Theme 2.3
Tools and Settings Supporting Mathematics Teachers' Learning in and from Practice

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1. Overview

The purpose of this chapter is to present a synthesis of the papers from Strand II which address the tools, dynamics, tasks, contexts, and learning settings that can be mobilized for pre-service and in-service mathematics teacher education. Within this focus, we have identified four topics around which our chapter is organized. In the first section, we deal with tasks for mathematics teacher education, including mathematical problems and activities, which are offered to teachers as opportunities for them to deepen their knowledge of what they have to teach to students and how they can teach this. These tasks are at the heart of mathematics teacher education and determine what teachers are learning, along with several working forms, dynamics, and contexts. Closely connected to the tasks is the topic that is addressed in the next section, the analysis of instructional episodes. These episodes include narrative cases, video cases, and lesson studies. They all provide opportunities for teachers to study and reflect on teaching-in-action. The last two sections tell us more about the context in which teachers' learning takes place. The former deals with *teachers' learning communities*, addressing teachers as learners in communities that constitute an environment in which the participants share experiences, meanings, knowledge, lessons, and stories about the school's practice. The latter describes e-learning in mathematics teacher education and confirms again the power of communities even when these communities are virtual.

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2. Features of Tasks for Mathematics Teacher Education

Teacher education aims at transforming prospective and practicing teachers from novice perspectives on teaching and learning mathematics to more professional perspectives for dealing with the challenges that teaching mathematics presents. This transformation occurs most advantageously through engagement in tasks that foster knowledge for teaching mathematics. Such tasks play a critical role in the learning offers that can be made to participants in various teacher education contexts and settings. We define these tasks as the problems or activities that are posed to teacher education participants. The tasks might be similar to tasks used in classrooms (e.g., the analysis of a graphing problem) or distinctive to teacher education (e.g., an analysis of a videotaped lesson or curriculum material).

Tasks play a significant role in teaching and learning (Krainer, 1993; Sullivan & Mousley, 2001; Zaslavsky, 2005). Hiebert & Wearne (1993) maintain that "what students learn is largely defined by the tasks they are given" (p. 395). This view may be extended to any learner, including prospective and practising teachers. According to Kilpatrick, Swafford, & Findell (2001), "[t]he quality of instruction depends, for example, on whether teachers select cognitively demanding tasks, plan the lesson by elaborating the mathematics that the students are to learn through those tasks, and allocate sufficient time for the students to engage in and spend time on the tasks" (p. 9). Sierpinska (2004) argues that the design, analysis, and empirical testing of mathematical tasks are among the most important responsibilities of mathematics education.

Stein & Smith (1998) suggest a three-phase framework of mathematical tasks used in classrooms for analyzing mathematics lessons. Their framework provides a tool for describing how tasks unfold during classroom instruction, as well as for highlighting the significant influences tasks have on what students actually learn (Henningsen & Stein, 1997).

There is a consensus that a key issue to be addressed in mathematics teacher education is the learning of mathematics. In several models accounting for learning and teaching (e.g., Jaworski, 1994; Steinbring, 1998; Zaslavsky & Leikin, 2004) engagement in meaningful and challenging mathematically related tasks is an important component. In Jaworski's (1994) teaching triad, as well as in Zaslavsky & Leikin's (2004) extension of the triad for mathematics teacher educators, the important role of the task is expressed in the demand for mathematical challenge. In Steinbing's (1998) model of teaching and learning mathematics as autonomous systems, as well as in Zaslavsky & Leikin's (2004) extension of it, solving problems and reflecting on them are critical elements. Zaslavsky (2005) points to the dual nature of tasks from a mathematics educator standpoint: on the one hand, tasks are tools for facilitating teacher learning; on the other hand, through a reflective process of designing and empirically testing tasks, they turn into means of enhancing learning of the facilitator. Zaslavsky (ibid.) offers a detailed account of how a task for mathematics teacher education, both pre-service and in-service, evolved through an iterative process of reflection and how her own learning as teacher educator evolved through this process.

Zaslavsky, Chapman, & Leikin (2003) make an attempt to articulate what may be considered productive tasks for mathematics teacher education. They consider most worthwhile in-service and pre-service activities as problem-solving situations—combining mathematics and pedagogy—which engage participants in "powerful tasks" fundamental for teacher development (e.g., Sullivan & Mousley, 2001; Krainer, 1993). Accordingly, tasks should offer in engaging and challenging ways mathematical and pedagogical problem-solving situations, in which relevant issues are addressed, including sensitivity to learners and reform-oriented approaches to management of learning (Jaworski, 1994). Zaslavsky et al. (ibid.) "expect powerful tasks to be open-ended, non-routine problems, in the broadest sense, that lend themselves well to collaborative work and social interactions, elicit deep mathematical and pedagogical considerations and connections, and challenge personal conceptions and beliefs about mathematics and about how one comes to understand mathematics" (p. 899). Moreover, worthwhile tasks should present challenges to the facilitator as well as to the learners (perhaps with some modifications and adaptations). By this, both educators and learners may have the opportunities to encounter and reflect on very similar experiences.

Many mathematics educators share the view that teaching is strongly influenced by a teacher's personal experiences as a learner (e.g., Stigler & Hiebert, 1999; Zaslavsky, 1995). Thus, many tasks offered for teacher education make affordances for teachers to experience the kind of mathematics and pedagogy that they are expected to offer their students. In-service teacher educators are usually rather flexible and "have a considerable latitude in terms of defining their curricula" (Cooney, 2001, p. 15) and do not face the same time and other constraints that teachers face in their practice. Consequently, tasks enhancing professional growth vary to a large extent with respect to their nature, content, and focus.

