Chapter 3.4.0

APPLICATIONS AND MODELLING FOR MATHEMATICS – OVERVIEW

Eric Muller and Hugh Burkhardt
Brock University, St. Catharines, Ontario, Canada, Email: emuller@brocku.ca / Shell Centre for Mathematical Education, Nottingham, UK, Email: Hugh.Burkhardt@nottingham.ac.uk

Abstract: These five chapters address important issues on mathematics teaching and learning. They include, amongst others, how Applications and Modelling help students learn mathematics in ways that result in a deep and holistic understanding; are central to the development of Mathematical Literacy, and; are enriched by the creative use of technology.

1. INTRODUCTION

Mathematics has long had a large slice of curriculum time in every country, mainly because of its perceived utility[1] in solving problems that people face in some other school subjects, and in life and work. We teach and we learn mathematics to develop:

- a powerful toolkit of mathematical strategies, concepts and skills, and
- competency in using it to tackle problems from the "real world" or as Henry Pollak (1979), a pioneer in this area, has suggested, the "rest of the world".

Modelling competence is essential for such problem solving and much of this book is about how it develops with appropriate teaching. In these five chapters we focus on the benefits that flow the other way – the contributions of modelling activities to the development of other mathematical competencies.

What are these other mathematical competencies?

As with other complex activities, there are many descriptions of mathematical performance; they have much in common but each has a different emphasis, particularly on the centrality they give to modelling. The Danish KOM model (KOM, 2002), described in Chapter 2.2 of this volume, identifies eight competencies of two types: (a) asking and answering mathematical questions (which requires mathematical thinking, problem tackling, modeling, and reasoning) and (b) dealing with mathematical language (through representation, symbols and formalism, communicating, and tools). The US National Council of Teachers of Mathematics (NCTM, 2000) also identifies two types of mathematical competencies: content (number and operations, algebra, geometry, measurement, data analysis and probability) and processes (problem solving, reasoning and proof, communication, connections, representation). The U.S. National Research Council (NRC, 2001) identifies five "interwoven and interdependent" strands of mathematical proficiency: conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and productive disposition. The complementary UK Tomlinson (2004) and Smith (2004) reports stress respectively the importance of "functional mathematics" and of "mathematics: for its own sake; for the knowledge economy; for science, technology and engineering; for the workplace; and for the citizen". These analyses all envisage the same range of mathematical practices that, developed in classrooms, will help students to learn mathematics in ways that result in deep understanding and the ability to use mathematics where it matters.

Characteristic of all these schemes of classification is a much broader view of "doing mathematics" than is shown in most traditional curricula. Each places emphasis on mathematical processes. In sharp and deliberate contrast, most school programs, most teachers and parents, and most official "high-stakes" school examinations treat mathematics as only the grammar and syntax of mathematical language, and often only a small procedural subset of these. The broader view requires that students engage with mathematics as a connected whole, not just as a succession of separate topics, chapters, and formulas. Time, of course, makes teaching sequential; in contrast, the multiple connections that are essential to a robust understanding of mathematics do not arise naturally – they require learning activities specifically designed to develop them.

Mathematical models of authentic situations do this well. They reveal more readily than do artificial textbook problems that, to be effective, mathematics must be approached holistically rather than as an accumulation of bits and pieces of de-contextualized knowledge. Although the development of mathematical expertise has traditionally been approached by decomposing problems into component skills that are taught separately, evidence from many sources (e.g. Schoenfeld, 1992, de Corte et al., 1996) shows that this is not an effective way to build expertise. In "doing mathematics" the whole is much more than the sum of the parts. Neither is it easier to learn in fragments. "It is harder, not easier, to understand something broken down into all the precise little rules than to grasp it as a whole" (Thurston, 1990).

In considering applications and modelling in relation to other mathematical competencies, it is important to distinguish two different types of application, shown in Fig. 3.4.0-1 (due to Malcolm Swan, see Chapter 3.4.1).

