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Upscaling: Effective Medium Theory, Numerical Methods and the Fractal Dream

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Abstract—Upscaling is a major issue regarding mechanical and transport properties of rocks. This paper examines three issues relative to upscaling. The first one is a brief overview of Effective Medium Theory (EMT), which is a key tool to predict average rock properties at a macroscopic scale in the case of a statistically homogeneous medium. EMT is of particular interest in the calculation of elastic properties. As discussed in this paper, EMT can thus provide a possible way to perform upscaling, although it is by no means the only one, and in particular it is irrelevant if the medium does not adhere to statistical homogeneity. This last circumstance is examined in part two of the paper. We focus on the example of constructing a hydrocarbon reservoir model. Such a construction is a required step in the process of making reasonable predictions for oil production. Taking into account rock permeability, lithological units and various structural discontinuities at different scales is part of this construction. The result is that stochastic reservoir models are built that rely on various numerical upscaling methods. These methods are reviewed. They provide techniques which make it possible to deal with upscaling on a general basis. Finally, a last case in which upscaling is trivial is considered in the third part of the paper. This is the fractal case. Fractal models have become popular precisely because they are free of the assumption of statistical homogeneity and yet do not involve numerical methods. It is suggested that using a physical criterion as a means to discriminate whether fractality is a dream or reality would be more satisfactory than relying on a limited data set alone.

Key words: Upscaling, Effective Medium, Numerical Methods, Fractals.

1. Upscaling and Effective Medium Theory

At all scales, from local to regional, rocks are heterogeneous. The influence of heterogeneity in rock, as long as it remains moderate, can often be dealt with using the ergodic assumption, considering that the medium is statistically homogeneous. That means that a representative elementary volume (REV) exists (Fig. 1) and that any part of the system with a volume considerably larger than the REV has identical physical properties. In this case, one can state that "upscaling" is equivalent to "homogenizing." The medium can be considered as invariant by translation (for any property

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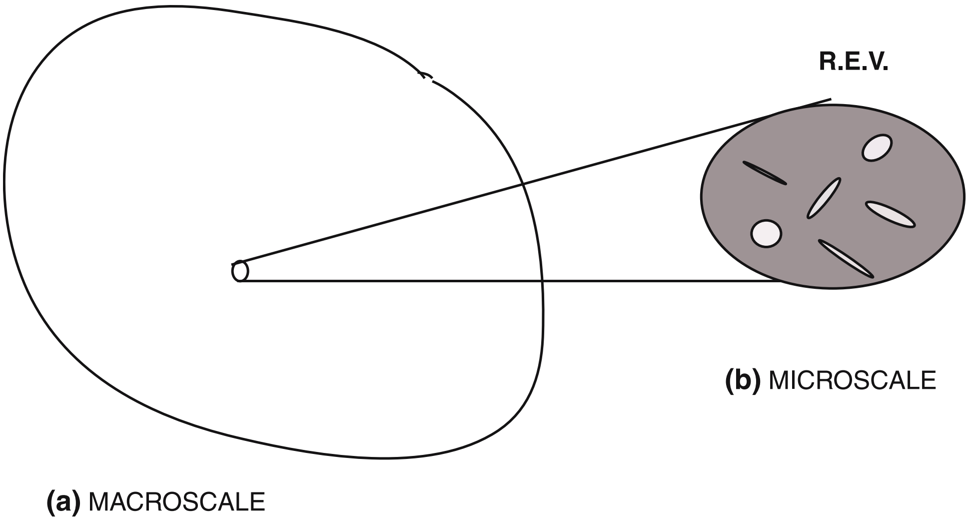

Figure 1
Representative Elementary Volume (REV): Statistical homogeneity (or equivalently translational invariance) assumes that any property averaged over a REV at any place gives the same result.

averaged over a REV). This is obviously applicable to geological objects substantially greater than a REV. In most cases, rock heterogeneity is mainly the result of variable composition and of the presence of pores and cracks. Effective Medium Theories (EMT) have been constructed that allow the successful prediction of rock properties as long as the degree of heterogeneity is not large. There exists in general some critical threshold above which such an approach does not apply. The limits of application of EMT to rocks has been examined by GUÉGUEN *et al.* (1997). We restrict ourselves in the following to the applicability field of EMT. Because there are many close but different EMT schemes, we present in the following a short introduction which is the common basis for all schemes. We then take as an example the elastic properties.

