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An Update on the Fracture Toughness Testing Methods Related to the Cracked Chevron-notched Brazilian Disk (CCNBD) Specimen

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Abstract—This paper reviews the use of the cracked Chevron-notched Brazilian disc (CCNBD) for fracture toughness testing. Theoretical and experimental backgrounds of the method are described. Some issues regarding the current development (i.e., recalibration) of the specimen geometry are presented and discussed. A number of geometries related to the CCNBD proposed recently for fracture toughness testing of rock are then introduced and commented on.

Key words: Fracture toughness testing, Brazilian disc, CCNBD, stress intensity factor.

1. Introduction

Due to the great popularity of using Brazilian disks in the rock mechanics community, the introduction of the CCNBD specimen for rock fracture toughness testing (Fowell and Xu, 1994; Chen, 1990) did not encounter any difficulty in gaining wide acceptance. Compared with the Chevron bend (CB) and short rod (SR) specimens, the CCNBD has numerous advantages which include easier sample preparation, much higher failure load, simpler testing procedure and it is easily adaptable for mixed-mode fracture toughness testing. Some selected publications about the use of the specimen include Chang *et al.* (2002), Dwivedi *et al.* (2000), Al-Shayea *et al.* (2000) and Krishnan *et al.* (1998).

2. Background

The basic CCNBD testing configuration is given in Figure 1. The relations between the geometrical entities are expressed in Equation (1) below.

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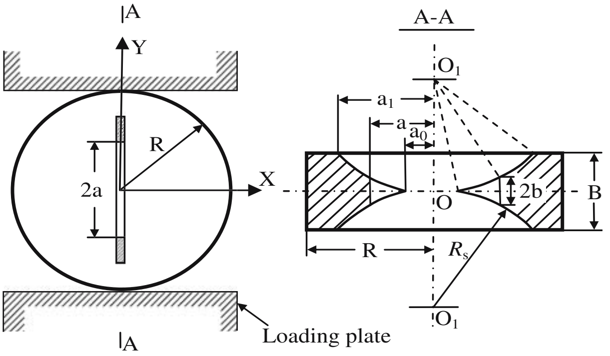

Figure 1
Basic configuration of CCNBD fracture toughness testing.

$$\left\{\begin{aligned} \alpha_s = R_s/R = \sqrt{\alpha_0^2 + \left(\alpha_1^2 - \alpha_0^2 + \alpha_B^2/4\right)^2 \div \alpha_B^2} \\ h_c = \left(\alpha_s - \sqrt{\alpha_s^2 - \alpha_1^2}\right) \cdot R = \left(\alpha_s - \sqrt{\alpha_s^2 - \alpha_0^2}\right) \cdot R + B/2 \\ \alpha_0 = \sqrt{\alpha_s^2 - \left(\sqrt{\alpha_s^2 - \alpha_1^2} + \alpha_B/2\right)^2} \\ \alpha_1 = \sqrt{\alpha_s^2 - \left(\sqrt{\alpha_s^2 - \alpha_0^2} - \alpha_B/2\right)^2} \\ \alpha_B = 2 \cdot \left(\sqrt{\alpha_s^2 - \alpha_0^2} - \sqrt{\alpha_s^2 - \alpha_1^2}\right) \end{aligned}\right. \tag{1}$$

where α_0 , α_1 and α_B are dimensionless and are calculated as: $\alpha_0 = a_0/R$, $\alpha_1 = a_1/R$ and $\alpha_B = B/R$. The ISRM suggested that standard specimen dimensions are given in Table 1 (ISRM, 1995).

In practical experiments, the dimensions of CCNBD specimens obtained will deviate from the standard figures. To obtain the stress intensity factor (SIF) for

Table 1

Standard CCNBD geometrical dimensions (Fig. 1)

