mathematical Framework of Fuzzy Logic" and its application to economics. The Author, Bernard Fustier exposes the principal of graduality, dealing with the counter position of the fuzzy set logic to the classical logic, characterized by the strict opposition true / false, according to the notions of fuzzy proposition, fuzzy set and fuzzy number. The connectors (links) in the fuzzy logic are defined with particular reference to the triangular norms and co-norms and to the Zadeh's operators (max, min and complement to one). The fuzzy optimization according to Bellmann and Zadeh follows and the fuzzy evaluation completes the mathematics of fuzzy set.

Satya R. Chakravarty is the Author of Chapter 3 of the Book on "An Axiomatic Approach to Multidimensional Poverty Measurement via Fuzzy Sets". Since the theory of fuzzy sets enables one to talk of imprecisions in a meaningful way, a proposal to extend multidimensional poverty measurement to a fuzzy environment is attempted. Suitable fuzzy analogues are proposed for various standard index properties. Possible fuzzy indices associated with some multidimensional poverty indices are also proposed.

In Chapter 4 Ehud Menirav examines the "Convergence of Various Unidimensional Approaches", referring to the experience of Israel in 1997. The Author performs a sensitivity analysis based on a comparison of 48 distributions derived from the *Household Expenditures Survey*; those distributions vary according to most relevant analytic elements of the unidimensional monetary approach (well-being variable, equivalence scale, weighting system). The most relevant conclusion is that poverty measures are very sensitive to the choices described above. Therefore the unidimensional monetary approach appears to be inadequate in correctly interpreting the complexity of the poverty phenomenon.

Enrica Chiappero Martinetti in Chapter 5 on "Capability approach and fuzzy set theory: description, aggregation and inference issues", underlines the already mentioned attitude of the fuzzy approach for representing the graduality rather than the dualisms in defining poverty. In particular, this is

in line with Amartya Sen's capability approach. It goes beyond a merely multidimensional reference framework with a gradual and non dichotomous interpretation of the complex phenomenon of poverty.

The most updated and complete version of the so called Totally Fuzzy and Relative (TFR) approach to the measurement of poverty is presented in Chapter 6 by Gianni Betti, Bruno Cheli, Achille Lemmi and Vijay Verma on "Multidimensional and longitudinal poverty: an integrated fuzzy approach". Two are the important aspects developed by the Authors: (i) the choice of membership functions i.e. quantitative specification of individuals' or households' degrees of poverty and deprivation, given the level and distribution of income and other aspects of living conditions of the population; and (ii) the choice of rules for the manipulation of the resulting fuzzy sets, rules defining their complements, intersection, union and aggregation. In relation to (i), the Authors propose an "Integrated Fuzzy and Relative" approach, showing a relationship with the Lorenz curve and the Gini coefficient.

The second Section of the Book mainly deals with applications of multidimensional fuzzy poverty analysis. The Countries examined are France, Israel, Romania, Switzerland, Poland and the United Kingdom, which constitute a sufficiently different and wide range of case studies; the examined time periods are quite recent and the underlying methodologies are either derived from the TFR approach or based on some other original contribution derived from the Fuzzy Sets theory.

Four Chapters of the second part of the Book refer to empirical studies based on cross sectional data sets. Three Chapters are devoted to the longitudinal aspects of multidimensional fuzzy poverty analysis. In each of these Chapters original contributions are present.

Valérie Berenger and Franck Celestini in Chapter 7 on "French Poverty Measures using Fuzzy Set Approaches", perform a sensitivity analysis according to the number of different empirical variables for the robustness of the fuzzy poverty index. On the basis of such analysis they propose a new method (based on the TFR approach) in order to deal with the possibility of extracting a law from multidimensional poverty scores analogous to the power law identified by Pareto from income data.

Joseph Deutsch and Jacques Silber in Chapter 8 present "The "Fuzzy Set" Approach to Multidimensional Poverty Analysis: Using the Shapley Decomposition to Analyze the Determinants of Poverty in Israel". The Chapter compares three "fuzzy set approaches" to multidimensional poverty measurement, the Totally Fuzzy Absolute (TFA), the TFR and the Vero and Werquin approaches on the basis of the 1995 Israeli Census. First various cross-tables are given to show the impact of different factors on poverty. Then logit regressions are used to determine, *ceteris paribus*,

the exact impact of these factors. Finally results based on a Shapley decomposition are presented to find out which of these determinants are really important.

Chapter 9 on "Multidimensional fuzzy set approach poverty estimates in Romania" by Maria Molnar, Filofteia Panduru, Andreea Vasile and Viorica Duma, consists of an empirical poverty estimation comparing two official unidimensional approaches with to a multidimensional fuzzy approach.

The Authors show how multidimensional analysis of poverty allows a more shading estimation of poverty degree, and outlines the core poverty profile, characterized by more symptoms and dimensions.

Analogous results are obtained by David Miceli in Chapter 10 on "Multidimensional and Fuzzy Poverty in Switzerland". In fact, using a multidimensional fuzzy approach, not only helps in giving a more complete picture of living conditions, but also provides a more accurate picture of poverty which is as near as possible to what may be perceived by simply observing reality.

In Chapter 11 Josiane Vero presents "A comparison of poverty according to primary goods, capabilities and outcomes. Evidence from French school leaver's surveys". In defining the multidimensional framework of poverty the Author considers three different ethical styles with particular reference to: i) social primary goods according to the Rawles' theory of justice; ii) social outcomes following Fleurbaey's definition; and iii) basic capabilities according to the theory of Sen. The three approaches have been tested on the CEREQ French data set; the most important conclusion is that different definitions of poverty identify different poverty profiles or segments of the population.

Tomasz Panek has presented "Multidimensional Fuzzy Relative Poverty Dynamic Measures in Poland" in Chapter 12. The estimation of poverty dynamics via the TFR method, is experimented on a Polish panel survey conducted from 1996 to 1999, during the transition of the Country to the market economy. Such a contribution gives an account of the weakness of the traditional approach, compared with the fuzzy approach, when several movements from poverty and non poverty (and vice-versa) occur.

Finally in Chapter 13 Gianni Betti, Antonella D'Agostino and Laura Neri present "Modelling fuzzy and multidimensional poverty measures in the United Kingdom with variance components panel regression". This last contribution of the Book deals with a panel regression approach for measuring the degree of poverty and living condition; this model is estimated considering as response variables, two fuzzy poverty measures, one based on the monetary indicator, and the other one based on multidimensional indicators. The empirical analysis is conducted on the basis of the waves 1-

12 of the British Household Panel Study, one of the most authoritative and complete longitudinal survey in Europe.

Referring to a non conventional and innovative use of regression models with variance components and TFR poverty estimates, the Authors show how those fuzzy measures can overcome a typical limitation of the traditional unidimensional approach: overestimation of individual mobility near the poverty line.

The topics presented here probably do not completely exhaust the complexity of the arguments contained in the fuzzy approach to the measurement of poverty. The Book should however allow the reader to become more familiar with additional, if not alternative, methods for analyzing poverty as well as living conditions in a correct and systematic way.

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