1.1 General Statistical Approach

In order to determine the macroscopic effective properties of a heterogeneous medium, some manner of ensemble averaging is usually performed. To the degree of the property of interest can be considered as a generalized susceptibility χ defined by a linear relation between a generalized force σ (for instance stress, electric field, etc.) and a generalized flux ϵ (for instance strain, electric flux, etc.), the general method is as follows. Let us point out that the properties considered here all belong to that category since elastic moduli, conductivities and permeability enter into that group. In each case, there is also a conservative flux equation (GUÉGUEN and PALCIAUSKAS, 1994) which makes the analogy complete. The flux (i.e., hydraulic, electric, thermal flux or stress) is assumed to be a linear function of the force (i.e., respectively fluid pressure gradient, electric potential gradient, temperature gradient, strain):

$$\sigma(\vec{r},t) = \int d\vec{r}' \int dt' \chi(\vec{r},\vec{r}',t-t')\epsilon(\vec{r}',t'). \tag{1}$$

Using the example of elasticity where σ stands for stress and ϵ stands for strain, equation (1) can be simply understood as follows. Any incremental strain $\delta\epsilon$ at a point of coordinates (x',y',z') and time t' produces a stress $\delta\sigma$ at a point of coordinates (x,y,z) and at a later time t. It is assumed that $\delta\sigma$ is a linear function of $\delta\epsilon$, the coefficient of proportionality being the generalized susceptibility χ (in practice the elastic constants, electrical conductivity, etc.). The susceptibility χ is considered to be a function of (x',y',z'); (x,y,z); $(t'-t)$. Summing all contributions $\delta\sigma$, equation (1) results. Let us note that the formal theory can be developed considering σ in terms of ϵ or, equivalently, the reverse.

The Fourier transform equation is

$$\sigma(\vec{q},\omega) = \int d\vec{q}' \chi(\vec{q},\vec{q}',\omega)\epsilon(\vec{q}',\omega). \tag{2}$$

If one can define a representative elementary volume which is large compared to any correlation lengths, but small compared to the macroscopic sample of interest, then it can be shown (GUBERNATIS and KRUMHANSL, 1975) that the equation (2) reduces to

$$\sigma(\vec{q},\omega) = \bar{\chi}(\vec{q},\omega)\epsilon(\vec{q},\omega) + \text{small fluctuations}. \tag{3}$$

Depending on the physical property which is considered, σ and ϵ can be respectively stress and strain, fluid flow and pressure gradient, etc. The calculation of the generalized susceptibility χ (which can be elastic modulus, permeability, etc.) can be performed when the specific field equations are introduced as shown later.

Let us point out that frequency effects can be important and in general, the susceptibility χ is frequency-dependent. When laboratory measurements are performed, for instance for elastic properties, ultrasonic waves are used and $\vec{q}$ and ω are non-zero. The elastic moduli may vary with ω. In that case, the physical process behind that is fluid relaxation in pores and cracks (LE RAVALEC and GUÉGUEN, 1996; SCHUBNEL and GUÉGUEN, 2003): high frequency (unrelaxed) moduli differ from low frequency ones (relaxed).

In the simple static case, both $\vec{q}$ and ω are zero. One can show that the volumic averages of static generalized force $\langle\sigma\rangle$ and flux $\langle\epsilon\rangle$ are

$$\langle\sigma\rangle = \sigma(0,0) \text{ and } \langle\epsilon\rangle = \epsilon(0,0) \tag{4}$$

so that the effective static susceptibility is

$$\bar{\chi}(0,0) = \frac{\langle\sigma\rangle}{\langle\epsilon\rangle}. \tag{5}$$

The above relations establish the formal theory.

1.2 Example of Elastic Properties

We will focus in this section on elastic properties. Then σ is the stress and ϵ the strain. Both are tensors. We restrict ourselves to the simple static case. However, the practical calculation of effective moduli $\bar{\chi}(0,0)$ remains far from straightforward. This is, because what can be determined usually is not the stress or strain distribution through the rock sample (which are required as input data in the above equation) but the composition and microstructure average distributions. At that point, as noted by GUBERNATIS and KRUMHANSL (1975), and focusing on static moduli, there are two options to determine the unknown fields of σ and ϵ. Interestingly enough, in the case of crustal rocks, the main source of heterogeneity is the presence of pores and cracks. The focus then becomes accountability for pore and crack effects on elastic moduli. In order to simplify the calculations, it can be assumed that either pores or cracks are dominant. The case of cracks is well taken into account by the first option and that of pores by the second one.

1.2.1 Inclusion models

The first option is more or less intuitive and can be split in several subgroups. Variational methods can provide rigorous bounds. HASHIN and SHTRIKMAN (1962) have contributed substantially to provide such useful bounds. We will not attempt to review these results here, and neither attempt to cover all other approaches which belong to that first group. We focus rather on methods following Eshelby's approach for an inclusion for practical reasons which will be explained below. Eshelby's solution provides the stress and strain fields due to a single inclusion. Such an approach has been extensively developed by KACHANOV (1993) within the so-called non-interacting scheme. In that case, the effects of many inclusions are obtained as the superposition of the effect of each individual inclusion. Basically, the elastic modulus M_0 of the uncracked rock is modified due to heterogeneities by ΔM

$$\langle \sigma \rangle = (M_0 + \Delta M)\langle \epsilon \rangle. \tag{6}$$

The unknown quantity ΔM is derived from Eshelby' solution for a single inclusion in a given field. It has been shown by KACHANOV (1992, 1993) that the non-interacting approximation is a very good one as regards cracks, to the degree their spatial distribution is random. This is due to compensating effects between positive and negative interactions. It can be also understood from the Mori-Tanaka scheme. In that scheme, any crack is considered to be located in an effective field which is that of the others cracks. If cracks are introduced in a medium with constant surface conditions, the average stress field remains constant. If we choose, as the simplest approximation, the effective field to be the spatially averaged field, the implication is that the effective field is unchanged whether cracks are present or not. That means that crack interactions cancel.

