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True Triaxial Stresses and the Brittle Fracture of Rock

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Abstract—This paper reviews the efforts made in the last 100 years to characterize the effect of the intermediate principal stress σ_2 on brittle fracture of rocks, and on their strength criteria. The most common theories of failure in geomechanics, such as those of Coulomb, and Mohr, disregard σ_2 and are typically based on triaxial testing of cylindrical rock samples subjected to equal minimum and intermediate principal stresses ($\sigma_3 = \sigma_2$). However, as early as 1915 Böker conducted conventional triaxial extension tests ($\sigma_1 = \sigma_2$) on the same Carrara marble tested earlier in conventional triaxial compression by von Kármán that showed a different strength behavior. Efforts to incorporate the effect of σ_2 on rock strength continued in the second half of the last century through the work of Nadai, Drucker and Prager, Murrell, Handin, Wiebols and Cook, and others. In 1971 Mogi designed a high-capacity true triaxial testing machine, and was the first to obtain complete true triaxial strength criteria for several rocks based on experimental data. Following his pioneering work, several other laboratories developed equipment and conducted true triaxial tests revealing the extent of σ_2 effect on rock strength (e.g., Takahashi and Koide, Michelis, Smart, Wawersik). Testing equipment emulating Mogi's but considerably more compact was developed at the University of Wisconsin and used for true triaxial testing of some very strong crystalline rocks. Test results revealed three distinct compressive failure mechanisms, depending on loading mode and rock type: shear faulting resulting from extensile microcrack localization, multiple splitting along the σ_1 axis, and nondilatant shear failure. The true triaxial strength criterion for the KTB amphibolite derived from such tests was used in conjunction with logged breakout dimensions to estimate the maximum horizontal *in situ* stress in the KTB ultra deep scientific hole.

Key words: Rock mechanics, brittle fracture, true triaxial stress, failure criterion.

Introduction

In this paper I review some of the most important contributions of the last century dealing with rock brittle fracture criteria, with particular emphasis on the effect of the intermediate principal stress σ_2. The influence of stress condition on brittle fracture of rock has been studied at various levels since the beginning of civilization. In the early days, making stone tools and mining minerals required understanding of how rock breaks. The spectacular Egyptian pyramids, and the

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Greco-Roman temples, aqueducts, and sculptures, are just a few examples of sophisticated quarrying and carving techniques developed from an intuitive knowledge of the process of fracturing. In recent times numerous theoretical predictions and highly specialized experimental techniques have been introduced in attempts to formulate a uniform criterion of brittle fracture, relating the acting stresses and the rock properties necessary to induce rock failure. Although a uniform criterion has not been established yet, considerable progress has been made in understanding the effect of the intermediate principal stress on rock strength. This paper summarizes the advances made, and submits substantial evidence that the most commonly used failure criteria, which neglect the effect of σ_2, do not reflect the mechanical behavior of rock under a general state of stress.

The nomenclature used in this paper follows that of JAEGER and COOK (1979). "Dilatancy" is defined as the formation and extension of open microcracks aligned with the direction of the largest principal stress throughout the volume of a rock specimen. The term "yield point", i.e., the stress level at which the transition between elastic and ductile behavior occurs, is only mentioned in conjunction with a special case in the second section of the chapter "True triaxial testing at the University of Wisconsin." Most work on brittle rocks has not been concerned with the yield point because it is difficult to discern accurately. The terms "failure," "fracture," and "strength" are used interchangeably in this paper to describe the same phenomenon, namely "brittle fracture," i.e., the complete lack of cohesion along a surface in a rock body, assumed to occur at the peak point of the stress-strain curve in experiments conducted in conventional testing machines (JAEGER and COOK, 1979, p. 81).

Brittle Fracture Theories

In terms of the three principal stresses in rock (σ_1, σ_2, and σ_3, where compression is positive and σ_1 is the maximum stress), the condition at failure can be expressed as:

$$\sigma_1 = f_1(\sigma_2, \sigma_3) \text{ or } f_2(\sigma_1, \sigma_2, \sigma_3) = 0, \tag{1}$$

where the nature of the functions f_1 or f_2 depends on the specific rock properties. For ductile materials there exists a single well-established triaxial criterion for plastic yield, which originated with von Mises (NADAI, 1950):

$$\tau_{\mathrm{oct}} \equiv \frac{1}{3}\sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2} = c. \tag{2}$$

Equation (2) states that the yield point is reached when the distortional energy, represented by the octahedral shear stress τ_{oct}, is equal to a constant c, which is material-dependent.

This is not the case, however, in brittle rock. One main reason is that rock strength varies considerably with confining pressure, σ_3. A number of brittle fracture criteria have been proposed over the years, of which the most known ones present the condition of failure in the form of:

$$\sigma_1 = F_1(\sigma_3) \text{ or } F_2(\sigma_1, \sigma_3) = 0, \tag{3}$$

i.e., by neglecting the effect of the intermediate principal stress σ_2. Here, F_1 and F_2 are functions controlled by the material properties. Equation (3) characterizes criteria such as those due to Coulomb, Mohr, Griffith, and McClintock and Walsh (JAEGER and COOK, 1979). For example, the Coulomb criterion can be expressed as:

$$\sigma_1 = C_0 + \sigma_3\left(\sqrt{\mu^2 + 1} + \mu\right)^2, \tag{4}$$

where the two constants, C_0 and μ, the uniaxial compressive strength and the coefficient of internal friction, respectively, are rock properties that can be determined in the laboratory (JAEGER and COOK, 1979, p. 97).

Another important criterion of the type shown in equation (3) was obtained by Griffith (JAEGER and COOK, 1979), who assumed that rock is pervaded by randomly oriented microcracks that give rise to stress concentration at their tips. Brittle fracture occurs when the maximum stress at the tip of the most favorably oriented microcrack reaches a critical value characteristic of the rock. With this assumption, the criterion developed by Griffith for a plane stress condition led to a parabolic relationship between the maximum shear stress and the mean stress:

$$(\sigma_1 - \sigma_3)^2 = 8T_0(\sigma_1 + \sigma_3) \quad \text{for} \quad \sigma_1 + 3\sigma_3 \geq 0, \tag{5}$$

where T_0 is the uniaxial tensile strength of the material, a property that can be determined in the laboratory.

