Chapter 13
Performance Analysis of ARQ Schemes in Self-Similar Traffic

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Abstract In this chapter, we present a new method to analyze the performance of Automatic Repeat reQuest (ARQ) schemes in self-similar traffic. Taking into account the self-similar nature of a massive-scale wireless multimedia service, we build a batch arrival queueing model and suppose the batch size to be a random variable following a Pareto(c, α) distribution. Considering the delay in the setting up procedure of a data link, we introduce a setup strategy in this queueing model. Thus a batch arrival Geom^X/G/1 queueing system with setup is built in this chapter. By using a discrete-time embedded Markov chain, we analyze the stationary distribution of the queueing system and derive the Probability Generation Functions (P.G.Fs.) of the queueing length and the waiting time of the system. We give the formula for performance measures in terms of the response time of data frames, setup ratios, and offered loads for different ARQ schemes. Numerical results are given to evaluate the performance of the system and to show the influence of the self-similar degree and the delay of the setup procedure on the system performance.

13.1 Introduction

With the rapid development of wireless applications, support for Internet services with excellent reliability is becoming more and more important [1]. In general, error control schemes in communication systems can be classified into two categories: Forward Error Correction (FEC) and Automatic Repeat reQuest (ARQ) schemes [2].

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In addition to FEC, ARQ schemes are in most cases used to ensure the transmission of packet data on higher layers, or are used as hybrid ARQ schemes on MAC/PHY layers.

It is a generally accepted view that discrete-time systems may be more complex to analyze than equivalent continuous-time systems. However, [3] has indicated that it would be more accurate and efficient to use discrete-time queueing models than continuous-time queueing models when analyzing and designing digital communication network systems.

The classical discrete-time queueing analyses have been presented in [3] and [4]. Extensive research of advanced ARQ schemes, as well as some performance analyses based on ARQ schemes have been conducted in [5]– [7]. In [5], an analysis of the ARQ feedback types was presented, but no algorithm to select the feedback was given. In [6], the ARQ mechanism were analyzed in the context of real-time flows of small packets. The key features and parameters of the ARQ mechanism were analyzed, and the ARQ block rearrangement, ARQ transmission window, and ARQ block size were researched in [7].

However, some simplifying assumptions considered in the above studies do not hold in practice. For example: self-similar behavior was neglected and the setting up procedure of a data link was omitted. This ignores both the influence of the self-similar degree as well as the delay of the setting up procedure on the system performance in such wireless networks.

In order to satisfy the demands of massive-scale wireless multimedia services and improve the performance of ARQ schemes, more accurate mathematical models that can faithfully capture the self-similar behavior of computer networks and the setting up procedure of a data link need to be constructed.

In this chapter, we avoid this unreal simplification to give a more constructed version, closer in nature to the actual system by considering the self-similar traffic shown in a service-oriented Internet [9]. Taking into account the delay in the setting up procedure of a data link, we build a batch arrival queueing model with a setup strategy. The results obtained in this chapter also include those in [8] for the system having arrivals of data frames. By using a discrete-time embedded Markov chain approach, we analyze the stationary distribution of the system, and present the stochastic decomposition of the queueing length and the waiting time. Based on numerical results, we evaluate the performance of ARQ schemes in terms of the response time of data frames, setup ratio, and the system's offered load. We also show the influence of the delay in the setup procedure and the self-similar degree on the system performance.

The chapter is organized as follows. In Sect. 13.2, the system model is described and some notation definitions are given. In Sect. 13.3, the stationary distribution of the system is derived. Correspondingly, performance measures for ARQ schemes are presented in Sect. 13.4. Numerical results are shown in Sect. 13.5 and conclusions are drawn in Sect. 13.6.

13.2 System Model and Notation

The system under analysis in this chapter consists of a pair of nodes, namely a transmitter and a receiver. When two adjacent nodes need to communicate with each other, a data link must be set up. We assume the time axis to be divided into slots of equal length and batch arrivals to follow a Bernoulli process. There are multiple data frames in a batch.

Self-similarity is the property we associate with one type of fractal, that is, an object whose appearance is unchanged regardless of the scale at which it is viewed [9]. A self-similar process may be constructed by superimposing many simple renewal reward processes, in which the rewards are restricted to the values 0 and 1, and the interrenewal times are heavy-tailed. The simplest heavy-tailed distribution is the Pareto(c,α) distribution [9]. We denote by Λ the number of data frames in a batch called batch size Λ (frames/slot), which is a random variable. The batch size follows a Pareto(c,α) distribution. When the transmission of all the data frames in the output buffer is finished, the data link should be released.

The system works as detailed below.

(1) When a batch arrives in the system, a setup period called "setup period U" is started, where the setup period U corresponds to a time period for setting up a new data link using a three-handshake signaling procedure.
(2) After the setup period U finishes, a busy period called "busy period Θ" begins. Here we define the busy period Θ to be a time period in which data frames are transmitted continuously until the transmitter buffer becomes empty.
(3) When there are no data frames in the output buffer of the transmitter to be transmitted, the data link is released and the system enters an idle period called "idle period I". A batch arriving during the idle period I makes the system enter a new setup period U again.

This process is repeated.

We define a transmission period B called "transmission period B" as being the time period taken to successfully transmit a data frame: that is, the time period from the instant for the first transmission of a data frame to the instant for the departure of the data frame from the transmitter buffer.

The transmission of a data frame only occurs after the correct reception of all data frames with a lower identifier, so we can assume that data frames in batches arriving in the buffer with an infinite capacity are transmitted using a common data link, one by one, in a First-Come First-Served (FCFS) discipline.

