# 1 Introduction

## 1.1 What is voltage stability?

Recently IEEE/CIGRE task force [1] proposed various definitions related to power system stability including voltage stability. Fig.1.1 summarizes these definitions.

In general terms, voltage stability is defined as the ability of a power system to maintain steady voltages at all the buses in the system after being subjected to a disturbance from a given initial operating condition. It depends on the ability to maintain/restore equilibrium between load demand and load supply from the power system. Instability that may result appears in the form of a progressive fall or rise of voltages of some buses. A possible outcome of voltage instability is loss of load in an area, or tripping of transmission lines and the other elements by their protection leading to cascading outages that in turn may lead to loss of synchronism of some generators.

This task force further classified the voltage stability into four categories: large disturbance voltage stability, small disturbance voltage stability, short-term voltage stability and long-term voltage stability. A short summary of these classifications is given below.

**Large-disturbance voltage stability** refers to the system's ability to maintain steady voltages following large disturbances such as system faults, loss of generation, or circuit contingencies. This ability is determined by the system and load characteristics, and the interactions of both continuous and discrete controls and protections. The study period of interest may extend from a few seconds to tens of minutes.

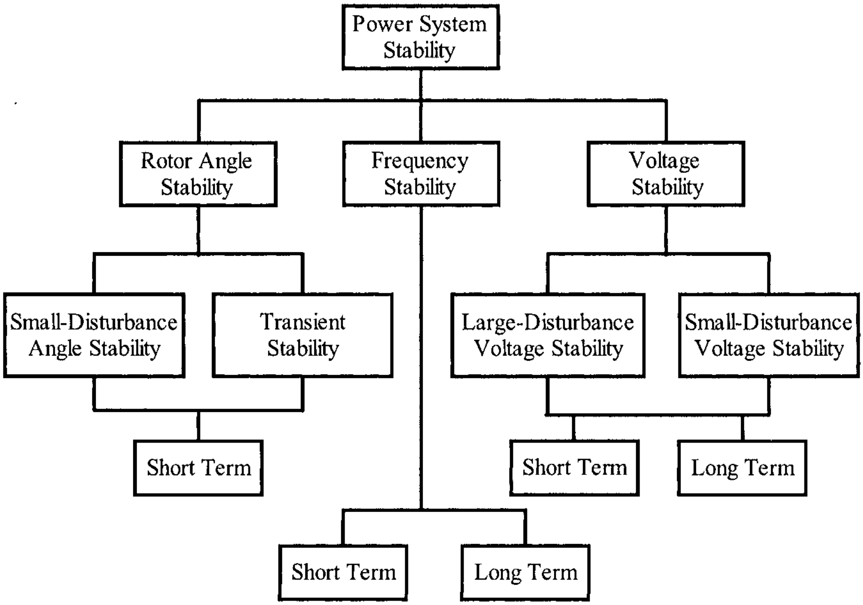


**Fig.1.1** Classification of power System stability [1]

**Small-disturbance voltage stability** refers to the system's ability to maintain steady voltages when subjected to small perturbations such as incremental changes in system load. This form of stability is influenced by the characteristics of loads, continuous controls, and discrete controls at a given instant of time.

**Short-term voltage stability** involves dynamics of fast acting load components such as induction motors, electronically controlled loads and HVDC converters. The study period of interest is in the order of several seconds, and analysis requires solution of appropriate system differential equations.

**Long-term voltage stability** involves slower acting equipment such as tap-changing transformers, thermostatically controlled loads and generator current limiters. The study period of interest may extend to several or many minutes, and long-term simulations are required for analysis of system dynamic performance. Instability is due to the loss of long-term equilibrium, post-disturbance steady-state operating point being small-disturbance unstable, or a lack of attraction towards the stable post-disturbance equilibrium. The disturbance could also be a sustained load buildup.

Voltage instability may be caused by various system aspects. Generators, transmission lines and loads are among the most important components.

Generators play an important role for providing enough reactive power support for the power systems. The maximum generator reactive power output is limited by field current limit and armature current limit. Even though reactive power plays an important role in voltage stability, the instability can involve a strong coupling between active and reactive power. When generator reactive capability is constrained by field current limit, the reactive output becomes voltage dependent. The maximum load power is severely reduced when the field current of the local generator becomes limited. Generator limits may also cause limit-induced bifurcation when voltage collapses occur right after the generator limits are reached [2].

Transmission networks are other important constraints for voltage stability. The maximum deliverable power is limited by the transmission network eventually. Power beyond the transmission capacity determined by thermal or stability considerations cannot be delivered.