MURRELL (1963), who was an early proponent of the need to include all three principal stresses in brittle fracture criteria, extended Griffith's theory to three dimensions, and obtained a criterion that can be described in terms of stress invariants:

$$\tau_{oct}^2 = 8T_0\sigma_{oct}, \tag{6}$$

where τ_{oct} is the octahedral shear stress, defined in equation (2), $\sigma_{oct} = \frac{1}{3}(\sigma_1 + \sigma_2 + \sigma_3)$ is the mean stress, also called the octahedral normal stress. MURRELL (1963) found this criterion to be in reasonable agreement with the limited experimental results available to him.

The exclusion of σ_2 from brittle fracture theories was also challenged by NADAI (1950), FREUDENTHAL (1951), DRUCKER and PRAGER (1952), BRESLER and PISTER (1957), WIEBOLS and COOK (1968), and others, all of whom included all three principal stresses in their respective criteria. NADAI (1950, p. 231) recommended that the von Mises yield criterion for ductile metals be adapted to pressure-dependent polycrystalline materials such as rocks by replacing the constant c in equation (2) with a monotonically rising function f_N of the octahedral normal stress σ_{oct}:

$$\tau_{oct} = f_N(\sigma_{oct}). \tag{7}$$

Equation (7) indicates that in general the shear stress (τ_{oct}) increases with the mean pressure (σ_{oct}) upon brittle fracture. A specific form of this criterion in the form of a linear relationship between the octahedral shear and octahedral normal stresses was suggested by FREUDENTHAL (1951), DRUCKER and PRAGER (1952), and BRESLER and PISTER (1957) for materials such as concrete and soil:

$$\tau_{oct} = c_1 + c_2\sigma_{oct}, \tag{8}$$

where c_1 and c_2 are positive material constants. FREUDENTHAL (1951) also proposed an alternative simplification of equation (7) in the form of a parabolic relationship, which may better fit the fracture strength of some materials:

$$\tau^2_{oct} = c_3 + c_4\sigma_{oct}. \tag{9}$$

Equation (9) is very similar to Murrell's extended Griffith criterion (equation 6).

An energy-based criterion for assessing rock strength under combined or true-triaxial (also called polyaxial) stresses was propounded by WIEBOLS and COOK (1968). The criterion postulates that rock can be considered an elastic material containing a multitude of uniformly distributed and randomly oriented closed microcracks. When stress is applied to a certain volume of rock, the strain energy in the material has two components: energy stored in the absence of cracks, and energy resulting from frictional sliding that occurs along some of the cracks. The cracks remain closed when the rock is subjected to three compressive principal stresses, and sliding within a crack occurs only if the shear and normal stress conditions are such that $|\tau| - \mu_s\sigma > 0$ (following Amonton's Law, see JAEGER and COOK, 1979), where μ_s is the coefficient of sliding friction between the surfaces of a crack. Wiebols and Cook call this inequality the effective shear stress τ_{eff}, and define the strain energy stored per unit volume around cracks as a result of frictional sliding as the effective shear strain energy W_{eff}. The effective shear stress on a given crack depends on the principal stress magnitudes and the orientation of the crack with respect to the principal axes. The amount of W_{eff} per unit volume is a function of τ_{eff} on each crack, the number of cracks and their size and shape. The assumption that the rock strength is reached when the effective strain energy attains a critical value implies that it is a function of both the material properties and all three principal stresses. Assessing W_{eff} for a general state of stress can only be done numerically. However, for a particular simple condition such as when $\sigma_2 = \sigma_3$, the relationship degenerates to:

$$W_{eff} = 2\pi N \int_{\theta_1}^{\theta_2} \tau^2_{eff} \sin\theta d\theta, \tag{10}$$

where θ is the angle between the normal to the crack plane and σ_1, and N is the number of cracks aligned with a narrow zone around θ.

The numerically derived Wiebols and Cook effective strain energy failure criterion has three significant characteristics: under conventional triaxial compression ($\sigma_1 > \sigma_2 = \sigma_3$) the strength increases linearly with the confining pressure, as in the Coulomb criterion (equation 4); under triaxial extension ($\sigma_1 = \sigma_2 > \sigma_3$) the strength increases linearly with σ_3 at the same rate as when $\sigma_2 = \sigma_3$, but the biaxial compressive strength ($\sigma_3 = 0$) is always greater than the uniaxial compressive strength ($\sigma_2 = 0$); for a constant value of σ_3, the strength as σ_2 is raised from its initial value of $\sigma_2 = \sigma_3$, first rises gradually to where it reaches a plateau and then declines in such a way that when $\sigma_2 = \sigma_1$ it is still higher than when $\sigma_2 = \sigma_3$. WIEBOLS and COOK (1968) presented their criterion in separate plots for selected coefficients of sliding friction μ_s in the form of families of curves corresponding to constant σ_3 and varying σ_2. A typical plot representing the failure criterion for $\mu_s = 0.25$ is shown in Figure 1. One limitation of this criterion is that there are no known laboratory techniques of measuring the coefficient of sliding friction on microcracks (μ_s should not be confused with μ). On the other hand experimental results described below generally confirm the qualitative adequacy of this theoretical criterion.

