Chapter 1.3.1
Mathematics Educators' Knowledge and Development

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This chapter focuses on mathematics educators. Generally, mathematics educators include educators who facilitate the learning of mathematics, as well as educators who facilitate the teaching of (or learning to teach) mathematics. There is much in common between the knowledge and development of mathematics teachers and that of their educators, yet there are some distinctive characteristics of teacher educators' knowledge which are worthwhile to examine (Zaslavsky & Peled, 2007).

1. Models of Educators' Development

Zaslavsky & Peled (2007) point to a number of trends in recent years with respect to studies related to teacher education. One of these trends has to do with entering mathematics teacher educators into the picture. Researchers have started to conceptualize teacher educators' role and knowledge for mathematics teacher education. Some papers deal with what is entailed in becoming a mathematics teacher educator, by self-reports and reflective accounts (e.g., Tzur, 2001), and others describe special courses for providers of professional development activities for mathematics teachers (e.g., Even, 1999) or programmes enhancing growth of mathematics teacher educators through their practice (e.g., Zaslavsky & Leikin, 2004).

Teacher educators can be seen both as learners and as facilitators of learning. The "content" related to their learning involves beliefs, knowledge, and practice as well as some meta-cognitive skills, such as exhibiting awareness and employing reflection. A major concern of providers of professional development activities is the need to foster teachers' reflective practice. Theories of reflective practice follow Dewey's emphasis on the reflective activity of both the teacher and the student as a means for advancing their thinking (Dewey, 1933; Schön, 1983, 1987). The notions of reflection on-action and reflection in-action have been acknowledged as significant components contributing to the development of teachers as well as teacher educators' knowledge and practice. The importance of reflecting on one's own practice and learning experiences is expressed by Lerman (2001): "Reflective practice offers a view of how teachers act in the classroom as informed, concerned professionals and how they continue to learn about teaching and about learning,

R. Even, D.L. Ball (eds.), *The Professional Education and Development of Teachers of Mathematics*, DOI 10.1007/978-0-387-09601-8_9,

about themselves as teachers, and about their pupils as learners" (p. 39). Thus, teachers and teacher educators can be seen as constant learners who should continuously reflect on their work and make sense of their experiences. From a social-practice theory, which stems from Vygotsky's theory on the social nature of the learning process (Vygotsky, 1978), mathematics teachers and mathematics teacher educators are often regarded as two interrelated communities of practice enhancing each other's development.

A key issue to be addressed in professional development of mathematics educators is the learning of mathematics. Cooney (1994, 2001) discusses two central constructs for mathematics educators—mathematical power and pedagogical power, which deal with teachers' abilities to draw on the knowledge needed to solve problems in context (mathematical or pedagogical). Jaworski (2001) adds a third construct—educative power, which characterizes the roles that teacher educators may play in the process of enhancing teachers' learning. This construct can be taken to include the ability of teacher educators to draw on knowledge that is needed for facilitating teachers' mathematical and pedagogical problem solving.

Various models have been suggested by scholars attempting to describe teacher practices as well as teacher learning. In this chapter I describe models that have been useful to me in my own work. A model that provides a lens through which to examine mathematics educators' practices is suggested by Jaworski (1992, 1994) in her Teaching Triad, which is consistent with constructivist perspectives of learning and teaching. Her triad includes three elements, which are often inseparable: the management of student learning, sensitivity to students, and the mathematical challenge. According to Jaworski, "this triad forms a powerful tool for making sense of the practice of teaching mathematics" (1992, p. 8). By substituting "students" with "learners", and "mathematical challenge" with "challenging tasks", a more general triad is obtained (Zaslavsky & Peled, 2007) that may be applied to other learners, for example, mathematics teachers, in the context of their learning.

Jaworski's Teaching Triad can be used not only for making sense of classroom practices, but also for highlighting the different kinds of knowledge teachers need for teaching mathematics, which concur to a large extent with some of Shulman's categories (1986).

Steinbring (1998) offers a model that provides insight to the mechanisms that facilitate learning of both students and teachers in the course of a mathematics lesson. His model looks at the learning of students and their teacher as two autonomous systems that build on each other. In this model, reflection plays a critical role in both student and teacher learning. While students learn by engaging in a task, interpreting and making sense of their solutions, and reflecting on and generalizing them, the teacher learns from observing the processes students encounter, varying the learning offers, and reflecting upon the entire process. Similar to the modification of Jaworski's model, by substituting in Steinbring's model "students" with "learners" and "teacher" with "facilitator", this mechanism can be useful for making sense of how various mathematics educators learn from their practice, including mathematics teachers, mathematics teacher educators, and mathematics teacher educator educators. As shown in **Fig. 1.3.1.2**, facilitators' learning occurs as an

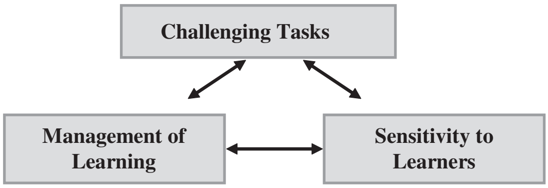

Fig. 1.3.1.1 Modification of Jaworski's Teaching Triad

outcome of their observations of learners' engagements in tasks and by reflecting on learners' work.