The third major factor that influences voltage instability is system loads. There are several individual load models due to variety of load devices. Static load models and dynamic load models are two main categories for load modeling. Constant power, constant current and constant impedance load models are representatives of static load models; while dynamic load models are usually represented by differential equations [3]. The common static load models include polynomial or constant impedance, constant current or constant power known as ZIP models. Induction motor is a typical dynamic load model. In real power systems, loads are aggregates of many different devices and thus parameters of load models may be the composite among individual load parameters. Another important load aspect is the Load Tap Changing (LTC) transformer which is one of the key mechanisms in load restoration. During the load recovery process, LTC tends to maintain constant voltage level at the low voltage end. Therefore, load behavior observed at high voltage level is close to constant power which may exacerbate voltage instability.

## 1.2 Voltage Collapse Incidents

Carson Taylor [4] in his book reported voltage collapse incidents up to the year 1987 (Table 1). Since then there have been additional incidents that are related to voltage collapse. On July 2nd 1996 the western region (WECC) of the United States experienced voltage collapse. The details of this incident are given in the reference [5]. During May, 1997 the Chilean power system experienced blackout due to voltage collapse that resulted in a loss of 80% of its load. The Chilean power system is mainly radial with prevalent power flows in south north direction. The system configuration is ideal for voltage stability related problems [6]. On July 12, 2004 Athens experienced a voltage collapse that resulted in the blackout of the entire Athens and Peloponnese peninsula [7]. The Hellenic system comprises of generation facilities in the North and West of Greece with most of its load concentrated near the Athens metropolitan region. This system has been prone to voltage stability problems due to the large electrical distance between the generation in the north and load in the Athens region [7]. Greece was then preparing for the Olympic Games that were to be held in Athens. A lot of upgrades and maintenance was scheduled for the system. Unfortunately, most of the planned upgrades were not in place when the event happened. The details of the event and the study are reported by Vournas in the reference [8].

**Table 1.1 Voltage Collapse incidents [4]**

| Date | Location | Time Frame |
|---|---|---|
| 11/30/86 | SE Brazil, Paraguay | 2 Seconds |
| 5/17/85 | South Florida | 4 Seconds |
| 8/22/87 | Western Tennessee | 10 Seconds |
| 12/27/83 | Sweden | 50 Seconds |
| 9/22/77 | Jacksonville, Florida | Few minutes |
| 9/02/82 | Florida | 1-3 Minutes |
| 11/26/82 | Florida | 1-3 Minutes |
| 12/28/82 | Florida | 1-3 Minutes |
| 12/30/82 | Florida | 2 Mintues |
| 12/09/65 | Brittany, France | ? |
| 11/20/76 | Brittany, France | ? |
| 8/04/82 | Belgium | 4.5 Minutes |
| 1/12/87 | Western France | 4-6 Minutes |
| 7/23/87 | Tokyo | 20 Minutes |
| 12/19/78 | France | 26 Minutes |
| 8/22/70 | Japan | 30 Minutes |
| 12/01/87 | France | ? |

There is an extensive literature available covering various aspects of voltage stability. There are excellent text books [4, 9-10], Bulk Power System Voltage Security workshop proceedings [11-16] and IEEE working group publications [17-20] that provide wealth of information that is related to voltage stability. Bibliography related to voltage stability up to 1997 is published in reference [21]. A web based voltage stability search engine is maintained at Iowa state university [22].

The next section provides basic concepts that relate to maximum power transfer through a simple two bus example.

## 1.3 Two Bus Example

Consider a generator connected to a load bus through a lossless - transmission line as show in Fig.1.2. If both voltages (E and V) are kept constant then the maximum power transfer occurs at an angle (θ) of $90^0$. The relation between θ and the power transfer (P) through the transmission line is shown in Fig.1.3.

