# Chapter 1.3.4
# Educators and the Teacher Training Context

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## 1. Introduction

There is increasing recognition in the United States of the need for mathematics departments and mathematicians to become involved in training mathematics teachers for primary and secondary schools. Certainly, issues that are valued by both mathematicians and mathematics educators could promote collaboration. For example, the Mathematical Sciences Research Institute, in Berkeley, California, has initiated a series of workshops entitled “Critical Issues in Mathematics Education,” which aims “to provide opportunities for mathematicians to cooperate with experts from other communities on the improvement of mathematics teaching and learning” (Thames, 2006, p. iii; see also, for example, Conference Board on Mathematical Sciences, 2001; McCallum, 2003). However, this area of the 15th study conference received no papers.

In this article, we discuss ideas that might be of interest to both mathematicians and mathematics educators, and we give examples of some projects that could represent ways to begin collaborations. The ideas that we discuss include the concept of the Knowledge Quartet (Rowland et al. 2005a, 2005b), the notion of a mathematical habit of mind, and the comparison of ways in which mathematicians present fractions to future teachers (McCrory, 2006). Finally, we look at preliminary results from a small international survey to consider whether and how mathematicians and mathematics educators might collaborate on content and pedagogy courses for intending teachers. It is important to find common grounds for discussion between mathematicians and mathematics educators on topics valued by both groups: when the communication between the two groups goes astray, difficulties such as the so-called “Math Wars” in the United States (Ralston, 2003) can result in the waste of much valuable energy.

## 2. Ideas of Mutual Interest

In the past, in many countries, the relationship between mathematics departments and teacher educators has been minimal, to say the least, especially at research-

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oriented universities. However, there is research concerning training of primary teachers that has proved to be of much interest to the mathematics community. For example, Liping Ma's work (1999) is well known and resonates well in the community of mathematicians regardless of whether they are interested in teacher training, because it shows why mathematics content is important for intending teachers.

Similarly, a recent project in the United Kingdom, "Subject Knowledge in Mathematics", explored how mathematics subject knowledge of trainee elementary school teachers influences their classroom teaching performance (see Rowland, Huckstep, & Thwaites, 2005a,b). Based on extensive analysis and coding of videotaped lessons taught by trainee teachers, the authors conceptualized a theoretical framework that describes certain aspects of trainee teachers' actions in the classroom and how these are influenced by their content knowledge. The framework is called the Knowledge Quartet and consists of foundation, transformation, connection, and contingency. Detailed description of the findings can be found, for example, in Rowland et al. (2005a). Further use of this framework has shown that it is a comprehensive tool for thinking about the ways in which subject knowledge comes into play in the classroom. In several publications by the Cambridge team (see, for example, Rowland et al., 2005b), there are examples of how the Quartet has been used to analyze the classroom actions of trainee teachers. Data-grounded research such as this is of potential interest to mathematicians, as it highlights how mathematics content knowledge relates significantly to the ways trainee teachers act in the classroom.

Equally influential, Hyman Bass's (2005) interpretation of mathematics education in the spirit of a field of applied mathematics has given credence to the importance of teacher preparation as a substantive field for application of mathematical study. McCallum's 2003 article adds depth to the conversation through the discussion of the similarities and differences between the nature of research in mathematics education and other mathematics disciplines. There are now some institutes (e.g., the Institute for Mathematics and Education and the Park City Mathematics Institute) that provide a variety of ways for doctoral-level mathematicians to become active in primary or secondary education.

In a review of some textbooks authored by mathematicians and meant for future teachers (McCrory, 2006), a key point is the comparing and contrasting of specific approaches to fractions from a variety of conceptual viewpoints. Given that a rigorous approach requires equivalence classes, which no one believes is the right approach to teaching future primary teachers (nor their pupils), each of the authors constructed fractions in different manners. Fractions are thus a natural place for mathematicians to see how deep the mathematics of the primary grades is. A joint seminar discussing this article would provide a venue to explore common interests between mathematics and mathematics education, as would a seminar to discuss the work of Liping Ma or the philosophy of Hyman Bass.

## 3. Mathematical Habit of Mind

It is difficult to trace the history of the idea of mathematical habit of mind (MHM) because it is something that mathematicians do all the time. It somehow becomes part of the internal thinking process. For example, it is present in all of Polya's writing on problem solving although he doesn't use the phrase. A fine example is Polya's 2007 article, which encourages students to reason "through analogy and verification in special cases" and to exercise "plausible reasoning" (inductive reasoning in more modern terms). The discussion of the area of a triangle in terms of the length of its sides is highly informative and delightful.

The Conference Board on Mathematical Sciences (CBMS) has had a strong influence on the courses taught in the United States by mathematicians to future teachers.