The setup period U and the transmission period B are independent and identical discrete-time random variables in slots, and are assumed to be generally distributed with probability distribution u_k and b_k, Probability Generation Functions (P.G.Fs.) $U(z)$ and $B(z)$ are as follows:

$$u_k = P\{U = k\}, \qquad k \geq 1, \qquad U(z) = \sum_{k=1}^{\infty} u_k z^k, \tag{13.1}$$

$$b_k = P\{B = k\}, \qquad k \geq 1, \qquad B(z) = \sum_{k=1}^{\infty} b_k z^k. \tag{13.2}$$

Let E$[U]$ and E$[B]$ be the averages of U and B in slots; we have that

$$\mathrm{E}[U] = \sum_{k=1}^{\infty} k u_k, \qquad \mathrm{E}[B] = \sum_{k=1}^{\infty} k b_k.$$

Let E$[\Lambda]$ be the average of the batch size Λ. We can give the probability λ_k, the P.G.F. $\Lambda(z)$, and average E$[\Lambda]$ of Λ as

$$\lambda_k = P\{\Lambda = k\}, \qquad k \geq 0, \qquad \Lambda(z) = \sum_{k=0}^{\infty} \lambda_k z^k, \qquad \mathrm{E}[\Lambda] = \sum_{k=0}^{\infty} k\lambda_k, \tag{13.3}$$

where λ_k is the probability that there are k data frames in a batch per slot. Specifically, $\lambda_0 = P\{\Lambda = 0\}$ is the probability that there is no batch ($\Lambda = 0$) arrival in a slot. From (13.1), we also know that the probability of no batch arrival during the transmission period B is $B(\lambda_0) = \lambda_0^B$. The ergodic condition is $\rho = \mathrm{E}[\Lambda]\mathrm{E}[B] < 1$, where ρ is called the offered load.

Let A_U and A_B be random variables representing the numbers of data frames arriving during U and B. We can then give the P.G.Fs. $A_U(z)$ and $A_B(z)$ of A_U and A_B as follows:

$$A_U(z) = \sum_{k=1}^{\infty} u_k (\Lambda(z))^k = U(\Lambda(z)),$$
$$A_B(z) = \sum_{k=1}^{\infty} b_k (\Lambda(z))^k = B(\Lambda(z)), \tag{13.4}$$

where $U(\Lambda(z))$ and $B(\Lambda(z))$ are composed functions of $U(z)$, $B(z)$, and $\Lambda(z)$.

We also define $\Lambda(B(z))$ to be the P.G.F. of the transmission time of a batch in slots. $\Lambda(B(z))$ can be given as

$$\Lambda(B(z)) = \sum_{k=0}^{\infty} \lambda_k (B(z))^k. \tag{13.5}$$

13.3 Performance Analysis

We assume that data frame arrivals and departures occur only at the boundary of a slot. Let $Q_n = Q(\tau_n^+)$ be the number of data frames in the system immediately after the nth data frame departure. Then $\{Q_n, n \geq 1\}$ forms an embedded Markov chain. We define the state of the system by the number Q of data frames in the system at the embedded Markov points as follows:

$$Q_{n+1} = \begin{cases} Q_n - 1 + A_B^{(n+1)}, & Q_n \geq 1 \\ \Lambda' + A_U + A_B^{(n+1)} - 1, & Q_n = 0, \end{cases} \tag{13.6}$$

where $A_B^{(n+1)}$ is the number of data frames arriving during the transmission time of the $(n+1)$th data frame, and Λ' denotes the number of data frames that arrive in a slot under the condition that there is at least one data frame arriving in that slot. Obviously, the P.G.F. $\Lambda'(z)$ of Λ' can be given as

$$\Lambda'(z) = \frac{\Lambda(z) - \lambda_0}{1 - \lambda_0}. \tag{13.7}$$

From (13.6), we can obtain the P.G.F. $Q(z)$ of Q as

$$\begin{aligned} Q(z) &= P\{Q \geq 1\}\mathrm{E}\left[z^{Q+A_B-1} | Q \geq 1\right] \\ &\quad + P\{Q = 0\}\mathrm{E}\left[z^{\Lambda' + A_U + A_B^{(n+1)} - 1} | Q = 0\right], \end{aligned} \tag{13.8}$$

where $P\{Q = 0\}$ is the probability that there are no data frames to be transmitted in the system at the embedded Markov points, and $P\{Q \geq 1\}$ is the probability that there is at least one data frame to be transmitted in the system at the embedded Markov points.

Substituting (13.7) to (13.8), we can give that

$$Q(z) = P\{Q = 0\} \times \frac{B(\Lambda(z))}{B(\Lambda(z)) - z} \times \left(1 - \frac{\Lambda(z) - \lambda_0}{1 - \lambda_0} U(\Lambda(z))\right). \tag{13.9}$$

Using the normalization condition and the L'Hospital principle in (13.9), we have that

$$P\{Q = 0\} = \frac{(1-\rho)(1-\lambda_0)}{\mathrm{E}[\Lambda](1 + \mathrm{E}[U](1 - \lambda_0))}. \tag{13.10}$$

Substituting (13.10) to (13.9), then the P.G.F. $Q(z)$ of Q can be obtained as

$$Q(z) = \frac{(1-\rho)(1-\Lambda(z))B(\Lambda(z))}{\mathrm{E}[\Lambda](B(\Lambda(z)) - z)} \times \frac{1 - \lambda_0 - (\Lambda(z) - \lambda_0)U(\Lambda(z))}{1 - \Lambda(z)}. \tag{13.11}$$

Equation (13.11) implies that Q can be decomposed into two parts (i.e., $Q = Q_0 + Q_U$), where Q_0 corresponds to the number of data frames for the classical queue Geom^X/G/1 and Q_U is the number of data frames added by the setup scheme considered in this chapter.

