# Theme 2.1
# Development of Teaching in and from Practice

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## 1. Overview

This chapter is principally concerned with the characteristics of the process of developing professional expertise in the teaching of mathematics in and from practice. The chapter comprises three sections, in which we discuss, respectively,

- Section A: Recognized Factors in Teacher Development,
- Section B: Recognized Benchmarks in Teacher Development, and
- Section C: Recognized Issues in Teacher Development.

In the discussion of "factors", we review the beliefs, experiences, and structures that have been identified as significant in studies and programs concerned with the development of mathematics teachers and teaching. This discussion frames the second section of the chapter, in which we delineate some of the markers or "benchmarks" of effective teaching practice, focusing in particular on how these have been used by researchers to study teacher development and effectiveness. Finally, in the "issues" section, we review some of the structures—conceptual, institutional, cultural, and so on—that enable and constrain research into and efforts to support teacher development.

Further to the established literature, we draw both on papers that were reviewed and accepted as contributions to the study conference and on notes distilled from our discussions during the conference. When the writing from a paper informs this writing, the relevant quotation from the paper is displayed in a box and attributed to the author. A list of authors, e-addresses and titles of the papers can be found at the end of the chapter.

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## 2. Section A: Recognized Factors in Teacher Development

Perhaps not surprisingly, the sorts of factors that tend to be identified as significant in discussions and studies of mathematics teacher development are focused on teachers themselves, including, for example, their beliefs about learning, their experiences with mathematics, and their attitudes towards formal education. This emphasis echoes the tendency among educationists to focus on the individual as the "site" of learning and knowing, although there are clear indications that the focus is being elaborated upon to include more collectivist interpretations of learning and knowledge. As such, although the first three of our four categories of "recognized factors" are concerned specifically with teachers, it should be understood that we are not identifying teachers as being positioned as either the problems or the solutions nor are they free of responsibility in the contexts in which they work. Our position is that what happens depends on the teachers but is not determined by them. Rather, the three categories of teacher beliefs, backgrounds, and positionings are identified as phenomena that must be interpreted and addressed, simultaneously, at personal, social, and cultural levels. This point is re-emphasized in our fourth category, in which structures for intervention are discussed.

### *2a. Factor 1: Teachers' Epistemological Frames*

Theories of knowing and knowledge have been among the most prominent issues in the mathematics education research literature over the past several decades. We have witnessed a proliferation of perspectives that, on the surface, seem to have little in common other than an explicit rejection of the cause-effect mechanisms, representationist assumptions, and rigid mental/physical (inner/outer) distinctions that are typical of behaviorist and cognitivist frames that prevailed through much of the twentieth century.

However, close examinations of constructivist, constructionist, socio-cultural, and emergentist theories reveal some important intersections, including a shared metaphoric commitment to "bodies" (e.g., physical bodies, bodies of knowledge, student bodies, the body politic), an explicit embrace of evolutionary dynamics, and a concern with adaptive coherence rather than absolute correspondence (Sierpinska & Lerman, 1996). Embraces of such notions appear to constitute an important factor in teacher development (see related boxes, Kidd and Cedillo & Santillàn), whether applied at the level of the individual, the social/collective, or some other level. These theories entail significant shifts from traditional emphases in mathematics classes.

For example, theories that focus principally on personal understanding (e.g., constructivism and socio-constructivism) posit the child as a coherence-seeking sense-maker, requiring the teacher to structure tasks that simultaneously interrupt current understandings while providing the tools needed to modify those understandings or to construe new, more viable interpretations (Steffe & Gale, 1995).

> The implication for teachers' pedagogical practices is that tasks need to challenge students to mentally organize new information into their existing knowledge. This construction of knowledge implies that knowledge is not static but dynamic and can be manipulated and moulded (Kidd, 2005, p. 2).

> [S]ocio-constructivist theories demand that the teachers change deeply their knowledge on learning and teaching mathematics.. . . [S]ocial constructivism assumes, among other premises, that each student gets to the classroom with his own ideas and that the teacher must provide him with new experiences that induce him to collect data to affirm them or to refute them (Cedillo & Santillàn, 2005, p. 1).

Theories of learning that are more concerned with interpersonal dynamics and social and cultural contexts offer compatible but different recommendations for teaching. More concerned with the patterns of interaction, relational dynamics, and group expectations of social bodies, these theories see knowledge as inextricably linked to social positioning and context-specific action (Lerman, 2000). Cognition in this frame is always a collective phenomenon: embedded in, enabled by, and constrained through the social phenomenon of language; caught up in layers of history and tradition; and confined by well-established boundaries of acceptability (see related boxes, Jaworski and Hodgen).

> Knowing and knowledge are *both* attributes and building blocks of the community *and* outcomes of reflective activity of individuals within the community. We have to account for development of practice both for *individual* teachers and educators and for the *communities* to which they belong. . . .[W]e can see knowledge as both situated and distributed within a community (Jaworski, 2005, p. 2).

