A Stochastic Trading Model of Wealth Distribution

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Summary. We develop a stochastic model where the poorer end of the society engage in two-party trading while the richer end perform trade with gross entities. Using our model we are able to capture some of the essential features of wealth distribution in societies: the Boltzmann-Gibbs distribution at the lower end and the Pareto-like power law tails at the richer end. A reasonable scenario to connect the two ends of the wealth spectrum is presented. Also, we show analytically how different power law exponents can be obtained. Furthermore, a link with the models in macroeconomics is also attempted.

1 Introduction

In countries like United States, Japan, United Kingdom, Germany, Switzerland, New Zealand, etc., where data for wealth distribution is readily available, it is observed that the wealth is very unequally distributed and is highly concentrated. In a wealthy country like the United States various surveys over the past 30 years (in particular the Survey of Consumer Finances) show that a lions share of the total wealth is concentrated in the richest percentiles: the richest 1% owns one third of the wealth, and the top 5% holds more than half. At the other extreme, the bottom 10% own little or nothing at all.

Income is also unequally distributed and inequality in income leads to unequal wealth distribution. Income is defined as revenue from all sources before taxes but after transfers and thus includes labor earnings and income generated by wealth. However, income distribution is less skewed (and hence less unequal) than wealth distribution. The 1992 Survey of Consumer Finances revealed that the income of the income-rich top 1% was 18.5% of the total sample, and that of the top 5% was one third of the total income. In the same survey, the Gini index (whose value 0 corresponds to equal distribution and value 1 to wealth entirely in the hands of the richest) was shown to be 0.78 for the wealth distribution and only 0.57 for the income distribution.

There are many possible measures of wealth. In this paper we will concentrate on total net worth which includes all assets held by the households (financial wealth, real estate, vehicles) and net of all liabilities (mortgages and other debts). The degree of concentration of net worth held by various wealth percentile groups in various years is given in Table 1.

Table 1. Percent of net worth held by various percentile groups of the wealth distribution [1].

Percentile group	Year 1989	1992	1995	1998	2001
0-49.9	2.7	3.3	3.6	3.0	2.8
50-89.9	29.9	29.7	28.6	28.4	27.4
90-94.9	13.0	12.6	11.9	11.4	12.1
95-98.9	24.1	24.4	21.3	23.3	25.0
99-100	30.3	30.2	34.6	33.9	32.7

Inequality in the distribution of wealth in the population of a nation has provoked a lot of political debate. The observations that the top few percentage own a lions share of the wealth has been mathematically formulated as a power law by Pareto at the turn of the 19th century [2]. It is important for both economists, econophysicists, and policy makers to understand the root cause on this inequality: whether social injustice is the main culprit for such a lop-sided distribution. Over the past, economists have developed two models, namely, the dynastic model and the life-cycle model, to explain wealth distribution. In the dynastic model, where bequests are vehicles of transmission of wealth inequality, people save to improve the consumption of their descendants. On the other hand, in the life-cycle model, where wealth of an individual is a function of the age, people save to provide for their own consumption after retirement. Both these models and their hybrid versions have had only limited success quantitatively [3]. However, one of the ingredients that goes into these models, i.e., uninsurable shocks or stochasticity in income [4], has been exploited by econophysicists with remarkable success in reproducing power law tails qualitatively. It appears that randomness may very well be enough to explain the skewed wealth distribution and that a loaded dice may not be the root cause.

The wealth distribution of the poor (0-90 wealth percentile group) is exponential or Boltzmann-Gibb's like [5, 6], while that of the higher wealth group has a power law tail with exponent varying between 2 and 3. The Boltzmann-Gibb's law has been shown to be obtainable when trading between two people, in the absence of any savings, is totally random [7, 8, 9]. The constant finite savings case has been studied earlier numerically by Chakraborti and Chakrabarti [7] and later analytically by us [8]. As regards the fat tail in the wealth distribution, several researchers have obtained Pareto-like behavior using approaches such as random savings [10], inelastic scattering [11], generalized Lotka Volterra dynamics [12], asymmetric interactions between agents

[13], nonextensive Tsallis statistics [14], analogy with directed polymers in random media [15], and three parameter based trade-investment model [16]. As regards an egalitarian solution, there has also been an interesting model (conservative exchange market model) based on the Bak-Sneppen model that takes measures to improve the lot of the poorest [17]. Within this model the authors obtain a Gibbs type of wealth distribution with almost all agents above a finite poverty line.