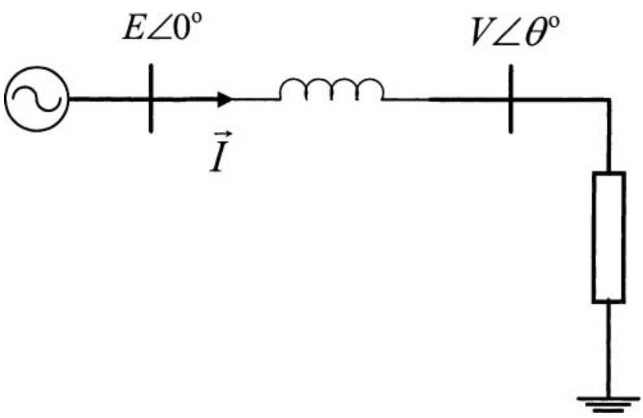


**Fig.1.2** A simple two bus system

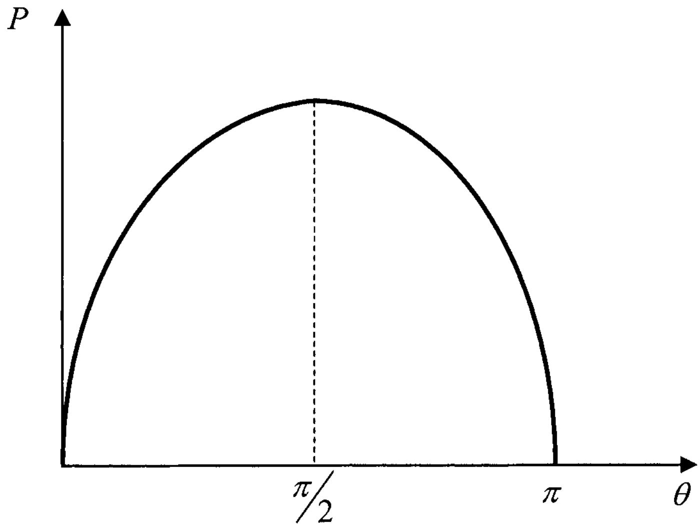


**Fig.1.3** The relationship between P and θ

Now consider the same generator with constant terminal voltage being connected to a load bus whose voltage is no longer constant. Then the relation between the load bus voltage and the power transfer through the transmission line is shown in Fig.1.4.

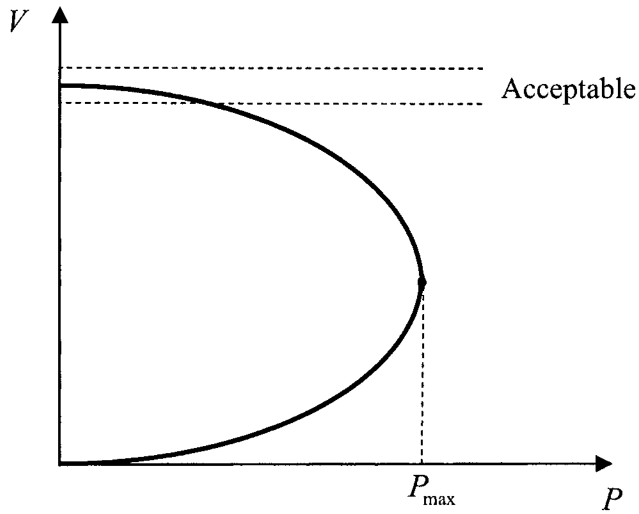


**Fig.1.4** Variation load bus voltage with P

With increase in load the voltage at the load bus decreases and reaches a critical value that corresponds to the maximum power transfer. In general this maximum power transfer is related to voltage instability if the load is constant power type. Beyond this point there is no equilibrium. However if the load is other than constant power then the system can operate below this critical voltage, but draws higher current for the same amount of power transfer.

### 1.3.1 Derivation for critical voltage and critical power

For this simple example, a closed form solution both for the critical voltage and corresponding maximum power can be derived. From Fig.1.2

$$\vec{V} = \vec{E} - jX\vec{I}$$

$$S = P + jQ = \vec{V}\vec{I}^* = \vec{V}\left(\frac{\vec{E}-\vec{V}}{jX}\right)^* = \vec{V}\left(\frac{\vec{E}^* - \vec{V}^*}{-jX}\right)$$

$$= V\angle\theta \frac{(E\angle 0 - V\angle -\theta)}{-jX} = -\frac{EV}{X}\sin\theta + j(\frac{EV}{X}\cos\theta - \frac{V^2}{X})$$

Separating real and imaginary parts

$$P = -\frac{VE}{X}\sin\theta \tag{1.1}$$

$$Q = \frac{EV}{X}\cos\theta - \frac{V^2}{X} \tag{1.2}$$

From Eqs.1.1 and 1.2

$$\sin\theta = -\frac{PX}{EV} \tag{1.3}$$

$$\cos\theta = \left(\frac{QX + V^2}{EV}\right)^2 \tag{1.4}$$

We know: $\sin^2\theta + \cos^2\theta = 1$ (1.5)