> [C]hange and learning are facilitated by a "combination of engagement and imagination" which enables identification "with an enterprise as well as. . .view[ing] it in context, with the eyes of an outsider. Imagination enables us to adopt other perspectives across boundaries and time. . .and to explore possible futures. . .[and] trigger new interpretations" (Hodgen, 2005 [citing Wenger, 1998, p. 217], pp. 2–3).

This manner of theorizing can be further elaborated by extending the notion of "learner" to encompass not just individuals and social collectives, but any self-maintaining, adaptive, coherence-seeking group or phenomenon (see related boxes, Brown and Davis & Simmt).

> [T]he teacher or the teacher educator learns about their students' strategies and behaviours in learning, as a learning culture develops in the group. Such learning cultures can also develop within a group of teachers working together in a school where the head of department is metacommenting on ways in which they want the group to work (Brown, 2005, p. 3).

> ["Learning" is used here to refer to] the co-specifying activities of agents whose collective activities exceeds the summed capacities of agents. . .includ[ing] neural activity, individual understanding, classroom collectives, the body of mathematical knowledge, and society (Davis & Simmt, 2006, p. 1).

Clearly, the range of perspectives encompassed by this "factor" in teacher development is broad. However, to reiterate, what appears to be critical is not what or who is identified as the specific site of learning/knowing but the fitness-oriented, coherence-seeking, self-maintaining, context-dependent dynamics that are understood to be at work in moments of learning/knowing.

## *2b. Factor 2: Teachers' Backgrounds*

The topic of teachers' backgrounds is often separated into the major categories of professional preparation and teaching experience. The former of these is typically further subdivided into such categories as pre-service education, in-service training, courses in mathematics, and courses in pedagogy.

Of course, there is clearly some value in these sorts of categories and subcategories. However, a consistent theme across the papers that served as the foundation for this chapter was that such fragmentation does not necessarily help to illuminate the critical factor of teachers' backgrounds, owing in part to the tendency to frame the discussion in terms of "required knowledge" and "necessary experiences".

For example, it is not difficult to find instances of effective mathematics teachers who have little formal preparation in the subject or ineffective teachers with extensive disciplinary knowledge. How, then, might one frame the topic of teachers' backgrounds in a manner that does not reduce the complexity of the phenomenon yet does not render it impossible to research and interpret?

Davis & Simmt provide at least a partial response to this question. In their study of teachers' disciplinary knowledge, they noted that a greater number of formal mathematics courses did not ensure more effective pedagogy. However, stronger backgrounds proved important when individuals were asked to deconstruct particular mathematical concepts. That is, when stronger disciplinary backgrounds were coupled with particular sorts of in-service experiences, teachers were able to generate much more profound and robust insights into specific concepts and, further, were able to suggest implications for pedagogy that were informed by their teaching

experiences but that had not previously been part of their teaching. In other words, mathematics background, in-service opportunities, and teaching experience were treated as three inextricably intertwined aspects of teachers' backgrounds.

Jaworski reaches a similar conclusion, although she articulates it in a somewhat different way. Further complicating the matter by implicating researchers in the development of teachers knowledge, Jaworski attends to such paired phenomena as knowledge and learning, insider and outsider, and individual and community (Jaworski, 2003). The underlying sensibility is similar to that articulated by Davis & Simmt: teachers' backgrounds should not be construed in terms of discrete elements but in terms of co-implicated, complementary, and mutually amplifying aspects (see related boxes, Davis & Simmt and Jaworski).

> [R]esearch focused on exclusively "questions of mathematics" or "questions of learning" is inadequate in efforts to understand teachers' mathematics-for-teaching (Davis & Simmt, 2006, p. 5).

> [L]earning is rooted in socio-cultural settings: school communities, educational communities, societal norms, project expectations, all contributed to activity and knowledge growth (Jaworski, p. 5).

Cedillo & Santillán make the same point in yet another way. Developing the metaphor of teaching algebra as a language, they point to the neurological, psychological, interpersonal, and cultural dimensions of knowing—which occur all at once. The lens provided by this metaphor prompts them to a provocative conclusion with regard to teachers' backgrounds as it pertains to teacher development. As they note, the "most resistant teachers to change were the more experienced" (p. 6) whereas "a characteristic noticed in the teachers with little experience and an insufficient subject knowledge was their good attitude towards the project" (p. 7).

On the surface, this conclusion might seem to conflict with that reported by Davis and Simmt. In one case, subject matter knowledge and teaching experience proved enabling; in the other, they were impediments. Such apparent tensions only underscore the complexity of the notion of "teachers' backgrounds", reaffirming the need to configure this construct in complex rather than reductive terms. To re-iterate, the "factor" of teachers' backgrounds comprises the disciplinary knowledge and other aspects of their formal preparation, their ongoing teacher experiences, and their opportunities for in-service work.