Zaslavsky et al. (2003) offer examples of six types of mathematical-related tasks which they consider "powerful tasks" and provide some indications for their potential contributions for teacher learning. Note that these types of tasks lend themselves particularly well to teacher workshops and settings in which a teacher-educator takes a leading role in designing activities and offering them to teachers. These examples involve the following features of tasks for teacher development:

- Dealing with uncertainty and doubt (e.g., resolving cognitive conflict)
- Rethinking mathematics and considering alternatives [Orit Zaslavsky][1] (e.g., considering alternative definitions)
- Engaging in multiple approaches to problem solving (e.g., open-ended problems)
- Identifying mathematical similarities and differences (e.g., through sorting tasks)
- Developing a critical view of the use of educational technology (e.g., learning to appreciate the merits and limitations of "free search" vs. "structure construction")
- Learning from students' thinking (e.g., by becoming familiar with and aware of students' potential responses and creative ideas)

[1] The references in brackets refer to the boxes that are included in the main text.

Orit Zaslavsky

Task: Considering alternative definitions of a square

(Based on Shir & Zaslavsky, 2001; Zaslavsky & Shir, 2005) This task served as the basis for several workshops with secondary mathematics teachers. Teachers received eight equivalent statements describing a square (note that they were not told that these statements were equivalent). They were first asked to individually determine, for each statement, whether they accepted it as a definition of a square. Then they formed groups and discussed their viewpoints and preferences from both mathematical and pedagogical perspectives.

Following are the statements the teachers were given:

1. A square is a quadrangle in which all sides are equal and all angles are 90°.
2. Of all the rectangles with a fixed perimeter, the square is the rectangle with the maximum area.
3. A square is a rhombus with a right angle.
4. A square is an object that can be constructed as follows: sketch a segment, from both edges erect a perpendicular to the segment, each equal in length to the segment. Sketch the segment connecting the other 2 edges of the perpendiculars. The 4 segments form a quadrangle that is a square.
5. A square is a quadrangle with diagonals that are equal, perpendicular, and bisect each other.
6. A square is a rectangle with perpendicular diagonals.
7. A square is the locus of points for which the sum of their distances from two given perpendicular lines is constant.
8. A square is a regular quadrangle.

This task evoked much debate regarding the nature and role of a mathematical definition that drew on teachers' numerous conceptions of necessary features and purpose of use. Clearly, it was a unique experience for many of them—the mere idea of having a choice of what definition to choose and relating it to context was new to them. In addition, considering the alternative statements led them to employ deep mathematical considerations in the course of examining whether the statements are equivalent. Teachers became aware of mathematical subtleties associated with conventions, arbitrariness and freedom of choice. They also appeared to develop an appreciation of the value of such (open) task, and its potential for classroom implementation (Zaslavsky, 2005).

In spite of the significant role tasks play, so far there have been relatively few publications focusing on (mathematical-related) tasks in mathematics teacher education. For example, an examination of articles published in *JMTE* in the past decade indicates seven articles that explicitly address in detail and offer descriptions and analyses of teachers' ways of dealing with tasks facilitated by teacher educators. Interestingly, of these seven articles, three deal with engagement of teachers in case-study or practice-based tasks involving analyses, construction, and/or reflections on real or imagined cases (such as described in the next section of this paper) (Lin, 2002; Masingila & Doerr, 2002; Walen & Williams, 2000); two engage teachers in sorting tasks calling for examination of deep structures—in which they were required to offer various ways of sorting of either mathematical objects (Zaslavsky & Leikin, 2004) or mathematical tasks for middle-school students (Arbough & Brown, 2005); and the other two articles use a specific classroom mathematical problem (or set of problems) to raise teacher awareness and enhance their content and pedagogical knowledge by implementing the task and considering alternative ways of unfolding it in the classroom—one used a sequence of modelling tasks (Doerr & English, 2006), while the other used a seemingly real-life geometrical task (Fried & Amit, 2005).

In the Study Conference, held in Brazil in 2005, there were two papers that provide examples of some of the types of mathematical-related tasks for teacher education indicated previously. One of these papers (Gilda de La Rocque Palis, 2005) describes an experience that concurs with the last type of tasks described previously, that is, learning from students' thinking [Gilda de La Rocque Palis]. The second paper (Zaslavsky & Lavie, 2005) can be seen as an attempt to make sense of teacher practice and consider alternatives, that is, alternative instructional examples for a given situation [Zaslavsky & Lavie].

Gilda de La Rocque Palis

Activities based on student work

This paper describes structured activities in which secondary teachers engaged collaboratively, focusing on real students' responses to a mathematical problem. Activity 1 is the following:

1. Individually: Answer the question and hand in your resolution: "Let r be the line $y = 3x - 1$ and P the point (1/2,3/5). Decide if P belongs to r, is below or above it".
2. Individually: Give grades to students whose copies were handed out.
3. Whole group: Construct a table showing how many teachers gave grade X to student Y, for all X and Y. Analysis of the table. Free discussion and justification of given grades.
4. In small group: Carry out a new analysis and grading of the same works.
5. Whole group: Compare criteria and grades given by the small groups.
6. Homework: Bring a written account of what you think you have learned through this activity.

Teachers were very much engaged in the activity, individually and collectively. The trust that was built within the group was essential to let the participants to expose their difficulties, sometimes basic ones. Teachers could give extremely distinct grades to the same work but group work took care of those differences. They said explicitly that they never have time to grade students' answers carefully; they have a model and check student resolutions against the model. It seems that some teachers do not even read what does not look similar to what they already expect to see.

Two similarly structured tasks are activities 2 and 3. Activity 2 deals with students' responses to "Choose 4 points (x,y), one in each quadrant, whose coordinates satisfy the inequality $y < x+1$" and asks for a lesson plan to address spotted difficulties. Activity 3 asks teachers to analyze students' responses to "Consider a circle C with given center and radius. Decide if a given point P belongs to C, is situated inside the region bounded by C or is outside this region" sorting their strategies and frame of work (algebraic, graphical). Then, teachers should compare students' strategies with some textbook approaches to the same topic.

Through the work with these two activities, we confirmed that although misconceptions about the graphical context abound, it seems that both teachers and students were not much aware of their possibilities and limits. This kind of work can contribute to the discussion of what content and pedagogical knowledge mathematics teachers should have constructed and represents an instantiation of how this construction may happen (Palis, 2005).