The P.G.F. $Q_0(z)$ of Q_0 can be given as

$$Q_0(z) = \frac{(1-\rho)(1-\Lambda(z))B(\Lambda(z))}{\mathrm{E}[\Lambda](B(\Lambda(z)) - z)}$$

and the P.G.F. $Q_U(z)$ of Q_U can be given as

$$Q_U(z) = \frac{1-\lambda_0-(\Lambda(z)-\lambda_0)U(\Lambda(z))}{1-\Lambda(z)}$$
$$= \frac{1}{1+(1-\lambda_0)\mathrm{E}[U]} \times U(\Lambda(z)) + \frac{(1-\lambda_0)\mathrm{E}[U]}{1+(1-\lambda_0)\mathrm{E}[U]} \times \frac{1-U(\Lambda(z))}{\mathrm{E}[U](1-\Lambda(z))}.$$

Obviously, $Q_U(z)$ equals the P.G.F. of the number of data frames arriving during the setup period U with the following probability as

$$\frac{1}{1+(1-\lambda_0)\mathrm{E}[U]}.$$

And $Q_U(z)$ equals the P.G.F. of the number of data frames arriving during the remaining setup period U with the following probability as

$$\frac{(1-\lambda_0)\mathrm{E}[U]}{1+(1-\lambda_0)\mathrm{E}[U]}.$$

Let $\mathrm{E}[X]$ and $X^{(2)}$ be the first and second factorial moments of a discrete-time random variable X by differentiating $X(z)$ with respect to z and evaluating the result at $z = 1$ as follows:

$$\mathrm{E}[X] = \left.\frac{dX(z)}{dz}\right|_{z=1}, \qquad X^{(2)} = \left.\frac{d^2X(z)}{dz^2}\right|_{z=1}.$$

Based on the above definition, we can give the average $\mathrm{E}[Q]$ of Q from (13.11) as

$$\mathrm{E}[Q] = \rho + \frac{\Lambda^{(2)} + B^{(2)}\mathrm{E}^3[\Lambda]}{2\mathrm{E}[\Lambda](1-\rho)} + \frac{\mathrm{E}[\Lambda]\left((1-\lambda_0)U^{(2)} + 2\mathrm{E}[U]\right)}{2\left(1+\mathrm{E}[U](1-\lambda_0)\right)}, \tag{13.12}$$

where $U^{(2)}$, $B^{(2)}$, and $\Lambda^{(2)}$ are the second factorial moments of the setup period U, the transmission period B, and batch size Λ.

Now, we begin to analyze the waiting time of a data frame. We focus on an arbitrary data frame in the system called "tagged data frame M". We note that the waiting time W of the tagged data frame M can be divided into two parts as follows. One is the waiting time W_g of the batch to which the tagged data frame M belongs. The other is the total transmission time J of the data frames before the tagged data frame M in the same batch. W_g and J are independent random variables, so we have the P.G.F. $W(z)$ of the waiting time W of the tagged data frame M as follows:

$$W(z) = W_g(z)J(z), \tag{13.13}$$

where $W_g(z)$ and $J(z)$ are P.G.Fs. of W_g and J.

Applying the analysis of the single arrival Geom/G/1 queue model to the setup in [8], we have that

$$W_g(z) = \frac{(1-\rho)(1-z)}{\Lambda(B(z))-z} \times \frac{\mathrm{E}[\Lambda]+(1-z-\mathrm{E}[\Lambda])\,U(z)}{(1+\lambda\mathrm{E}[U])\,(1-z)}. \tag{13.14}$$

Referencing [3], with $\Lambda(B(z))$ given in (13.5), we have that

$$J(z) = \frac{1-\Lambda(B(z))}{\mathrm{E}[\Lambda](1-B(z))}. \tag{13.15}$$

Substituting (13.14) and (13.15) to (13.13), then the P.G.F. $W(z)$ and the average $\mathrm{E}[W]$ of W can be obtained as

$$W(z) = \frac{(1-\rho)(1-z)}{\Lambda(B(z))-z} \times \frac{1-\Lambda(B(z))}{\mathrm{E}[\Lambda](1-B(z))} \times \frac{\mathrm{E}[\Lambda]+(1-z-\mathrm{E}[\Lambda])\,U(z)}{(1+\lambda\mathrm{E}[U])\,(1-z)},$$

$$\mathrm{E}[W] = \frac{\Lambda^{(2)}\mathrm{E}^2[B]+\mathrm{E}[\Lambda]B^{(2)}}{2(1-\rho)} + \frac{\mathrm{E}[\Lambda]U^{(2)}+2\mathrm{E}[U]}{2(1+\mathrm{E}[\Lambda]\mathrm{E}[U])} + \frac{\Lambda^{(2)}\mathrm{E}[B]}{2\mathrm{E}[\Lambda]}. \tag{13.16}$$

Next, we define the busy cycle called "busy cycle R" as a time period from the instant in which a busy period Θ is completed to the instant in which the next busy period Θ ends. Obviously, a busy cycle R is composed of three parts: a setup period U, a busy period Θ, and an idle period I. Denoted by $\mathrm{E}[R]$, $\mathrm{E}[\Theta]$, and $\mathrm{E}[I]$ the averages of the busy cycle R, the busy period Θ, and the idle period I, respectively, we give that

$$\mathrm{E}[R] = \mathrm{E}[U]+\mathrm{E}[\Theta]+\mathrm{E}[I], \tag{13.17}$$

where $\mathrm{E}[U]$ is defined in (13.1), and $\mathrm{E}[\theta]$ and $\mathrm{E}[I]$ are given below.