Use Eq.1.5 to combine Eqs.1.3 and 1.4 into

$$\left(\frac{-PX}{EV}\right)^2+\left(\frac{QX+V^2}{EV}\right)^2=1$$

The above expression can be put into the following form

$$\frac{V^4}{E^4}+\frac{V^2}{E^4}(2QX-E^2)+\frac{X^2}{E^4}(P^2+Q^2)=0 \tag{1.6}$$

Let $v=\frac{V}{E}$, $p=\frac{PX}{E_2}$, and $q=\frac{QX}{E^2}$, then Eq.1.6 becomes (where $v$, $p$ and $q$ are normalized quantities.)

$$v^4+v^2(2q-1)+p^2+q^2=0 \tag{1.7}$$

Let $\phi$ be the power factor angle of the load, substitute $q=p\tan\phi$ in Eq.1.7 and simplify

$$v^4+v^2(2p\tan\phi-1)+p^2\sec^2\phi=0 \tag{1.8}$$

Eq.1.8 is quadratic equation in $v^2$, where

$$v^2=-(2p\tan\phi-1)\pm\frac{\sqrt{(2p\tan\phi-1)^2-4p^2\sec^2\phi}}{2} \tag{1.9}$$

$v$ has four solutions out of which two are physically meaningful. These two physical solutions correspond to high voltage and low voltage solution as shown in Fig.1.5. For example from Eq.1.9, at $p=0$, $v=0$ or 1.

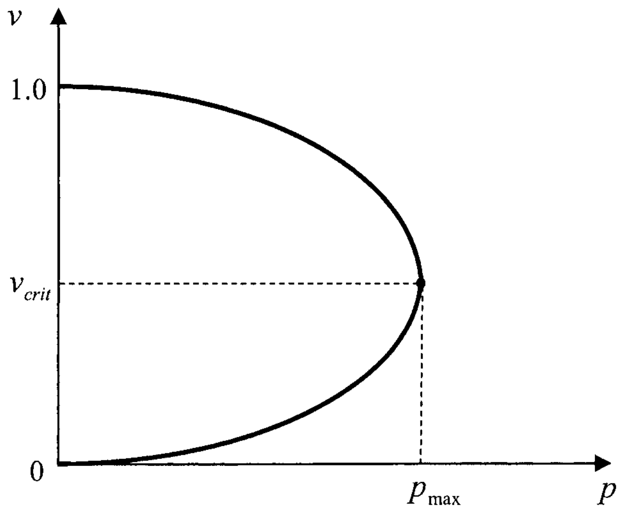


**Fig. 1.5** Variation of v with respect to p

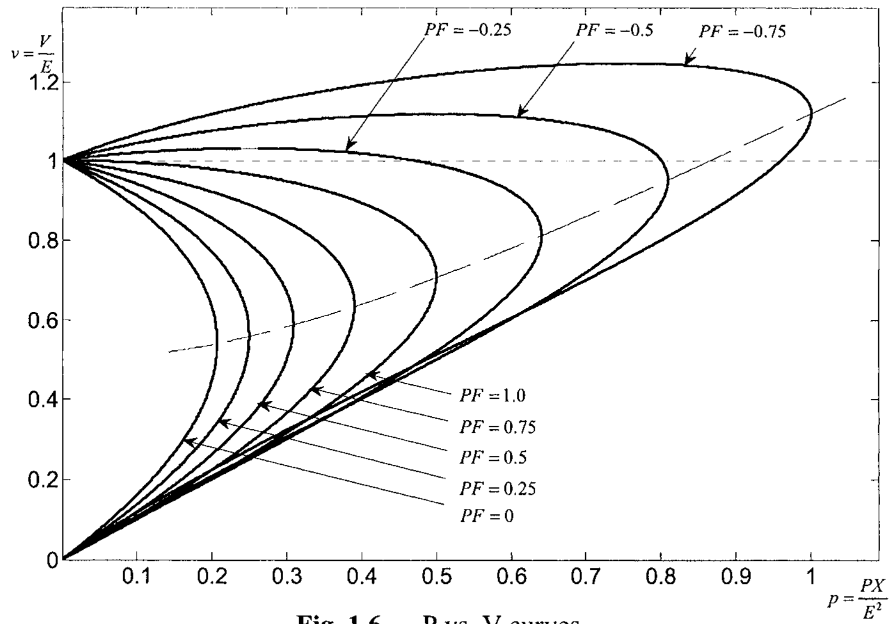


**Fig. 1.6** P vs. V curves

At the maximum power point the term inside the square root in equation Eq.1.9 is zero. With this condition, we can show

$$p_{\max} = \frac{\cos\phi}{2(1+\sin\phi)} \tag{1.10}$$

$$v_{crit} = \frac{1}{\sqrt{2} * \sqrt{1+\sin\phi}} \tag{1.11}$$

At unity power factor $\phi = 0.0$; $p_{\max} = \frac{1}{2} = 0.5$; $v_{crit} = \frac{1}{\sqrt{2}} = 0.707$.