## *2c. Factor 3: Teacher Positioning*

Mathematics teaching, of course, is about more than personal beliefs and histories. It is a radically contextualized phenomenon and must thus be considered in light of

educational cultures, professional interactions, local expectations, and so on. Further, as Hodgen develops, the relationship between context and teaching cannot be understood in unidirectional or causal terms (see related box, Hodgen).

> An individual's identity, and ultimately legitimacy, within a community depends not simply on their acceptance by the community, but on the individual's identification with it (Hodgen, 2005, p. 4).

Hodgen draws on Lacanian psychoanalytic theory to introduce matters of desire, motivation, and identity to the discussion of teacher action. In Lacanian theory, pleasure is seen as dialectically linked with pain. It thus provides a way of locating the motivation to sustain change in the face of the very real difficulty of this for teachers. Lacan conceives of identity in terms of unattainable completeness. As with the aspects that contribute to teachers' backgrounds, these matters cannot be isolated and treated separately but must be considered as part of an evolving constellation of identifications that affect and are affected by the contexts of teachers' work. For example, as Hodgen elaborates, there is not just "a personal and emotional investment in professional change but also a compulsion to change" (p. 4). Hence, Hodgen concludes, "a potentially productive strategy in teacher education would be an explicit focus on such affective issues and the facilitation of the desire" (p. 5).

Also interested in matters of motivation, Brown focuses more on "purposes", which she describes in terms of ideas or aims that serve to orient efforts. Brown's "purposes" have much in common with Hodgen's "desires", contributing to student teachers "gaining a sense of the teacher they want to become" (p. 2). "Purposes" (Brown & Coles, 2000) figure into student teachers' evolving teacher identities, as articulated choices of strategies and emphases closely enough linked to this new world of the classroom that they see the need for action. They cannot be reduced to private dispositions but must be understood in terms of complex unfoldings of agent-in-context (see related box, Brown). For instance, a common problem for teachers when meeting a new group of students is, "How do I know what they know?", but experienced teachers have a range of teaching strategies that they use implicitly to find out. Seeing the need for finding out prior knowledge allows a range of strategies to be discussed and tried out by student teachers in awareness of them. Philosophical descriptions of practice do not link so easily with actions.

> Purposes seemed to be in the middle position between philosophical attitudes and teaching behaviours in the classroom (Brown, 2005, p. 1).

Kidd's research into what happens to the teacher when a curriculum changes might be considered as a counterpoint to Hodgen's and Brown's discussions. She notes that the extent to which teachers embrace curriculum change varies across regions and appears to be linked to their expectations. This observed simultaneity of situational and affective influences provides support for the need to attend to matters

of desire and purpose, understood in radically contextual terms. We might ask, how are notions of expectation and desires/purposes linked or counterposed?

## *2d. Factor 4: Structures for Intervention*

Processes, models, and tools to support learning in and from practice are the foci of other chapters in this section, and so we will limit our comments in this chapter—linking our remarks in particular to some of the emphases developed in the discussions of the preceding three factors.

In particular, and re-emphasizing the concurrency of these phenomena, a consistent theme across discussions of the structures of pre-service and in-service experiences is the need to consider structural issues (e.g., patterns of interactivity, opportunities for collaboration, blurring the roles of teachers and researchers) and conceptual matters (e.g., epistemology, disciplinary knowledge, teacher motivation) as co-implicated, mutually affective phenomena. More colloquially, one cannot separate what one knows from the structures in which one comes to know. With that point in mind, we look across some of the structures for intervention that were presented in the various research papers, underscoring how they might complement and enable the factors already identified.

Perhaps the most consistent structural feature among the research projects is some manner of joint action in which individual needs and interests were addressed in ways that supported more collective aims. This point was articulated in terms of the emergence of "inquiry communities" (Jaworski), "adaptive (or learning) systems" (Davis & Simmt), "collective participation" (Siemon), "sustained conversations" (Dawson), and "egalitarian dialogue" (Bairral & Giménez). One might characterize these collaborative moments in terms of a "collective we" rather than a "collection of me's", in which roles are blurred, intentions elaborated, and possibilities enlarged (see related boxes, Jaworski and Dawson). For instance, a community of practice becomes a community of inquiry when participants engage together in "critical alignment" (Jaworski, 2006).

> [T]he project meetings developed a small inquiry community in which teachers came to see themselves as researchers (Jaworski, 2005, p. 5).

> The mentors. . .melded themselves together into functioning professional development teams, they are engaging others from their local educational community. . . (Dawson, 2005, p. 6).

An important aspect of these moments of collectivity is that they arise around focal practices—that is, specific, focused, and sustained engagements in specific activities or with particular topics. Dawson's teams of mentors, for example, cohere

around the shared work of providing professional development opportunities to teachers in Micronesia. This project is sustained over years and involves intense five-day institutes to delve into the sorts of issues discussed previously, as well as through more extended practices, such as lesson studies. Phrased differently, these collectives are articulated not around who they are but around what they are doing.