Let Q_Θ be the number of data frames at the beginning of a busy period Θ. The P.G.F. $Q_\Theta(z)$ of Q_Θ is then given by

$$Q_\Theta(z) = \frac{\Lambda(z)-\lambda_0}{1-\lambda_0}U(\Lambda(z)). \tag{13.18}$$

Each data frame at the beginning of a busy period Θ will introduce a subbusy period θ. A subbusy period θ of a data frame is composed of the transmission period B of this data frame and the sum of the subbusy period θ incurred by all the data frames arriving during the transmission period B of this data frame. All the subbusy periods brought by the data frames at the beginning of the busy period combine to make a system busy period Θ, so we have that

$$\theta = B + \underbrace{\theta+\theta+\cdots+\theta}_{A_B}, \qquad \Theta = \underbrace{\theta+\theta+\cdots+\theta}_{Q_\Theta},$$

where A_B is the number of data frames arriving during the transmission period B presented in Sect. 13.2.

Considering the Bernoulli arrival process in this system, the P.G.F. $\theta(z)$ of θ can be obtained as follows:

$$\theta(z) = B(z(\Lambda(\theta(z)))),$$

which yields the average $\mathrm{E}[\theta]$ of θ as follows:

$$\mathrm{E}[\theta] = \frac{\mathrm{E}[B]}{1-\rho}. \tag{13.19}$$

From (13.18), we can obtain the P.G.F. $\Theta(z)$ of Θ as

$$\Theta(z) = Q_{\Theta}(z)|_{z=\theta(z)} = \frac{\Lambda(\theta(z)) - \lambda_0}{1-\lambda_0} U(\Lambda(\theta(z))). \tag{13.20}$$

Differentiating (13.20) with respect to z at $z = 1$ and using (13.19), the average $\mathrm{E}[\Theta]$ of Θ is then obtained as

$$\mathrm{E}[\Theta] = \frac{\mathrm{E}[\Lambda](1+(1-\lambda_0)\mathrm{E}[U])}{(1-\lambda_0)} \times \frac{\mathrm{E}[B]}{(1-\rho)}. \tag{13.21}$$

The idle period I is a residual interarrival; due to the memoryless geometrically distributed interarrival time, we can obtain the average $\mathrm{E}[I]$ of I as

$$\mathrm{E}[I] = \frac{1}{1-\lambda_0}. \tag{13.22}$$

Substituting (13.21) and (13.22) to (13.17), the average $\mathrm{E}[R]$ of the busy cycle R can be given as

$$\begin{aligned} \mathrm{E}[R] &= \mathrm{E}[U] + \frac{\mathrm{E}[\Lambda](1+(1-\lambda_0)\mathrm{E}[U])}{(1-\lambda_0)} \times \frac{\mathrm{E}[B]}{(1-\rho)} + \frac{1}{1-\lambda_0} \\ &= \frac{1+(1-\lambda_0)\mathrm{E}[U]}{(1-\lambda_0)(1-\rho)}. \end{aligned} \tag{13.23}$$

13.4 Performance Analysis for Different Kinds of ARQ Schemes

Based on the analysis presented in Sect. 13.3, we can obtain the following performance measurements of the system.

13.4.1 Performance Measures

Response time T is defined as the total delay of a data frame. In our analysis, T is subdivided into two parts. One is the waiting time W of this data frame, which is the time spent in the buffer before its transmission. The other is the corresponding transmission period B of this data frame. The average $\mathrm{E}[T]$ of T is given as follows:

$$\mathrm{E}[T] = \mathrm{E}[W] + \mathrm{E}[B]. \tag{13.24}$$

Substituting (13.16) to (13.24), we have that

$$\mathrm{E}[T] = \frac{\Lambda^{(2)}\mathrm{E}^2[B] + \mathrm{E}[\Lambda]B^{(2)}}{2(1-\rho)} + \frac{\mathrm{E}[\Lambda]U^{(2)} + 2\mathrm{E}[U]}{2(1+\mathrm{E}[\Lambda]\mathrm{E}[U])} + \frac{\Lambda^{(2)}\mathrm{E}[B]}{2\mathrm{E}[\Lambda]} + \mathrm{E}[B]. \tag{13.25}$$

The setup ratio γ is defined as the number of times that the system goes into the setup period U in a slot. There is a setup period U in the busy cycle R. The setup ratio γ can be given by

$$\gamma = \frac{1}{\mathrm{E}[R]}. \tag{13.26}$$

Substituting (13.23) to (13.26), we have that

$$\gamma = \frac{(1-\lambda_0)(1-\mathrm{E}[\Lambda]\mathrm{E}[B])}{1+(1-\lambda_0)\mathrm{E}[U]}. \tag{13.27}$$

We define the offered load ρ as the average number of data frames actually transmitted during a transmission period B, so the offered load ρ is given by

$$\rho = \mathrm{E}[\Lambda]\mathrm{E}[B]. \tag{13.28}$$

13.4.2 Performance Analysis for ARQ Schemes

In this subsection, we present the performance analysis for ARQ schemes. There are three kinds of basic ARQ schemes: Stop-and-Wait ARQ scheme, Go-Back-N ARQ scheme, and Selective-Repeat ARQ scheme. The principles and the differences among the different ARQ schemes are shown in Figs. 13.1–13.3.

To give the formulas for the performance measures for different kinds of ARQ schemes, the following assumptions and notions are introduced.

(1) The transmissions of the ACK frame and the NACK frame are error-free, and the lengths of the ACK frame and the NACK frame are omitted.

