Experimental Study on Control of Redundant 3-D Snake Robot Based on a Kinematic Model

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Abstract. In this paper, we derive a kinematic model and a control law for 3D snake robots which have wheeled link mechanism. We define the redundancy controllable system and find that introduction of links without wheels makes the system redundancy controllable. Using redundancy, it becomes possible to accomplish both main objective of controlling the position and the posture of the snake robot head, and sub-objective of the singular configuration avoidance. Experiments demonstrate the effectiveness of the proposed control law.

1 Introduction

Unique and interesting gait of the snakes makes them able to crawl, climb a hill, climb a tree by winding and move on very slippery floor [1]. Snake does not have hands and legs, however it has many functions. It is useful to consider and understand the mechanism of the gait of the snakes for mechanical design and control law of snake robots.

Snake robots are active code mechanism and useful for search and rescue operation in disaster. Utilization of autonomous intelligent robots in Search and Rescue is a new challenging field of robotics dealing with tasks in extremely hazardous and complex disaster environments. Intelligent, biologically-inspired mobile robots, and, in particular, snake-like robots have turned out to be the widely used robot type, aiming at providing effective, immediate, and reliable response to many strategic planning for search and rescue operations. Design and control of the snake-like robot have recently been receiving much attention, and many locomotion modes for snake-like robot have been proposed [2–5].

Hirose has long investigated snake robots and produced several snake robots, and he models the snake by a wheeled link mechanism with no side slip [2]. Some other snake-like mechanisms are developed in [3–5]. The present research [2,4,5] is looking for other variety of possible locomotion modes "Ring mode", "Inching mode", "Wheeled Locomotion mode" and "Bridge mode" as shown in Fig. 1.

1. Ring mode: The two ends of the robot body are brought together by its own actuation to form a circular shape. The drive to make the uneven circular shape to rotate is expected to be achieved by proper deformation and shifting the center of gravity as necessary.

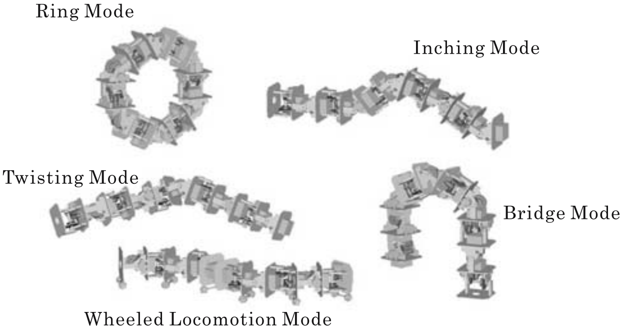


Fig. 1. Variety of possible locomotion of snake robot

2. Inching mode: This is one of the common undulatory movements of serpentine mechanisms. The robot generates a vertical wave-shape using its units from the rear end and propagates the 'wave' along its body - resulting a net advancement in its position.
3. Twisting mode: In this mode the robot mechanism folds certain joints to generate a twisting motion within its body, resulting in a side-wise movement [5] .
4. Wheeled locomotion mode: This is one of the common wheeled locomotion mode where the passive wheels (without direct drive) are attached on the units resulting low friction along the tangential direction of the robot body line while increasing the friction in the direction perpendicular to that [2].
5. Bridge mode: In this mode the robot configures itself to "stand" on its two end units in a bridge like shape. This mode has the possibility of implementation of two-legged walking type locomotion. The basic movement consists of left-right swaying of the center of gravity in synchronism with by lifting and forwarding one of the supports, like bipedal locomotion. Motions such as somersaulting may also be some of the possibilities.

The snake robots which have many functions, locomotion modes and 3D motion have been developed, but in the study of controller design for the snake robots the movement is restricted to 2D motion. Construction of a controller which accomplishes 3D motion of 3D snake robots is one of challenging and important problems.