The relationship between $\theta$ and $\phi$ at the maximum power condition can be derived as follows. We know

$$\cos^2\theta = 1 - \sin^2\theta \tag{1.12}$$

From Eq.1.1, at the maximum power conditions, $\sin\theta = -p_{\max} / v_{crit}$. Substituting $\sin\theta$ in Eq.1.12 with $p_{\max}$ from Eq.1.10 and $v_{crit}$ from Eq.1.11.

$$\cos\theta = \sqrt{1+\sin\phi} = \frac{1}{2v_{crit}} \tag{1.13}$$

**Table 1.2 Values of various variables at the critical point**

| $p_{crit}$ | $v_{crit}$ | $q$ | $\phi$ | $\theta$ |
|---|---|---|---|---|
| 0.5 | 0.707 | 0 | 0 | 45 |
| 0.288 | 0.577 | 0.166 | 30 | 30 |
| 0.207 | 0.541 | 0.207 | 45 | 22.49 |
| 0.1339 | 0.5175 | 0.232 | 60 | 15 |
| 0 | 0.5 | -0.25 | 90 | 0 |
| 0.866 | 1 | -0.5 | -30 | 60 |
| 1.206 | 1.306 | -1.206 | -45 | 67.48 |
| 1.86 | 1.93 | -3.22 | 60 | 75 |
| . | . | . | . | . |
| . | . | . | . | . |
| . | . | . | . | . |

### 1.3.2 Q-V curves

Similar to PV curves one can also obtain QV curves. For each PV curve the power factor is constant, whereas for each QV curve the p is kept constant. From Eq.1.7

$$v^2 = \frac{-(2q-1) \pm \sqrt{(2q-1)^2 - 4(p^2+q^2)}}{2} \tag{1.14}$$

If we keep $p$ constant in Eq.1.14, then for each $p$ the relation between $q$ and $v$ is shown in Fig.1.7.

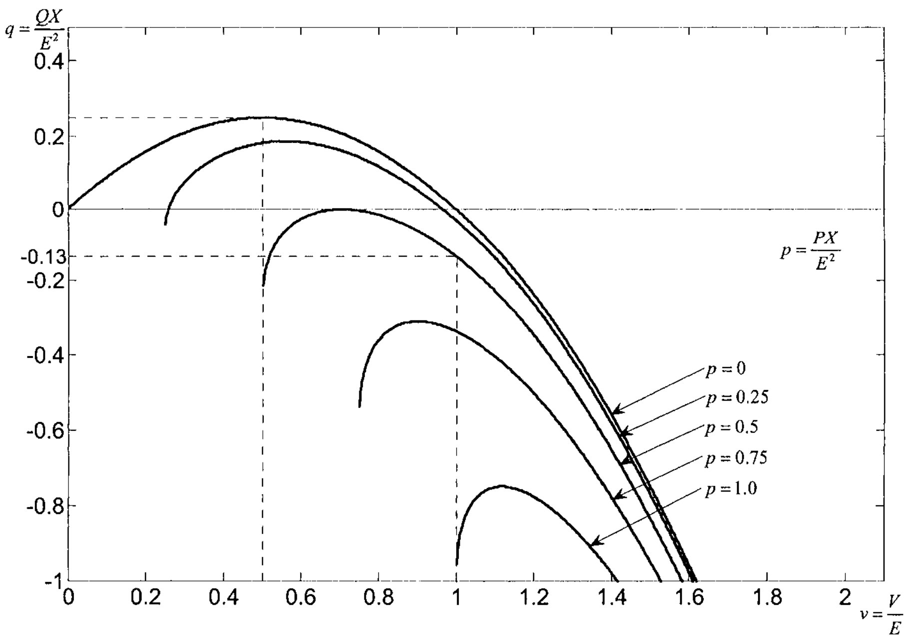


**Fig. 1.7** q vs. v curves

We can get $q_{crit}$ by equating the term inside the square root sign to zero in equation Eq.1.14. Then

$$q_{crit} = \frac{1}{4} - p^2 \tag{1.15}$$

and

$$v_{crit} = \sqrt{\frac{1}{2}(1 - 2q_{crit})} \tag{1.16}$$

at $p = 0 \rightarrow q_{crit} = 0.25$; $v_{crit} = 0.5$.