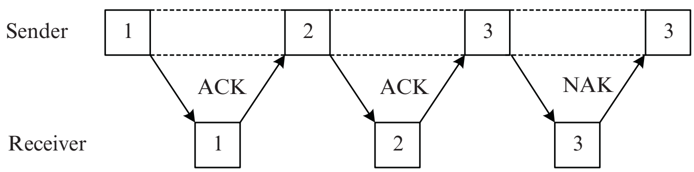

Fig. 13.1 The principle for a Stop-and-Wait ARQ scheme.

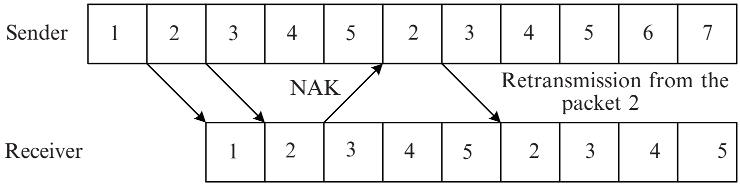

Fig. 13.2 The principle for a Go-Back-N ARQ scheme.

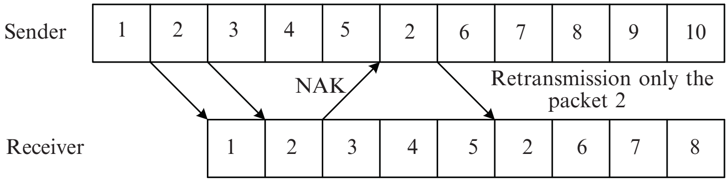

Fig. 13.3 The principle for a Selective-Repeat ARQ scheme.

(2) The rate of the transmission error is e $(0 \leq e \leq 1)$. Each data frame is correctly transmitted with probability $v = 1 - e$ $(0 \leq v \leq 1)$, and each data frame will be transmitted or retransmitted until correct reception is achieved.
(3) The round-trip time is assumed to be d slots as a system parameter.

Let N be the number of times of transmission needed for a data frame to be received correctly. Then the probability distribution and the P.G.F. $N(z)$ of N can be given as follows:

$$P\{N = n\} = (1-v)^{n-1}v, \qquad n = 1, 2, \ldots,$$
$$N(z) = \sum_{n=1}^{\infty} P\{N = n\} z^n = \frac{vz}{1-(1-v)z}. \tag{13.29}$$

In the system with a Stop-and-Wait ARQ scheme, we denote by $B_{SW}(z)$, $\mathrm{E}[B_{SW}]$, and $B_{SW}^{(2)}$ the P.G.F. $B(z)$, the average $\mathrm{E}[B]$, and the second factorial moment $B^{(2)}$ of the transmission period B, respectively. From (13.25), we can give the average response time $\mathrm{E}[T]$ denoted by $\mathrm{E}[T_{SW}]$ for a Stop-and-Wait ARQ scheme as follows:

$$\mathrm{E}[T_{SW}] = \frac{\Lambda^{(2)}\mathrm{E}^2[B_{SW}] + \mathrm{E}[\Lambda]B_{SW}^{(2)}}{2(1-\rho)} + \frac{\mathrm{E}[\Lambda]U^{(2)} + 2\mathrm{E}[U]}{2(1+\mathrm{E}[\Lambda]\mathrm{E}[U])}$$
$$+ \frac{\Lambda^{(2)}\mathrm{E}[B_{SW}]}{2\mathrm{E}[\Lambda]} + \mathrm{E}[B_{SW}]. \tag{13.30}$$

Each transmission in a Stop-and-Wait ARQ scheme will take $1+d$ slots, no matter whether the transmission is correct or not. So, $B_{SW}(z)$ [3], $\mathrm{E}[B_{SW}]$, and $B_{SW}^{(2)}$ are given as follows:

$$B_{SW}(z) = N(z^{1+d}) = \frac{vz^{1+d}}{1-(1-v)z^{1+d}}, \tag{13.31}$$

$$\mathrm{E}[B_{SW}] = \frac{1+d}{v}, \tag{13.32}$$

$$B_{SW}^{(2)} = \frac{(1+d)(vd+2(1-v)(1+d))}{v^2}. \tag{13.33}$$

Substituting (13.32) to (13.27) and (13.28), we can give the setup ratio γ_{sw} and the offered load ρ_{sw} as follows:

$$\begin{aligned}\gamma_{sw} &= \frac{(1-\lambda_0)(1-\mathrm{E}[\Lambda]\mathrm{E}[B_{SW}])}{1+(1-\lambda_0)\mathrm{E}[U]} \\ &= \frac{(1-\lambda_0)(v-\mathrm{E}[\Lambda](1+d))}{v(1+(1-\lambda_0)\mathrm{E}[U])}, \\ \rho_{sw} &= \mathrm{E}[\Lambda]\mathrm{E}[B_{SW}] = \frac{\mathrm{E}[\Lambda](1+d)}{v}.\end{aligned}$$

In the system with a Go-Back-N ARQ scheme, we denote by $B_{GBN}(z)$, $\mathrm{E}[B_{GBN}]$, and $B_{GBN}^{(2)}$ the P.G.F. $B(z)$, the average $\mathrm{E}[B]$, and the second factorial moment $B^{(2)}$ of the transmission period B, respectively. From (13.25), we can give the average response time $\mathrm{E}[T]$ denoted by $\mathrm{E}[T_{GBN}]$ for a Go-Back-N ARQ scheme as follows:

$$\begin{aligned}\mathrm{E}[T_{GBN}] = {} & \frac{\Lambda^{(2)}\mathrm{E}^2[B_{GBN}] + \mathrm{E}[\Lambda]B_{GBN}^{(2)}}{2(1-\rho)} + \frac{\mathrm{E}[\Lambda]U^{(2)} + 2\mathrm{E}[U]}{2(1+\mathrm{E}[\Lambda]\mathrm{E}[U])} \\ & + \frac{\Lambda^{(2)}\mathrm{E}[B_{GBN}]}{2\mathrm{E}[\Lambda]} + \mathrm{E}[B_{GBN}].\end{aligned} \tag{13.34}$$