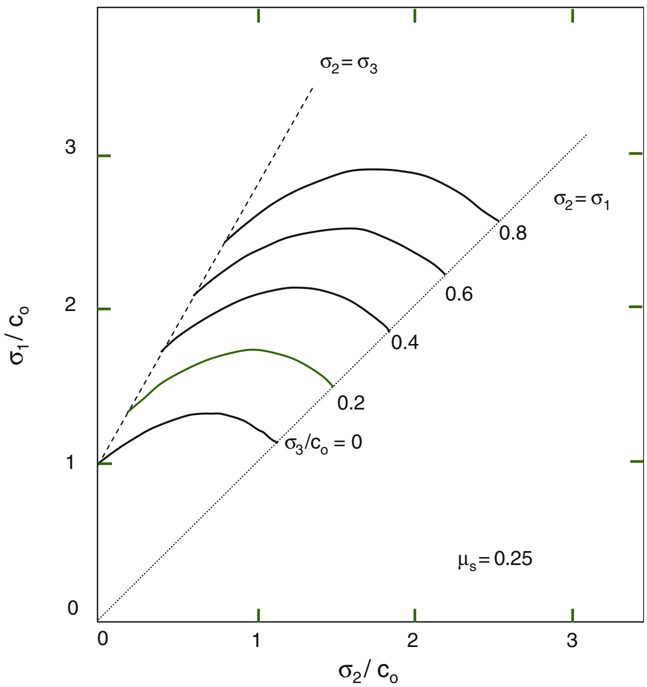

Figure 1
True triaxial failure criterion for a rock with a sliding coefficient of friction $\mu_s = 0.25$, as predicted by WIEBOLS and COOK (1968) theory. All principal stresses are normalized by the uniaxial compressive strength C_0.

Early Observations of the Effect of σ_2 on Brittle Fracture

Experiments that revealed the potential effect of the intermediate principal stress σ_2 on brittle fracture were performed as early as the turn of the last century by VON KÁRMÁN (1911) and BÖKER (1915). VON KÁRMÁN conducted so-called 'triaxial tests' on Carrara marble in which a confining fluid pressure around the cylindrical surface of a rock specimen was kept constant while the axial compressive load was raised until brittle fracture occurred. In these tests the maximum principal stress, σ_1, is aligned with the specimen axis, while the least and intermediate principal stresses, σ_3 and σ_2 respectively, are both equal to the confining pressure. Böker carried out similar triaxial tests on Carrara marble, except that he first applied an axial stress and kept it constant (σ_3), and then raised the confining pressure ($\sigma_1 = \sigma_2$) until failure. The compressive strength of the marble for the same σ_3 in one set of experiments was different than that in the other (Fig. 2), with the difference attributed to the variation of σ_2 from one extreme ($\sigma_3 = \sigma_2$) to the other ($\sigma_1 = \sigma_2$).

Although these results became widely known, they were not seen as a challenge to commonly used failure criteria such as those attributed to Coulomb, Mohr, and Griffith (JAEGER and COOK, 1979), which neglected the effect of σ_2 on rock strength. MURRELL (1963), however, expressed concern about the lack of a generalized

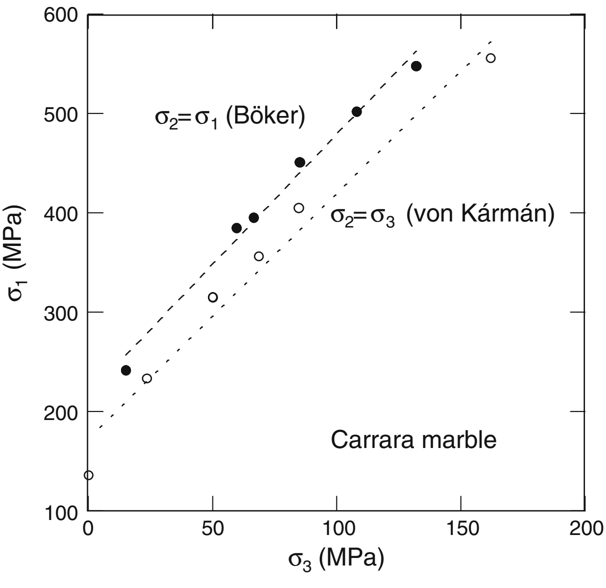

Figure 2
Compressive strength of Carrara marble under conventional triaxial compression ($\sigma_2 = \sigma_3$) and under triaxial extension ($\sigma_2 = \sigma_1$) based on experiments conducted by VON KÁRMÁN (1911) and BÖKER (1915), respectively, as reported by MURRELL (1963).

criterion for brittle fracture of rocks under a truly three-dimensional state of stress, similar to the von Mises criterion for plastic yield. He was among the first to point out that commonly conducted conventional triaxial tests in compression, like Von Kármán's, in which the intermediate principal stress σ_2 is equal to the least principal stress σ_3, do not lead to a general failure criterion. To demonstrate this point, he reproduced Von Kármán's and Böker's results, which are also recaptured in Figure 2 in the form of σ_1 at failure as a function of σ_3. The effect of the intermediate principal stress is obvious: Carrara marble is stronger when σ_2 equals σ_1, at any level of σ_3 tested. At $\sigma_3 = 100$ MPa, for example, the ultimate strength increases by some 15% as the stress condition changes from one in which $\sigma_2 = \sigma_3$ to one where $\sigma_2 = \sigma_1$.

HANDIN *et al.* (1967) conducted similar conventional triaxial compression ($\sigma_2 = \sigma_3$) and triaxial extension ($\sigma_2 = \sigma_1$) tests on a limestone, a dolomite and glass. Their results mirrored those of Von Kármán and Böker. Figure 3, showing the results obtained in specimens cored from the same Solnhofen limestone block, is representative of their findings. The ultimate strength in extension tests was clearly higher than that under compression conditions at any level of σ_3. Taking for example the case of $\sigma_3 = 100$ MPa, compressive strength when $\sigma_2 = \sigma_1$ was about 15% larger than that under $\sigma_2 = \sigma_3$. Handin *et al.* experiments on Blair dolomite, as well as on pyrex glass mimicked those on Solnhofen limestone. Since the mechanical properties