In this article, we try to model the processes that produce the wealth distribution in societies. Our model involves two types of trading processes – tiny and gross [19]. The tiny process involves trading between two individuals while the gross one involves trading between an individual and the gross-system. The philosophy is that small wealth distribution is governed by two-party trading while the large wealth distribution involves big players interacting with the gross-system. The poor are mainly involved in trading with other poor individuals. Whereas the big players mainly interact with large entities/organizations such as government(s), markets of nations, etc. These large entities/organizations are treated as making up the gross-system in our model. The gross-system is thus a huge reservoir of wealth. Hence, our model invokes the tiny channel at small wealths while at large wealths the gross channel gets turned on. Our two types of trading model is motivated by the fact that a kink seems to be generic in the wealth/income distributions in real populations (as borne out by the empirical data in Fig. 9 of Ref. [5] and Fig. 1 of Ref. [18]) indicating that two different dynamics may be operative in the poor and the wealthy regimes.

2 Two Types of Trading Model

2.1 Model for Tiny-Trading

The model describes two-party trading between agents 1 and 2 whose respective wealths y_1 and y_2 are smaller than a cutoff wealth y_c. The two agents engage in trading where they put forth a fraction of their wealth $(1 - \lambda_t)y_1$ and $(1-\lambda_t)y_2$ [with $0 \leq \lambda_t < 1$]. Then the total money $(1-\lambda_t)(y_1+y_2)$ is randomly distributed between the two. The total money between the two is conserved in the two-party trading process. We assume that probability of trading by individuals having certain money is proportional to the number of individuals with that money.

We will now derive the equilibrium distribution function $f(y)dy$ which gives the probability of an agent having money between y and $y + dy$. We assume that, irrespective of the starting point, the system evolves to the equilibrium distribution after sufficient number of trading interactions. We will now consider interactions after the system has attained steady state. The joint probability that, before interaction, money of 1 lies between x and $x+dx$ and money of 2 lies between z and $z + dz$ is $f(x)dxf(z)dz$. Since the total money is conserved in the interaction, we let $L = x + z$ and analyze in terms of L. Then the joint probability becomes $f(x)dxf(L-x)dL$. We will now generate equilibrium distribution after interaction by noting that *at steady state the distribution is the same before and after interaction.* Probability that L is distributed to give money of 1 between y and $y + dy$ is

$$\frac{dy}{(1-\lambda_t)L} f(x)dxf(L-x)dL, \tag{1}$$

with $x\lambda_t \leq y \leq x\lambda_t + (1-\lambda_t)L$. Thus we see that $x \leq y/\lambda_t$ and $x \geq [y-(1-\lambda_t)L]/\lambda_t$. Actually x should also satisfy the constraint $0 \leq x \leq L$ because the agents cannot have negative money. Thus the upper limit on x is $\min\{L, y/\lambda_t\}$ (i.e., minimum of L and y/λ_t) and the lower limit is $_{\max}\{0, [y-(1-\lambda_t)L]/\lambda_t\}$. Now, we know that the total money L has to be greater than y so that the agents have non-negative money. Thus we get the following distribution function for the money of 1 to lie between y and $y + dy$

$$f(y) = \int_y^{\infty} dL \int_{a(y,L,\lambda_t)}^{b(y,L,\lambda_t)} dx \mathcal{F}(x, L, \lambda_t), \tag{2}$$

where

$$a(y, L, \lambda_t) \equiv_{\max} [0, \{y-(1-\lambda_t)L\}/\lambda_t],$$

$$b(y, L, \lambda_t) \equiv \min[L, y/\lambda_t],$$

and

$$\mathcal{F}(x, L, \lambda_t) \equiv \frac{f(x)f(L-x)}{(1-\lambda_t)L}.$$

The above result was obtained earlier by using Boltzmann transport theory [19].