Descriptions	Values	Dimensionless Expression
Diameter **D** (mm)	75.0	
Thickness **B** (mm)	30.0	$\alpha_B = B/R = 0.80$
Initial chevron-notched crack length $\mathbf{a_0}$ (mm)	9.89	$\alpha_0 = a_0/R = 0.2637$
Final chevron-notched crack length $\mathbf{a_1}$ (mm)	24.37	$\alpha_1 = a_1/R = 0.65$
Saw diameter $\mathbf{D_s}$ (mm)	52.0	$\alpha_s = D_s/R = 0.6933$
Cutting depth $\mathbf{h_c}$ (mm)	16.95	
$\mathbf{Y^{*}_{min}}$ (dimensionless)	0.84	
Critical crack length $\mathbf{a_m}$ (mm)	19.31	$\alpha_m = a_m/R = 0.5149$

different dimensions, the specimen geometry is analyzed theoretically using the combination of compliance method, dislocation method and superimposition technique. The SIF for a CCNBD specimen with crack length α can be calculated as (Xu and Fowell, 1993):

$$Y^{*}(\alpha)=\left[\frac{\alpha_B^4\cdot g_3(\alpha)}{8\cdot g_1(\alpha)\cdot g_2(\alpha)}\right]^{\frac{1}{2}}, \tag{2}$$

where

$$\begin{cases} g_1(\alpha)=(\alpha_1^2-\alpha_0^2)+\dfrac{\alpha_B^2}{4}-\sqrt{(\alpha_1^2-\alpha^2)\cdot\alpha_B^2+\left(\alpha_1^2-\alpha_0^2-\dfrac{\alpha_B^2}{4}\right)^2} \\ g_2(\alpha)=\left[\dfrac{1}{B'E}\cdot\dfrac{g_1(\alpha)}{C(\alpha)}+\dfrac{1}{B'E}\cdot\displaystyle\int_{\alpha}^{\alpha_1}\dfrac{g_4(\xi)}{C(\xi)}\cdot d\xi\right]^2 \\ g_3(\alpha)=\dfrac{2}{(B'E)^2}\cdot\dfrac{Y(\alpha)\cdot g_1(\alpha)}{C^2(\alpha)}-\dfrac{(1-c_k)}{B'E}\cdot\dfrac{g_4(\alpha)}{C(\alpha)} \\ g_4(\alpha)=\dfrac{\alpha_B^2\cdot\alpha}{\sqrt{(\alpha_1^2-\alpha^2)\cdot\alpha_B^2+(\alpha_1^2-\alpha_0^2-\alpha_B^2/4)^2}} \end{cases} \tag{3}$$

The results are then validated using the finite element method (FEM) and the boundary element method (BEM) (Xu and Fowell, 1993). For fracture toughness testing, the important SIF value for the specimen is the minimum SIF, denoted as Y^{*}, as it corresponds to the failure load recorded during the testing. From them the fracture toughness value can then be calculated as:

$$K_C=\frac{P_{\max}}{B\cdot\sqrt{R}}\cdot Y_m^{*}. \tag{4}$$

Note the above calculation only depends on the compliance $C(\alpha)$ of the corresponding cracked straight-through Brazilian disc (CSTBD) and is independent of fracture mode, i.e., if the correct compliance is supplied the above fracture toughness is equally applicable to mode I, II, III or mixed mode fracture testing.

However, due to the plane strain constraint, not all geometries of the CCNBD are valid to be used for fracture testing. Studies (Xu and Fowell, 1993, 1994) showed that CCNBD geometries must fall within the range outlined in Figure 2 to yield valid fracture toughness results. These ranges can also be expressed below:

$$\begin{cases} \alpha_1\geq 0.4, & \text{Line 0} \\ \alpha_1\geq \alpha_B/2, & \text{Line 1} \\ \alpha_B\leq 1.04, & \text{Line 2} \\ \alpha_1\leq 0.8, & \text{Line 3} \\ \alpha_B\geq 1.1729\cdot(\alpha_1)^{5/3}, & \text{Line 4} \\ \alpha_B\geq 0.44, & \text{Line 5} \end{cases}$$

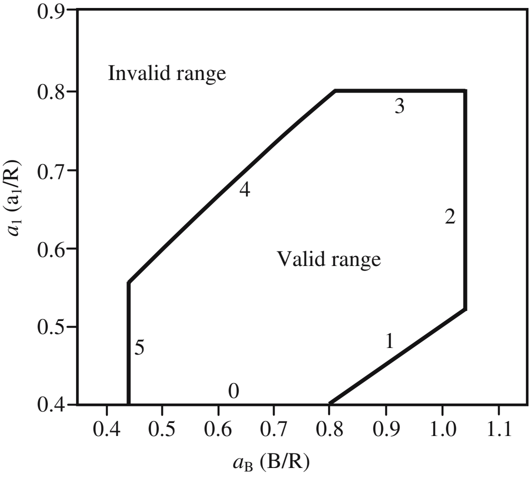

Figure 2
Valid geometrical ranges for fracture toughness testing.