Orit Zaslavsky & Orna Lavie

From a teacher's use of an instructional example to a task for teachers

As a result of a genuine classroom event concerning a mismatch between a teacher's intention in using a mathematical example in her lesson and a student's response to it, indicating such mismatch, the teacher had to alter her example to accommodate the student's thinking. This authentic classroom event was used by the authors as a trigger for in-service teachers to engage in a discussion of what would be an appropriate example in such a case. The teachers generated several examples, and considered each one along its merits and limitations. Through this process they became aware of and began articulating the complex web of considerations underlying choice of instructional examples (Zaslavsky & Lavie, 2005).

There is now both a growing body of literature and substantial interest in tasks presented to prospective and practicing mathematics teachers by teacher educators that have been found to be effective in addressing specific aspects of mathematics teacher education. This is manifested in a special (triple) issue of *JMTE* (Vol. 10, 4-5-6) dealing with the nature and role of tasks for mathematics teacher education. This issue as well as an AERA SIG/RME symposium on this theme (held at the 2007 meeting in Chicago) indicate a shift towards a recognition of this field as a noteworthy part of teacher education. In the next section we will discuss two particularly powerful kinds of professional learning tasks—video cases and lesson studies.

3. Analysis of Instructional Episodes

Teacher educators, professional developers, and researchers have recently taken great interest in the development and facilitation of practice-based approaches to mathematics teacher education (Smith, 2001). In this approach, teachers engage in activities that are deliberately situated in "the work of teaching"—activities that resemble or replicate components of teachers' daily work, such as planning for mathematics instruction, analyzing student work, and viewing and discussing instructional episodes. At the heart of practice-based approaches to mathematics teacher education is an attempt to assist teachers to develop the knowledge they need in their classroom practice by engaging them in tasks and situations that embody the complex interactions that occur in their classrooms. Towards this end teachers engage with tasks that embody authentic aspects of instructional practice and that allow teachers to access, utilize, and develop knowledge of mathematics content, pedagogy, and student learning simultaneously (Ball & Cohen, 1999). Advocates for this approach argue that learning experiences that are highly connected to and contextualized in professional practice can better enable mathematics teachers to make the kinds of complex, nuanced judgments required in teaching (Ball & Bass, 2003; Gal, 2005; Gal & Linchevski, 2005; Little & McLaughlin, 1993).

Using the work of teaching as a central resource, practice-based approaches attempt to coordinate and link different facets of teacher knowledge to each other and to the settings in which the knowledge is used. In traditional approaches to teacher education, the knowledge domains of mathematics content, mathematics pedagogy, and student thinking tend to be treated separately. In particular, teachers often take some specific courses to learn mathematics, different ones to learn pedagogy, and others to gain information about how students learn. Moreover, the content of the mathematics courses is often provided apart from any deep consideration of its use in the work of teaching. One consequence of such a treatment of knowledge is that the learner is burdened with the responsibility for making the needed connections across domains and recognizing the settings in which the knowledge could be appropriately used. In contrast, in practice-based approaches, the knowledge domains are treated as intertwined, and they are tied to settings in which they appear in the work of teaching.

Mathematics teacher educators have developed and utilized several different kinds of stimuli to support practice-based professional development. Of interest here are those that entail the analysis of instructional episodes. Among the most popular approaches of this kind are those involving narrative or video cases of mathematics instruction and those involving the planning and enactment of specific lessons (often dubbed "lesson study"). These resources for teacher learning make the actual work of teaching available for investigation and inquiry (Smith, 2001).

There is considerable variation in how teacher educators have structured and conveyed instructional episodes in their work, but narrative and video cases generally present an entire lesson (or some significant portion of a lesson), providing an edited account of the actions and interactions of a teacher with students as they work on the mathematics at stake in the lesson. In lesson study, teachers typically engage in collaborative lesson planning, followed by observation and analysis of the enacted lesson.

Video and narrative cases can be deliberately constructed to provoke discussion regarding interactions among the teacher, the students, and the mathematical task in the case, and the way in which those interactions affect students' opportunities to learn mathematics. In addition, cases can provide opportunities for teachers to consider, and sometimes learn, mathematics content as they examine the mathematics featured in the case, participate in analytic discussions about mathematics, and consider how mathematics develops in the lesson depicted in the case.

Video cases have been used effectively in different ways. For example, Gal & Linchevski (2000) were specifically interested in video cases which present identifiable situations that arise in the course of teaching when *the teacher* is unsuccessful in helping the *students* overcome difficulties encountered during studying. These double-foci class situations were named "problematic learning situations" (PLS) and were widely used in pre- and in-service teachers' yearly academic programme, which provided tools to enlarge teachers' awareness of and being able to analyze and cope with such PLS [Gal & Linchevski].

Hagar Gal & Liora Linchevski

Changes in teachers' ways of coping with problematic learning situations in geometry instruction

This paper describes the findings of a study (Gal, 2005) that examines changes in teachers' ways of coping with problematic learning situations (PLS, after Gal & Linchevski, 2000) in geometry instruction. PLS refer to situations in which the student faces difficulty and the teacher has difficulty in helping the student. The intervention was planned as a yearly academic course for pre-service and in-service junior high school teachers of mathematics. The course aimed to enhance teachers' ability to identify, analyze and cope with PLS, expand and deepen their awareness and understanding of students' ways of thinking, and to enhance their ability to retrieve and utilize relevant knowledge while making instructional decisions.

The course combined theoretical pedagogical knowledge with its practical application (using videotaped classroom events involving PLS) to reveal the difficulties of both students and teachers and to illustrate how the theories learned in the course could explain these difficulties. The course was spiral in nature; the participants in the course were first presented with PLS events for which the theoretical material they had already learned could provide an explanation. Then, new theoretical material was presented, after which some of these PLS examples were presented a second time so that they could be re-analyzed in light of the new approaches and so that participants could track their own progress by comparing their earlier interpretations to new ones. At the same time, new PLS examples were presented which could be analyzed in terms of either the old or new theoretical material and over again.