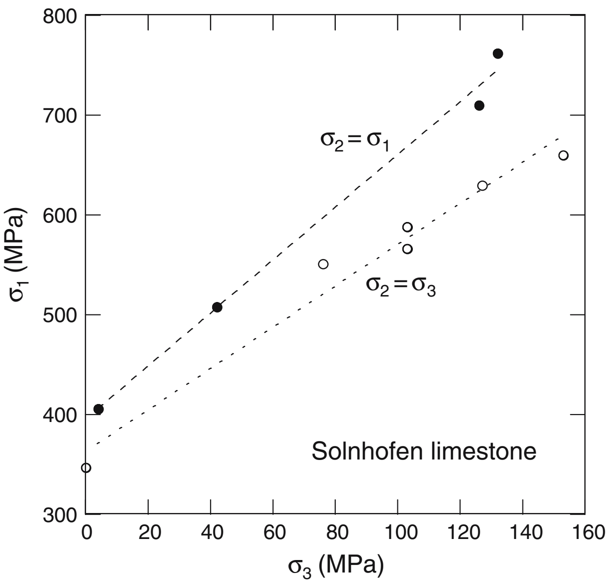

Figure 3
Compressive strength of Solenhofen limestone based on conventional triaxial compression ($\sigma_2 = \sigma_3$) and extension ($\sigma_2 = \sigma_1$) results (HANDIN *et al.*, 1967).

of these materials differ widely, the authors' reasonable conclusion was that the discrepancy in strength could only be the result of the different σ_2 between the two types of tests. Handin *et al.* also found that σ_2 caused the angle between the fault plane at brittle fracture and the direction of σ_1, to decrease between triaxial compression and triaxial extension. Similarly, the brittle-ductile transition stress in the limestone increased considerably between triaxial compression and triaxial extension.

Concurrent with Handin *et al.*'s work, JAEGER and HOSKINS (1966) conducted a series of innovative experiments they termed 'confined Brazilian tests', in which the typical Brazilian test consisting of a rock disc subjected to an increasing line load along the diametral plane until failure, was extended by jacketing the disc and applying a constant confining pressure to the specimen. This created a state of triaxial stress acting on the disc with three unequal principal stresses. However, these stresses were not independent of each other. Nevertheless, tests in Bowral trachyte, Gosford sandstone, and Carrara marble, supported previous findings suggesting that the magnitude of the intermediate principal stress was a factor influencing the conditions for failure in all three rocks.

Perhaps the greatest contribution to the study of brittle fracture and its dependence on the intermediate principal stress was made by MOGI (1967, 1971). He embarked on a long-term project of studying rock failure under the most general state of stress by first conducting experiments in a conventional triaxial cell. By nearly eliminating stress concentration due to end effects and other test refinements, he was able to demonstrate even more convincingly than previous workers the indisputable influence that σ_2 has on brittle fracture (MOGI, 1967). Figures 4 and 5 summarize his results in Westerly granite and Dunham dolomite. The substantial discrepancy between the strength of these two rocks in triaxial compression and triaxial extension is convincingly demonstrated, and the only variable that could have led to the difference in strength was σ_2.

True Triaxial Experiments and Strength Criteria

After conducting precision conventional triaxial compression and extension experiments, MOGI (1971) concluded that in order to observe more closely the role played by the intermediate principal stress on brittle fracture, he had to design and utilize a high-pressure true triaxial stress apparatus for testing hard rocks under three independently applied principal stresses ($\sigma_1 > \sigma_2 > \sigma_3$). His apparatus consisted of a pressure vessel that accommodated a rectangular prismatic rock sample (size $15 \times 15 \times 30$ mm) and two sets of pistons independently actuated to apply two of the principal stresses. The minimum principal stress, which has a major effect on fracture strength, was always provided by the confining pressure in the vessel, thus ensuring a uniform distribution of σ_3. Mogi minimized friction on the sample faces subjected to

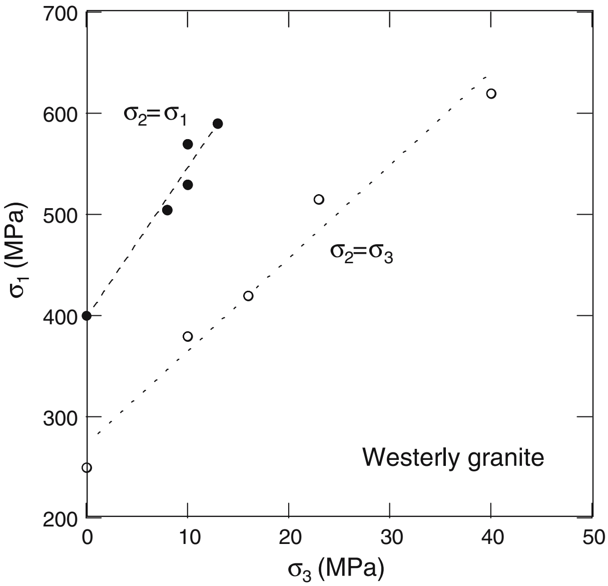

Figure 4
MOGI's (1967) improved conventional triaxial testing results in Westerly granite subjected to relatively low confining pressure σ_3, under both compression and extension.

piston loading by using lubricants such as copper sheet jacketing and Teflon or thin rubber sheets between specimen faces and pistons. Stress concentration at specimen ends was found to be greatly reduced by the high confining pressure (MOGI, 1966). Silicon rubber jacketing of specimen sides subjected to confining pressure and all around the edges of the piston-specimen contacts prevented the confining fluid from permeating the rock. Pressure generation was supplied by high-capacity hydraulic cylinders. A minimum principal stress reaching 800 MPa could be applied.

Using his ingenious apparatus, MOGI (1971) tested several carbonate and silicate rocks, and presented detailed true triaxial fracture strength results for Dunham dolomite and Mizuho trachyte. He discovered that σ_2 has indeed a significant effect on fracture strength at all levels in-between $\sigma_2 = \sigma_3$ and $\sigma_2 = \sigma_1$. Moreover, the largest effect on strength is reached at a level well inside this range. For example, when Dunham dolomite is subjected to $\sigma_3 = 45$ MPa and $\sigma_2 = 250$ MPa, its strength increases to 650 MPa, which is 160 MPa higher than the strength when $\sigma_2 = \sigma_3$, and 90 MPa higher than the strength when $\sigma_2 = \sigma_1$ (compare with Fig. 5 above). Mogi plotted his results in the form of σ_1 at failure versus σ_2 for different families of tests in which σ_3 was kept constant. A typical example is shown in Figure 6 for Dunham dolomite. Fracture strength increases steadily with the magnitude of σ_2.until a plateau is reached following which strength tends to decline as σ_2 approaches σ_1. The best-fitting curve to experimental data for any given σ_3 is downward concave, but