For CCNBD specimens within these ranges, an easier version (compared to Equation 3) to calculate the minimum dimensionless SIF values is given in (ISRM, 1995):

$$Y^*_{\min} = u \cdot e^{v \cdot \alpha_1}, \tag{6}$$

where constants u and v are listed in ISRM (1995). A selected portion for some common configurations is given in Table 2 below:

Wang (1998) introduced a correction factor to account for the compliance of uncracked disc in the SIF evaluation of CCNBD specimens and the results of full-scale calibration using FEM are presented in Wang *et al.* (2004).

3. Geometry Related to CCNBD: 1 – Flattened Brazilian Disc (FBD)

The uncracked Brazilian disc was used by Guo *et al.* (1993) directly for mode I fracture toughness testing. This configuration was revisited by Wang *et al.* (2004) by introducing two parallel flat loading planes as shown in Figure 3(a). The geometry is analyzed by Wang *et al.* (2004) using the FEM and Figure 3(b) shows the dimensionless SIF for the configuration for different stages of the crack propagation. The experiment using this testing method can only be performed in a displacement controlled loading system and a typical load-displacement curve is given in Figure 3(c). The mode I fracture toughness K_{IC} can then be calculated as:

$$K_{IC} = \frac{P_{\min}}{\sqrt{R} \cdot t} \cdot \Phi^*_{\max}, \tag{7}$$

Table 2

Selected valuesof u and *v*

α_0	0.200	0.250	0.275	0.300	0.325	0.350	0.375	0.400
				u				
α_B								
0.680	0.2667	0.2704	0.2718	0.2744	0.2774	0.2807	0.2848	0.2888
0.720	0.2650	0.2683	0.2705	0.2727	0.2763	0.2794	0.2831	0.2871
0.760	0.2637	0.2668	0.2693	0.2719	0.2744	0.2781	0.2819	0.2860
0.800	0.2625	0.2657	0.2680	0.2706	0.2736	0.2772	0.2811	0.2845
0.840	0.2612	0.2649	0.2672	0.2699	0.2727	0.2763	0.2801	0.2831
0.880	0.2602	0.2642	0.2668	0.2691	0.2723	0.2754	0.2793	0.2816
0.920	0.2598	0.2634	0.2658	0.2684	0.2716	0.2747	0.2782	0.2811
0.960	0.2593	0.2633	0.2655	0.2685	0.2710	0.2746	0.2767	0.2799
1.000	0.2591	0.2630	0.2653	0.2679	0.2709	0.2738	0.2768	0.2786
				v				
0.680	1.7676	1.7711	1.7757	1.7759	1.7754	1.7741	1.7700	1.7666
0.720	1.7647	1.7698	1.7708	1.7722	1.7693	1.7683	1.7652	1.7617
0.760	1.7600	1.7656	1.7649	1.7652	1.7662	1.7624	1.7593	1.7554
0.800	1.7557	1.7611	1.7613	1.7603	1.7596	1.7561	1.7525	1.7512
0.840	1.7522	1.7547	1.7551	1.7548	1.7535	1.7499	1.7469	1.7473
0.880	1.7487	1.7492	1.7478	1.7487	1.7463	1.7452	1.7403	1.7434
0.920	1.7423	1.7446	1.7443	1.7432	1.7411	1.7389	1.7360	1.7363
0.960	1.7370	1.7373	1.7372	1.7346	1.7344	1.7309	1.7343	1.7331
1.000	1.7308	1.7307	1.7306	1.7297	1.7273	1.7270	1.7258	1.7302

where $P_{\min}$ is the local minimum load reading as shown in Figure 3(c), $\Phi^*_{\max}$ is the maximum dimensionless SIF as shown in Figure 3(b). The main attraction of this method is the even simpler sample preparation as no slot needs to be cut at the center of the disc. However extensive analytical, numerical and experimental validation of the method is needed. A sensitive displacement loading requirement and low success rate for tests are also some of the disadvantages of this specimen geometry.