Interestingly, both models (**Figs. 1.3.1.1 and 1.3.1.2**) can be applied to various kinds of learning settings, differing in the specific facilitator-learner identities (i.e., mathematics teacher–students, mathematics teacher educator–mathematics teachers, mathematics teacher educator educator–mathematics teacher educators) and in the nature and content of the learning. In both models, tasks play a critical role.

Zaslavsky & Leikin (2004) suggest a recursive look at these two models that conveys similarities between the different mathematics educators yet also suggests that the type and complexity of tasks changes considerably for the different kinds of mathematics educators (mathematics teachers, mathematics teacher educators, and mathematics teacher educator educators). Thus, the knowledge and practice required of mathematics teacher educators include engaging teachers in tasks that address all three elements of Jaworski's teaching triad while exhibiting sensitivity to teachers as learners and managing the entire activities.

Similarly, a recursive look at **Fig. 1.3.1.2** (as in Zaslavsky & Peled, 2007) conveys the mechanism of learning through practice and at the same time illustrates the

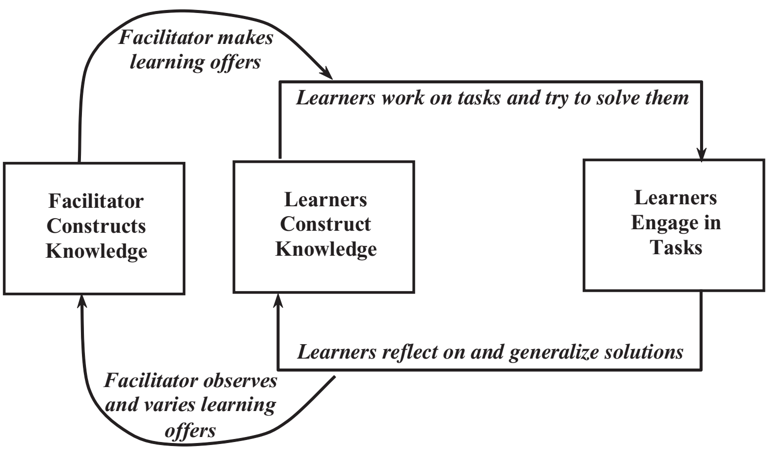

Fig. 1.3.1.2 Modification of Steinbring's Model of Teaching and Learning Mathematics (based on *Zaslavsky & Peled, 2007)*

increasing complexity of the cycles of reflections that are required of the various kinds of mathematics educators.

As mentioned earlier, these models have been useful in thinking about mathematics educators' knowledge and development (Zaslavsky, Chapman, & Leikin, 2003; Zaslavsky & Leikin, 2004; Peled & Hershkovitz, 2004). They are particularly helpful in trying to make sense of how one becomes a teacher educator. Unlike the many kinds of institutionalized (pre-service and in-service) teacher education programmes, there are hardly any teacher educator education programmes; thus, becoming a mathematics teacher educator occurs over time, through ongoing reflection on one's own experiences in facilitating teachers' learning (e.g., Tzur, 2001). Teacher educators' responsibilities usually involve both teaching and research (Adler, Ball, Krainer, Lin, & Novotna, 2005; Zaslavsky, 2007). Mostly, their research contributes to their own professional development.

The above models convey the recognition that teacher learning is highly influenced by tasks in which teachers engage. Increasing attention is given to the nature of tasks that promote teacher learning and to task design (e.g., Arbaugh & Brown, 2005). This trend is presented in detail in Zaslavsky (2003), in which they stress the importance of the type of mathematical tasks, which teachers are offered by their educators in the course of professional-development programmes.

2. An Example of a Task Stemming from a Teacher Educator's Research

Zodik & Zaslavsky (2007) present the following task, which emerged from their study on mathematics teachers' choice and treatments of examples. Consider the following problem (the Rhombus Problem):

> A rhombus $\square BDEF$ is inscribed in a triangle ΔABC. Its diagonal BE is perpendicular to the side of the triangle, i.e., $BE \perp AC$.
> Prove that ΔABC is an isosceles triangle.