Similar to p vs. v curves one can generate q vs. v curves for a given p. In the above formulation, we assumed q to be positive for inductive reactive power. However, if we assume q as negative for inductive reactive power, then q vs. v curves can be shown in Fig.1.8. In general in power system literature q is negative for inductive reactive power.

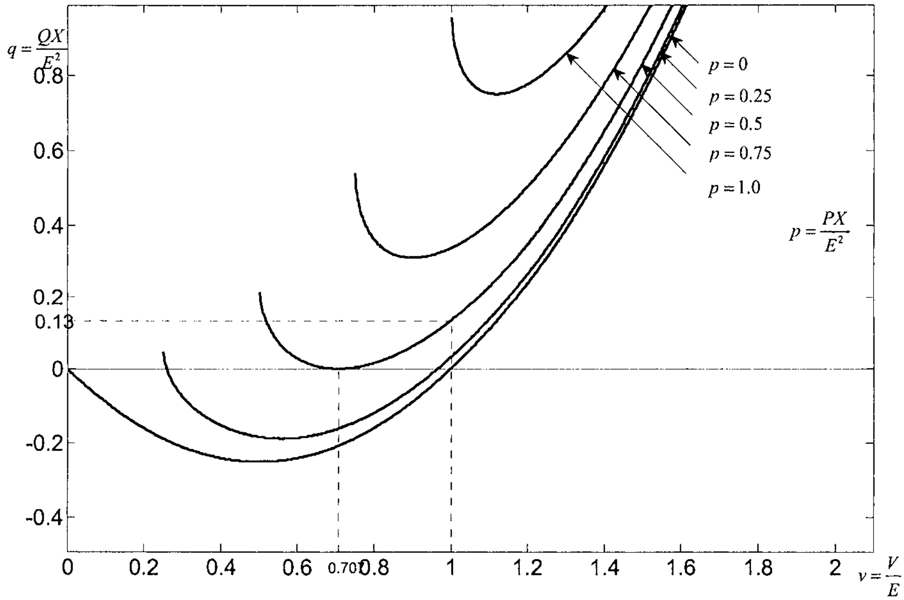


**Fig. 1.8** Relationship between voltage and the reactive power

### 1.3.3 Discussion on PV and QV Curves

**PV curves**: As mentioned before p vs. *v* curves can be obtained from Eq.1.7. These curves are shown in Fig.1.6. Each curve corresponds to a particular power factor. There is a maximum transferable power. For any given value of "p" there are two possible voltages (higher voltage with lower current or lower voltage with higher current). The normal operation corresponds to high voltage solution. With capacitor compensation (leading power factor) the maximum power increases. However the corresponding critical voltage also increases. From Fig.1.6, one can see that with highly compensated transmission line, normal voltages become critical voltages.

**QV curves**: These curves give the relation between q and *v* for a given real power transfer p. They provide reactive requirement at a given bus to maintain a certain voltage. For example in Fig.1.7 or Fig.1.8 in the p=0.5curve,to maintain the voltage at 1.0 p.u., a capacitive reactive power injection of q=0.13 p.u. is needed. If this reactive power injection is lost, the voltage will be decreased to 0.707p.u. which is a critical value ( the q= 0 axis just touches the q vs. v curve corresponding to p=0.5). For p=0.5 there is no solution if the net injection is inductive reactive power and this may result in voltage instability. For critical buses, QV curves can be generated from power flow.

Power through transmission lines introduces both real and reactive power loss. These losses strongly depend on the amount of power through the line. Transmission lines are mainly dominated by inductive and capacitive characteristics of the line.

At light loads it acts like a capacitor (supply reactive power to the system). At heavy loads it acts like an inductor (absorb reactive power). The loading at which the inductive and capacitive affects cancel each other is called surge-impedance loading “SIL.” “SIL” = approximately 40% to 50% of the line’s thermal capacity.

Fig.1.9 [23] shows the relation between line loading and losses.