In a Go-Back-N ARQ scheme, each error transmission occupies $1+d$ slots, and the last correct transmission takes one slot. So, $B_{GBN}(z)$ [3], $\mathrm{E}[B_{GBN}]$, and $B_{GBN}^{(2)}$ are given as follows:

$$B_{GBN}(z) = \frac{N(z^{1+d})}{z^d} = \frac{vz}{1-(1-v)z^{1+d}}, \tag{13.35}$$

$$\mathrm{E}[B_{GBN}] = \frac{1+(1-v)d}{v}, \tag{13.36}$$

$$B_{GBN}^{(2)} = \frac{(1-v)(1+d)(2+2d-vd)}{v^2}. \tag{13.37}$$

Substituting (13.36) to (13.27) and (13.28), we can give the setup ratio γ_{GBN} and the offered load ρ_{GBN} as

$$\gamma_{GBN} = \frac{(1-\lambda_0)(1-\mathrm{E}[\Lambda]\mathrm{E}[B_{GBN}])}{1+(1-\lambda_0)\mathrm{E}[U]}$$
$$= \frac{(1-\lambda_0)(v-\mathrm{E}[\Lambda](1+(1-v)d))}{v(1+(1-\lambda_0)\mathrm{E}[U])},$$
$$\rho_{GBN} = \mathrm{E}[\Lambda]\mathrm{E}[B_{GBN}] = \frac{\mathrm{E}[\Lambda](1+(1-v)d)}{v}.$$

In the system with a Selective-Repeat ARQ scheme, we denote by $B_{SR}(z)$, $\mathrm{E}[B_{SR}]$, and $B_{SR}^{(2)}$ the P.G.F. $B(z)$, the average $\mathrm{E}[B]$, and the second factorial moment $B^{(2)}$ of the transmission period B, respectively. From (13.25), we can give the average response time $\mathrm{E}[T]$ denoted by $\mathrm{E}[T_{SW}]$ for a Stop-and-Wait ARQ scheme as follows:

$$\mathrm{E}[T_{SR}] = \frac{\Lambda^{(2)}\mathrm{E}^2[B_{SR}]+\mathrm{E}[\Lambda]B_{SR}^{(2)}}{2(1-\rho)} + \frac{\mathrm{E}[\Lambda]U^{(2)}+2\mathrm{E}[U]}{2(1+\mathrm{E}[\Lambda]\mathrm{E}[U])}$$
$$+\frac{\Lambda^{(2)}\mathrm{E}[B_{SW}]}{2\mathrm{E}[\Lambda]} + \mathrm{E}[B_{SR}].$$

Each transmission in a Selective-Repeat ARQ scheme, no matter whether it is correct or not, takes, one slot. So, $B_{SR}(z)$, $\mathrm{E}[B_{SR}]$, and $B_{SR}^{(2)}$ are given as follows:

$$B_{SR}(z) = N(z) = \frac{vz}{1-(1-v)z}, \tag{13.38}$$

$$\mathrm{E}[B_{SR}] = \frac{1}{v}, \tag{13.39}$$

$$B_{SR}^{(2)} = \frac{2(1-v)}{v^2}. \tag{13.40}$$

Substituting (13.39) to (13.27) and (13.28), we can also give the setup ratio γ_{SR} and the offered load ρ_{SR} as follows:

$$\gamma_{SR} = \frac{(1-\lambda_0)(1-\mathrm{E}[\Lambda]\mathrm{E}[B_{SR}])}{1+(1-\lambda_0)\mathrm{E}[U]}$$
$$= \frac{(1-\lambda_0)(v-\mathrm{E}[\Lambda])}{v(1+(1-\lambda_0)\mathrm{E}[U])},$$
$$\rho_{SR} = \mathrm{E}[\Lambda]\mathrm{E}[B_{SR}] = \frac{\mathrm{E}[\Lambda]}{v}.$$

13.5 Numerical Results

In line with prevalent wireless network applications, we let the transmission rate be 50 Mbps. To ensure that the latest conflict signal is sensed by the transmitter before a data frame is completely sent out, we assume the size of a data frame to be 1,250 bytes and the round-triptime to be 0.1 ms. The setup period U follows a geometrical distribution with an average value of 0.2 ms.

At the same time, taking into account the burst data shown in Internet traffic, we suppose the batch size Λ to be a Pareto(c,α) distribution with $\lambda_k = ck^{-(\alpha+1)}$, $k = 0,1,\ldots$, where c is a normalization factor for $\sum_{k=1}^{\infty}\lambda_k = 1$, and the parameter α is related to the Hurst factor H by $H = (3-\alpha)/2, 0.5 < H < 1, 1 < \alpha < 2$. The smaller the result of α is, the more the burst is shown in Internet traffic. Especially, there is no self-similarity when $\alpha = 2$. Some research shows that the transmission mode of the browser shows self-similarity [9] with $\alpha = 1.16-1.5$ and the data of each signal source are self-similar [10] with $\alpha = 1.2$.

With these parameters, we show the setup ratio γ and offered load ρ as functions of the batch arrival rate $\lambda_g = 1-\lambda_0$ (batches/slot) with the rate of the transmission error $e = 0.1$ under the burst degree of $\alpha = 1.2, 1.6, 2.0$, respectively. For different kinds of ARQ schemes in Figs. 13.4–13.9, where $\alpha = 2.0$ means that there is actually no self-similarity.