1.2.2 T matrix model

If, in the case of cracks, the non interacting scheme is optimal, it can be shown (GUÉGUEN *et al.*, 1997) that in the case of round pores, an excellent approximation is obtained with the differential self-consistent method, which is an improved form of the self-consistent scheme. This last scheme can be understood as derived from the so-called T matrix model which represents the second option to determine the unknown fields. This model is very general indeed. It uses integral equations, assuming that stress and strain obey

$$\frac{\partial \sigma_{ij}(\vec{r})}{\partial x_j} = \frac{\partial [C_{ijkl}(\vec{r})\epsilon_{kl}(\vec{r})]}{\partial x_j} = F_i. \tag{7}$$

An important result is that the general solution for the static case can be obtained from equation (3) using integral equations with Green's function kernels. The solution of the integral equation can itself be systematically approximated. Writing the elastic stiffnesses tensor as

$$C_{ijkl} = C^0_{ijkl} + \delta C_{ijkl}, \tag{8}$$

and introducing the tensor Green's function $G_{ijkl}(\vec{r}, \vec{r}')$ for strain, the following equation results (GUBERNATIS and KRUMHANSL, 1975)

$$\epsilon_{ij}(\vec{r}) = \epsilon^0_{ij}(\vec{r}) + \int d\vec{r}' G_{ijkl}(\vec{r}, \vec{r}')\delta C_{klmn}(\vec{r}')\epsilon_{mn}(\vec{r}'). \tag{9}$$

This provides, using the various possible approximations, the solutions for the static field. The iterated solution of equation (9) is

$$\epsilon = \epsilon^0 + G\delta C\epsilon^0 + G\delta CG\delta C\epsilon^0 + \dots \tag{10}$$

where δC stands for the elastic stiffnesses tensor. The strain field is obtained as a series which can be formally summed so that

$$\epsilon = \epsilon^0 + G\left(\delta C \frac{I}{I - G\delta C}\right)\epsilon^0 = \epsilon^0 + GT\epsilon^0 \tag{11}$$

where $T = \delta C(I - G\delta C)^{-1}$ is called the T matrix in the scattering theory. For a constant strain field ϵ^0, and using the definition of T, the effective susceptibility, i.e., the tensor C^{eff} is obtained

$$C^{\text{eff}} = C^0 + \langle T \rangle \frac{I}{I + \langle GT \rangle}. \tag{12}$$

This gives the exact solution provided that $\langle T \rangle$ and $\langle GT \rangle$ can be computed. At this point various approximations are used. For instance, a particularly important approximation is the self consistent approximation. It assumes $\langle T \rangle = 0$. Because T is a function of C^0, it is possible to make such an approximation which provides a set of implicit equations for C^0. In that particular case, $C^0 = C^{\text{eff}}$. Let us emphasize that C^0 here is no longer the elastic stiffness tensor of the intact (non-porous) rock. It is the effective medium itself. The method is called ''self-consistent'' precisely because it assumes the effective properties (to be calculated) known from the beginning. The method consists in calculating the effective properties in terms of a set of implicit equations.

2. Upscaling and Numerical Methods

In many cases, the situation is that of a strong heterogeneity. Then one cannot define any REV. The objects of interest are of size smaller than a hypothetical REV. An example of great importance is the oil reservoirs. Any model of fluid flow (or any other process) in such a case cannot rely on the EMT approach. Numerical methods are required to simulate flow at the reservoir scale. This implies that a discrete model of the geological object considered must be constructed. Because such methods can be highly CPU-time consuming, there exists various attempts which aim at the optimum compromise between the quality of the simulation and the required amount of CPU time. In that case, ''upscaling'' refers to the techniques used to transform a fine-grid model into a more practical, coarser one.

Aquifers or oil reservoirs models are discretized over fine grids populated by permeability values. These models generally contain too many gridblocks to be used directly for flow simulation. Some manner of averaging or upscaling is therefore required to reduce the number of gridblocks and make flow simulation tractable in terms of CPU time.

2.1 From Effective to Equivalent Permeability

As discussed in section 1, the term effective refers to media which are statistically homogeneous on the large scale. In the case of transport properties, the effective permeability is defined from an averaged form of Darcy's law as:

$$K^{\text{eff}} = -\mu \frac{\langle \vec{U} \rangle}{\vec{\nabla} P}. \tag{13}$$

K^{eff} is the effective permeability, μ fluid viscosity, $\langle \vec{U} \rangle$ the mean over the domain of Darcy velocity and $\vec{\nabla} P$ the pressure gradient. This simple expression refers to isotropic media. Once the effective permeability is computed, the finely gridded reservoir model is replaced by a single gridblock with a constant permeability (Fig. 2).