> Two general themes of this report are: (i) the intellectual substance in school mathematics; and (ii) the special nature of the mathematical knowledge needed for teaching. We owe to mathematics education research of the past decade, or so, the realizations that substantial mathematical understanding is needed even to teach whole number arithmetic well (CBMS, 2001, p. xi).

The notion of the mathematical habit of mind is mentioned explicitly in one of the CBMS recommendations:

> Recommendation 4: Along with building mathematical knowledge, mathematics courses for prospective teachers should develop the habits of the mind of a mathematical thinker and demonstrate flexible, interactive styles of teaching. ...*Teachers need to learn to ask good mathematical questions, as well as find solutions, and to look at problems from multiple points of view* (p. 8, italics in original).

There are other possible descriptions of MHM. One is contained in Thames: "Throughout this document, the term knowledge is used broadly to include the knowledge, skill, dispositions, and habits of mind that support doing mathematics" (2006). Another example is contained in the preface to Long et al. and describes MHM as follows:

> One of the ways to enhance conceptual understanding is for future teachers to develop a deeper way of thinking about mathematical concepts and problem solving, to explore mathematical ideas, to formulate questions, and to ask themselves whether there is "something more" (a generalization) in the mathematics on which they are working. This trait is called a "mathematical habit of the mind" in some recent writings or, more simply, a habit of the mind (MHM). MHM includes open ended or vaguely worded problems in which you will have to examine how you would model a situation (2008).

Here are two examples that have been used in teacher preparation courses. The students in course Math for Future Primary Teachers at the University of Kentucky have found that a traffic-jam problem (Peter-Koop, 2005) is quite intriguing as a group exercise. The students were astounded by the success of the German fourth-graders who tried the problem, as reported by Peter-Koop.[1] Another example of MHM is found in a course for intending teachers of middle school pupils (children ages 12–14, grades 6–8, or 7–9 in the United States). After investigating a number of examples exploring the sum of even integers and the product of odd numbers, the instructor would ask the students to prove that the product of even numbers is even. The students were asked if "something more is going on." After some discussion, they realized that the product is divisible by four as well as two. This also is an

[1] There is a 3-km traffic jam on the motorway. How many vehicles are caught in this traffic jam?

example of a MHM that can be introduced to primary-grade students and to their teachers.

MHM should be reserved for ideas and problems in which significantly more is going on than meets the eye. On the other hand, problem solving, communicating, reasoning, and making connections, all principles of the National Council of Teachers of Mathematics, can also be described as habits of mind.

The previous argument advocates the inclusion of MHM in the curriculum of courses for intending teachers. To our knowledge, in spite of Recommendation 4 of CBMS as quoted previously, there is not yet a research base for this approach. Furthermore, MHM is now in the process of being constructed for a curriculum in an American high school. The articles (Cuoco, 2001) and (Cuoco, Goldenberg, & Mark, 1996) are position papers setting out a direction for the mathematics curriculum and drawing upon a curriculum-development project, Connected Geometry (for more details, see www.phschool.com/cme). However, the ideas promoted in these papers are familiar to those who have been involved in innovative curriculum developments in mathematics over the last fifty years. The thinking is implicit in Polya's books on problem solving (written by a mathematician) and in more recent publications, such as Mason, Burton, and Stacey (1985) (written by mathematicians and mathematics educators). In many countries, some of this thinking has found its way into national curricula but with limited influence. In England and Wales, the development of a strand of the mathematics national curriculum entitled "Using and Applying Mathematics" was intended to develop awareness of some of the things that Cuoco calls MHM, and the introduction of assessed coursework for public assessment at age sixteen was intended to promote this, but assessed coursework has recently been abandoned in the face of fears about widespread plagiarism. Even now, twenty two years after the introduction of the national curriculum in England, with "Using and Applying Mathematics" at its heart, many of those teaching mathematics in English schools are not confident about encouraging their pupils to develop mathematical habits of mind. The role of MHM or the development of mathematical thinking in school mathematics is fertile ground for joint work between mathematicians and mathematics educators but, given space considerations, we only have dealt with, and advocated, MHM in the curriculum for future teachers.