A geometry closely related to the flattened Brazilian disc is the modified ring (MR) configuration reported in Fischer *et al.* (1996). The geometry is depicted in Figure 4 and the mode I fracture toughness value is obtained from the test graph shown in Figure 5. Finite-element analysis of the geometry is also presented by Fischer *et al.* (1996).

4. Geometry Related to CCNBD: 2 – Semi-circular Specimen under Three-point Bend (SCB)

This geometry was proposed by Chong (1987) and received extensive study by Lim *et al.* (1994 a,b,c) and experimental attention from Krishnan *et al.* (1998) and Funatsu *et al.* (2004). The specimen and configuration for testing are shown in

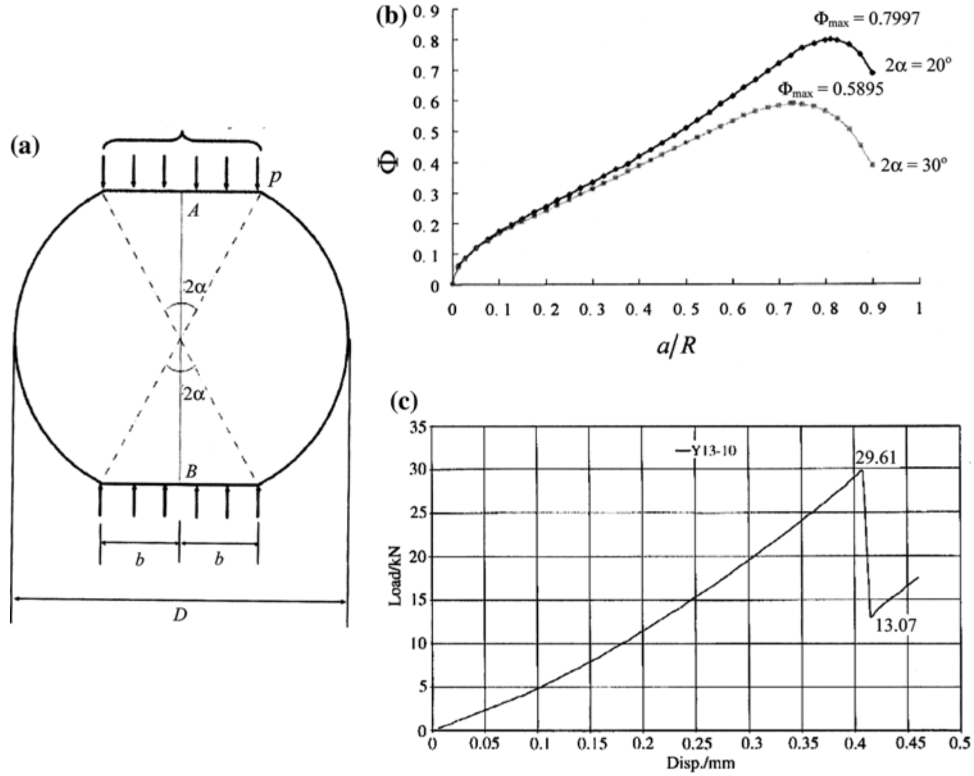

Figure 3
Flattened Brazilian disk, a) Geometrical dimensions, b) SIF c) L-D curve (Wang *et al.*, 2004).

Figure 6(a). The fracture toughness for mode I and II is then obtained by Equation (8) below (Lim *et al.*, 1995).

$$\begin{cases} K_{IC} = \dfrac{P}{2rt} \cdot \sqrt{\pi a} \cdot Y_I^* \\ K_{IIC} = \dfrac{P}{2rt} \cdot \sqrt{\pi a} \cdot Y_{II}^* \end{cases}, \tag{8}$$

where Y_I^* and Y_{II}^* are dimensionless SIF values for the specimen with crack length a, t is the thickness, r is the radius and P is the recorded failure load. Figure 6 (b) gives the dimensionless SIF Y_I for pure mode I loading fixture and the dimensionless SIF Y_I and Y_{II} for mixed conditions are given in Figure 6(c). A typical load-displacement curve for the testing is shown in Figure 6(d).