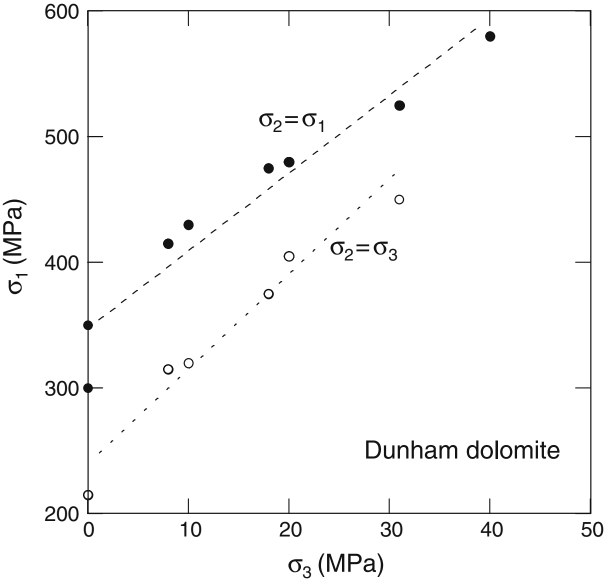

Figure 5
Substantial discrepancy between Dunham dolomite brittle strength under compression ($\sigma_2 = \sigma_3$) and extension ($\sigma_2 = \sigma_1$), using MOGI's (1967) improved conventional triaxial loading apparatus.

strength as σ_2 nears σ_1 remains higher than that when $\sigma_2 = \sigma_3$. Remarkably, this behavior is also predicted by the theoretical model proposed by WIEBOLS and COOK (1968).

In fact, WIEBOLS and COOK (1968) attempted to verify experimentally their own theoretical findings by employing a true triaxial cell in which loading was provided by pressurizing flat jacks inserted between the rectangular prismatic specimen sides and three pairs of rigid anvils. Flat jacks are inflatable bladders consisting of two thin metal sheets that are soldered together around the edges, and which allow hydraulic oil to be pumped in and pressurized against the specimen sides. However, while flat jacks are superior to direct piston loading in terms of the uniformity of the applied stresses, the magnitudes of the stresses must be kept low because of the limited internal pressure accommodated by the soldered sheets. Thus, Wiebols and Cook's experiments in Karroo dolerite lead only to a rather incomplete comparison with their theoretical criterion. It was not until MOGI (1971) that a more convincing agreement between Wiebols and Cook's model and experimental results was achieved in principle for several rocks.

MOGI (1971) also attempted to derive a fitting true triaxial criterion of brittle fracture based entirely on his experimental results. He observed that plotting all his test data for Dunham dolomite and Mizuho trachyte in the NADAI (1950) domain (τ_{oct} versus σ_{oct}, equation 7) did not lead to a uniform criterion. He then rationalized

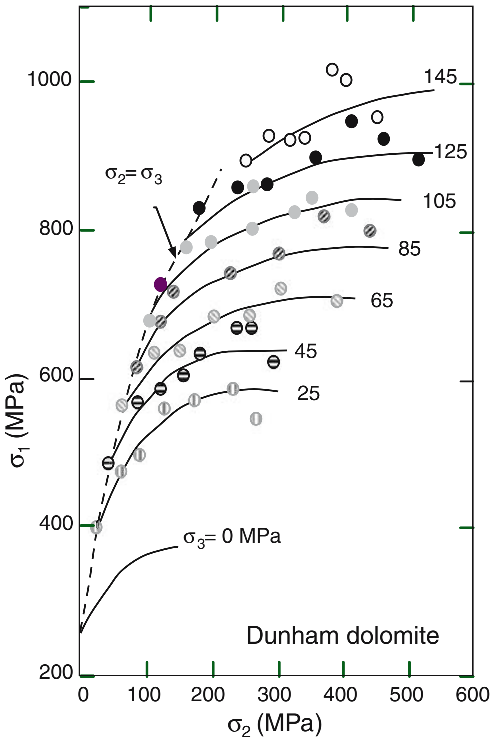

Figure 6
Results of the first comprehensive true triaxial tests conducted by MOGI (1971) using his new apparatus for testing rectangular prismatic specimens. Seven groups of tests are plotted, each for a different σ_3. Within each group a series of tests data are shown, each which is for a different σ_2. Dunham dolomite fracture strength increases in each case when σ_2 is raised above σ_3.

that since brittle fracture was in the form of shearing along one single plane striking in the σ_2 direction, it was more realistic to degenerate the mean normal stress $\sigma_{\rm oct}$ to just the mean stress on planes striking along σ_2. He found that in both rocks plotting the experimental results as τ_{oct} versus $\sigma_{\rm m,2}$ yielded single monotonically rising curves, best expressed as power functions f_M (Fig. 7):

$$\sigma_{\rm oct} = f_M \sigma_{m,2} \quad \text{where} \quad \sigma_{m,2} = \frac{1}{2}(\sigma_1 + \sigma_3). \tag{11}$$

Contemporaneous with the observations, testing, and theoretical advances related to the role of the intermediate principal stress in rocks, such as those by MURRELL (1963), HANDIN *et al.* (1967), WIEBOLS and COOK (1968), MOGI (1967, 1971) and others, similar developments were taking place in the field of soil, concrete, and soft-rock mechanics. A series of true triaxial testing machines were developed, typically based on the application of hydraulic pressure through three pairs of flexible