In Figs. 13.4–13.6, we show how the setup ratio γ changes with the batch arrival rate λ_g with the rate of the transmission error $e = 0.1$ and with the parameter of burst degree $\alpha = 1.2, 1.6, 2.0$ for different ARQ schemes. It should be noted that for all the burst degree parameters, the setup ratio γ experiences a two-stage trend. In the first stage, the setup ratio γ will increase along with the batch arrival rate λ_g. During this stage, the greater the batch arrival rate λ_g is, the higher the number of data frames arriving in the idle period I will be, and the greater the number of times needed for the setup procedure will be. In the second stage, the setup ratio γ will decrease with the incremental batch arrival rate λ_g. During this period, the greater the batch arrival rate λ_g is, the higher the number of data frames arriving in the busy period Θ will be, and these data frames can be transmitted directly without any setup procedure.

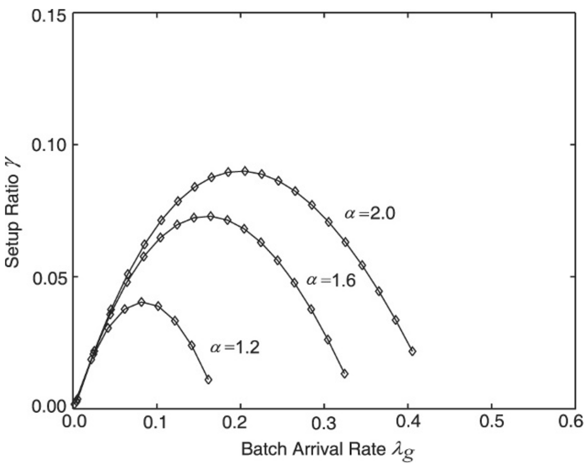

Fig. 13.4 Setup ratio γ for a Stop-and-Wait ARQ scheme.

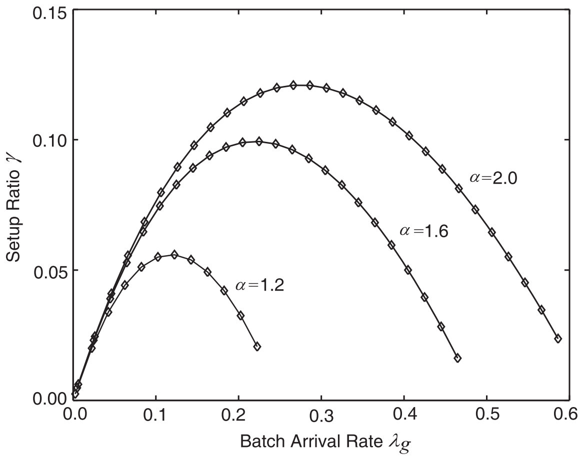

Fig. 13.5 Setup ratio γ for a Go-Back-N ARQ scheme.

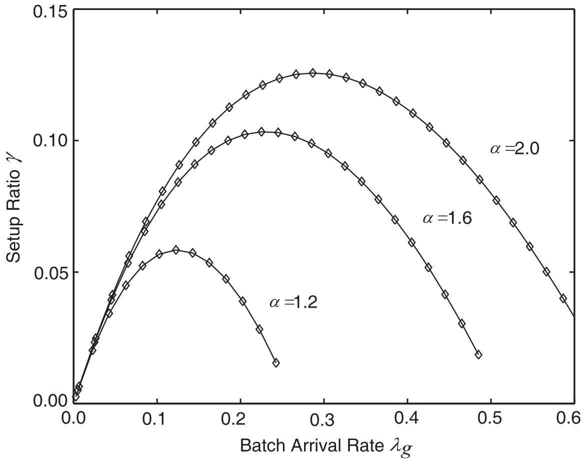

Fig. 13.6 Setup ratio γ for a Selective-Repeat ARQ scheme.

There is a maximal setup ratio γ for all the burst degree parameters, and it can also be observed that the larger the burst degree parameter α is, the greater the maximal setup ratio γ will be, and we can conclude that if we omitted any self-similar Internet traffic, the setup ratio γ would be overevaluated.

In Figs. 13.7–13.9, we compare the offered load ρ with the rate of the transmission error $e = 0.1$ versus batch arrival rate λ_g for the parameters of burst degree $\alpha = 1.2, 1.6, 2.0$ for different ARQ schemes. It can be found that with an increasing batch arrival rate λ_g, the offered load ρ increases also for all the ARQ schemes and all the parameters of burst degree. It should be noted that for the same batch arrival rate λ_g, the lower the parameter of burst degree α is, the larger the offered

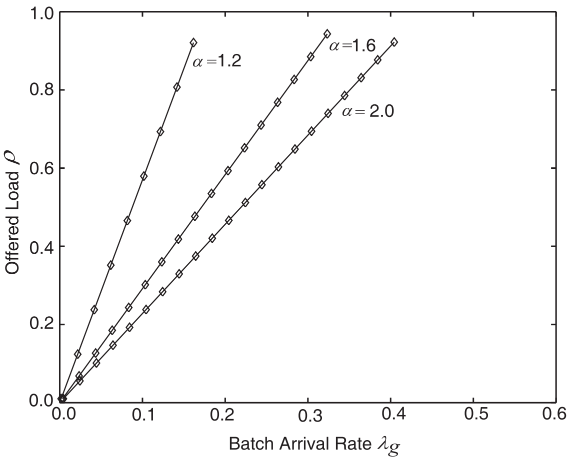

Fig. 13.7 Offered load ρ for a Stop-and-Wait ARQ scheme.