This geometry, although retaining some merit such as being easily adaptable for mixed mode testing, loses some advantages of the CCNBD specimen. The most notable one is the low failure load which can sometimes be difficult to implement in practical experiments and is error prone, as it will be more difficult to obtain accurate

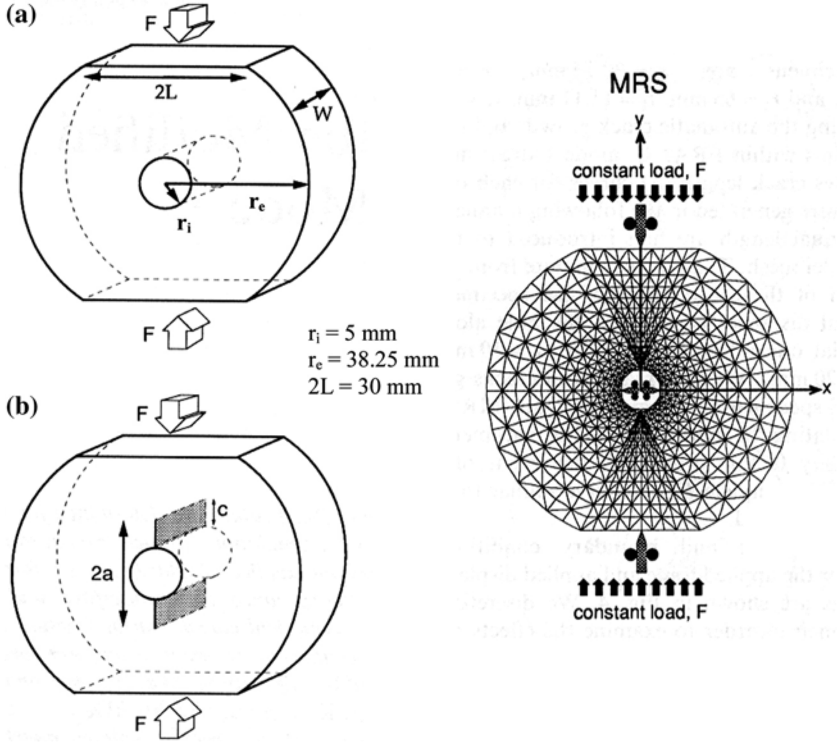

Figure 4
Modified ring (MR) specimen geometry (FISHER *et al.*, 1996).

load-displacement recordings. Another disadvantage will be the possible need for a pre-cracking process prior to testing although this can be easily overcome by introducing a Chevron-notch instead of a straight-through crack.

5. Geometry Related to CCNBD: 3–Double-edge Cracked Brazilian Disc (DECBD)

This geometry has been recently proposed by CHEN *et al.* (2001) for mode I fracture toughness testing. The specimen and loading configuration is as shown in Figure 7(a). The specimen is studied in CHEN *et al.* (2001) using a weight function and the FEM method. The dimensionless mode I SIF for the specimen for different crack inclination angles is given in Figure 7 (b).

In the authors' opinion, this geometry has great potential. An initial impression of the specimen is that it will retain most of the merit of the CCNBD specimen, with the improvement of an easier sample preparation. The specimen, if configured properly (i.e., certain crack inclination angle), will act like a shear box which will