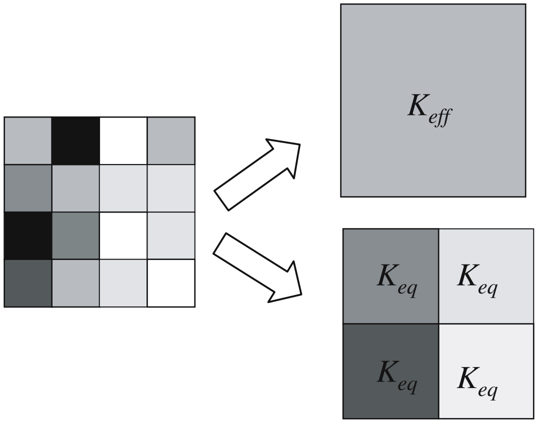

Figure 2
From fine gridblocks to coarse ones.

However, in most engineering situations involving water flow or oil flow, conditions for the emergence of an effective permeability are not met. The studied domains are not large enough to ensure statistical homogeneity. Instead, one focuses on equivalent or block permeabilities. An equivalent permeability is assigned to a group of neighboring fine gridblocks (Fig. 2). In this case, the fine reservoir model is replaced by a set of permeability values instead of a single one. The equivalent permeability must be selected so that flow is possibly least perturbed when replacing the permeabilities of the fine gridblocks by this equivalent permeability. This intuitive concept needs to be mathematically expressed, which entails the definition of equivalence criteria.

As permeability is not an additive property, the equivalent permeability cannot be derived from a simple algebraic expression as the arithmetic mean. Therefore, one instead considers flow rate (CARDWELL and PARSONS, 1945; WARREN and PRICE, 1961; RUBIN and GOMEZ-HERNÁNDEZ, 1990) or dissipated energy (MATHERON, 1967; INDELMAN and DAGAN, 1993). In the first case, the equivalence between the grouped fine gridblocks and the resulting coarse gridblock is expressed in terms of flow rates: Flow rates at the boundaries of the aggregate of fine gridblocks must be the same as those of the coarse gridblock. In the second case, the dissipated energy must be identical for the aggregate of fine gridblocks and the corresponding coarse gridblock. Flow or dissipated energy provide equivalence criteria. Stating equivalence following one of these criteria leads to an estimation of the equivalent permeability of the coarse gridblock. Although both criteria involve distinct formulations, they come down to identical results with periodic conditions applied at the boundaries of the coarse gridblock (BOE, 1994).

The equivalent permeability is generally not a scalar, but a second-order tensor. Its shape motivated many research works. For instance, GELHAR and AXNESS (1983) stated that it is symmetric. This property was actually evidenced by MATHERON (1967) for stationary porous media; that is media with unchanged two-order statistics regardless of the considered location. On the other hand, according to ABABOU (1988) or KING (1993), the equivalent permeability tensor is not symmetric. For engineering problems, permeability is not stationary at the scale of the coarse gridblock. Hence, the equivalent permeability tensor can be considered as non-symmetric. Last, if the coarse gridblock is large enough, the equivalent permeability tends towards the effective permeability, when it exists.

2.2 Numerical Upscaling Techniques

In order to estimate the equivalence criteria, we need to perform flow simulations over the target coarse gridblock. The problem to solve is based upon the combination of Darcy's law and the continuity equation under steady-state conditions

$$\vec{u} = -\frac{k}{\mu}\vec{\nabla}p \quad \text{and} \quad \vec{\nabla}\cdot\vec{u} = 0, \tag{14}$$

where $\vec{u}$ is the filtration or Darcy velocity, k permeability, μ fluid viscosity and p pressure. In this particular case, the continuity equation is written for an incompressible fluid. We also assume that there are no source terms. Combining both equations gives

$$\vec{\nabla}\cdot(k\vec{\nabla}p) = 0. \tag{15}$$

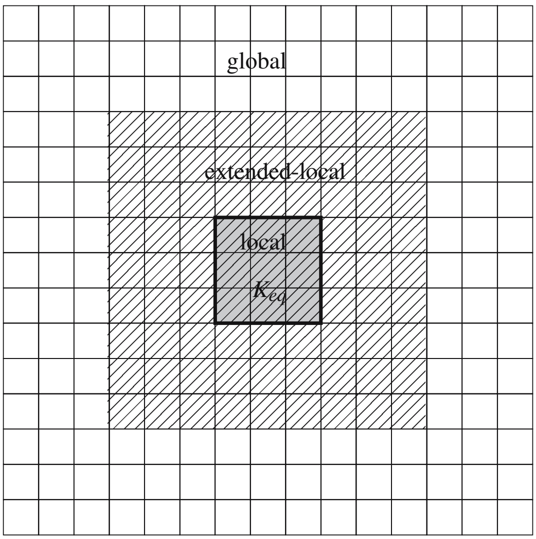

Figure 3
Classification of the upscaling techniques depending on the size of the flow simulation domain.