The study was based on three groups: (1) nine B.Ed. students, (2) seven students with a B.Sc. degree in mathematics studying towards a junior and senior high school teaching diploma, (3) twenty-three M.A. students, all in-service teachers. The study analyzed changes in: (1) teachers' awareness of difficulties and ability to identify them, (2) teachers' ability to analyze difficulties by means of cognitive theories, and (3) teachers' ability to suggest effective ways of coping with difficulties.

The findings show a very significant change in the teachers' ability to identify and analyze difficulties, both in laboratory settings and during classroom instruction. Treatment of PLS became an inherent part of their instruction. They were trying to follow their students' thinking processes in real time. Teachers' ability to cope with difficulties in laboratory settings underwent a marked change. The teachers also exhibited an ability to cope with difficulties during classroom instruction, and could analyze and suggest ways to cope with them retrospectively.

The main conclusions were that: (1) the intervention was long enough for most participants to be able to identify, analyze, and suggest appropriate solutions to PLS in laboratory settings; (2) one academic year was not enough in terms of classroom instruction; and (3) overall, there were no essential differences between changes in experienced teachers and novices, or between teachers with a strong or weaker background in mathematics. There were between-participant differences in the extent of change (Gal & Linchevski, 2005).

Another approach is evident in the work of Bao & Huang (2007), who describe how teachers can learn from using multimedia technology to create hypermedia video cases, which record, evaluate, and integrate the crucial elements of exemplary lessons. According to Huang & Bao (2006a) the process of developing an exemplary lesson in the form of a video case study has several advantages: emphasizing professional learning and upgrading teaching, learning ideas, and theories; supporting reflection on or being in action; encouraging revision of design and enactment of new action; and engaging teachers with the process of choosing episodes and creating cases or narratives. Video case studies can include the analysis of lesson episodes from different perspectives and by different agents (acting teacher, colleges, master teacher, students), and this multiplicity of views enhances teachers' awareness and ability to reflect on and improve their own practice (Huang & Bao, 2006b).

A key element in practice-based professional development anchored by video or narrative records of classroom teaching episodes is a well-designed, well-facilitated professional learning task (Ball & Cohen, 1999). Professional learning tasks are complex tasks that create opportunities for teachers to ponder pedagogical problems and their potential solutions through processes of reflection, knowledge sharing, and knowledge building. See the previous section of this chapter for more on tasks used in teacher professional development; tasks associated with narrative and video cases can be similar to those discussed there but with a specific focus on the particular features of the lesson illustrated in the case.

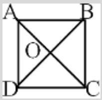
In lesson study, the professional learning task focuses on the design of the lesson and the development of an analysis scheme to determine the extent to which the lesson meets expectations. Gal (2005) studied a spiral method, consisting of teachers' observations of a short video case, followed by an iterative process of reflection on them, study of a theory that can help in analyzing the difficulty presented in the instructional episode, more reflection, study of a new theory, and so on [Gal].

Hagar Gal

Problematic learning situation: Perpendicular lines

(Based on Gal & Vinner, 1997; Gal & Linchevski, 2005).

The following dialogue took place between a (pre-service) teacher (T) and a female student (S) during a geometry class for 9th graders (weak group), where students were asked to examine the properties of a square. The assignment was to check if the diagonals of a square were perpendicular to one another. In the Hebrew mathematical jargon this is expressed by the phrase: the diagonals "cut each other" at a right angle. "Cut" in Hebrew means: divide into two parts, intersect, split! The students were shown the following drawing of a square with its diagonals.



S: (pointing at AC) They "cut" each other here, right? So here it's 90 (points at <ADC).
(She probably means that the diagonal splits the square into two congruent parts. In the triangles obtained as a result, the right angle of the triangle, which is also an angle of the square, is a quite dominant figure).

T: When we speak about perpendicular diagonals...show me the diagonals.
(The teacher tries to locate the source of the difficulty)

S: (Points at AC and BD)

T: That's right. And where do they "cut" each other?

S: (points at diagonal AC and shows that it forms two triangles, ABC and ADC.)
(She probably means that the diagonals split the square in two)

S: (After a brief hesitation): Oh, no, they "cut through" here (points at the four vertices)
(Here, she probably thinks that the question is about the intersection points of the diagonals with the square)

T: Where do these two diagonals "cut each other"?

S: Here (points to O)
T: Well done. Now, when I ask if they are perpendicular to one another, what I mean is that as they "cut each other", are 90° angles formed there (points to O)?
S: Yes! (points to the right angle in each of the two triangles ABC and ADC, i.e., angles <ACB and <ADC.)
(After a while):
Oh, no, these (points to the two other triangles and their angles, i.e., <BAD and <BCD)
T: When I speak about diagonals which are perpendicular to one another, their point of intersection has 90°. Where is that point?
S: (points to O)
T: Now tell me if an angle of 90° is formed between this diagonal (points to AC) and this diagonal (points to BD).
S: (points to <ABC!).

Assignment for the teacher

a. What is your reaction to this section? What are the things that caught your attention?
b. What difficulties for the student arise during the dialogue? Can you explain them?
c. What did you like while observing the teacher?
d. According to the dialogue, what are the difficulties for the teacher?
e. Are you familiar with such a situation? Has something similar ever happened to you?
f. How would you lead the situation if you were this student's teacher? What would you do to overcome the difficulty? (Gal, 2005)

Although there are several similarities between the use of cases and the use of lesson study, there is at least one important difference. In lesson study, the focus is typically on the specific lesson itself, and the aim is the perfection (or "polishing") of that lesson. In contrast, narrative and video cases focus on a specific lesson as an exemplar of a larger class of lessons or as an instance of a more general instructional issue, and the aim is identification and analysis of the more general issue. Thus, learning from lesson study is immediately applicable to practice but its applicability is limited, whereas learning from cases is less immediately connected to practice but its generality affords broad potential applicability.