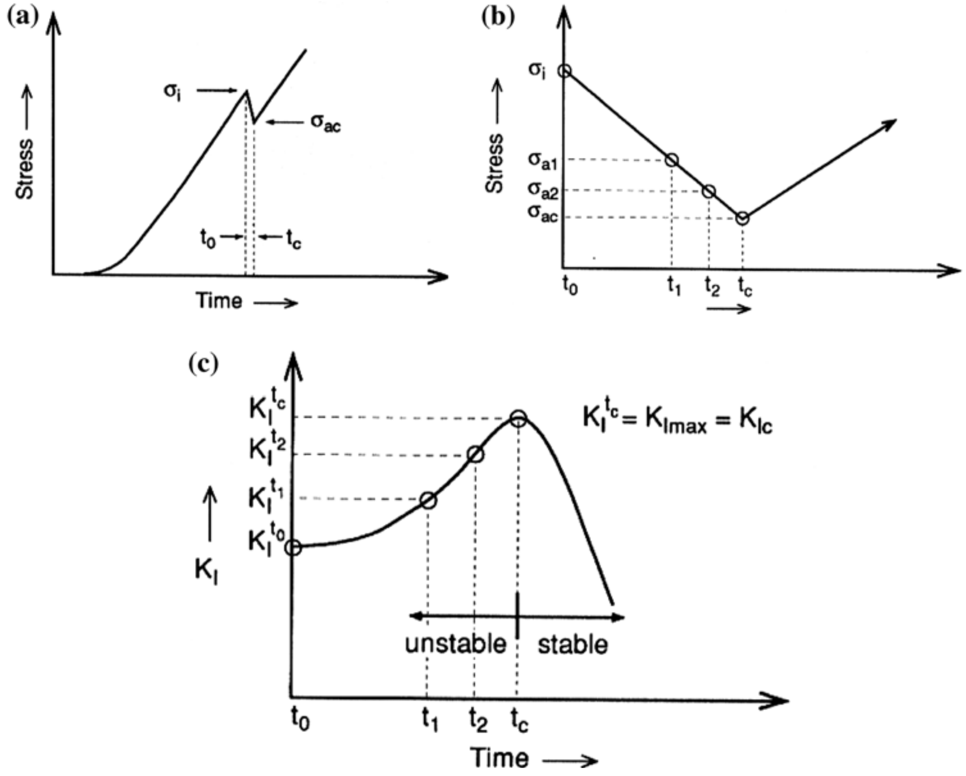

Figure 5
Fracture toughness determination using MR testing (Fisher *et al.*, 1996).

make it extremely suitable for mixed modes I and II fracture toughness testing. Certainly the geometry is still in its early stages of development and we cannot be sure at this stage if these claims will be correct. Some work for this geometry is imminent, for example, mode II SIF evaluation and extensive experimental testing validation.

6. Conclusion

Since its introduction, fracture toughness testing using the CCNBD specimen has attracted considerable attention in the rock fracture research community. Advantages of using the method have started to be realized, which include, easier sample preparation, much higher failure load, and hence simpler and less error-prone testing procedure. Another superb advantage of the specimen is that it can easily be adapted for pure Model II and mixed Modes I and II fracture toughness testing, which has