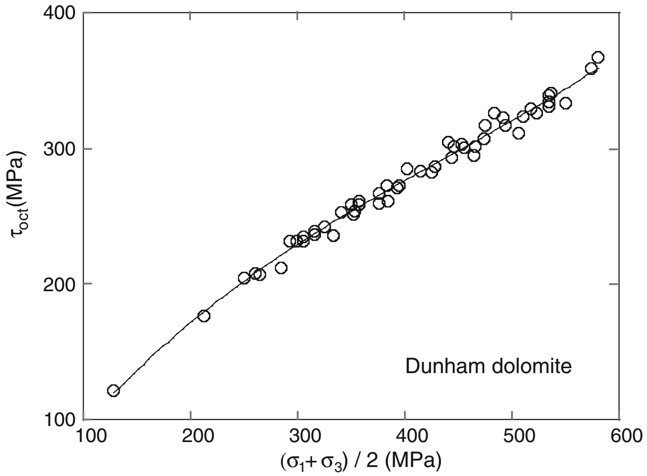

Figure 7
Generalized strength criterion for Dunham dolomite, in which all the experimental data points in Figure 6 align themselves closely with a power function in the octahedral shear stress τ_{oct} versus $(\sigma_1 + \sigma_3)/2$ domain (after MOGI, (1971).

membranes or fluid cushions to cubical test specimens. KO and SCOTT (1967), ATKINSON and KO (1973), LADE and DUNCAN (1973), STURE and DESAI (1979), and DESAI *et al.* (1982) are among those who developed different designs of true triaxial testing machines capable of testing soils and weak rocks. The capacity of these machines was typically less than 200 MPa, and thus not appropriate for loading hard rocks to failure under high intermediate and least principal stresses.

MICHELIS (1985) modified the flexible membrane design, in order to increase pressure capacity of the true triaxial cell. His apparatus consisted of a hollow cylinder that accommodated a prismatic specimen of size 50 × 50 × 100 mm. The axial (maximum) load was provided by two rigid pistons, while the intermediate and minimum pressures were supplied by two prismatic PVC bags between specimen and cylinder walls filled and pressurized by hydraulic oil. The cell allowed pressures reaching 300 MPa in the lateral directions, and an axial load extending to 1500 MPa. The initial results in tests conducted on Naxos marble for just one constant σ_3 (13.8 MPa) and different σ_2, showed a clear and substantial effect of the intermediate principal stress on strength (Fig. 8). As σ_2 was raised from 13.8 MPa to 113.2 MPa, the true triaxial strength increased from (approximately) 130 MPa to 280 MPa, which is a 115% rise over the commonly used conventional triaxial strength.

Mogi's pioneering true triaxial testing of hard rocks subjected to high pressures was followed by TAKAHASHI and KOIDE (1989), who designed and fabricated a near replica of MOGI's (1971) true triaxial cell. Their equipment accommodated much larger specimens (reaching 50 × 50 × 100 mm), and was meant specifically for testing

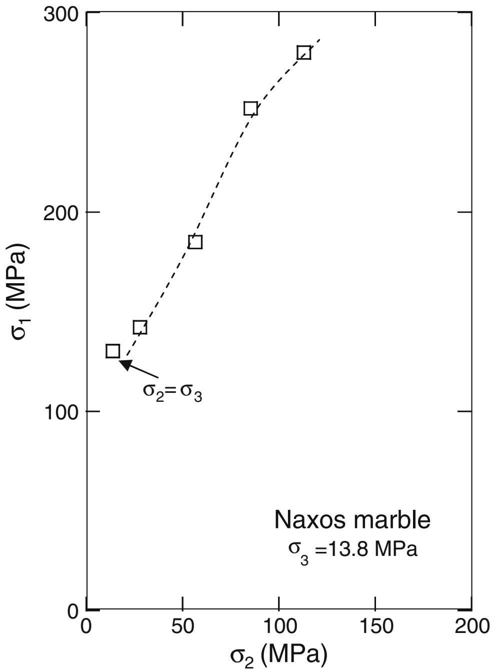

Figure 8
Strength of Naxos marble as a function of σ_2 for a constant and relatively low least principal stress σ_3 in a novel true triaxial apparatus for testing large prismatic rock specimens (after MICHELIS, 1984).

weak rocks such as sandstones and shales. The applied minimum principal stress range was limited to 0–50 MPa, simulating shallow depths, because the interest of these researchers was in rock engineering applications. Again, the strengthening effect of σ_2 as it increased beyond that of σ_3 was evident. Figure 9 reproduces Takahashi and Koide's results in Yuubari shale, which reveals an increase of up to 20% in resistance to brittle fracture as a function of σ_2, for each of the two σ_3 levels tested. TAKAHASHI and KOIDE (1989) also extended WIEBOLS and COOK's (1968) theoretical failure criterion, by rendering it more applicable to true triaxial testing. Their suggested model prediction of the variation of σ_1 as a function of σ_2 for various applied σ_3 matched well their experimental results in Shirahama sandstone.

In the mid-1990s a radically different design of a true triaxial cell was implemented in the form of a conventional triaxial cell in which cylindrical (and not prismatic as in most other designs) rock specimens are subjected to vertical loading applied through pistons at both ends of the cell, and two unequal lateral stresses provided by differential radial pressures (SMART, 1995). Radial pressure variation is achieved through an array of 24 PVC tubes aligned with the specimen long axis and trapped between the specimen and the triaxial cell inner wall. Each tube

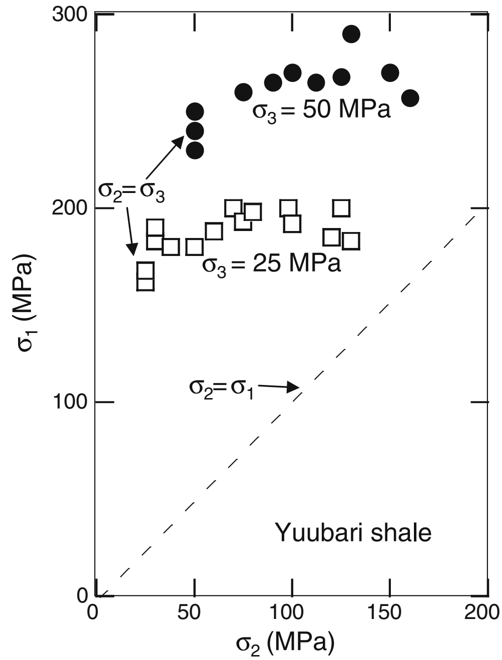

Figure 9
True triaxial testing in a Mogi designed apparatus modified to accept larger samples, shows that the strength of rocks such as shales is also controlled by the level of σ_2, not just σ_3 (after TAKAHASHI and KOIDE, 1989).