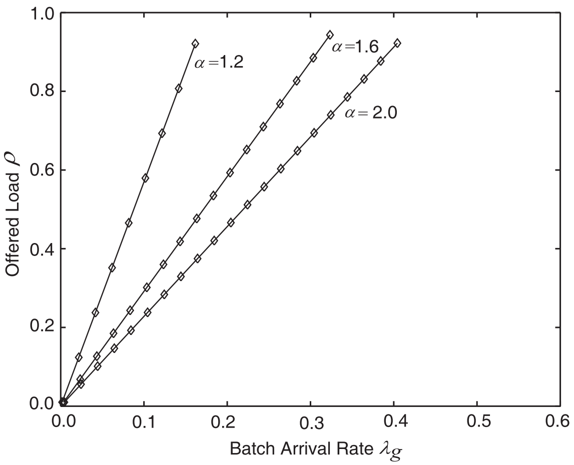

Fig. 13.8 Offered load ρ for a Go-Back-N ARQ scheme.

load ρ will be for all the ARQ schemes. Therefore, we can conclude that if the self-similarity is not considered, the offered load ρ would be underevaluated.

Due to the finite first factorial moment and the infinite second moment of a Pareto distributed stochastic variable, some other performance measures such as the average response time $\mathrm{E}[T]$ in (13.24) are difficult to calculate analytically. So we present the change trend of average response time $\mathrm{E}[T]$ by using the method of simulation.

There is no ready Pareto function in most simulation tools such as Matlab to be used, so we use an inverse function method to generate random number sequences following the Pareto distribution.

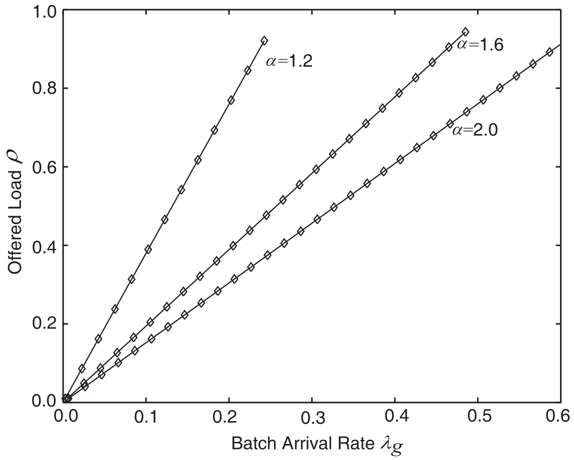

Fig. 13.9 Offered load ρ for a Selective-Repeat ARQ scheme.

Table 13.1 Response time $\mathrm{E}[T]$ of different ARQ schemes for various λ_g with $\alpha = 1.2$ and $e = 0.1$.

Batch Arrival Rate λ_g	0.001	0.02	0.04	0.06	0.08	0.10
Stop-and-Wait ARQ	2.96	39.17	41.30	517.66	1414.30	14087
Go-Back-N ARQ	5.22	77.13	113.14	731.88	4655.90	6130
Selective-Repeat ARQ	5.44	413.99	2671.40	3453.50	6645.60	24328

The general discrete distribution is characterized as follows:

$$\begin{aligned} p_k &= P\{X = k\}, \qquad k \geq 0, \\ F(m) &= \sum_{k=0}^{m} p_k, \qquad m \geq 0. \end{aligned} \tag{13.41}$$

By using a random numbers generation function, we generate random numbers of a $1 \times n$ vector named M whose elements are uniformly distributed in the interval (0,1). On the other hand, following the inverse function method, we introduce another $1 \times n$ vector named N whose elements are set by $N(i) = \min\{m : F(m) > M(i)\}$, where $F(m)$ is given in (13.41) and $m \geq 1$, $i \geq 1$. In this way, the data in the vector N will be Pareto distributed.

The change trend of average response time $\mathrm{E}[T]$ for different ARQ schemes when $\alpha = 1.2$ and $e = 0.1$ with various λ_g is presented in Table 13.1. The measurement of average response time $\mathrm{E}[T]$ behavior for different ARQ schemes when $\alpha = 1.2$ and $\lambda_g = 0.04$ with various error rates e is shown in Table 13.2.

From Tables 13.1 and 13.2, we can observe that with an increasing batch arrival rate λ_g or an increasing error rate e, the average response time $\mathrm{E}[T]$ increases also and tends to be infinite for all the ARQ schemes. This is because of the self-similarity shown in the size of the data frame batch, which is in fact the reason why network performance deteriorates in self-similar traffic.

Table 13.2 Response time E$[T]$ of different ARQ schemes for various e with $\alpha = 1.2$ and $\lambda_g = 0.04$.

Error Rate e	0.02	0.06	0.10	0.14	0.18	0.22
Stop-and-Wait ARQ	17.065	26.081	41.30	64.139	2256.7	7957.9
Go-Back-N ARQ	15.268	63.604	113.14	173.91	686.78	778.46
Selective-Repeat ARQ	70.924	228.49	2671.40	2699.6	7813.5	9924.8

13.6 Conclusions

In this chapter, we presented a new method to analyze the performance of high-reliability Internet systems in self-similar traffic with ARQ schemes. Considering the self-similar nature widely shown in Internet traffic and the setting up procedure of a data link, we built a batch arrival Geom^X/G/1 queue model with a setup strategy. We analyzed the stationary distribution of the system, derived the Probability Generation Functions (P.G.Fs.) of the queueing length and the waiting time of the system. Correspondingly, we gave the formula for performance measures in terms of response time, setup ratio, and offered load for different kinds of ARQ schemes. We presented numerical results to evaluate and compared these performance measures, and to show the influence of the burst degree in self-similar traffic and the delay in the setup procedure on the system performance.

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