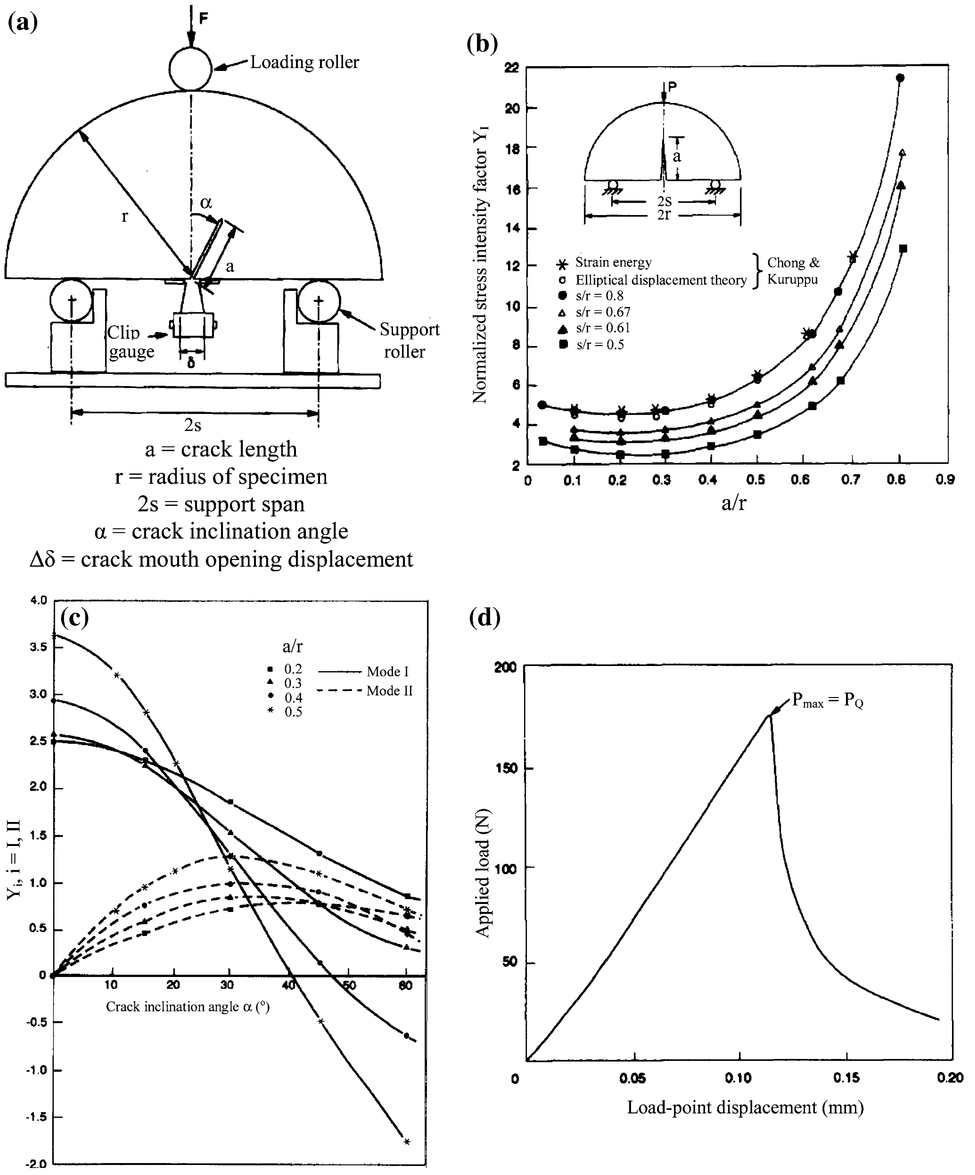

Figure 6
SCB a) Configuration, b) Mode I SIF c) Mixed-mode SIF d) L-D curve (Lim *et al.*, 1994a,b).

attracted extensive research in the past decade and is expected to remain as a very active research topic for the foreseeable future. It may be necessary to revise the dimensionless SIF values for a future release of the suggested method to incorporate some recent developments. More research and input from different sources need to be coordinated.

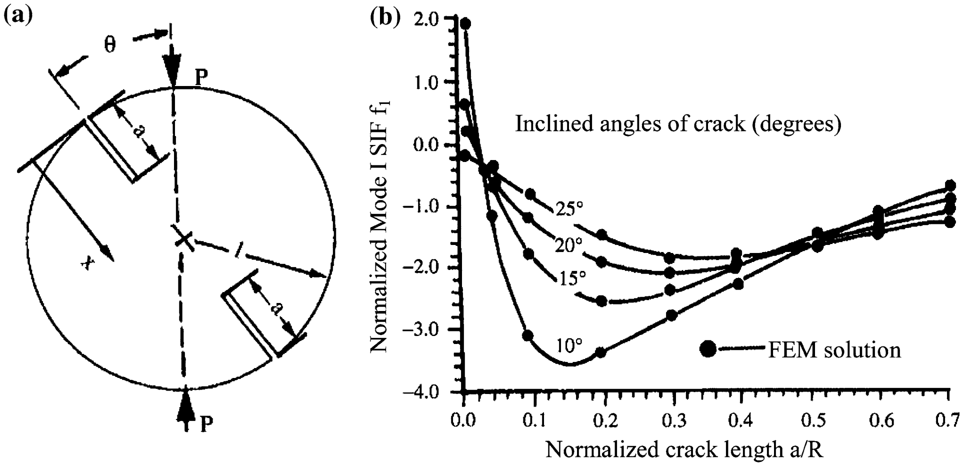

Figure 7
Double edge Brazilian disk - DECBD a) Geometry b) Mode I SIF (Chen *et al.*, 2001).

Several specimen geometries closely related to the CCNBD have also been developed and used, however in the authors' opinion, none of them have the same unique desirable combinations of features and advantages as the CCNBD geometry. The DECBD specimen, however, is believed to have great potential for further development.

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