On introducing an upper cutoff y_c for the two-party trading, the contribution to the distribution function $f(y)$ from the tiny channel becomes

$$\gamma \int_y^{\infty} dL \int_{a(y,L,\lambda_t)}^{b(y,L,\lambda_t)} dx \mathcal{F}(x, L, \lambda_t)\mathcal{H}(x, L, y_c). \tag{3}$$

In the above equation

$$\mathcal{H}(x, L, y_c) \equiv [1-\theta(x-y_c)][1-\theta(L-x-y_c)],$$

with $\theta(x)$ being the unit step function and $\gamma = 1/\int_0^{y_c} dxf(x)$ is a normalization constant introduced to account for the less than unity value of the probability of picking a person below y_c.

2.2 Model for Gross-Trading

Next, we will analyze the contribution to the distribution function $f(y)$ from gross-trading. An individual possessing wealth y_1 larger than a cutoff wealth (y_c) trades with a fraction $(1-\lambda_g)$ of his wealth y_1 with the gross-system. The latter puts forth an equal amount of money $(1-\lambda_g)y_1$. The trading involves the total sum $2(1-\lambda_g)y_1$ being randomly distributed between the individual and the reservoir. Thus on an average the gross-system's wealth is conserved. The probability that the individuals money after interaction lies between y and $y+dy$ is

$$\frac{dy}{2(1-\lambda_g)y_1} f(y_1)dy_1, \tag{4}$$

where $\lambda_g y_1 \le y \le (2-\lambda_g)y_1$. Then the distribution function $f(y)$ is given by

$$f(y) = \int_{y/(2-\lambda_g)}^{y/\lambda_g} \frac{dy_1 f(y_1)}{2y_1(1-\lambda_g)}. \tag{5}$$

Now it is interesting to note that the solution of the above equation is given by $f(y) = c/y^n$. Then, to obtain n one solves the equation

$$(2-\lambda_g)^n - \lambda_g^n = 2n(1-\lambda_g), \tag{6}$$

and obtains $n = 1, 2$. Only $n = 2$ is a realistic solution because it gives a finite cumulative probability. Surprisingly, the solution is *independent of* λ_g. Also, clearly the distribution function makes sense only for $y > 0$. On taking into account an upper cutoff y_c, the contribution to the distribution function $f(y)$ from the gross channel is

$$\int_{y/(2-\lambda_g)}^{y/\lambda_g} \frac{dy_1 f(y_1)}{2y_1(1-\lambda_g)} \theta(y_1 - y_c). \tag{7}$$

2.3 Hybrid Model

Here an individual possessing wealth larger than a cutoff wealth y_c does trading with the gross-system, while individuals possessing wealth smaller than y_c engage in two-party tiny-trading. Hence from Eqs. (3) and (7), the distribution function is obtained to be

$$\begin{aligned} f(y) = {} & \gamma \int_y^\infty dL \int_{a(y,L,\lambda_t)}^{b(y,L,\lambda_t)} dx \mathcal{F}(x,L,\lambda_t)\mathcal{H}(x,L,y_c) \\ & + \int_{y/(2-\lambda_g)}^{y/\lambda_g} \frac{dx f(x)}{2x(1-\lambda_g)} \theta(x-y_c). \end{aligned} \tag{8}$$

Now, it must be pointed out that when the savings $\lambda_t = 0$, $\lambda_g \neq 0$, and $y \to 0$, Eq. (8) yields (up to a proportionality constant) the following same result as the purely tiny-trading case without an upper cutoff [8]:

$$f'(y) \propto -f(y)f(0). \tag{9}$$

In obtaining the above equation we again assumed that the function $f(y)$ and its first and second derivatives are well behaved. Then the solution for small y is given by

$$f(y) \propto f(0)exp[-yf(0)]. \tag{10}$$

3 Results and Discussion

The distribution function $f(y)$ can be obtained by solving the nonlinear integral Eq. (8). To this end, we simplify Eq. (8) for computational purposes as follows:

$$\begin{aligned} f(y) = \ & \gamma\mathcal{G}(y,\lambda_t,y_c)\int_y^{2y_c} dL \int_{a(y,L,\lambda_t)}^{b(y,L,\lambda_t)} dx\mathcal{F}(x,L,\lambda_t)\mathcal{H}(x,L,y_c) \\ & +[1-\theta(y-y_{as})]\int_{y/(2-\lambda_g)}^{y/\lambda_g} \frac{dx f(x)}{2x(1-\lambda_g)}\theta(x-y_c) \\ & +\theta(y-y_{as})f(y_{as})\frac{y_{as}^2}{y^2}, \end{aligned} \tag{11}$$

where $\mathcal{G}(y,\lambda_t,y_c) \equiv 1-\theta[y-(2-\lambda_t)y_c]$ and $y > y_{as}$ gives the asymptotic behavior $f(y) \propto 1/y^2$. In our calculations, we have taken y_{as} to be at least $20y_c$ and obtained $f(y)$ for all y less than 2000 times the average wealth per person y_{av}. We solved Eq. (11) iteratively by choosing a trial function, substituting it on the RHS (right hand side) and obtaining a new trial function and successively substituting the new trial functions over and over again on the RHS until convergence is achieved. The criterion for convergence was that the difference between the new trial function f_n and the previous trial function f_p satisfies the accuracy test $\sum_i |f_n(y_i) - f_p(y_i)|/\sum_i f_p(y_i) \leq 0.002$ [20].

In Fig. 1, using a log-log plot we depict the distribution function $f(y)$ for the constant savings case $\lambda_t = \lambda_g = 0.5$ with the average money per person y_{av} being set to unity and with the values of the wealth cutoff $y_c = 3, 5, 10$. As expected, for larger values of y_c, the Pareto-like $1/y^2$ behavior sets in later. The transition to purely gross-trading occurs at $(2-\lambda_t)y_c$, while below $\lambda_g y_c$ it is purely two-party tiny-trading. Thus the transition from purely tiny-trading to purely gross-trading occurs in Fig. 1 over a region of width y_c. However, all the tails merge irrespective of the cutoff. At smaller values of y the behavior of $f(y)$, depicted in the inset, is similar to the purely two-party trading model studied earlier (see Ref. [8]). The curves in the inset appear to be close because here the trading is two-party and is governed by the same savings. Next, in Fig. 2 we plot $f(y)$ with the cutoff $y_c = 5$, $y_{av} = 1$, and for values of savings fraction $\lambda_t = \lambda_g = \lambda = 0.1, 0.5, 0.8$. Here the power-law behavior $(1/y^2)$ takes over for $y > (2-\lambda)y_c$ and hence at lower savings it sets in later. In the power-law region the curves merge together. As shown

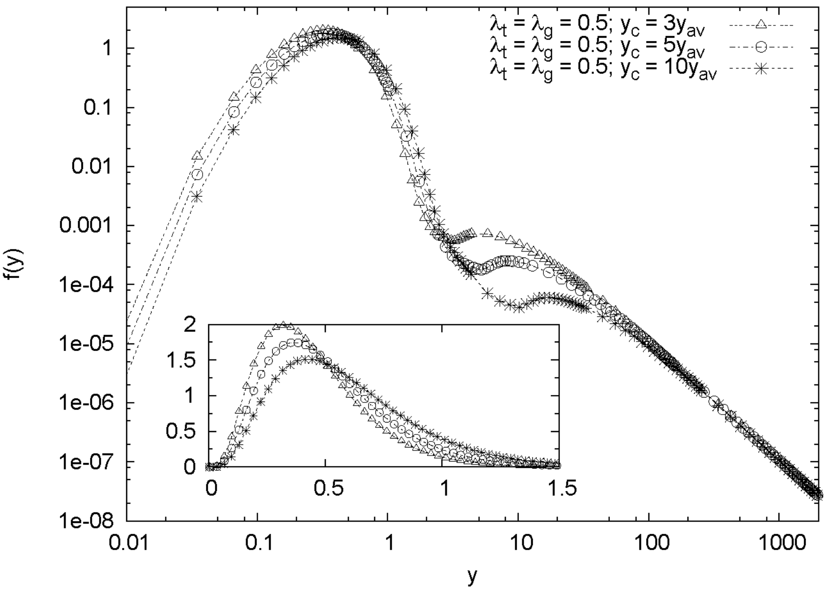

Fig. 1. Plot of the wealth distribution function for savings $\lambda_t = \lambda_g = 0.5$ and various wealth cutoff values $y_c = 3, 5, 10$. The average money per person y_{av} is set to unity. The dotted lines are guides to the eye.