A variety of upscaling techniques have been developed. They can be classified depending on the size of the domain, over which the above equation is solved (Fig. 3). When this domain is limited to the target coarse gridblock, upscaling techniques are said to be local. When it includes the immediate neighboring gridblocks, upscaling techniques are said to be extended-local. Last, when it extends to the entire fine grid, upscaling techniques are said to be global. In addition, solving the above equations calls for boundary conditions. Most of the time, they are unknown. Subsequently, arbitrary boundary conditions are selected: They are often of the no-flow type or uniform or periodic. Clearly, the computed equivalent permeabilities depend on the boundary conditions considered.

2.2.1 Local Techniques

Local upscaling techniques derive the equivalent permeability of a coarse gridblock from the permeabilities of the fine gridblocks included in this coarse gridblock. First, we assume that we extract the fine gridblocks inside the coarse one from the entire fine grid. Second, we solve equation (15) over the finely gridded target coarse gridblock and we use the solution to compute its equivalent permeability. A flow direction has to be selected: it may be axis X, Y or Z. Boundary conditions must also be prescribed. For instance, flow can be simulated with no-flow (or permeameter) conditions applied at the boundaries of the coarse gridblock (WARREN and PRICE, 1961). In such a case, constant pressures are imposed at inlet and outlet faces and no flow is allowed across the other faces. Let L be the distance between the two faces where pressures are imposed, A the surface of the section crossed by fluid, $\triangle P$ the pressure difference and Q the flow rate across the coarse gridblock. In this case, we consider the equivalence criterion based upon flow equality. Flow rate Q results from the addition of the elementary flow rates at the outlet face. These elementary flow rates are obtained when solving equation (15). Reverting to Darcy's law yields the equivalent permeability of the coarse gridblock

$$K^{\mathrm{eq}} = -\frac{Q}{A}\mu\frac{L}{\triangle P}. \tag{16}$$

When flow is driven in the X direction, the computed equivalent permeability is K^{eq}_{XX}. Repeating the computations for flow along axes Y and Z provides the diagonal terms of the equivalent permeability tensor. Extra-diagonal terms cannot be determined, because no flow is allowed in the direction transverse to the pressure differences.

The major differences in local upscaling techniques are due to their treatment of the boundary conditions. Detailed reviews were published by RENARD and DE MARSILY (1997) and WEN and GOMEZ-HERNÁNDEZ (1996). For instance, GALLOUËT and GUÉRILLOT (1994) used uniform boundary conditions while DURLOFSKY (1991) considered periodic boundary conditions. We note that a complete permeability tensor cannot be computed when assuming no-flow boundary conditions as mentioned above. Also, the upscaled permeability tensor is non-symmetric with uniform boundary conditions and symmetric with periodic ones. PICKUP *et al.* (1994) showed that periodic boundary conditions provide the most reliable results in the example problems considered. The key point is that actual boundary conditions change with time and are unknown.

Local techniques focus on the fine gridblocks inside the coarse gridblock only. They do not account for the effects of the neighboring regions in the estimation of the equivalent permeability. In addition, the computed equivalent permeability tensor strongly depends on the type of boundary conditions. There is no unique equivalent permeability.

Local upscaling techniques provide reliable estimation for reservoir models with smooth permeability variations, but not for reservoir models with complex geological features and strong permeability contrasts.

2.2.2 Extended-local Techniques

The development of extended-local upscaling techniques has been motivated, at least partially, by the work of HOU and WU (1997). These authors obtained improved results by incorporating the effects of neighboring gridblocks in their computations. Extended-local upscaling techniques are very similar to local approaches. However, instead of using the coarse gridblock solely to derive the equivalent permeability, we consider a slightly larger domain consisting of the coarse gridblock plus local border regions (GOMEZ-HERNÁNDEZ and JOURNEL, 1994; HOLDEN and LIA, 1992; WU *et al.*, 2002; WEN *et al.*, 2003). The objective is to obtain more accurate estimations by capturing the effects of larger-scale permeability connectivity and by reducing the influence of boundary conditions into the calculation of the equivalent permeability. It is reasonable to expect that the effect of the boundary specification on the upscaled equivalent permeability will be less when increasing the border regions and that results obtained with different types of boundary conditions will tend to converge.

2.2.3 Global Techniques

Extended-local upscaling techniques yield refined estimations of equivalent permeabilities of coarse gridblocks compared to local techniques. However, in some cases, particularly when applied to reservoirs with complex and tortuous geological features such as channels, extended-local upscaling techniques still need improvement. The leading idea is to better capture the influence of the neighboring geological bodies and to reduce further the dependency on boundary conditions, which are arbitrarily selected. This motivated the development of global upscaling techniques.