is filled with oil and pressurized, and by coupling opposing groups of tubes to independent hydraulic circuits, the application of different lateral pressures to the cylindrical specimen is enabled. The practicality of this cell is obvious, since it does not require tedious six-sided specimen preparation and makes use largely of existing equipment, with some modifications. The limitations of the design are the relatively low absolute and differential pressures that the PVC tubes can tolerate (50 and 7 MPa, respectively) and the intuitive uncertainty regarding the uniformity of the three principal stresses throughout the specimen. The cell cannot be used for a complete series of true triaxial tests since the maximum difference between σ_2 and σ_3 cannot exceed 14 MPa, but important information can still be drawn from tests performed at relatively low intermediate and minimum principal stresses. CRAWFORD *et al.* (1995) tested two sandstones in this apparatus and found that their fracture strengths were strongly affected by the magnitude of σ_2. Observations of failed sandstone specimens revealed that the brittle fracture mechanism involved the development of stress-induced dilatant microcracks extending parallel to σ_2 and opening against σ_3, progressively coalescing as σ_1 rose, and forming a through-going fault when peak σ_1 was reached.

Another major development in true triaxial testing was the design and fabrication of the Sandia loading facility, which incorporates a conventional triaxial pressure vessel and high-pressure hydraulic rams inside the vessel (WAWERSIK *et al.*, 1997). The cell resembles Mogi's, in that two of the principal stresses are applied through steel pistons, with the third one applied by the confining pressure in the vessel. The innovation here is that the pistons and the hydraulic jacks for the lateral intermediate stress are contained inside the vessel and react against the vessel's inside wall. Bending moments generated by uneven loading are minimized by the use of rubber sheets between the specimen faces and the pistons. Specimen size accommodated by the cell is $57 \times 57 \times 25$ mm or $76 \times 76 \times 178$ mm. The cell is intended specifically for testing high-porosity rocks that undergo large deformations during loading. Unfortunately, beyond some initial true triaxial testing in Castlerock sandstone, no published data on additional experimental results are available.

True Triaxial Testing at the University of Wisconsin

KTB Amphibolite and Westerly Granite

In 1994 the rock mechanics group at the University of Wisconsin embarked on a multi-year true-triaxial research project. This was motivated at first by the need to determine the true triaxial strength criterion of the KTB amphibolite, in order to enable the estimation of the maximum horizontal *in situ* stress from the logged breakout dimensions in the KTB ultra deep hole. The KTB hole was drilled to a depth of 9100 m in Bavaria by the German Continental Deep Drilling Program to study the properties and processes of the lower continental crust (EMMERMANN and LAUTERJUNG, 1997). One of the main objectives was the measurement of the *in situ* stress. The vertical stress (σ_v) was assessed from the weight of the overlying strata, and the two horizontal principal stresses (σ_h and σ_H) were estimated using the hydraulic fracturing method. However, below 3000 m only two partially successful hydraulic fracturing tests were carried out (at 6000 m and 9000 m), which yielded estimates of just the least horizontal stress magnitude (BRUDY *et al.*, 1997). Fortunately, borehole breakouts were logged in the amphibolite zone of the KTB hole (3000–7000 m depth). They provided the required information on *in situ* stress directions. Moreover, the dimensions of the logged breakouts at the borehole wall, together with knowledge of the vertical and least horizontal stresses, were used in an attempt to estimate the maximum tectonic horizontal stress. The model assumed that the stress condition at the points where breakouts intersect the borehole reached the strength criterion of the rock. However, use of criteria based on conventional triaxial tests was considered inadequate for deriving reasonable maximum stress estimates because the principal stresses at the borehole wall are noticeably differential (VERNIK and ZOBACK, 1992). BRUDY *et al.* (1997) employed the theoretical true triaxial strength criterion suggested by WIEBOLS and COOK (1968) to estimate σ_H. Subsequently, we undertook to re-estimate σ_H by first deriving an empirical true triaxial strength criterion. In the process we conducted a comprehensive experimental study of the true triaxial mechanical behavior of the KTB amphibolite, emphasizing strength and deformability.

The first step was to design and fabricate a true triaxial testing system capable of applying three mutually perpendicular high compressive loads to rectangular prismatic rock specimens (HAIMSON and CHANG, 2000). Our goal was to construct a cell similar to the one used by MOGI (1971), but keep it compact, and make use of existing equipment as much as possible, since our funding was limited. What emerged was a true triaxial loading system, which consists of a polyaxial pressure vessel inside an existing biaxial loading apparatus (Fig. 10). Three independent and mutually orthogonal pressures are generated in this system and applied to a rectangular prismatic specimen. The biaxial apparatus facilitates the application of two pressures, one in the axial (σ_1) and the other in one of the two lateral directions (σ_2) of the specimen. These two pressures are transmitted from the biaxial cell to the rock specimen via two perpendicular pairs of pistons mounted in the pressure vessel. The third pressure (σ_3) is applied directly to the second pair of specimen lateral faces by the confining hydraulic pressure inside the pressure vessel. The loading system was thoroughly calibrated using a strain-gaged aluminum sample of known elastic properties (HAIMSON and CHANG, 2000). The maximum stresses that this loading system can apply to a rock specimen of dimensions $19 \times 19 \times 38$ mm are 1600 MPa in the two piston-loading directions and 400 MPa in the third direction. This high capacity enables the equipment to bring very strong rocks to failure under realistic stress conditions of the order encountered in the earth's crust at depths exceeding 10 km.