Chirikijian and Burdick discuss the sidewinding locomotion of the snake robots based on the kinematic model [6]. Ostrowski and Burdick analyze the controllability of a class of nonholonomic systems, that the snake robots are included, on the basis of the geometric approach [7]. The feedback control law for the snake head's position using Lyapunov method has been developed by Prautesch et al. on the basis of the wheeled link model [8]. They point out the controller can stabilize the head position of the snake robot to its desired value, but the configuration of it converges to a singular configuration. We find that introduction of links without wheels and shape controllable points in the snake robot's body makes the system redundancy controllable.

In this paper we consider the singular configuration avoidance of the redundant 3D snake robots. Using redundancy, it becomes possible to accomplish both the main objective of controlling the position and the posture of the snake robot head and the sub-objective of the singular configuration avoidance. Experimental results by using a 13-link snake robot (ACM-R3 [9]) are shown.

2 Redundancy controllable system

In our previous paper we define the redundancy controllable system and propose structure design methodology of redundant snake robots based on the wheeled link model [10].

Let $\boldsymbol{q} \in R^{\bar{n}}$ be the state vector, $\boldsymbol{u} \in R^{\bar{p}}$ be the input vector, $\boldsymbol{w} \equiv S\boldsymbol{q} \in R^{\bar{q}}$ be the state vector to be controlled, S be a selection matrix whose row vectors are independent unit vectors related to generalized coordinates, $A(\boldsymbol{q}) \in R^{\bar{m}\times\bar{q}}, B(\boldsymbol{q}) \in R^{\bar{m}\times\bar{p}}$, where $\bar{m}$ is number of equations. We define that the system

$$A(\boldsymbol{q})\dot{\boldsymbol{w}} = B(\boldsymbol{q})\boldsymbol{u}, \qquad \boldsymbol{u} = \boldsymbol{u}_1 + \boldsymbol{u}_2 \tag{1}$$

is redundancy controllable if $\bar{p} > \bar{q}$ (redundancy I), $\bar{p} > \bar{m}$ (redundancy II), [1] the matrix A is full column rank, B is full row rank, and following two conditions are satisfied.

1. There exists an input $\boldsymbol{u}_1$ which accomplishes the main objective of the convergence of the vector $\boldsymbol{w}$ to the desired state $\boldsymbol{w}_d$ ($\boldsymbol{w} \to \boldsymbol{w}_d, \dot{\boldsymbol{w}} \to \dot{\boldsymbol{w}}_d$).
2. There exists an input $\boldsymbol{u} = \boldsymbol{u}_1 + \boldsymbol{u}_2$ which accomplishes the increase (or decrease) of a cost function $V(\boldsymbol{q})$ related to the sub-objective compared to the input $\boldsymbol{u}_1$ and does not disturb the main objective.

For a snake robot based on the wheeled link model we discuss the condition that the system is redundancy controllable [10].

3 Kinematic model of snake robots

Let us consider a redundant n-link snake robot on a flat plane. We introduce a coordinate frame Σ_A which is fixed on the head of the snake robot. The tip point of the snake head is taken as the origin of Σ_A. The reference configuration is set as a straight line configuration on the ground as shown in Fig. 2. The ${}^{A}x$ axis is set as the central body axis of the snake robot taking the

[1] In the case of $\bar{m} = \bar{p}$, if the state vector to be controlled $\dot{\boldsymbol{w}}$ in (1) is given, the input $\boldsymbol{u}$ is determined uniquely. In this sense the system is not redundant, so we introduce the redundancy II.

reference configuration. All joints rotate around y axis or z axis in the reference configuration. Let ${}^{A}\hat{\boldsymbol{l}}_i = [l_i, 0, 0]^T$ be a link vector from the i-th joint to the $(i-1)$-th joint with respect to Σ_A in Fig. 2. Let ϕ_i be the relative