With global approaches, the intent is to solve equation (15) on the entire fine grid. The solution is then used to estimate the boundary conditions to be applied to coarse gridblocks when computing equivalent permeabilities. We hope that this choice is

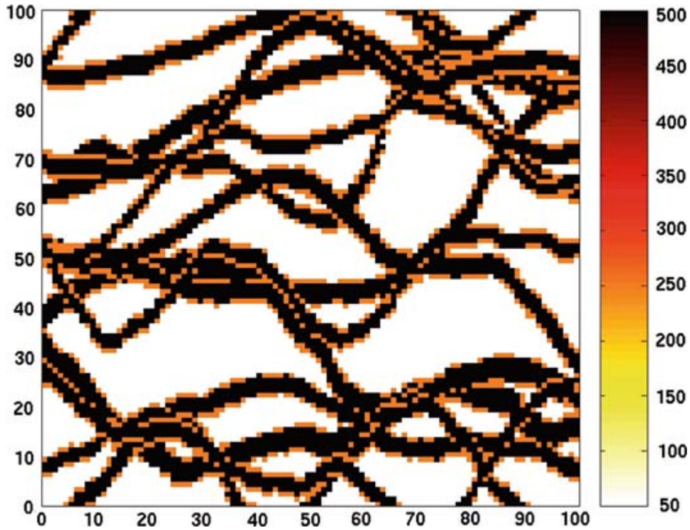

Figure 4
Fine channelized reservoir model.

more realistic than the boundary conditions used with local and extended-local upscaling techniques. Also, considering the flow simulation over the entire fine grid should help to better catch the connectivity of neighboring geological bodies. Because purely global methods are computationally intensive (HOLDEN and NIELSEN, 2000), recent works mainly focus on quasi-global techniques, which decrease the computational overburden by substituting some type of approximate global information in place of global fine scale results. There are two kinds of quasi-global approaches: local-global and global-local.

Local-global methods attempt to estimate the effects of the global flow without actually solving a global fine scale problem. CHEN *et al.* (2003) proposed to first perform global coarse scale flow simulations and then local fine scale flow simulations. The boundary conditions considered for the global flow simulations are arbitrary. These flow simulations provide pressures at the centers of the coarse gridblocks. Then, the domain defined by the target coarse gridblock surrounded by half a coarse

Table 1

Facies properties

Facies	Volume fractions (%)	Permeability (mD)
Foodplain shale	50	50
Channel margin	15	250
Channel sand	35	500

gridblock is extracted. An interpolation process yields the pressures in the fine gridblocks at the boundaries of this extended domain. Thereafter, extended-local flow simulations can be run to derive the equivalent permeability tensor of the target coarse gridblock. The overall method is iterative. The procedure is repeated until the upscaled permeability tensor no longer varies, that is until the result is self-consistent. The suitability of the obtained results is not warranted: Negative terms may appear on the diagonal of the permeability tensor (DURLOFSKY *et al.*, 2003).

Global-local approaches call for global fine scale flow simulations and then local fine scale flow simulations. Unlike local-global methods, the flow problem is first solved over the entire fine grid. This initial global fine scale flow simulation is run with appropriate boundary conditions, that is boundary conditions as consistent as possible with the actual ones. It allows the boundary conditions to be considered in the subsequent local flow simulations. In this special case, no interpolation is required: Pressures are computed at once in the fine gridblocks at the boundaries of the coarse gridblocks. Then, local flow simulations are