Although cases and lesson study are certainly different kinds of stimuli for teachers' professional learning, they are in many ways complementary. As Silver et al. (2005, 2006) argue, each approach has strengths and limitations, and the strengths of each address the weaknesses of the other. In particular, case analysis and discussion can be used to build teachers' proficiency with all of the following intellectual practices and professional dispositions that are needed for successful use of lesson study:

- Treat classroom instruction as an object of inquiry in discussions with colleagues
- Adopt analytic stance toward teaching in general
- Learn to make claims based on evidence rather than opinion
- Attend to general instructional goals and issues
- Consider a classroom lesson as a unit of analysis

Similarly, lesson study may complement and enhance the effects of case analysis and discussion by assisting teachers in becoming more proficient in all of the following intellectual practices and professional dispositions that are needed for instructional improvement:

- De-privatize classroom instruction within a professional community so that others can learn from it
- Adopt an analytic stance toward one's own teaching
- Commit to the steady improvement of teaching
- Analyze general instructional issues in relation to one's teaching
- Consider a classroom lesson as a unit of *improvement*

Both approaches to the analysis of lessons have great potential as the basis for professional learning tasks in practice-based professional development endeavors that are focused on enhancing the knowledge of mathematics, pedagogy, and students that teachers use in their classrooms. In the next section we discuss the nature of teachers' learning communities in which activities such as video cases and lesson studies maximize their potential as learning settings.

4. Learning Communities

The role of the mathematics teacher is to create conditions to support mathematics learning in his or her students. However, the professional development of the mathematics teacher may also be seen as a learning process in itself. Teachers learn in practice, for practice, and from practice. They learn as they design their instruction, looking for new ideas, educational materials, and tasks; as they listen to their pupils' answers, questions and comments; and as they reflect on what happened in the classroom and the suitability of their planning and their actions. They learn as they get involved in projects and all sorts of other activities (Ponte & Chapman, 2006).

Seeing the professional development of the mathematics teacher as a learning process helps in mobilizing current views about students' learning and adults' learning to the learning of teachers regarding their professional roles, and this may bring in new ideas about professional development and learning. It is now widely recognized that learning is inextricably an individual and social phenomenon that stands on the activity of the individual carried out in a given context (Lerman, 2001). Teachers learn from themselves, from their activity and their reflection on their activity, but such learning takes place in a particular (school) context and in a social environment, interacting with others, notably, students, colleagues, administrators, parents, and other members of the community.

Teachers learn professional knowledge (including knowledge about teaching in general, didactics of mathematics, and knowledge about mathematics as a school subject), learn professional values, learn about their professional roles, and so forth in close connection with other teachers. They begin their learning during pre-service teacher education, when they interact with teacher educators and also with teachers in schools during field work. They continue their professional learning during their professional career, interacting with other teachers in an informal way in their schools, attending professional meetings, reading the professional literature, attending in-service courses and specialized training programs, or getting involved in projects, study groups, or inquiry groups. Learning communities of teachers are special contexts in which teachers learn. These learning communities can be a class of a pre-service, in-service, or specialized teacher education program, or may be a group of teachers from one school who developed habits of working together, or any other group that was constituted especially with the purpose of learning, developing, or inquiring.

The notion of learning community, thus, refers to a group of people with some sort of stability in terms of membership involved in some kind of activity that learn together and, more importantly, learn from each other (Jaworski, 2004). These learning communities may be homogeneous, formed by people with similar professional roles and backgrounds—for example, teachers from the same grade levels from one single school with similar professional experience. However, such learning communities may be heterogeneous—for example, including both primary and secondary teachers, experienced and new teachers, teachers from different schools, or even teachers and other professionals such as teacher educators. Diversity and heterogeneity create more difficulty in finding a common language and adjusting purposes and ways of working but may be of strong value to the work of the group. Different viewpoints, different experiences, and different expertise may make the group more powerful to identify and deal with issues, thus leading to stronger and deeper learning from their members.

There are four key issues in learning communities. One is the purpose of the group and its relation to the individual purpose of its members. The most important condition for one person to learn is that person wanting to learn. Similarly, the most important condition for one group to learn is its willingness to learn. Therefore, we have to pay attention to the purposes with which the groups are constituted, to what brings people to one group and how they identify with the purposes of the group and assume their personal purposes.

This is important since the very beginning but may evolve with time within that group [Fiorentini et al.].

Dario Fiorentini et al.

Learning through collaboration from professionals with different knowledge

The paper addresses an implicit LC, formed by a heterogeneous group of mathematics teachers (the number varied from 7 to 12) and teacher educators (3). The group begun in 1999 and keeps going on. “Each subject takes part in collaborative work with one’s own purposes and personal needs and, by means of interchange, also learns more about oneself, about the others and about life in general” (p. 2). The purpose of the group “was to face the challenge of changing school practices towards the construction of the teacher current society demands, which cannot be achieved by school teachers and university educators independently” (p. 1). The interest of the mathematics teachers was in studying and improving their teaching practice and evolve professionally and the interest of the teacher educators was to investigate the teachers’ development process in a collaborative context (p. 1).

The dominant topic in this work is the interaction of two different groups of professionals: mathematics school teachers and university math educators and how this interaction is fruitful, originating new knowledge in both parts. This process of generating knowledge involves teachers in “assuming an investigative stance on their practice, developing investigations themselves” (p. 5).

Learning in the group happens through “collaborative work among professionals with different views and knowledge, as well as critical interlocution with studies produced in an academic environment and may contribute not only to addressing such challenges but also to developing teacher autonomy in curriculum management and production of knowledge” (p. 6). This LC shares reflections on: (i) Produced meanings of participants of the group from readings/discussions of papers that discuss new alternatives of teaching in the scholar practice in mathematics; (ii) Narratives/inquiries that had been written by some participants of the group about their own innovative experiences of the practice of classroom. The conditions that made the group have a more fruitful activity, include; “The resignification of participants knowledge took place more effectively when the group studied classroom situations in which student’s thoughts when learning mathematics emerged, and they expressed the meaning attributed to mathematics activities” (p. 5). This work shows that shared reflections and meanings, among professionals with different views and backgrounds, contribute to the development of all participants, once they (1) co-produce new significations and knowledge on mathematics teaching and learning and (2) understand their work better, as well as the curriculum, their students and their own role as educators (Fiorentini et al., 2005).