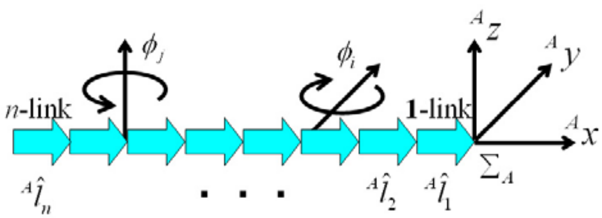


Fig. 2. Reference configuration of 3D snake robot

joint angle between link i and $i+1$. The link vector ${}^{A}l_i$ with respect to Σ_A is expressed as

$$
{}^{A}\boldsymbol{l}_i = R_{\phi_1} \cdots R_{\phi_{i-1}} {}^{A}\hat{\boldsymbol{l}}_i \;\; (i = 1, \cdots, n) \tag{2}
$$

where R_{ϕ_i} is $Rot(y, \phi_i) = R^{j\phi_i}$ or $Rot(z, \phi_i) = R^{k\phi_i}$. The 3D snake robot divided two parts. One is the base part and the other is the head part. We define that the first n_h links (head part) are not contact with the ground, and the residual n_b links (base part) are on the same plane which is parallel to the ground. In the base part wheeled links are contact with the ground. Let us introduce inertial Cartesian coordinate frame Σ_W and the coordinate frame Σ_B which is fixed on the end point of the base part ((n_h+1)-th link) as shown in Fig. 3. We introduce following three assumptions.
[assumption 1]: All joints of the base part rotate around z axis.
[assumption 2]: Environment is flat.
[assumption 3]: The robot is supported by the wheels of the base part and the head part is not contact with the ground.

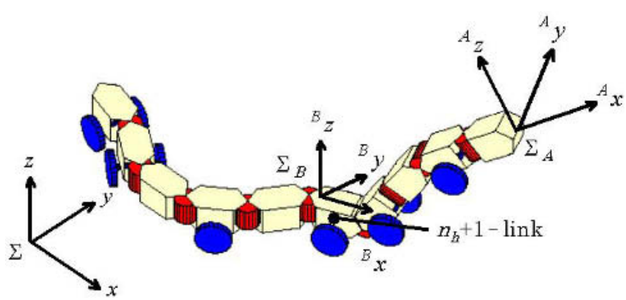


Fig. 3. Coordinate systems of 3D snake robot

The rotation matrix from Σ_B to Σ_A is given as

$$ {}^{A}R_B = R_{\phi_1} \cdots R_{\phi_{n_h}}. \tag{3} $$

Let ψ be the absolute attitude angle of the head of the base part about z axis, then the rotation matrix from Σ_B to Σ is expressed as

$$ {}^{W}R_B = R^{k\psi} \tag{4} $$

where $R^{k\psi} = Rot(z, \psi)$. The rotation matrix ${}^{W}R_A$ from Σ_A to Σ is expressed as

$$ {}^{W}R_A = {}^{W}R_B({}^{A}R_B)^{-1} = R^{k\psi} R_{-\phi_{n_h}} \cdots R_{-\phi_1}. \tag{5} $$

Using (5) and (2) gives the link vector $\boldsymbol{l}_i$ with respect to Σ

$$ \begin{aligned} \boldsymbol{l}_i &= R^{k\psi} R_{-\phi_{n_h}} \cdots R_{-\phi_i} {}^{A}\hat{\boldsymbol{l}}_i \quad (i = 1, \cdots, n_h) \\ \boldsymbol{l}_{n_h+1} &= R^{k\psi} {}^{A}\hat{\boldsymbol{l}}_{n_h+1} \\ \boldsymbol{l}_i &= R^{k\psi} R_{\phi_{n_h+1}} \cdots R_{\phi_{i-1}} {}^{A}\hat{\boldsymbol{l}}_i \quad (i = n_h+2, \cdots, n). \end{aligned} \tag{6} $$

Let (R, P, Y) be roll, pitch, yaw angles, then we obtain

$$ \begin{aligned} R &= \mathrm{atan2}(\pm\tilde{R}_{32}, \pm\tilde{R}_{33}) \\ P &= \mathrm{atan2}(-\tilde{R}_{31}, \pm\sqrt{\tilde{R}_{11}^2 + \tilde{R}_{21}^2}) \\ Y - \psi &= \mathrm{atan2}(\pm\tilde{R}_{21}, \pm\tilde{R}_{11}) \end{aligned} \tag{7} $$

where $\widetilde{R} = R_{-\phi_{n_h}} \cdots R_{-\phi_1}$. Using (6) and (7) yields