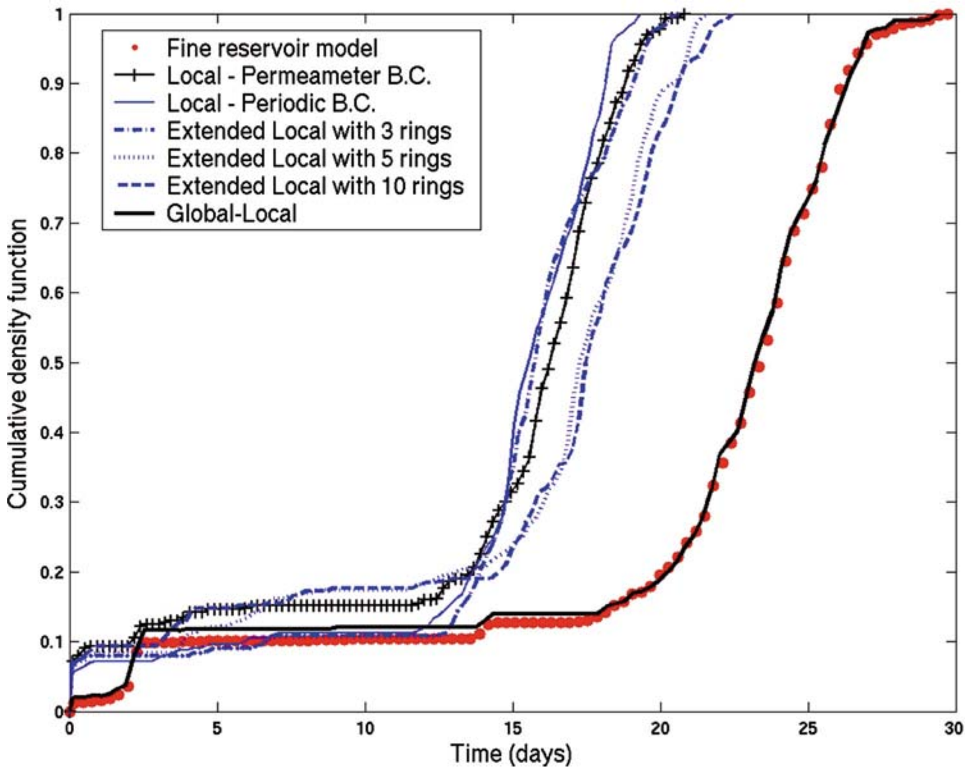

Figure 5
Cumulative density function of the times needed by particles to cross the reservoir.

performed for each coarse gridblock, first with the computed boundary conditions and second with perturbed boundary conditions (PICKUP *et al.*, 1992; RICARD *et al.*, 2005). This results in the estimation of the equivalent permeability tensors of the coarse gridblocks. Contrary to the local-global approach of CHEN *et al.* (2003), these global-local upscaling methods are not iterative. They provide at once the coarse scale permeability tensors and give no negative diagonal terms. Despite their interesting features, global-local upscaling approaches are not very popular due to the CPU-time required to run a flow simulation over the entire fine grid.

2.3 A Numerical Example

For underground reservoirs with smooth permeability variations, local and extended-local upscaling techniques provide satisfactory approximations. However, they can fail in the case of more complex geological systems such as channelized reservoirs. In the example depicted in Figure 4, the two-dimensional reservoir model is discretized over 100×100 gridblocks. This horizontal layer is made of three facies: A floodplain shale, a channel margin and a channel sand. Their volume fractions and permeabilities are reported in Table 1. For simplicity, porosity is assumed to be 0.25 everywhere. The reservoir is bounded by two horizontal wells: An injector at $x = 1$ and a producer at $x = 100$. A pressure difference of 1 kPa is set between the two wells. Next, inert particles are injected into the injector. Due to pressure difference, they move from the injector to the producer. For the example studied, fluid viscosity is assumed to be 1 cP and the size of each gridblock 1×1 m^2. This numerical tracer test experiment allows for computing the times needed by the particles to cross the reservoir. The cumulative density function of these times is reported in Figure 5. This fine reservoir model is upscaled to 20×20 gridblocks on the basis of an aggregation rate of 5×5. We illustrate the influence of large-scale connectivity on the computation of equivalent permeabilities by applying local, extended-local and global methods. The objective is to perform the same numerical tracer experiment as the one described above for the upscaled reservoir models and to compare the resulting travel times to those determined for the fine reservoir model. Therefore, we hope to assess the quality of the upscaled models. Various upscaling techniques are considered:

1— a local upscaling with permeameter (or no-flow) boundary conditions;
2— a local upscaling with periodic boundary conditions;
3— an extended-local upscaling with target gridblocks surrounded by 3 rings of fine gridblocks and with periodic boundary conditions;
4— an extended-local upscaling with target gridblocks surrounded by 5 rings of fine gridblocks and with periodic boundary conditions;

5— an extended-local upscaling with target gridblocks surrounded by 10 rings of fine gridblocks and with periodic boundary conditions;
6— a global-local upscaling.

The computed cumulative density function of travel times is reported in Figure 5. Clearly, the results derived from local and extended-local upscaling methods are very different from the ones pointed out for the reference finely gridded reservoir. Particles move much faster in the upscaled reservoir than in the fine one. Two main features can be stressed for the example presented here. First, in the case of purely local approaches, the determined cumulative density functions are very close, regardless of the boundary conditions considered. Second, in the case of extended-local approaches, the larger the border region, the better the agreement between the upscaled and fine results. Now, if we examine the cumulative density function of travel times derived from a global-local upscaling technique (RICARD *et al.*, 2005), the improvement is obvious. The cumulative density function of travel times estimated for the upscaled reservoir model fits almost perfectly that one identified for the fine reservoir. This example illustrates the benefit in using global computations, although they depend on flow simulations on the whole fine grid.

3. Upscaling and the Fractal Dream

A particular situation which corresponds to media where statistical homogeneity does not apply, and yet can be handled without the time consuming numerical simulations discussed in the previous section, is that of a fractal geometry. Fractal objects have the property of scale invariance. This obviously makes "upscaling" an irrelevant question. The properties of a fractal medium such as transport properties are scale-dependent through power laws. This makes the situation considerably simpler than that examined in section 2. For all these reasons, the scale invariance property is a very appealing characteristic of fractal media. One could list an impressive number of geological features which have been considered to be fractal and characterized by power laws: Fault length, fault roughness, gouge particle size, slip displacement, pore surface, etc. This raises indeed two questions. First, could one put most geological objects into one of the two following groups: (1) statistical homogeneity, or (2), fractal geometry? The first case corresponds to translational invariance and the second one to scale invariance. Such an alternative would offer a way out of the time-consuming techniques described in section 2. Second, if the answer to the previous question is positive, and pervasive fractality is real and not a dream, what is the reason for a such universal behavior? More specifically, since fractality is a geometrical feature, the basic question is: What are the physical processes which could lead to a fractal geometry? and what are those relevant to geology?