A group that is artificially formed by institutional processes, such as in pre-service mathematics teacher education, may develop as a learning community as its members superimpose on the institutional roles their own learning goals about practice from practice and by reflecting on practice [Chapman]. A group that starts with weak purposes and relationships may evolve and become stronger, more ambitious, and more productive, and a group with a very good start may decline over time. Any group has its ups and downs—the issue is not how to keep things up permanently but how to renew things when needed.

Olive Chapman

Stories of practice: A tool in pre-service secondary mathematics teacher education

A learning community (LC) is implicit in the activity of a mathematics education course. This LC consisted of the instructor and a homogeneous group of 12 preservice secondary mathematics teachers in one year and 14 in another year. The activity involved writing and analysing stories of secondary school mathematics teaching. The goals of the activity for the participants included: (i) to learn about and from practice through the use of stories of practice, and (ii) to learn how to reflect on practice by unpacking self-stories of practice.

The participants learnt from each other's thinking, written stories, and orally shared experiences. The community provided the voice of the other in order to allow the individual preservice teacher to be questioned and to question or validate through consensus his or her thinking. The stories collectively provided alternatives or possibilities for the preservice teachers to unpack. After initially reading relevant theory and analyzing the story, in the LC, the preservice teachers' role involved sharing their thinking then reflecting on, discussing, comparing/contrasting, challenging, and validating/resonating in their thinking and that of others. For example, they shared their thinking through their analyses of the stories, others reacted to this based on their story by sharing a related aspect of their story to support or counter it, or by offering new stories. They also shared their revised stories of practice with each other and received and provided instructional suggestions. The learning outcome was better understanding of self and practice, for example, as reflected in their revised stories of practice (Chapman, 2005).

The second issue concerns the knowledge that develops from the activity of the learning community based on its shared practices or shared common actions. What do the participants really learn? How do they learn it? In a learning community the negotiation of meanings is a complex process that takes time. Only by being attentive to and engaging with what the others do, feel, and question can participants share a significant learning. The learning process in learning, and their own role as educators [Fiorentini et al.]. Sharing practices and developing together new practices (or new roles in school), learning in close relationship with others, is very recognizable when teachers have to assume new responsibilities at school [Van den Heuvel-Panhuizen & De Goeij].

Marja van den Heuvel-Panhuizen & Erica de Goeij

Offering primary-school teachers a multi-approach experience-based learning setting to become a mathematics coordinator in their school

This paper concerns an explicit LC, formed by a homogeneous group of trainees for elementary school mathematics coordinator. The project had two phases. First a module was developed for future primary school mathematics coordinators (this is a job that did not exist when we started with this module). The module had two focus points: a math-didactical (in this case: gender differences in mathematical knowledge and strategies; what are these differences; what are the consequences for teaching your students; how can you adapt your teaching to the needs of the students) and a professional (how to function as a mathematics coordinator in your school; how to give support to your colleagues). The module was developed by a group consisting of us (university staff) and future "in-service trainers" of the module (staff from teacher education colleges and teacher advisory centres). The draft module was piloted with three groups of future mathematics coordinators who had to comment to the module and had to bring in their own ideas and experiences. After the pilot the draft module was revised. Among other things this meant that many examples from practice were included in the module. Next, there was a schooling for other "in-service trainers" (staff from teacher education colleges and teacher advisory centres) who wanted to give the module at their own location. In the second phase of the project, the organizers of this activity worked together with three "in-service trainers" who had formed each a new group of teachers to give the module. In this second phase of the project the focus was on the future mathematics coordinator in his or her school context. Moreover the mathematics coordinator met each other and discussed their practice. In this case, the purpose of the participants is to learn the objectives of the course — "The course is aimed at enhancing the teachers' domain-specific didactical expertise and the teachers' coaching skills" (p. 2) (van den Heuvel-Panhuizen, & Goeij, 2005).

The third issue is how the learning happens in the group. There are myriad ways, including conducting lesson studies, carrying out collaborative work with other professionals with different views and knowledge, undertaking critical interlocution with studies produced in an academic environment, writing and discussing narratives about classroom practice, reflecting on classroom episodes presented orally or through video records, or conducting small-scale or extended projects. The macro forms of learning communities may vary widely, but the micro activities involve a lot of reading, studying, discussing, reflecting, negotiating, arguing, adjusting, writing, and sharing.

The fourth issue concerns the roles and relationships of the members of the group. The mutual involvement and commitment of the participants to the progress of the group is a vital condition in a learning community. The learning community is stifled if some members do not feel confident enough to expose their concerns, do not ask for help, and refrain from participation in the group, or if, on the contrary, other members participate "too much", occupying all space, helping others too much or in an improper way, and so forth. A proper style of leadership is a critical element to the working of any group [Ponte & Serrazina]. Leadership concerns the establishment of the group's purposes, plans and the daily conduction of activities. There are always participants that play a more prominent role in one stage or another of any group, but the group itself may establish a collective leadership, assuming the most important decisions after a thorough discussion of the issues, and a distributed leadership for practical activities, assigning specific group members the conduction of a particular activity.

João Pedro da Ponte & Lurdes Serrazina

Understanding and transforming practice: A Portuguese experience

This paper describes an implicit LC formed by ten mathematics teachers and teacher educators, whose work lasted for two years. The group begun as a study group, studying a topic of common interest and later transformed itself in a working group, centred in the production of a book. The members of the group learn from each other as they "write papers and collaborate in discussing their colleagues' papers. The successive drafts were to be sent by e-mail to everyone to be discussed in the following meeting, a process that was used up to the final stage of production of each paper. In this way, the study group transformed itself into a working group (p. 2).

"When the activity was completed, the group carried out a collective reflection addressing what participants thought they had learned, their difficulties, and the aspects that they regarded as most important in the work of the group. The participants indicated that they developed their knowledge and competences, and felt they were growing professionally. They indicate that this activity contributed in a significant way towards knowing better what is involved in the activity of a teacher who researches his or her own professional practice. They also feel that they developed their competency in doing collaborative work and in their communication ability (especially in writing), as well as their self-confidence. Some of them expressed a feeling of professional growth and reinforcement of their reflexive attitude" (p. 3).