$$ \begin{aligned} \boldsymbol{l}_i &= R^{k(Y - \mathrm{atan2}(\pm\tilde{R}_{21}, \pm\tilde{R}_{11}))} R_{-\phi_{n_h}} \cdots R_{-\phi_i} {}^{A}\hat{\boldsymbol{l}}_i \\ &\quad (i = 1, \cdots, n_h) \\ \boldsymbol{l}_{n_h+1} &= R^{k(Y - \mathrm{atan2}(\pm\tilde{R}_{21}, \pm\tilde{R}_{11}))} {}^{A}\hat{\boldsymbol{l}}_{n_h+1} \\ \boldsymbol{l}_i &= R^{k(Y - \mathrm{atan2}(\pm\tilde{R}_{21}, \pm\tilde{R}_{11}))} R_{\phi_{n_h+1}} \cdots R_{\phi_{i-1}} {}^{A}\hat{\boldsymbol{l}}_i \\ &\quad (i = n_h+2, \cdots, n). \end{aligned} \tag{8} $$

The middle position $\boldsymbol{P}_i = [x_i, y_i, z_i]^T$ of the rotational axis of two wheels attached on the link i is expressed as

$$ \boldsymbol{P}_i = \boldsymbol{P}_h - \boldsymbol{l}_1 - \boldsymbol{l}_2 - \cdots - \boldsymbol{l}_{i-1} - \frac{l_{wi}}{l_i}\boldsymbol{l}_i \tag{9} $$

where $\boldsymbol{P}_h$ is the position vector of the snake head and l_{wi} is the distance between the joint i and the attached position of the wheel of the link i. As the wheel does not slip to the side direction, the velocity constraint condition should be satisfied. If the i-th link is wheeled and contact with the ground, the constraint can be written as

$$ \dot{x}_i \sin\theta_i - \dot{y}_i \cos\theta_i = 0 \tag{10} $$

where θ_i is the absolute attitude of the i-th link about z-axis and it is expressed as

$$\theta_i = \psi + \sum_{j=n_h+1}^{i-1} \phi_j$$
$$= Y - \mathrm{atan2}(\pm\tilde{R}_{21}, \pm\tilde{R}_{11}) + \sum_{j=n_h+1}^{i-1} \phi_j. \quad (11)$$

From the assumption 3 z-element of the position vector of the first link of the base part is constant. We set it as h, then we obtain

$$(\boldsymbol{P}_h - \boldsymbol{l}_1 - \boldsymbol{l}_2 - \cdots - \boldsymbol{l}_{n_h})^T \boldsymbol{e}_z = h \quad (12)$$

where $\boldsymbol{l}_z = [0\ 0\ 1]^T$. Using time derivative of the geometric relation (7)(9)(12) and the velocity constraint condition (10) yields kinematic model of 3D snake robot.

4 Condition for redundancy controllable system

We consider control of position and posture of the snake head. Let $\boldsymbol{w} = [\, x_h \quad y_h \quad z_h \quad R \quad P \quad Y \,]^T$ be the vector of the position and the posture of the snake head, $\boldsymbol{\theta} = [\, \phi_1 \quad \cdots \quad \phi_{n-1} \,]^T$ be the vector of relative joint angles, $\boldsymbol{q} = [\, \boldsymbol{w}^T \quad \boldsymbol{\theta}^T \,]^T \in R^{n+5}$ be the generalized coordinates. The angular velocity of each joint is regarded as the input of the robot system. If a wheel free link is connected to the tail, the movement of the added link does not contribute to the movement of the snake head. So we assume that wheel is attached at the tail link. If all links are wheeled, then we obtain

$$\bar{A}(\boldsymbol{q})\dot{\boldsymbol{w}} = \bar{B}(\boldsymbol{q})\boldsymbol{u}\ ,\ \boldsymbol{u} = \dot{\boldsymbol{\theta}} \quad (13)$$