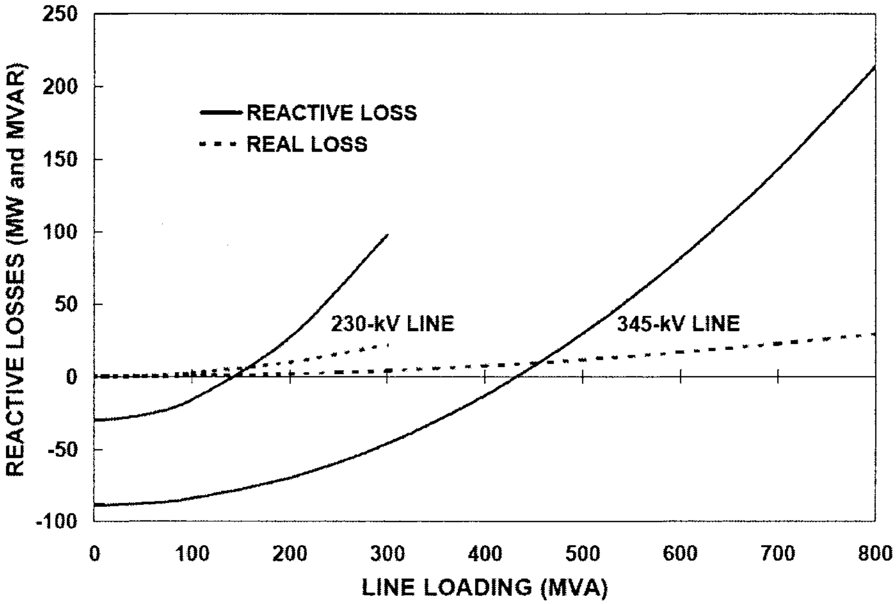


**Fig.1.9** Real and reactive power loss vs. line loading for a 100 mile line with the voltages supported at both ends [23]

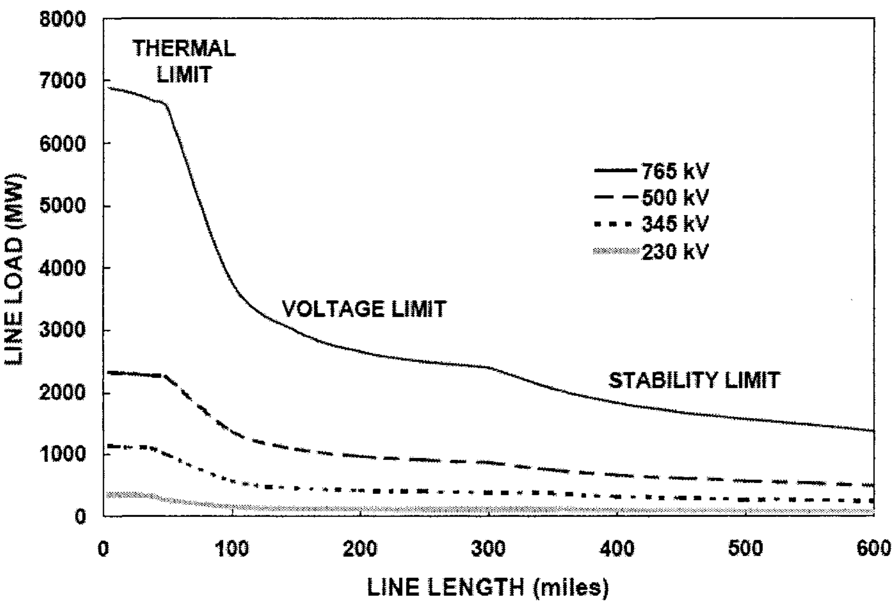


**Fig.1.10** Line loading as limited by thermal, voltage and stability [23]

From Fig.1.9 at full line loading reactive losses are 5 times greater than real loss for 230kV line and 9 times greater than real loss for 345kV line.

Fig.1.10 shows the relation between transmission line capacity and the length of the transmission line. Limitations for short, medium and long transmission lines are thermal, voltage drop and stability limits respectively. The above limitations are without any control.

### 1.3.4 Maximum power and power flow Jacobian

For the two bus example, the power flow equations are:

$$0 = P_0 + \frac{EV}{X}\sin(\theta) = f_1(E,V,\theta)$$

$$0 = Q_0 - \frac{EV}{X}\cos(\theta) + \frac{V^2}{X} = f_2(E,V,\theta)$$

The Jacobian (J):

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial \theta} & \frac{\partial f_1}{\partial v} \\ \frac{\partial f_2}{\partial \theta} & \frac{\partial f_2}{\partial v} \end{bmatrix} = \begin{bmatrix} \frac{EV}{X}\cos\theta & \frac{E}{X}\sin\theta \\ \frac{EV}{X}\sin\theta & \frac{-E}{X}\cos\theta + \frac{2V}{X} \end{bmatrix}$$

The determinant of this Jacobian is $\frac{-E^2V}{X^2} + \frac{2EV^2}{X^2}\cos\theta$.