Several conditions were critical elements for the success of this LC: "For the participants, the activity was successful because of the collaborative environment, the personal relationships, the group dynamic and the methodology. They also indicate that such collaborative environment and dynamic developed from the style of leadership, largely shared by the group, and the emerging nature of its objectives and working processes" (p. 3) (da Ponte & Serrazina, 2005).

Some natural questions to ask concerning teachers' learning communities in general are the sort of activities related to the work of teachers that help their natural establishment. Two rather important elements of the teachers' work are planning and reflecting on teaching, and this gives rise to interest in models such as lesson study and collective reflection such as the use of video cases, addressed in the previous section of this chapter.

An important variety of a learning community occurs when that community establishes itself as a community of inquiry, that is, when inquiring on some issue becomes part of the purpose of the whole group and of each individual member. Inquiring is a very powerful form of constructing knowledge and therefore of learning [Fiorentini et al.] [Van den Heuvel-Panhuizen & De Goeij] [Ponte & Serrazina]. In fact, it is more and more common to see professionals researching on their own practice, often in collaboration with other professionals and social actors (Jaworski, 2001, 2004; Llinares & Krainer, 2006; Ponte, 2002).

5. Teacher's Learning in Virtual Communities

As has been discussed in this chapter, learning communities cannot be forced, as one cannot impose on the other the need or the desire to learn. In particular, if we talk about professionals, be they pre-service teachers or in-service teachers, it is unlikely that an artificial imposition from the outside will last long. Willingness to learn something is also socially bound, as we are always interacting with others and are always embedded in a culture. A didactical example could be that kids who are born in the United States are more likely to play baseball than kids in Brazil, while the latter are more likely to play soccer, the explanation for this being cultural rather than biological. Likewise, teachers may be drawn to different learning communities depending on the environment at the school where they teach and its surroundings, which may include support from local universities and from the school district.

However, Internet access in settings where teachers find themselves—schools, homes, universities—opens up new possibilities for the formation of communities. With the advent of the Internet, one's interests (in a sport or a topic, for example) need no longer be restricted to one's birthplace, physical location, or culture. As Borba & Penteado (2001) and Borba & Villarreal (2005) argue communities can also be built around common interests as opposed to communities formed for geographical reasons. The transformation of the notions of time and space brought by the Internet have affected teacher education as well and is portrayed in the 15th ICMI Study, albeit somewhat timidly. At this conference, Bairral & Giménez (2005) and Bairral & Zanette (2005) pointed out how teachers from different schools who are interested in geometry use the Internet as a means of building communities to support each other in the teaching and learning of this mathematics topic. Teachers who felt isolated in schools where no one else shared this interest found a "community" in the virtual world [Bairral & Giménez].

Marcelo Bairral & Joaquin Giménez

Dialogic use of teleinteractions for distance geometry teacher training (12–16 years old) as an equity framework

This paper presents a virtual geometric environment for an in-service dialogic course. Such an environment was structured around 6 hypertextual axes/scenarios:

(a) activities which introduce the use of materials forcing the teachers to review their own knowledge on geometry and professional activity,
(b) observations of the role that everyday life plays in the different geometric activities,
(c) reconstruction of cognitive processes of students in class,
(d) observation of the role of manipulative aids for each subject,
(e) organization of summaries of contents, and
(f) continuous self-regulation.

The geometric content was developed in didactical units. A group of mathematics teachers worked on a 50-hour Internet course over six months, using a range of online interactive tools and materials: e-mail conversations, geometric "authentic" tasks, self-regulation inquiries, discussion forum and distributed chats. Three different experiments were done. Semi-structured interviews, text writings and videotaped experiences of teachers' classrooms were used to recognize changes-in-action in geometry by means of their a-synchronous productions (Bairral, 2002). The observations and results are organized in three parts: about the enactive role of interactions, the formative moments, and the new role for trainers (Bairral & Giménez, 2005).

Another paper, Dawson (2005), reports on the challenges of starting mathematics teacher education programs in the Pacific Islands. During his presentation at the conference and in papers that followed this meeting, this author shows how the Internet helped to bridge the gaps between the few face-to-face meetings that could be arranged between different groups of teachers, since travel time between many of the islands is counted in days and not in hours.

The last paper presented at the 15th ICMI Study regarding online education reports on research developed since 1999, focusing on extension courses offered to teachers that involve extensive use of chat for synchronous sessions and e-mail for asynchronous interaction. In this paper, Borba (2005) stresses the social impact that the Internet can have, like in the Pacific Islands, making it possible to connect remote areas of Brazil to mathematics education centers such as UNESP-Rio Claro, in Sao Paulo. The possibility of having a social impact in poor areas, and of taking people out of isolation, seems to be a theme that emerges even in papers outside the ICMI study [Borba].

Marcelo Borba

Internet-based continuing education programs

This paper presents some preliminary research findings regarding the nature of interactions and learning that take place during distance courses in mathematics offered to teachers. Predicated on the belief that knowledge is generated by collectives of humans-with-media, and that different technologies modify the nature of the knowledge generated, the authors sought to understand how the Internet modifies interactions and knowledge production in the context of distance courses. The research was conducted over a period of several years, during distance courses proffered annually from the mathematics department at UNESP, Rio Claro, SP, to teachers throughout Brazil, conducted mainly via weekly chat sessions.

Specific difficulties emerge with the learning of mathematics in such environments. For instance, prior to a scheduled chat meeting with all twenty teachers participating on one of the courses, a problem was posed to them regarding Euclidean geometry. Different solutions and questions were raised by all the participants, but one of the teacher's reflections called our attention. During the discussion, Eliane Cristovão, said: "I confess that, for the first time, I felt the need for a face-to-face meeting right away. . .it lacks eye-to-eye contact." She then followed up, explaining that discussing geometry made her want to see people and to share a common blackboard. While some preliminary findings were presented, more questions were raised: How to provide continuing support to teachers following the course? What mathematics should be taught, once the Internet becomes part of the humans-with-media collective? What are the implications for preparing teachers to teach via the Internet? Others were invited to help seek answers to these questions, as distance education offers new possibilities for teachers to interact with each other and with university professors and researchers over great distances, thus helping to address the disproportionate concentration of knowledge production in certain regions of Brazil (Borba, 2005).