Siemon describes a completely different approach designed for a different context and framed by different concerns, which also embodies notions of collectivity and focal practice. She comments on a "behind-the-screen" approach, by which video technologies are used to enable unobtrusive observations of classroom activity. These observations frame discussions of student learning, and group members develop strategies and vocabularies to describe practices in ways that resonate with teachers' actual experiences of teaching.

Within these sorts of collective structures and focal practices, the role of the teacher/leader appears to be configured not in terms of directing or overseeing but in more participatory ways. For example, Brown (see related box) uses the term "metacommenting" (adapted from the work of Bateson, 2000, p. 137), to describe an aspect of the role through which she foregrounds the teacher/leader's responsibilities in setting the ethos of the collective. The notion also highlights that the work and learning of the collective is dependent on the teacher/leader, but it is certainly not determined by that person.

> The students are learning through [their teacher's] metacomments that "it's all right to be wrong" and that part of the culture of their classroom is "sharing different responses and methods" (Brown, 2005, p. 5).

Bairral & Giménez echo a similar sensibility. In their report, they point to a new vocabulary for describing this transformation in the role as they offer such terms as "Burning-Animator," "open provocateur," "supporter," and "team player" (these and other terms are developed in more detail in Section 3). These terms are particularly provocative when considered in the context of their study of the use of Internet space to enable dialogic learning across distances in Brazil—underscoring that the critical aspects seem to be a means and a focus for collective action.

## *2e. Summary Comments on Section A*

In some cultures of contemporary educational research, there seems to be a strong temptation in a discussion of factors to identify the most basic and fundamental elements that constitute a phenomenon and, once identified, to consider those aspects in isolation. This tendency is a troublesome one when it comes to enterprises as complex as the development of mathematics teaching expertise. Drawing on a mathematical analogy of sorts, the factors discussed here should be understood as composite ones that are curiously irreducible to more primary elements.

Indeed, as we review the descriptions of the four composite factors described previously, we note that they cannot be considered apart from one another. The constructs of "teachers' epistemological frames", "teachers' backgrounds", and "teacher positionings" are little more than interpretive tools that entail artificial distinctions and temporary ignorances. They would likely not lend themselves to checklists and surveys but must be understood as inextricably intertwined with one another and complexly co-implicated with grander social and cultural structures.

The value of these constructs, then, lies not in the possibility for dissecting the process of becoming an effective teacher but for supporting appreciations, through taking multiple perspectives, of the contingencies of teaching and of learning to teach.

## 3. Section B: Recognized Benchmarks in Teacher Development

In this section, we take on the matter of what it might mean to operationalize the factors identified in the previous section—that is, to translate them into tools of observation, structures of intervention, and markers of transformation.

The section is organized into three categories of benchmarks. The first two of these pertain to teacher beliefs—or, more specifically, to the manners in which teachers are able to articulate their convictions (Benchmark 1: Explicit beliefs) and the ways that, and the extents to which, those convictions play out in their teaching practices (Benchmark 2: Enacted beliefs). These discussions are organized in parallel manners, as we delve into beliefs about the learning of mathematics, beliefs about the nature of mathematics learners, beliefs about the nature of mathematics, and beliefs about the teaching of mathematics.

Our third category, Teacher attitudes (Benchmark 3), cuts across the two other categories. In it, we discuss the sorts of dispositions that may or may not be explicit and may or may not be readily discernible in practices, but are nonetheless likely to have a profound shaping influence on "the way teachers are" with mathematics, learners, and formal education.

As in our discussions of factors, we think of benchmarks as emergent, evolving, and intertwining terms. Although we attempt to delve into aspects of these complex markers in the discussions that follow, our examinations should not be interpreted as attempts to pry these categories into subcategories.

### *3a. Benchmark 1: Explicit Beliefs*

Given the prominence of the topic of teachers' epistemological frames in the contemporary research literature, it is not surprising that the topic of teachers' explicit beliefs about the learning of mathematics was a central theme in many of the papers considered in this strand.

For example, Brown's discussions of purposes (in reference to "the sorts of guiding principles that student teachers found energising when learning from their own experience," p. 1) and metacommenting (in effect, a means to render explicit "students' strategies and behaviours in learning," p. 3) are examples of pedagogical strategies that are intended to enable and compel prospective teachers to be explicit about what they believe about the learning of mathematics. Cedillo & Santillán echo the same sensibility. Also situating their research in the context of pre-service teacher education, they emphasize the importance of requiring "that the teacher know the state of development of mathematical thought of his students" (p. 1).

Although located differently, experienced teachers choosing to explore their own practice, Jaworski's research emphasis on "[l]earning in practice from a study of practice" is suggestive of similar emphases on finding ways to articulate what and how one learns. In this case, practising teachers work together to render explicit what is believed about learning and how those beliefs give shape to their teaching practices and research efforts. In a parallel manner, Chiocca focuses specifically on teachers examining their practices in relation to how they become aware of and work with pupils' need for correction as a means to become explicitly aware of the model of learning that informs pedagogical decisions.