where

$$\bar{A} = \begin{bmatrix} a_{11} & a_{12} & 0\,0\,0 & a_{16} \\ a_{21} & a_{22} & 0\,0\,0 & a_{26} \\ \vdots & \vdots & \vdots\,\vdots\,\vdots & \vdots \\ a_{n_b1} & a_{n_b2} & 0\,0\,0 & a_{n_b6} \\ 0 & 0 & 1\,0\,0 & 0 \\ 0 & 0 & 0\,1\,0 & 0 \\ 0 & 0 & 0\,0\,1 & 0 \end{bmatrix}$$

$$\bar{B} = \begin{bmatrix} b_{11} & \cdots & b_{1n_h} & 0 & \cdots & 0 \\ b_{21} & \cdots & b_{2n_h} & -l_{w(n_h+2)} & \cdots & 0 \\ \vdots & & \vdots & & \ddots & \vdots \\ b_{n_b1} & \cdots & b_{n_bn_h} & b_{n_b(n_h+1)} & \cdots & -l_{wn} \\ b_{(n_b+1)1} & \cdots & b_{(n_b+1)n_h} & 0 & \cdots & 0 \\ b_{(n_b+2)1} & \cdots & b_{(n_b+2)n_h} & 0 & \cdots & 0 \\ b_{(n_b+3)1} & \cdots & b_{(n_b+3)n_h} & 0 & \cdots & 0 \end{bmatrix}$$

In (13), the first n_b equations are obtained from (9) (10), the (n_b+1)-th from the derivative of (12), and the (n_b+2)-th and (n_b+3)-th equations from derivative of the first two equations of (7).

Let m be the number of wheeled links of the base part. The kinematic model is expressed as

$$A(\boldsymbol{q})\dot{\boldsymbol{w}} = B(\boldsymbol{q})\boldsymbol{u}\ ,\ \boldsymbol{u} = \dot{\boldsymbol{\theta}} \tag{14}$$

We consider conditions so that the n-link snake robot system can be regarded as the redundancy controllable system which is defined in the section 2. To satisfy the redundancy II the inequality

[condition 1] : $3 \le m < n - 4$

should be satisfied. To satisfy the full row rankness of the matrix B we should introduce following conditions.

[condition 2] :
In the case that the (n_h+1)-th link is wheel free : $n_h \ge 3$
In the case that the (n_h+1)-th link is wheeled : $n_h \ge 4$

[condition 3] :
All joints of the head part do not have same direction of rotational axes.

These three conditions are sufficient condition so that the system is redundancy controllable [10].

The necessary and sufficient condition for the existence of the solution of the system (14) is

$$\mathrm{rank}[A, B\boldsymbol{u}] = \mathrm{rank}A. \tag{15}$$

5 Controller design for main-objective

Let us define the control input as

$$\begin{aligned}\boldsymbol{u} = \dot{\boldsymbol{\theta}} = B^{+}A\{\dot{\boldsymbol{w}}_d - K(\boldsymbol{w} - \boldsymbol{w}_d)\}\\ + (I - B^{+}B)\boldsymbol{k}\end{aligned} \tag{16}$$

where B^{+} is a pseudo-inverse matrix of B, $\boldsymbol{k}$ is an arbitrary vector and $K > 0$. The first term of the right side of (16) is the control input term to accomplish the main objective of the convergence of the state vector $\boldsymbol{w}$ to the desired value $\boldsymbol{w}_d$. As the second term $(I - B^{+}B)\boldsymbol{k}$ belongs to the null space of the matrix B, we obtain

$$B\boldsymbol{u} = A\{\dot{\boldsymbol{w}}_d - K(\boldsymbol{w} - \boldsymbol{w}_d)\}. \tag{17}$$

As the vector $B\boldsymbol{u}$ can be expressed as a linear combination of column vectors of the matrix A, the condition (14) of the existence of the solution (14) is satisfied. The second term in (16) does not disturb the dynamics of the controlled vector $\boldsymbol{w}$. As there is no interaction between $\boldsymbol{w}$ and $\boldsymbol{\theta}$, we find that the control law (16) accomplishes the sub-objective.