However, the main point that Borba makes is that technology is not neutral at a cognitive level. Similar to the way geometry software transformed the nature of the mathematics generated in classrooms, as shown by research in the 1990s and in this century, the Internet has changed the way teachers interact and the way that mathematics is communicated. Orality and the blackboard shape mathematics in certain ways, and chats and asynchronous interaction through e-mail do so in different ways. In the study paper, Borba (2005) presents the voice of a teacher who complains about the difficulty of doing geometry online due to the lack of a common figure to share with other participants in the course. In a paper presented at Psychology of Mathematics Education 29, which followed the ICMI study paper, Borba (2005b) expands the discussion regarding the influence of a given Internet interface in the production of knowledge, pointing out that writing, and the kind of writing done in a chat, led in-service teachers participating in a course to develop a kind of linearity associated with their mother tongue, in this case, Portuguese. Santos (2006) takes this argument even further to show how demonstrations about space geometry developed by teachers in chats are developed in parts, in a dynamic way that is strongly shaped by the medium, reinforcing the notion that knowing is developed by humans and by the media that surround them in a given historical moment.

As could be noticed during the 15th ICMI Study, studies of the role of online interactions in teacher education were just beginning. This area of inquiry seems to be flourishing. For instance, Ponte et al. (2007) have studied how different pre-service teachers have different experiences with online supervision of their teacher training. Pre-service teachers felt that online interaction could intensify collaboration but at the same time complained about technical difficulties when using the Internet; they also did not feel comfortable with the way everything is recorded in the "virtual world" and with the time spent writing. Similarly, Ponte & Santos (2005) found that some in-service teachers feel comfortable dealing with mathematics and professional learning tasks in online environments while others do not.

Such a difference could be understood based on studies like those that examine how teachers' identities may change in an Internet environment (Rosa, in progress). This author, employing the notion of humans-with-media, proposes that identity is also shaped by different media that are part of a given collective producing knowledge. In a recently published book, Borba, Malheiros, & Zulatto (2007) raise the conjecture that learning styles are also shaped by different media (Internet, orality in the classroom) and have detailed different needs that different teachers have as they teach and learn in different environments. These authors also illustrate how different abilities are necessary depending on the different Internet interface that is used. Video conferences or chat rooms used as the main interface for a course change the nature of interactions among participants and require different abilities, according to Borba and colleagues.

Other authors do not emphasize the role of the environment as much. For example, in a recent study, McGraw et al. (2007) analyze how non-homogenous groups—composed of in-service mathematics teachers, pre-service teachers, mathematics teacher educators, and mathematicians—analyze multimedia cases. These authors built into their research design possibilities for online forum interactions as well as face-to-face interactions. Participants were able to interact in real time only in their face-to-face meetings and not in the virtual world. In the analysis presented in the paper—in which they focused on the role of different participants as they were exposed to a multimedia case—the role of medium is not treated as relevant.

It is too early to draw conclusions regarding the role of the Internet in teacher education. The 15th ICMI Study showed just a few reports, and work in this area seems to be in its beginning stages even for research groups that are focusing on this topic. It can be said that, at this point, some researchers appear to view the Internet as "transparent", or as not playing any specific role in cognition. For instance, some of the authors view writing in a chat as similar to writing with paper and pencil. Others believe that the medium is so important that different "units" of collective knowers are formed depending on the medium that is used. It is very possible that there are intermediary positions, but it will be necessary to wait longer to see whether the above distinction is one that will divide the studies on virtual communities, e-learning, and other terms that are being coined. The 15th ICMI Study and the studies that have been carried out on the theme since the beginning of this century show, however, that the Internet can no longer be ignored because, at least, of its social impact.

6. Conclusion

This chapter discusses the role and nature of tasks, contexts, and learning settings in mathematics teacher education. It indicates what may be considered as powerful tasks for teacher development and suggests what may be their desirable features. Those tasks offer mathematical and pedagogical problem-solving situations that address relevant issues in engaging and challenging ways. Particularly noteworthy kinds of professional learning tasks are discussions of video cases that consider the lesson as a unit of analysis and tend to attend to general instructional issues. Another important kind of professional learning task is conducting lesson studies that see the classroom as a unit of improvement and analyse instructional issues in relation to one's teaching in a professional learning context. The measure to which such professional learning tasks promote teachers' learning depends on the activity that teachers generate from them. Such activity depends on a number of factors, some internal to the teacher (his/her interests, concerns, previous knowledge, willingness to get involved), some depending on the wider setting (curriculum and instruction frameworks, nature of contract and school conditions, time available), and some depending on the professional development setting created by the teacher educator (including tasks, resources, schedules, size and composition of the groups, forms of work, and of interaction). A key element of this setting is the learning community—face to face, based in virtual interactions, or a combination of both—that supports each teacher in opening new perspectives about mathematics teaching and learning, challenges their beliefs and conceptions, provides security in attempting new approaches and activities, and promotes effective and reflective change in teacher practice.

It is taking into account this sensitive combination of factors that teacher educators have to carry out their job, identifying what may be important learning goals, assessing participants' readiness, designing tasks, and negotiating working procedures and social relationships to maximize learning opportunities. Designing appropriate professional learning settings for a group of teachers and conducting a professional development activity requires the ability of collecting all the necessary information about the teachers and their contexts, as well as a sensitivity in dealing with often-contradictory emergent phenomena during the activities. Such sensitivity is part of the professional preparation of teacher educators and requires years of reflecting and researching their own practice to develop. However, this sensitivity may be clearly supported by a general understanding of the relationships among the purposes of the educational activities, tasks, dynamics, and contexts and the roles that different participants (teachers and teacher educators) may assume in these educational contexts.

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