Explicit beliefs about mathematics learning, of course, are tangled together with explicit beliefs about the nature of mathematics learners. Several of the reports highlighted the importance of bringing these beliefs out into the open. Cedillo & Santillán (see related box), for example, identify such articulations as an integral element in effecting teacher change.

> [I]t is required that the teachers make evident. . .that they understand [their students to be] intellectually creative, able to make non-trivial questions, to solve problems and to construct theories and reasonable knowledge (Cedillo & Santillàn, 2005, p. 1).

Similarly, in Kidd's study of varied mathematics teaching strategies, the perceptions or beliefs teachers had about their students and of how they learn were indicative of their teaching practices. In particular, Kidd notes, these beliefs come to be articulated in terms of expectations of what students will and will not be able to do—which in turn contribute to decisions about classroom tasks and pedagogical approaches.

Davis and Simmt take the discussion of learners in a different direction, complexity science, which is principally concerned with phenomena that spontaneously arise in the interactions of relatively autonomous agents and that, in this spontaneous emergence, demonstrate traits and capacities that are not manifest by any of the agents on their own (Davis & Sumara, 2006). Invoking complexity science, which they define as the study of learning systems, they argue that any self-organizing, self-maintaining, and self-referencing collective can be regarded as a learner—opening the door to the possibility of treating classrooms and other social groupings as coherent learners in and of themselves. Although introducing quite different sets of entailments, the suggestion is useful here insofar as it underscores how teachers'

abilities to be explicit about their beliefs about learners can enable and orient their teaching practices.

Davis and Simmt actually extend their definition to encompass systems of knowledge, such as mathematics. This move prompts them to investigate the figurative and experiential underpinnings of mathematical concepts, casting these as fluid and interactive aspects that open new interpretive possibilities when combined. That is, they demonstrate how being explicit about beliefs on the nature of mathematics is an important benchmark in the development of teacher knowledge, given the profound influence that such beliefs can have on pedagogy. Kidd confirms this point with reference to an interview question, not cited in her paper, "What do you think of the body of mathematics as a whole?", one of the prompts used to make sense of teachers' practices and their attitudes towards change (p. 3).

Kidd's larger research focus is actually explicit beliefs about the teaching of mathematics—which comprises explicit beliefs about learners, learning, and mathematics. It is also a topic of broad shared interest among researchers. For example, both Dawson's work with novice and experienced teachers and teacher mentors and Siemon's work with groups of practising teachers are organized around the entangled issues of mathematical content, conceptions of learning, and associated pedagogies. These examples serve as important reminders that explicit beliefs should not be treated as independent strands of knowing.

## *3b. Benchmark 2: Enacted Beliefs*

There is a tendency, within discussions of formal education, to focus on explicit, stated knowledge. However, prompted by psychoanalytic, structuralist, post-structuralist, pragmatist, enactivist, and other interpretive frames, there has been a recent surge in interest in tacit knowledge (Polanyi, 1958)—that is, in interpretations and beliefs that are so deeply inscribed in a teacher's being that they are difficult or impossible to articulate, even though they are embodied in moment-to-moment actions. Indeed, such tacit knowings might conflict with explicit statements about what is known and believed (see related box, Brown).

> In the case of student teachers beginning the process of entering a new world of the classroom, without a range of effective behaviours, what seems important is that a structure for learning is put in place by the teacher-educator that supports the students in the move to implicit effective behaviours (embodied actions) (Brown, 2005, p. 3).

Of course, explicit and enacted knowledge cannot and should not be treated as distinct categories. They are, rather, dynamic complements. Explicit understandings are rooted in embodied knowings, while they orient and fade into enacted knowledge. Brown's work with pre-service teachers is organized around this notion, as she works with student teachers to render explicit otherwise tacit beliefs, focusing in particular on "purposes", such as needing to find strategies to find out what their

students know and "basic-level categories", which are linked to action and are the most simple ways in which we come to see the world or learn (Lakoff, 1987).

Unfortunately, although acknowledged as important by many, few of the papers considered in this chapter actually discuss or present strategies for tracking and interpreting enacted beliefs. For example, around the topic of enacted beliefs about the learning of mathematics, Kidd discusses the importance of setting intellectually rich tasks, Cedillo and Santillán argue that the profoundly embodied competence of using language is an apt metaphor for mathematics learning, Bairral and Giménez highlight the dialectic of explicit experimentation and implicit knowledge and beliefs, and Davis & Simmt point to the profound differences that arise when learning tasks are organized around assumptions of the vibrant sufficiency of knowledge, good enough to do what is needed (versus a belief that teaching is about correcting deficiencies). The two phrases, "vibrant sufficiency" and "static deficiency" are used to draw attention to the sorts of dynamics assumed to be operating in knowledge-producing systems. "Vibrant sufficiency" is rooted in an assumption of evolutionary dynamics, calling up notions of ongoing adaptation, mutual affect (with other systems), and coping. The measures of "truth" in this frame are adequacy and utility—that is, so long as something continues to work it is likely to persist. By contrast, theories of knowledge organised around the principle of "static deficiency" tend to assume that there is an ultimate, unchanging truth (usually thought to be located outside the knower or knowing system) that the agent must strive to attain. That is, truth is held to be static, and the knower is regarded as necessarily deficient in relation to that truth.