in the inset of Fig. 2, at smaller values of y the $f(y)$s become zero with the higher peaked curves (corresponding to larger λs) approaching zero faster similar to the case of the purely two-party trading model in our earlier work [8]. Here the transition from purely tiny- to purely gross-trading at higher λ is sharper because the transition occurs over a region of width $2(1 - \lambda)y_c$. Lastly, in Fig. 3, we show the distribution function $f(y)$ for the zero savings case in the tiny-channel ($\lambda_t = 0$) and for various savings $\lambda_g = 0.2, 0.5, 0.9$ in the gross-channel with $y_{av} = 1$ and $y_c = 5$. The distribution, as expected, decays exponentially (or Boltzmann-Gibbs-like) for small values of y and has power-law ($1/y^2$) behavior at large values. The curves merge in the Pareto-like region and, in fact, $f(y) \approx 0.1/y^2$ in all the three figures at large values of y. In Fig. 3 too, for reasons mentioned earlier, the transition is sharper at larger values of λ_g. Fig. 3 takes into account the fact that, in societies, the rich tend to have higher savings fraction (λ) compared to the poor. Actually, if the savings fraction were to increase gradually with wealth, one can expect a more gradual change in the transition region of the distribution rather than the sharp local maxima (around $y \approx 6.5$) shown by the $\lambda_g = 0.9$ curve.

In all the figures anomalous looking kinks/shoulders appear at the cross over between the Boltzmann-Gibbs-like and the Pareto-like regimes. This is due to the sharp cut-off at y_c that we introduced using a step function. However, as mentioned in the introduction, such kinks do occur in real in-

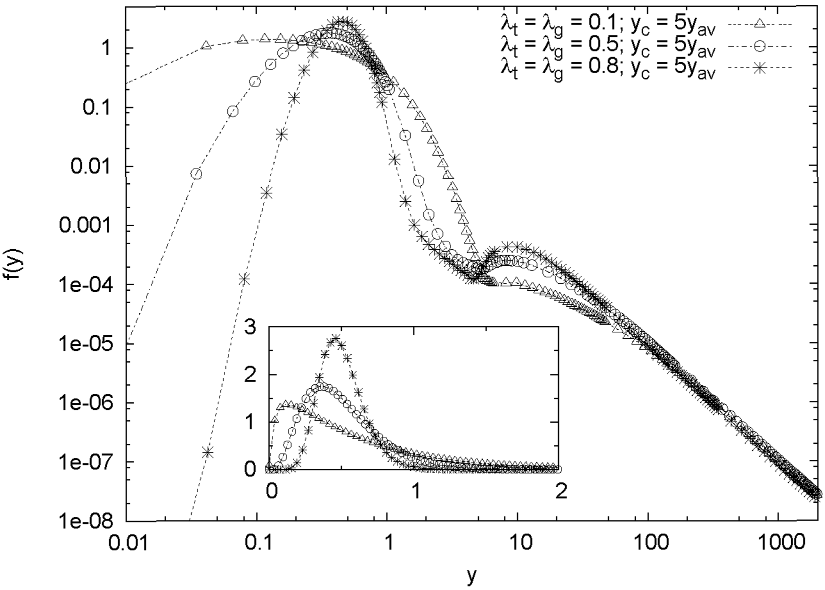

Fig. 2. Wealth distribution $f(y)$ at average wealth $y_{av} = 1$, wealth cutoff $y_c = 5$, and various values of savings $\lambda_t = \lambda_g = 0.1, 0.5, 0.8$.