The closed-loop system is expressed as

$$A\{(\dot{\boldsymbol{w}} - \dot{\boldsymbol{w}}_d) + K(\boldsymbol{w} - \boldsymbol{w}_d)\} = 0. \tag{18}$$

If the matrix A is full column rank, the uniqueness of the solution is guaranteed. The solution of (18) is given as

$$\dot{\boldsymbol{w}} - \dot{\boldsymbol{w}}_d + K(\boldsymbol{w} - \boldsymbol{w}_d) = 0$$

and we find that the controller ensures the convergence of the controlled state vector to the desired value ($\boldsymbol{w} \longrightarrow \boldsymbol{w}_d$). A set of joint angles which satisfies rank$A < q$ (A is not full column rank) means the singular configuration, for example a straight line ($\phi_i = 0, i = 1, \cdots, n-1$).

6 Controller design for sub-objective

We consider the controller design for the sub-objective. In the control law (16), $\boldsymbol{k}$ is an arbitrary vector. Let us introduce the cost function $V(q)$ which is related to the sub-objective. If we set the vector $\boldsymbol{k}$ as the gradient $\boldsymbol{k}_1$ of the cost function $V(\boldsymbol{q})$ with respect to the vector $\boldsymbol{\theta}$ related to the input vector $\boldsymbol{u}$, we obtain

$$\boldsymbol{k}_1 = \nabla_{\boldsymbol{\theta}} V(\boldsymbol{q}) = \left[\begin{array}{ccc} \frac{\partial V}{\partial \theta_1} & \cdots & \frac{\partial V}{\partial \theta_{n-1}} \end{array}\right]$$

and we find that the second term of (16) accomplishes the increase of the cost function V. Actually we can derive

$$\begin{aligned} \dot{V}(\boldsymbol{q}) &= (\partial V/\partial \boldsymbol{w})\dot{\boldsymbol{w}} + (\partial V/\partial \boldsymbol{\theta})\dot{\boldsymbol{\theta}} \\ &= (\partial V/\partial \boldsymbol{w})\dot{\boldsymbol{w}} + \boldsymbol{k}^T B^+ A\{\dot{\boldsymbol{w}}_d - K(\boldsymbol{w} - \boldsymbol{w}_d)\} \\ &\qquad + \boldsymbol{k}_1^T (I - B^+ B)\boldsymbol{k}_1. \end{aligned} \tag{19}$$

As $I - B^+B \geq 0$ [11], we find that the second term of the input (16) accomplishes the increase of the cost function V.

In the case that the sub-objective is the singularity avoidance, we set $B^+ = B^T(BB^T)^{-1}$ and

$$V = \alpha(\det(A^T A)) + \beta(\det(BB^T)) \tag{20}$$

where $\alpha, \beta > 0$. The first term of the right side of (20) implies the measure of the singular configuration. The second term of the right side of (20) is related to the manipulability of the system.

7 Experiments

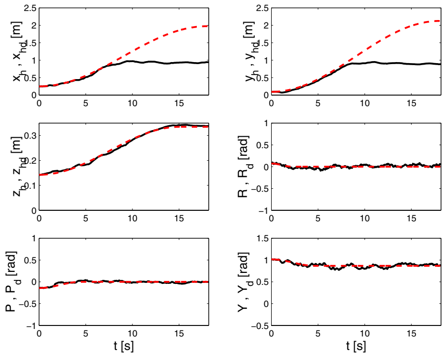
To demonstrate the validity of the proposed control law experiments have been carried out. The snake robot that we use for the experiments is ACM-R3 [9] as shown in Fig. 4. The snake robot has 13 links and the 2, 6, 8, 9, 10, 12, 13-th links are wheeled. The length $l_i (i = 1, \cdots, 13)$ of the links are as follows: $l_1 = l_7 = l_8 = l_9 = l_{10} = l_{11} = l_{12} = 0.16$[m], $l_2 = l_3 = l_4 = l_5 = l_6 = l_{13} = 0.08$[m]. We set $K = I$, $\alpha = 0.2, \beta = 2.0 \times 10^6$. The initial position and posture of the head of the snake robot and initial relative joint angles are set as $w(0) = [0, -0.1, 0.142, 0.0715, -0.143, \pi/10]^T, \theta(0) = [0, \pi/18, \pi/30, \pi/18, \pi/12, -\pi/9, \pi/6, \pi/6, -\pi/9, -\pi/6, -\pi/10, \pi/30]^T$.