We suggest in this last section that a precise investigation of each situation in which a geological object is considered to be fractal should be conducted. Because most data on possibly fractal objects cover a narrow scale range, and because such geological data are difficult to obtain the desired accuracy, we suggest that a physically based and more restrictive criterion should be used to attest fractality. Only a clear positive answer to the previous question, i.e., identification of the physical mechanism explaining fractality, should be used to identify fractal objects. A multitude of observations and models has been devoted to fractal systems in the past 30 years. We do not attempt to review all of these investigations, however concentrate instead on the few cases where it appears that there are well identified processes which could explain the existence of fractal geological objects. Two examples appear to be good candidates for inclusion into that category. One at macroscopical scale, and the other at microscopical scale. We do not know of possible processes which could explain similarly the hypothetical fractality of other objects, nonetheless we suggest that further investigations should be conducted to seek out physical processes in the other cases.

3.1 Percolation Theory and Fractal Geometry

The first situation pertains to fractal networks of fractures and percolation theory. This is very relevant to upscaling since percolation theory deals with strongly heterogeneous media. As stated in section 1, EMT does apply to statistically homogeneous media only. However, beyond some degree of heterogeneity, i.e. a large number of fractures in the medium in our case, EMT is no longer valid. This is easy to understand as clustering effects, clusters are connected fractures in our case, become increasingly more important. The aim of percolation theory is precisely to investigate such clustering effects and account for the behavior of the medium in that state. The concept of percolation threshold, here it means overall interconnection of fractures providing a connected path, is a key one in this theory (MADDEN, 1983; STAUFFER and AHARONY, 1992). It is well known that the circumstances of such a percolative system at threshold are those of a phase transition. As shown by WILSON (1971), renormalization group theory can explain the particular behavior of physical systems in those conditions. Fluctuations at all scales are present or, stated in different terms, the system can be described as a fractal one. This implies that percolation models provide a simple explanation for the existence of a fractal set. For that reason, it has been suggested that fractal set of cracks, or fault networks, could exist if they were linked to a percolation effect. A possible model is that a fractal set of fractures would be the result of fluid pressure driven mechanisms (GUÉGUEN *et al.*, 1991). If the driving force for fracture propagation is assumed to be fluid pressure, it will be turned off as soon as the percolation threshold is reached because a strong permeability increase would develop then, relaxing the fluid pressure. It follows that the network, in proximity to percolation threshold, is fractal. In terms of phase transition, the system stays very close to its critical point.

Although this mechanism provides a clear explanation for the possible existence of fractal sets of fractures, it remains to be established whether such situations exist or not. At least in such cases, one would expect a fractal distribution. There does not seem to exist other clear processes implying a fractal geometry for fractures.

3.2. Aggregation Processes and Fractal Geometry

Another geological object which can be fractal is pore surface (KATZ and THOMPSON, 1985). Interestingly, it ensues that various growth models can explain the development of fractal surfaces. In particular, Diffusion Limited Aggregation models (DLA theory) provide a means to finish with fractal objects. A common point to those models is the Laplace equation. In rocks, diagenesis, healing and sealing processes could be the mechanisms responsible at the microscopical scale for growth. Although it has been well established that some pore surfaces (KATZ and THOMPSON, 1985), or rough surfaces (KARDAR *et al.*, 1986) are fractal, this is not the general state in geology. Again, it remains for investigation to determine if this is more the exception or the rule.

4. Conclusion

Geological media can be viewed as disordered media. In the simple case where the real system can be approximately described as a homogeneous embedding medium containing various inclusions, Effective Medium Theory (EMT) is an appropriate tool to derive any property of interest (a generalized susceptibility) at large scales. This provides a first possible method for upscaling. However, EMT becomes itself ineffective when the degree of heterogeneity is too large. This is so because basically, EMT is a perturbation method which fails beyond some point. Despite this drawback, the advantage of EMT is that it relies on the assumption of translational invariance: The medium is considered to be statistically homogeneous. That means that, beyond the scale of a Representative Elementary Volume (REV), the averaged susceptibility has the same value everywhere. When this breaks down, one may be tempted to shift from translational invariance to scale invariance: Any kind of invariance is of great interest because it does simplify the description of the system. By itself, scale invariance suppresses the upscaling question. It is a manner of unexpected answer, that we could dream of, to the question examined here. Scale invariance becomes the specific property of fractal systems. This is why fractal models have become so popular. As regards geological systems, fractal geometry appears however to be possible in a small number of specific situations only. For most cases, when Effective Medium Theory becomes ineffective, the only way to address upscaling is to use numerical methods. Considerable progress has been reached in that direction and global techniques allow the calculation of upscaled properties, at the expense of a certain amount of CPU-time.

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