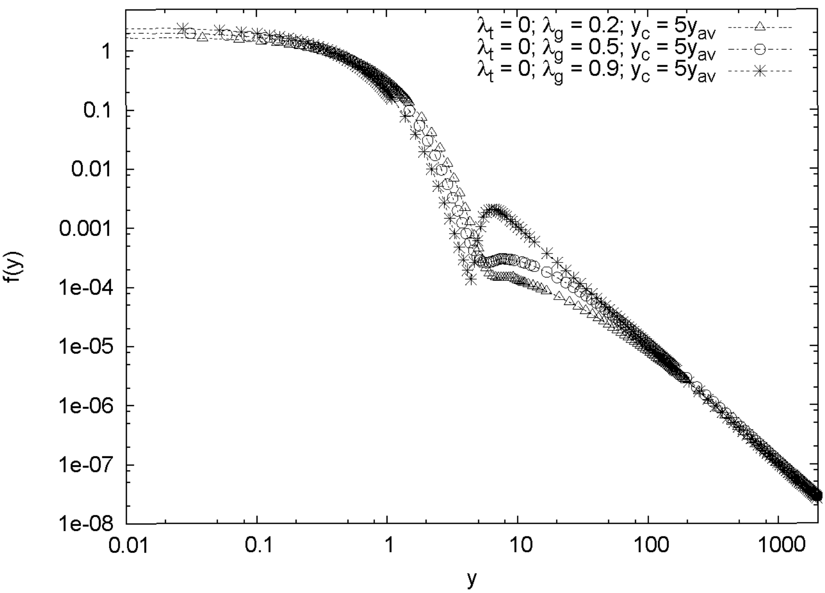

Fig. 3. Money distribution function at zero savings for tiny-trading and various savings values $\lambda_g = 0.2, 0.5, 0.9$ for the gross-trading. The average money $y_{av} = 1$ and the wealth cutoff $y_c = 5$.

come/wealth distributions [5, 18]. Different societies have the onset of Pareto-like behavior at different wealths which is indicative that the cut-off has to be obtained empirically based on various factors like the social structure, welfare policies, type of markets, form of government, etc. It is of interest to note that the analysis carried out on income classes in USA during 1983-2001 in Ref. [21] revealed that the Boltzmann-Gibbs part is quite stationary while the Pareto tail swells and shrinks (and thus changes with time). This is perhaps indicative that the poorer section corresponds to a system at equilibrium while the richer society represents a steady state system that is far from equilibrium. Thus perhaps some sort of a self-organized criticality is operative in the wealthier society where wealth generating ideas or new technology may be responsible for driving the system away from equilibrium.

In Japan the wealth/income distribution vanishes at zero wealth/income and then rises to a maximum (see Ref. [5]). In US the distribution seems to be a maximum at zero wealth/income (see Ref. [5]). Both these aspects can be covered in our model as the poor in general are known to save very little. If their savings are zero, one gets the Boltzmann-Gibbs behavior at the poor end. On the other hand, if the savings are small one gets a maximum close to zero and the distribution vanishes at zero wealth.

It would be interesting to deduce the savings pattern from the wealth distribution. While it has been observed that the rich tend to save more than the poor, how gradually the savings change as wealth increases can perhaps be inferred from the change in slope. However, as explained below, the middle region (involving the middle-class) has been modeled quite crudely by us and needs to be refined before a serious connection with savings pattern can be attempted.

We will now further discuss the motivation for using two different mechanisms to model the observed wealth distribution. The model is an approximation where the direct wealth exchange occurs between people who are in economic proximity. At the bottom of the spectrum, the poor, who have limited economic means and avenues, come in contact with a few poor and their economic activity is modeled in terms of two-party trading. At the other end of the wealth spectrum, the rich have access to various economic avenues (e.g., markets, know-how, work force, capital, credit facilities, contacts, wealthy society, etc.) due to which they can trade with huge organizations and are thus modeled to interact with a reservoir. As regards the middle-class that is between the rich and the poor, they trade amongst themselves as well as with the poor and the reservoir. As a first step towards realizing this scenario, we included in our earlier work [19] only the two extreme cases of interaction. What we had not taken into account is the interaction of the middle class with the reservoir. To rectify this, we have chosen the cutoff y_g for the interaction with the reservoir such that y_g lies below the two-party trading cutoff y_t. However, this did not seem to alter the calculated curves significantly [22]. Thus, we believe that our model is a reasonable one at the poor and rich ends and is a crude approximation for the middle class. In order to model the wealth distribution of the middle class better, one needs to produce a gradual transition from a two-party trading at the poorer end to the gross trading at the richer end.