Fig. 4. A research platform robot (ACM-R3)

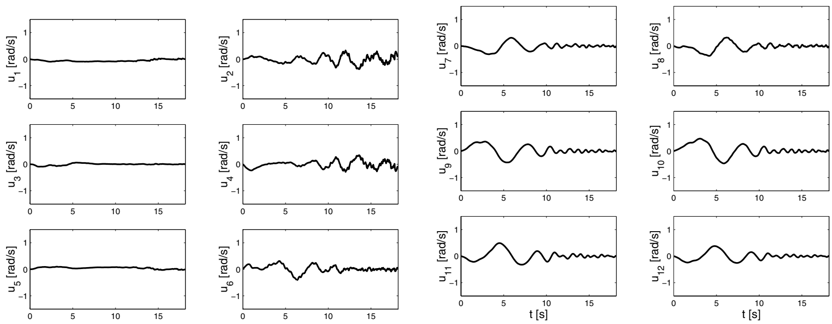
In experiments, to measure the position and the posture of the snake head we use Quick MAG IV stereo vision system with two fixed CCD cameras. The desired trajectory w_d corresponding to w is represented as the broken lines in Figs. 5 and 6. Fig. 5 shows the transient responses for the controller (16) without using redundancy ($\boldsymbol{k} = 0$). From Fig. 5(a) and (c) we find that the snake robot can not track the desired head trajectory because of the convergence to the singular configuration of a straight line. Fig. 6 shows the transient responses for the controller (16) with using redundancy ($\boldsymbol{k} = \boldsymbol{k}_1$). From Fig. 6 (a) and (c) we find that the snake robot avoids the singular configuration of the straight line. Experimental results show the effectiveness of the proposed controller.

8 Conclusion

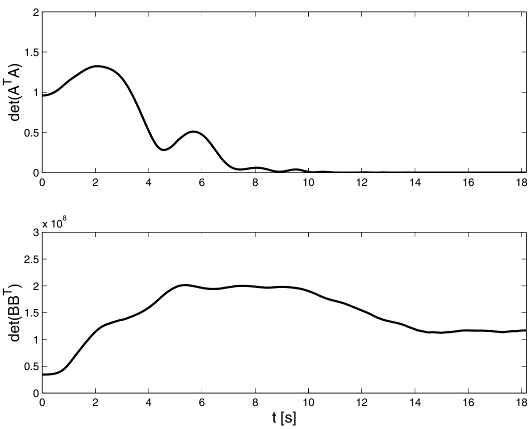
We have considered control of redundant 3D snake robot based on kinematic model. We derived conditions so that the snake robot system is redundancy controllable. We propose controller that the snake head tracks the desired trajectory and the robot avoids singular configurations by using redundancy. Experimental results ensure the effectiveness of the proposed control law.



(a) w(——) and w_d(− − −)