Similarly, on the topic of enacted beliefs about the nature of mathematics learners, Brown uses the construct of "metacommenting" to foreground the assertion that learners are embodied knowers who move back and forth between explicit and tacit actions. Hodgen's interests in motivation, identity, and desire are oriented by a similar assumption—that is, that the non-conscious dimensions of mathematics learners and mathematics learning vastly exceed the conscious dimensions.

As for enacted beliefs on the nature of mathematics knowledge, Davis and Simmt highlight some of the problems that arise when certain beliefs are allowed to remain implicit. In particular, they comment on the issues that can arise around the literalization of metaphors—that is, when teachers treat figurative aspects of images, analogies, gestures as though they were isomorphic with associated mathematical concepts and immutable. They also comment on possible pedagogical implications of enacted beliefs around the emergence of mathematical knowledge, drawing a distinction between recursive elaborative and accumulative processes. In a more pragmatic vein, Brown comments on the use of "purposes" as a useful "way of influencing [student-teachers'] developing images of mathematics. . .at the superordinate [philosophical] level" (p. 4).

As with explicit beliefs, implicit beliefs about learning, learners, and mathematics all seem to coalesce in enacted beliefs about the teaching of mathematics, including teachers' inclinations to attend to student sense-making (e.g., Brown; Cedillo & Santillán; Kidd), their attitudes towards errors (e.g., Chiocca; Kidd), the extent and nature of their textbook usage (Kidd), the likelihood of group work or student-led

discussions (Brown; Kidd), and the tendency to delve into the implicit structures of mathematical concepts versus treating concepts as sites for technical mastery (Davis & Simmt).

## *3c. Benchmark 3: Teacher Attitudes*

Teachers' attitudes and predispositions are topics of emergent interest, as indicated in an *Educational Studies in Mathematics* special issue on the topic "Affect in mathematics education: Exploring theoretical frameworks", although these matters are currently difficult to locate in the matrix of established research in mathematics education. The dearth of research is likely related to the tendency to regard the individual student as the locus and fundamental unit of learning. With that assumption in place, matters of the teacher's affect or inclinations might seem relatively unimportant.

However, with the noted shift towards thinking in terms of collectivity, situated action, and contextual ethos, teacher attitude is coming to be understood as a vital element in occasioning and sustaining interest among students. Brown's remarks on teacher curiosity and Davis & Simmt's commentary on the pleasure that teachers seemed to derive in moments of joint mathematical inquiry are thus germane.

However, the most frequently mentioned topic within this benchmark is critical reflection (see related boxes, Dawson and Siemon). Bairral and Giménez comment that "Enactive and reflective processes are the basis for growing professional development" (p. 1), and Jaworski foregrounds the role of critical reflection, coupling it with inquiry as two mutually amplifying tendencies. Dawson also indicates how groups of teachers can affect another's attitude towards critical reflection.

> The ability of the project to achieve its stated purpose of nurturing effective mathematics instruction in novice and experienced teachers is dependent upon the achievement of a number of goals, including but not limited to: . . .developing novice and experienced teachers' and mentors' abilities to reflect critically on their practices and on their growth as mathematics teachers and educators (Dawson, 2005, p. 3).

> A disposition to reflect on practice and work collaboratively with peers to review and explore the teaching of mathematics is recognized as. . .crucial. . .. An emergent professional language. . .alongside other forms of peer observation and review, appear to offer powerful means of supporting and engaging teachers in ongoing, professional learning based on insightful and informed reflection (Siemon, 2005, p. 2).

Critical reflection, in other words, is broadly regarded not only as an important attitude, but also a responsibility—and thus constitutes a vital part of this particular benchmark.

### *3d. Summary Comments on Section B*

At the risk of being too repetitive, once again we emphasize that the benchmarks presented here—that is, explicit beliefs, enacted beliefs, and attitudes—should not be treated as elements that can be isolated but as components of a more complex whole. The overarching concern here is how a mathematics teacher is present and presents (self, knowledge, purpose, etc.) in the mathematics classroom.

It is notable that the research into explicit beliefs far outweighs research into enacted beliefs and teacher attitudes. This disparity might point to a need to develop operational definitions of these constructs (i.e., of noted aspects of enacted beliefs and teacher attitudes) in order to enable observation and interpretation.

## 4. Section C: Recognized Issues in Teacher Development

There is a risk with framing factors and benchmarks, as we have, in terms of multi-faceted and evolving forms. It is important to observe that these sorts of complex constructs do not lend themselves to reliable and replicable observations, measurements, predictions, and manipulations.