## 4. Content/Pedagogy Course Survey

We now report briefly on some preliminary findings emerging from the content/pedagogy survey. In the survey, which consisted of an online questionnaire, we asked mathematicians and mathematics educators for their views about the roles of content and pedagogy courses in the context of initial teacher training, about the actual and desired role of mathematics educators in the content courses, and the actual and desired role of mathematicians in the pedagogy courses. We received thirty-four responses from fourteen countries; twenty of the respondents described themselves as mathematics educators, four as mathematicians, and ten as both. We used the institutional frame of the University of Kentucky (where the first author works) as an example of a possible structuring of teacher training, and we asked respondents to comment. The first emergent finding is that, in some institutions (four in our sample), there is no division between content courses and pedagogy courses. Mathematics content is taught together with its pedagogical content. Other respondents, from institutions where this is not the case, also expressed a preference for this way of structuring the course. One respondent commented:

> It is not very healthy to separate content courses from methods courses. Content courses (for future or practicing teachers) should have a strong methods component (or at least a component that links the content to research in mathematics education); and methods courses should have a very strong content component.

For the role of content courses, there is agreement among the respondents that future teachers need to have the "big picture" of mathematics and that they should be able link the mathematics that they will eventually teach to other parts of the subject. This would seem to relate to the finding of Askew, Brown, Rhodes, Wiliam, and Johnson (1997), in a study of effective primary mathematics teachers, that effective teaching involves a connectionist approach to mathematics. The affective dimension is also mentioned in the responses; the case is proposed that trainee teachers (especially at primary level) often come to content courses with their own "mathematics anxiety", which needs to be addressed if they are to teach mathematics confidently and not transmit their own anxiety to their students. As for pedagogy courses, respondents suggest that such courses should aspire to teach theoretical constructs in mathematics education and how these bear on teaching practice. However, the findings most relevant to our chapter emerged from the final questions.

## *4a. The Actual and Desired Role of Mathematics Educators in Content Courses*

Across the responses there is an almost equal split between institutions where mathematics educators teach content courses and where mathematics educators have no input on such courses. There are however a few institutions where such courses are taught jointly by mathematicians and mathematics educators. As for what mathematics educators can bring to these courses, there is overwhelming agreement that this is "knowledge of mathematics for teaching". This is articulated in several ways ranging from the application of teaching and learning theories in problem-solving activities to knowledge of new development in mathematics education.

## *4b. The Actual and Desired Role of Mathematicians in Pedagogy Courses*

The most common response is that mathematicians have no role in teaching pedagogy courses. As for the desired role for mathematicians in such courses, there is a spectrum from respondents who believe strongly that mathematicians should indeed have no role in pedagogy courses (with some very strong support for this thesis) to respondents who think this role should, for example, involve mathematicians sharing their own experience as researchers and learners, offering the "big picture" of mathematics beyond the curriculum taught in content courses and offering epistemological ideas on what is mathematics. One respondent wrote:

> The best thing would be that mathematicians (I intend: mathematicians with no competence in mathematics education) do not teach those courses! Otherwise it would be like if they would teach specific medical disciplines to prospective medical doctors, based on their experiences of ill people and auto-diagnosis and auto-therapy...

Another wrote:

> I think mathematicians and mathematics educators should collaborate in planning and teaching both the content and pedagogy courses. Mathematics education is an interdisciplinary field and both courses benefit from the different perspectives and expertise of both mathematicians and mathematics educators.

In conclusion, the main findings of this small survey point to the following:

1. A split between pedagogy courses and content courses might not be productive. It is possible to teach courses that address both at the same time.
2. The main role of mathematicians in a content course is perhaps the ability to offer to student teachers the "big picture" of mathematics and develop students' epistemology of mathematics. This will help students form their pedagogy.
3. The role of mathematics educators in content courses is to provide theoretical frameworks within which the teaching of mathematics can be conceptualized.
4. There is no agreement on what the role of mathematicians in pedagogy courses should be. There is some strong support for the thesis that mathematicians will not be able to contribute to pedagogy courses, as pedagogy is not their area of expertise. On the other hand, many responses pointed to the opportunity to capitalize on mathematicians' knowledge of mathematics and "how mathematics works" and their experiences as learners and teachers of mathematics.

In considering these points, it is important to recognize that nearly all of the participants considered themselves to be either mathematics educators or both mathematician and mathematics educator. In particular, points 3 and 4, stated previously, might suggest a positive perception of mathematics educators and a rather critical image of mathematicians among respondents. There is another interesting issue connected to the results of our survey that comes from the first question, when the respondents were asked to describe themselves as mathematicians, mathematics educators, or both. In the survey we omitted a clear definition of what qualifies someone to be a mathematician or a mathematics educator. Eight of our respondents felt the need to describe their academic qualifications and the type of work they do in response to this question, for example, indicating that what makes someone a mathematician can be open to discussion. An investigation into the decision processes used by respondents to determine whether they considered themselves as mathematicians, mathematics educators, or both might shed light into why, for example, mathematicians are viewed rather critically by the participants in the survey.

Our small survey suggests that there is much common ground as well as some significantly different views between mathematicians and maths educators. We hope that this short paper might encourage more dialogue between the different communities.

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