Although it is true that the poor also come in market contact with wealthy organizations like a soft-drink company, nevertheless the contact is an indirect one mediated through intermediaries. For example, the poor person deals with a richer shop-keeper selling the drink who in turn deals with a richer local distributor who in turn deals with the big soft-drink company. Thus the middle-class act as intermediaries between the rich and the poor. Next, we will examine the validity for our type of two-party trading. We feel that in any trading there is a random fluctuation of the price around its true value. The total money put forth for trading corresponds to the amount of random fluctuation. However the poorer of the two puts forth less and makes the trading biased in his/her favor. This can be justified from the fact that the poor people are constantly looking for bargains to make ends meet.

Compared to other types of analysis involving two-party trading to explain Pareto law (see Ref. [10]), our gross-trading mechanism can make contact with the standard approach in macroeconomics as will be shown below. In macroeconomics, the objective is to maximize a cumulative utility function subject to a wealth constraint [23]. Mathematically this is formulated as

$$\max_{c_{t+i},y_{t+i}} E_t \sum_i \beta^i u(c_{t+i}), \tag{12}$$

subject to the constraint

$$y_{t+i} = (1+r)y_{t+i-1} + e_{t+i} - c_{t+i}, \tag{13}$$

where c_t, y_t, and e_t are consumption, wealth, and labor earnings respectively at time t, r is the interest rate on wealth y, $0 < \beta < 1$ is the time-discount factor, $u(c_t)$ is the concave utility function, E_t is the expectation value based on the available information at time t. Using the method of Lagrange multipliers, the conditions of optimality yield

$$E_t[u'(c_t) - (1+r)\beta u'(c_{t+1})] = 0, \tag{14}$$

where $u'(c_t)$ is the derivative of $u(c_t)$ with respect to c_t. From the above equation we see that consumption at different times are related. In our work [see Eq. (4)], we introduced the stochasticity

$$y_{t+1} - y_t = \epsilon(1-\lambda_g)y_t, \tag{15}$$

where ϵ is a random number such that $-1 \leq \epsilon \leq 1$, which implies that

$$ry_t + e_{t+1} - c_{t+1} = \epsilon(1-\lambda_g)y_t. \tag{16}$$

The above equation can be made consistent with the optimal consumption relation given by Eq. (14). In fact if it is assumed that $(1+r)\beta = 1$, which is anyway approximately true, then consumption smoothing of the form $c_{t+1} = c_t$ [which is consistent with Eq. (14)] implies that all the stochasticity given by the RHS of Eq. (16) lies in the income only. Thus our model (for the power-law tail) is consistent with the standard approaches in macroeconomics using uninsurable shocks in income.

It is of interest to note that if we modify the stochasticity as

$$y_{t+1} - y_t = \epsilon(1 - \lambda_g) y_t^{1-\delta}, \tag{17}$$

with $0 \leq \delta \leq 1$, then the asymptotic behavior of the distribution function has two power-law solutions with exponents $2 - 2\delta$ and $1 - 2\delta$ [24]. Such solutions are obtained by solving the integral equation

$$f(y) = \int_{x_-}^{x_+} \frac{dx f(x)}{2(1 - \lambda_g) x^{1-\delta}}, \tag{18}$$

where the limits of integration $x_\pm$ are obtained iteratively in terms of y, from the equation

$$x_\pm = y \pm (1 - \lambda_g) x_\pm^{1-\delta}, \tag{19}$$

as a power series with a typical term in the series being $y^{1-n\delta}$ with $n = 0, 1, 2, ...$ Thus one can obtain different exponents for the power-law tail.

In conclusion, we introduced interaction of the rich with huge entities (a model that is consistent with main models in macroeconomics) and obtained a Pareto-like power-law. On the other hand, the Boltzmann-Gibbs-like wealth distribution, corresponding to the bulk of the society, is understood through a two-party trading mechanism. All in all, we show that stochasticity can explain the observed skewness in the wealth distribution in societies.

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