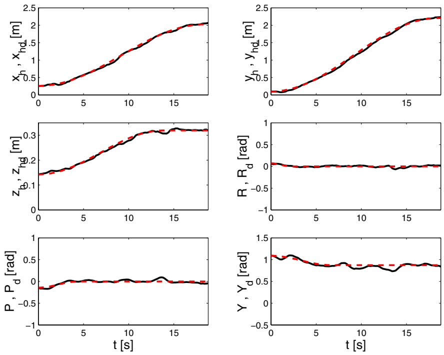


b) Input $u_1, \cdots, u_{12}$

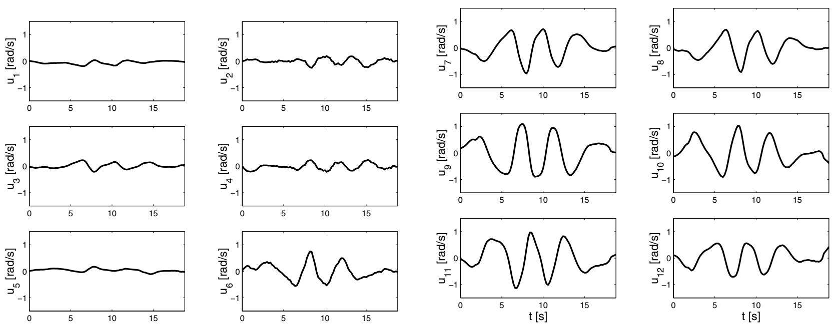


(c) $\det(A^T A)$ and $\det(BB^T)$

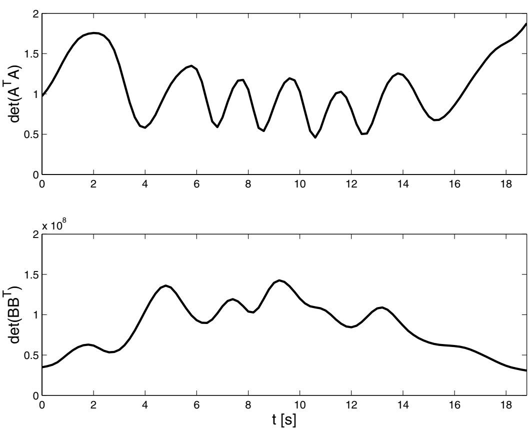
Fig. 5. Transient responses for controller without using redundancy ($\boldsymbol{k} = 0$)



(a) w(——) and $w_d(- - -)$



(b) Input $u_1, \cdots, u_{12}$



(c) $\det(A^T A)$ and $\det(BB^T)$

Fig. 6. Transient responses for the controller with using redundancy ($\boldsymbol{k} = \boldsymbol{k}_1$)

References

1. J. Gray, *Animal Locomotion*, pp. 166-193, Norton, 1968
2. S. Hirose, *Biologically Inspired Robots (Snake-like Locomotor and Manipulator)*, Oxford University Press, 1993
3. B. Klaassen and K. Paap, GMD-SNAKE2: A Snake-Like Robot Driven by Wheels and a Method for Motion Control, *Proc. IEEE Int. Conf. on Robotics and Automation*, pp. 3014-3019, 1999.
4. M. Yim, D. Duff and K. Poufas, PolyBot: a Modular Reconfigurable Robot, *Proc. IEEE Int. Conf. on Robotics and Automation*, pp. 514-520, 2000.
5. T. Kamegawa, F. Matsuno and R. Chatterjee, Proposition of Twisting Mode of Locomotion and GA based Motion Planning for Transition of Locomotion Modes of a 3-dimensional Snake-like Robot, *Proc. IEEE Int. Conf. on Robotics and Automation*, pp. 1507-1512, 2002.
6. G. S. Chirikijian and J. W. Burdick, The Kinematics of Hyper-Redundant Robotic Locomotion, *IEEE Trans. on Robotics and Automation*, Vol. 11, No. 6, pp. 781-793, 1995
7. J. Ostrowski and J. Burdick, The Geometric Mechanics of Undulatory Robotic Locomotion, *Int. J. of Robotics Research*, Vol. 17, No. 6, pp. 683-701, 1998
8. P. Prautesch, T. Mita, H. Yamauchi, T. Iwasaki and G. Nishida, Control and Analysis of the Gait of Snake Robots, *Proc. COE Super Mechano-Systems Workshop'99*, pp. 257-265, 1999
9. M. Mori and S. Hirose, Development of Active Cord Mechanism ACM-R3 with Agile 3D Mobility, *Proc. IEEE/RSJ Int. Conf. on Intelligent Robots and Systems*, pp. 1552-1557, 2001.
10. F. Matsuno and K. Mogi, Redundancy Controllable System and Control of Snake Robot with Redundancy based on Kinematic Model, *Proc. IEEE Conf. on Decision and Control*, pp. 4791-4796, 2000.
11. Y. Nakamura, H. Hanafusa and T. Yoshikawa, Task-Priority Based Redundancy Control of Robot Manipulators, *Int. J. of Robotics Research* , Vol. 6, No. 2, pp. 3-15, 1987