In practice, such a problem is usually presented with an accompanying diagram. In Zodik and Zaslavsky's study, the authors report on a classroom observation in which the teacher shared with one of the researchers her dilemmas regarding the kind of diagram to provide. The teacher raised some subtle concerns that increased the researchers' awareness to this problematic aspect of learning and teaching (Mason, 1998). Namely, each diagram has some advantages and some limitations. This classroom event seemed to the researchers a basis for creating a genuine learning opportunity for other practising as well as prospective teachers.

Thus, the task for both prospective and practising teachers, designed by the teacher educators based on their research findings, was as follows:

Consider the diagrams–a, b, c, d in **Fig. 1.3.1.3**–which one would you choose to accompany the problem?

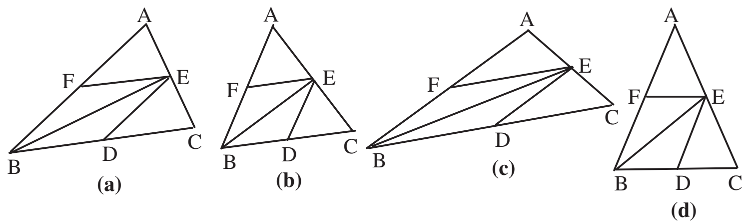

Fig. 1.3.1.3 Possible diagrams accompanying the Rhombus Problem (based on Zodik & Zaslavsky, 2007)

Diagrams a, b, c, and d in **Fig. 1.3.1.3** depict possible examples representing the givens of the problem. This task emerged from a classroom observation and was followed by an interview with the teacher, who expressed perplexity regarding what diagram to choose. By using this with prospective and practising teachers, we created a dilemma for them to consider, similar to the natural one the observed teacher experienced.

Note that the formulation of the problem does not indicate which pair of sides of VABC are equal. Thus, it may not be clear at first sight what the specific goal of the problem is, that is, which of the following equalities should be proven: $BA = BC$, $CA = CB$, or $AB = AC$. A capable student could analyse the problem and reach the conclusion that an attempt to prove that $BA = BC$ makes more sense than the other options (e.g., for symmetrical considerations). However, most students do not approach this problem in such a way. Moreover, they tend to rely to a large extent on the accompanying diagram. If no diagram is provided, students are likely to act in a prototypical manner by sketching the problem situation, as in Diagram d and attempting to prove that $AB = AC$, since many students think of an isosceles triangle with a horizontal basis.

Diagram a is a rather accurate illustration of the given case. It conveys the two sides of the triangle that are equal ($BA = BC$), making it easier for the student to decide how to proceed. Diagram b is a special case of the given in which VABC appears to be an equilateral triangle. Thus, it conceals the direct outcome of the given (i.e., that $BA = BC$) and may leave the student helpless regarding where to focus and what to prove. Diagrams c and d are distortions of the possible cases, as they are both "impossible": in Diagram c, VABC seems like a "generic" triangle, not an isosceles one (each side is of different length). This can be perceived as a "general case" that does not disclose any hints regarding which two sides are equal. Diagram d can be seen as a misleading sketch that contradicts the given and may lead the student to an attempt to prove that $AB = AC$, which in fact cannot be inferred from the given.

As seen previously, the specific sketch accompanying the geometric problem may influence the way a student approaches the problem and the extent to which the student is successful in proving it. One may argue that the more accurate the figure the better. However, a counter argument could be that by disclosing the full picture (as in Diagram a), the task for the student changes. He or she no longer needs to analyse the situation and make an educated choice what to prove. For a student who relies on the visual appearance, it becomes straightforward that s/he should focus on proving that $BA = BC$.

3. Concluding Remarks

Teachers' deliberations surrounding the Rhombus Problem drew their attention to subtleties they were not aware of before, including ambiguities they had transmitted to their students with respect to the role of diagrams in geometry. They had different views on what the best diagram would be and drew on their experiences and knowledge of students to support their views. The arguments they brought were both mathematical and pedagogical. This task illustrates how teacher educators' research may influence their teaching.

For mathematics educators, tasks serve a dual purpose (Zaslavsky, 2005, 2007). On the one hand, tasks are the means and content by which learning is facilitated. On the other hand, through a reflective process of designing, implementing, and modifying tasks, they turn into a means of the facilitator's (e.g., teacher educator's) learning. Moreover, in many cases research informs and enhances teaching and vice versa through tasks.

There is a fruitful interplay between teacher educator's roles as researchers, facilitators of teacher learning, and designers of tasks for teacher education. This interplay is a driving force for effective professional development of all mathematics educators—mathematics teachers, teacher educators, and educators of teacher educators.

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