Equating this determinant to zero, we get

$$\cos\theta = \frac{1}{2\left(\frac{V}{E}\right)} = \frac{1}{2v}$$

This corresponds to the condition of the critical voltage derived in previous section (Eq.1.13).

In general the power system Jacobian becomes singular at the maximum power point.  This may leads to convergence problems if one applies the traditional Newton-Raphson method to solve power flow equations.

In this book, computational techniques based on bifurcation and continuation methods will be described to avoid singularities and convergence problems.

## References


[1] Kundur, P., Paserba, J., Ajjarapu, V., Anderson, G., Bose, A., Canizares, C., Hatziargyriou, N., Hill, D., Stankovic, A., Taylor, C. W., Van Cutsem, T., Vittal, V., Definitions and classification of power system stability, IEEE/CIGRE Joint Task force on Stability Terms and Definitions , IEEE transactions on Power Systems, vol. 19, no. 3, pp. 1387-1401, Aug. 2004

[2] Dobson, I. L., Lu, L., Voltage collapse precipitated by the immediate change in stability when generator reactive power limits are encountered, IEEE Transactions on Circuits and Systems I: Fundamental Theory and Applications, vol. 39, no. 9, pp.762-766, Sept. 1992

[3] Hill, David J., Nonlinear dynamic load models with recovery or voltage stability studies, IEEE Transactions on Power Systems, vol. 8, no. 1, pp. 166-176, Feb. 1993

[4] Taylor, C. W., Power system voltage stability, McGraw Hill, 1994

[5] NERC, 1996 system disturbances: review of selected electric system disturbances in North America, ftp://www.nerc.com /pub/sys/all_updl/oc/dawg/disturb96.pdf, pp. 22-33, 2002

[6] Vargas, L., Quintana, V. H., Voltage collapse scenario in the Chilean interconnected system, IEEE Transactions on Power Systems, vol. 14, no. 4, pp. 1415-1421, Nov. 1999

[7] Vournas, C., Technical summary on the Athens and southern Greece blackout of July 12, 2004

[8] Vournas, C. D., Manos, G. A., Kabouris, J., Van Cutsem, T., Analysis of a voltage instability incident in the Greek power system, IEEE Power Engineering Society Winter Meeting, vol. 2, 23-27, pp. 1483-1488, Jan. 2000

[9] Kundur, P., Power system stability and control. New York: McGraw-Hill, 1994

[10] Van Cutsem, T., Vournas, C., Voltage stability of electric power systems, Kluwer, 1998

[11] Fink, L. H., Proc. of bulk power system voltage phenomena-I, voltage stability and security, Potosi, MO: ECC/NSF/EPRI Workshop, 1989

[12] Fink, L. H., Proc. of bulk power system voltage phenomena-II, voltage stability and security, Deep Creek Lake, MD, ECC/NSF Workshop, ECC Inc, 1991

[13] Fink, L. H., Proc. of bulk power system voltage phenomena-III, voltage stability and security, Davos, Switzerland: ECC/NSF Workshop, 1994

[14] Fink, L. H., Proc. of bulk power system dynamics and control-IV, restructuring, Santorini, Greece: IREP Workshop, 1998
[15] Fink, L. H., Proc. of bulk power system dynamics and control-V, security and reliability in a changing environment, Onomichi, Japan: IREP Workshop, 2001
[16] Proceedings of bulk power system dynamics and control –VI, Managing complexity in power systems: from micro-grids to mega interconnections, Italy, 2004
[17] IEEE Working Group on Voltage Stability, Voltage stability of power systems: concepts, analytical tools, and industry experience, 1990
[18] IEEE Working Group on Voltage Stability, Suggested techniques for voltage stability analysis, 1993
[19] IEEE Power System Relaying Committee Working Group K12, System protection and voltage stability, IEEE publication No. 93, THO 596-7 PWR.
[20] Cañizares, C. A., Editor, Voltage stability assessment: concepts, practices and tools, IEEE-PES Power Systems Stability Subcommittee Special Publication, Product Number SP101PSS (ISBN 0780378695), 2003.
[21] Ajjarapu, V., Lee, B., Bibliography on voltage stability, IEEE Transactions on Power Systems, vol. 13, no. 1, pp. 115-125, Feb. 1998
[22] Iowa State University's Web Based Voltage Stability Search Engine, http://design-2.ece.iastate.edu/biblio/
[23] Kirby, B., Hirst, E., Ancillary service details: voltage control, Oak Ridge National Laboratories/ORNL, http://www.ornl.gov/sci/btc /apps/Restructuring/con453.pdf, Dec. 1997