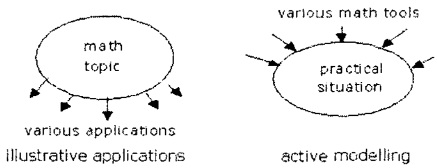

Figure 3.4.0-1. Goal types for applications

Most curricula offer *illustrative applications*; there the focus is on a specific mathematical topic, showing the various practical domains where it can be useful and practising its use in those contexts. The student has no doubt as to the mathematics to be used – it is the topic just taught. In contrast, in *active modelling* the focus is on the practical situation and understanding it better. Usually, a variety of mathematical tools will be useful for different aspects of the analysis. (This is a good indication as to the real goal.) Choosing and using tools appropriately is a major part of the challenge to the student. Both types of activity are important in learning mathematics. Both provide connections between mathematics and practical situations. However only active modelling, as opposed to learning models, involves the full range of mathematical competencies. Modelling is all about *applied power.*

Thus context-based mathematical modelling provides ideal settings to blend content and process so as to produce flexible mathematical competence. The iterative self-correcting cycle of asking questions, using tools, producing answers, and then asking new questions helps *students develop the cognitive connections* required to understand mathematics as a discipline. Concrete, contextualized models can be especially effective as a glue that binds together in the minds of students the many abstract and otherwise disconnected facets of mathematics.

Another complementary role played by modelling in developing mathematical competence is *enhancing student motivation.* Students confronted with appealing applications and models will learn, from direct experience, convincing answers to the universal question that plagues mathematics teachers everywhere: "Where am I going to use this?"

Although connecting mathematics to authentic contexts helps make mathematics meaningful, it demands delicate balance. On the one hand, contextual details camouflage broad patterns that are the essence of mathematics; on the other hand, these same details offer associations that are critically important for many students' long-term learning. Few can doubt that the tradition of de-contextualized mathematics instruction has failed the many students who leave high school with neither the usable mathematical skills nor the quantitative confidence required for today's society. The tradition of formal mathematics, used mainly as a 'gatekeeper' to future academic study, leaves many able students both innumerate and undereducated. However, when the traditional symbol-intensive curriculum is anchored in authentic applications and modelling by the students, many will reveal aptitude for mathematics that was previously undeveloped. A diverse curriculum featuring both abstract and applied mathematics can help break the rigidity of traditional expectations and enable more students to achieve higher levels of mathematical competence. In this chapter we discuss how, with appropriate teaching, modelling competence can support the development of other mathematical competencies in the learning process.

2. PAPER SUMMARIES

Applications and Modelling for Mathematics is structured in five chapters. After this introduction, Swan, Turner and Yoon describe, analyse, and provide examples on ways modelling encourages the asking and the answering of mathematical questions, and how it promotes the use of mathematical language. They highlight the fact that, in modelling situations, students develop mathematical expertise based on an integrated field of knowledge, make multiple connections both within and outside mathematics, and not only reinforce their mathematical understanding but also develop new mathematical knowledge. The third chapter is on *Mathematical Literacy*. Steen and Turner describe what ML means, the kinds of problem it involves and how it is developed, along with an outline of some contentious issues. In the fourth chapter, Antonius, Haines, Jensen and Niss discuss the pattern of classroom activities needed, and the roles of the teacher, in supporting the learning of other mathematical competencies through modelling and applications. The fifth chapter explores uses and possibilities of various technologies in mathematical modelling while it focuses on the development of other mathematical competencies. Pead provides multiple examples from the secondary school level while Ralph describes a university modelling program that is technology-centred.

We conclude this overview by drawing attention to a few of the key questions that need further research in depth, and associated development:

- In what ways do concrete applications and active modelling build understanding of mathematical concepts?
- How far do the extended chains of reasoning involved in modelling real

situations encourage students to improve the reliability of their technical skills in mathematics?

- How far does modelling improve the performance of students in pure mathematical problem solving, and in which aspects?
- How can typical mathematics teachers be enabled to effectively help their students in their classroom to learn to handle real world problems?
- In what ways can technology help enough with all these goals, so as to encourage teachers and schools to make the necessary investments
- How can mathematical literacy be 'sold' to teachers and to school systems, as a prime goal of mathematics education?

Reliable answers to these questions will need warrants for their generality from replication in many parallel but diverse projects and school systems (Burkhardt & Schoenfeld, 2003).

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[1] Other less "useful" subjects with comparable cultural importance and intellectual challenge, such as music, get far less time in schools.