In some ways, that is precisely the point being made. Such ideals are anchored in long-standing practices of isolating, de-contextualizing, and otherwise separating or reducing phenomena. In this section, we look across some of the ways that these tendencies manifest themselves in teaching and research practices, endeavoring to articulate attitudes that are more attentive to the evolving characters of the phenomena we study and seek to affect.

### *4a. Issue 1: Making and Unmaking Distinctions*

Throughout the process of writing this chapter, we have been confronted with the conflicting needs, on the one hand, to use such distinctions as individual/collective, knower/knowledge, explicit/enacted, and teacher/researcher and, on the other hand, to be suspicious of these sorts of distinctions.

For the most part, we have resolved the apparent conflicts by framing these dyads as complementarities rather than dichotomies—a move that might have some important conceptual and practical advantages. For example, conceived as a dichotomy, the individual/collective dyad prompts attentions to competing interests and incompatible goals. There seems to be an implicit win/lose, good/bad judgemental position associated with the separation.

By contrast, thinking in terms of complementarities—that is, in terms of a win-win, mutually beneficial logic—prompts attentions to the manners in which individual self-interest can serve the collective good and how collective structures can enable individual growth. This sort of thinking certainly seems to underlie the

emphasis on collectivity that is so prominent in many of the research reports (as noted in the discussion of Factor 4: Structures for intervention).

Other dyads that might be similarly re-construed (and some of the reports in which they are invoked) include:

- explicit (formal) knowledge and enacted (implicit, tacit) knowledge (Brown; Davis & Simmt);
- educational researcher and teacher educator (Brown; Davis & Simmt; Jaworski; Siemon);
- researcher and teacher (Davis & Simmt; Jaworski);
- theory and practice (Brown; Jaworski);
- formal mathematics and school mathematics (Davis & Simmt); and
- insider and outsider (Dawson; Jaworski).

This said, some dyads, which appear to be read as strong distinctions, were invoked in the reports, including:

- vibrant sufficiency versus static deficiency (Davis & Simmt); and
- constructivist pedagogy versus traditional pedagogy (Kidd).

## *4b. Issue 2: Contextual and Situational Matters*

Another possible entry to the list of troubling dyads would be action-and-context, which is a topic that has figured prominently in discussions of mathematics, education, and mathematics education informed by culturalist, sociological, and anthropological literatures.

Dawson flags some of the dimensions of context, including geography and culture, that are often allowed to slip into the backdrop of contemporary discussions. Geographical matters are rarely raised in discussions of mathematics education. By contrast, culture is much more prominently represented in the literature, but it is perhaps given short shrift in mainstream discussions. The "simple" matter that the language of expression for this volume is English—which might be argued to be co-implicated in the Eurocentric biases that are knitted through most of formal mathematics—is a case in point. Dawson's discussion presents a number of cautions, rooted in his experiences of learning to be mindful of where one is, whom one is with, and how one is with them.

Kidd's examination of some of the regional differences in mathematics pedagogy in the United States also provides a cautionary tale of sorts. The variations in attitudes, emphases, and outcomes further underscore that care must be taken to be attentive to the nuances of context.

In addition, of course, such attention must be sustained. A prominent theme across the research reports was the need to study practices, beliefs, and transformations over extended periods—not only because individuals change and collectives evolve, but also because they and their contexts/situations are enfolded in and unfold from one another. Davis and Simmt express this sensibility in their

acknowledgement that they affect what they are studying in their efforts to study phenomena associated with mathematics pedagogy—the point being that sensitivity to context entails an attunement to one's role in defining and triggering changes to context.

## *4c. Issue 3: Embracing Complexity*

This point amounts to a re-iteration of one that we have made throughout this chapter: when seeking to understand phenomena that are contextually sensitive and dynamically specified, one must not construe "factors", "benchmarks", "issues", or any other tool to enable observation and interpretation in isolated, reductive terms. The question emerges of how to focus research on specifics while maintaining a hold on complexity.

For researchers and teacher educators, we might argue that certain ethical issues present themselves here. The following are among the issues that merit consideration:

- How do we study a phenomenon that is not only constantly changing but that changes in part because it is being studied? In particular, how do we track our own complicity in the phenomena that we study?
- How are assumptions concealed and/or exposed by our languaging practices?
- What is the role of new vocabularies in helping us to think/act differently (including, e.g., "virtual monologue", "metacommenting", "vibrant sufficiency") with regard to our roles as researchers, teacher educators, teachers, and mathematics knowers.

## *4d. Summary Comments on Section C*

An unfortunate aspect of educational research is that, as an academic enterprise, it was first articulated in an era when inquiries were to be framed by well-articulated questions, unambiguous definitions, and pre-selected methods—and those inquiries were expected to provide correspondingly robust, replicable, and universally applicable results.

An emergent issue in research in the social sciences and humanities is that desires for validity and reliability, drawn from analytic science, have given way to desires for viability and reasonableness, rooted in more complex awarenesses. Mathematics, education, pedagogy, and research are evolving forms—ones that are entangled in, but not able to be reduced to, the actions of those who engage in or identify with them. However, there are also moves to pull back from vague qualitativity to provide clear definitive constructs and warrants for evidence (Shavelson, Phillips, Towne, & Feuer, 2003).

## 5. Concluding Remarks

What is being said in the research papers presented to this strand of the study group recognizes the complexity of teacher development. Knowledge is situated in the contexts of the practices of countries, within both mathematics learning and teaching practices. An individual teacher's practice and development is difficult to talk about in isolation without considering the local nature of the practices involved at different focal lengths of the lens viewing those practices ranging from governmental, school, particular class culture, and achievements of students. To be able to consider working with teachers who are themselves learning about the teaching and learning of mathematics working together in groups across schools (Jaworski) or across islands of Micronesia (Dawson) seems like an important construct. Co-observation and co-teaching—being able to act in different ways through observing a different reality, planning together, or developing a culture within a department or within a classroom that itself becomes a learning community or classroom (Davis & Simmt, Brown) gives us a way of thinking about professional development in which the teachers share thoughts and practices rather than a particular way of doing things. Teachers learn, and those who teach teachers learn correspondingly.

## References

Bateson, G. (2000 [1972]). *Steps to an ecology of mind*. Chicago: University of Chicago Press.

Brown, L., & Coles, A. (2000). Complex decision-making in the classroom: The teacher as an intuitive practitioner. In T. Atkinson & G. Claxton (Eds.), *The intuitive practitioner: On the value of not always knowing what one is doing* (pp. 165–181). Buckingham: Open University Press.

Davis, B., & Sumara, D. (2006). *Complexity and education: Inquiries into learning, teaching, and research*. Mahwah, N.J.: Lawrence Erlbaum.

Jaworski, B. (2003). Research practice into/influencing mathematics teaching and learning development: Towards a theoretical framework based on co-learning partnerships. *Educational Studies in Mathematics*, 54(2–3), 249–282.

Jaworski B. (2006). Theory and practice in mathematics teaching development: Critical inquiry as a mode of learning in teaching. *Journal of Mathematics Teacher Education*, 9(2), 187–211.

Lakoff, G. (1987). *Women, fire and dangerous things*. Chicago: University of Chicago Press.

Lerman, S. (2000). The social turn in mathematics education research. In J. Boaler (Ed.), *Multiple perspectives on mathematics teaching and learning*. London: Ablex.

Polanyi, M. (1958). *Personal Knowledge*. London: Routledge and Kegan Paul.

Shavelson, R. J., Phillips, D. C., Towne, L., & Feuer, M. J. (2003). On the science of education design studies. *Educational Researcher*, 32(1), 25–28.

Sierpinska, A., & Lerman, S. (1996). Espistemologies of mathematics and mathematics education. In A. J. Bishop, K. Clements, C. Keitel, J. Kilpatrick, & C. Laborde (Eds.), *International handbook of mathematics education* (pp. 827–876). Dordrecht: Kluwer Academic Publishers.

Steffe, L. P., & Gale, J. (1995). *Constructivism in education*. Hillslade, N.J.: Lawrence Erlbaum.

Wenger, E. (1998). *Communities of practice: Learning, meaning and identity*. Cambridge: Cambridge University Press.

## Original titles of papers sunmitted to ICMII5, Strand II, Theme 1

All papers presented at the conference of the 15th ICMI Study on the Professional Education and Development of Teachers of Mathematics, Águas de Lindóia, Brazil (available at http://stwww.weizmann.ac.il/G-math/ICMI/log_in.html).

Marcelo Bairral (mbairral@ufrrj.br) & Joaquin Giménez. Dialogic use of teleinteractions for distance geometry teacher training (12–16 years old) as an equity framework.

Laurinda Brown (laurinda.brown@bris.ac.uk). Purposes, metacommenting and basic-level categories: Parallels between teaching mathematics and learning to teach mathematics.

Tenoch Cedillo-Avalos (tcedillo@upn.mx) & Marcela Santillán. Algebra as a language in use: A promising alternative as an agent of change in the conceptions and practices of the mathematics teachers.

Catherine-Marie Chiocca (catherine-marie.chiocca@educagri.fr). Functions of writing for the consideration of pupils' learning by trainee teachers.

Brent Davis (brent.davis@ubc.ca), Elaine Simmt, Dennis Sumara. Mathematics-for-teaching: An ongoing investigation of the mathematics teachers (need to) know.

A. J. (Sandy) Dawson (dawsona@hawaii.edu). Mathematics education in Micronesia: Building local capacity to provide professional development for teachers of mathematics.

Jeremy Hodgen (jeremy.hodgen@kcl.ac.uk). Motivation, desire, and teacher change in primary mathematics.

Barbara Jaworski (b.jaworski@lboro.ac.uk). Learning in practice from a study of practice.

Margaret L. Kidd (mkidd@fullerton.edu). What factors help or hinder a change in teaching practices when the curriculum changes?

Dianne Siemon (siemon@rmit.edu.au). Learning in and from professional practice through peer observation and review: A case study.