In the first series of tests on KTB amphibolite, dry jacketed specimens were loaded to failure under the most general state of stress ($\sigma_1 > \sigma_2 > \sigma_3$). These experiments were intended to provide fundamental data on the mechanical behavior of the amphibolite. The immediate observation in our true triaxial tests was that brittle fracture in the form of a steeply inclined through-going fault plane is generally similar to that observed in conventional triaxial tests. However, the true triaxial loading confirmed the expectation that the dip direction of the fault plane is aligned with σ_3 (CHANG and HAIMSON, 2000). Test results showing the state of stress at failure were plotted graphically as the true triaxial strength σ_1 versus the applied σ_2 for different constant values of σ_3 (Fig. 11). The dependence of σ_1 at failure on the intermediate principal stress σ_2 is obvious. As σ_2 increased beyond σ_3 magnitude, the compressive strength was always higher than that when $\sigma_2 = \sigma_3$. Despite some considerable scatter (the amphibolite tested came from a depth of 6300 m and may have some inhomogeneity and anisotropy), the increase in strength as a function of σ_2 for constant σ_3 is substantial, and for example, as much as 50% or more over the commonly used conventional triaxial strength at $\sigma_3 = 30$ MPa. The effect of σ_2 on

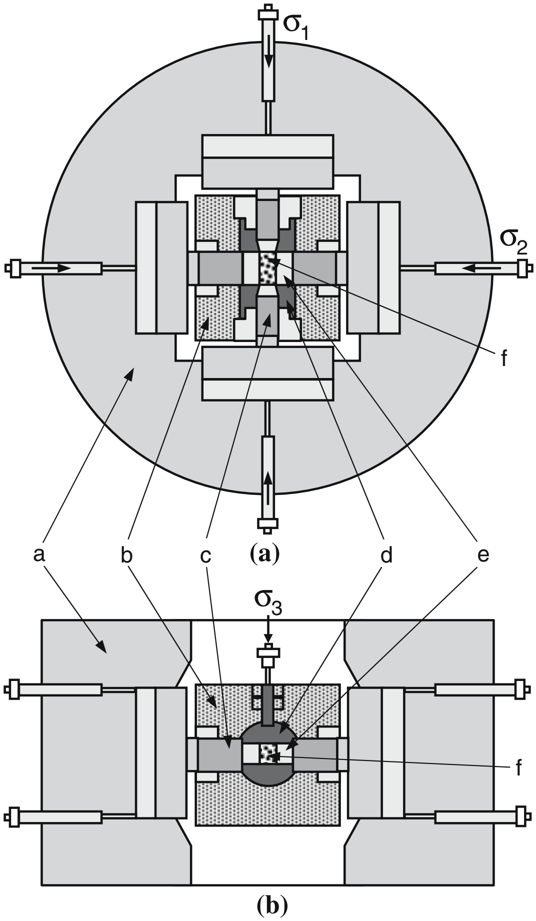

Figure 10
The University of Wisconsin true triaxial testing system: (a) Cross section, (b) profile. In this diagram *a* is the biaxial loading apparatus; *b* is the polyaxial pressure vessel; *c* is the loading pistons; *d* is the confining fluid; *e* is the metal anvil; and *f* is the rock specimen.

rock strength appears to weaken as the level of σ_3 rises, but remains significant (10% higher at $\sigma_3 = 150$ MPa). Thus, conventional triaxial tests provide only a lower bound of strength for a given least principal stress in KTB amphibolite. It is also noted that at some level of σ_2 a plateau is reached where strength appears to level off. This is similar to previous findings in softer rocks by MOGI (1971) and TAKAHASHI and KOIDE (1989), and in support of WIEBOLS and COOK (1968) theoretical prediction.

In order to define a fitting strength criterion to the experimental results in Figure 11, we replotted the data in the domains used by NADAI (1950), MURRELL

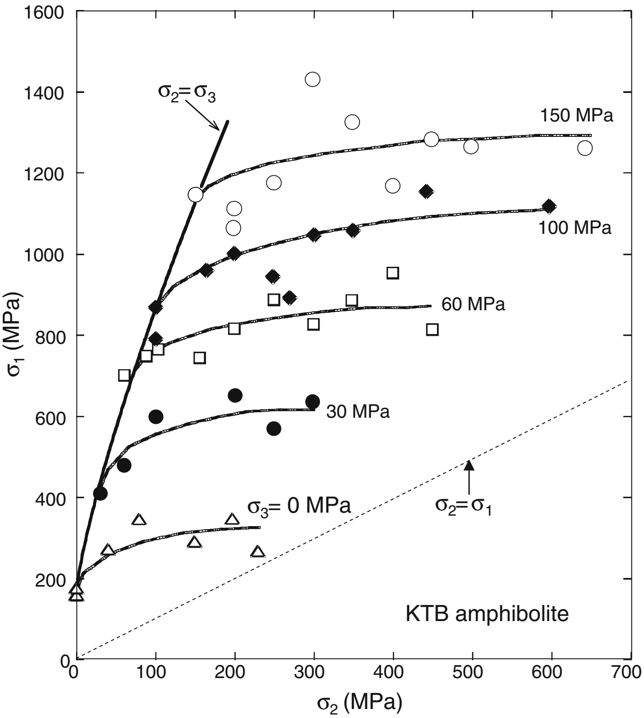

Figure 11
Variation of peak compressive stress σ_1 as a function of σ_2 for different constant values of σ_3 in KTB amphibolite (after CHANG and HAIMSON, 2000).

(1963), and MOGI (1971) for their respective strength criteria (equations 6, 7, 11). The only domain in which our experimental data points fit along one single curve with minimum deviation was that suggested by MOGI (1971) for brittle failure, i.e., the octahedral shear stress τ_{oct} as a function of the mean normal stress acting on the failure plane $\sigma_{m,2}$ (equation 11). The best fitting curve for KTB amphibolite was a monotonically increasing power function of the form (Fig. 12):

$$\tau_{oct} = A\sigma_{m,2}^{b}, \tag{12}$$

where A and b are two parameters that are rock-specific.

Our testing system enabled the measurement of strain in all three principal directions (HAIMSON and CHANG, 2000). The stress-strain behavior observed, revealing accelerated extension in the σ_3 direction as σ_1 increased, suggests that most of the induced and reopened microcracks eventually leading to brittle failure are aligned with the $\sigma_1 - \sigma_2$ plane and open up in the σ_3 direction. For each set of stress-strain curves we calculated the stress-volumetric strain and marked the onset of dilatancy, the stress level at which the constant rate of volumetric decrease starts

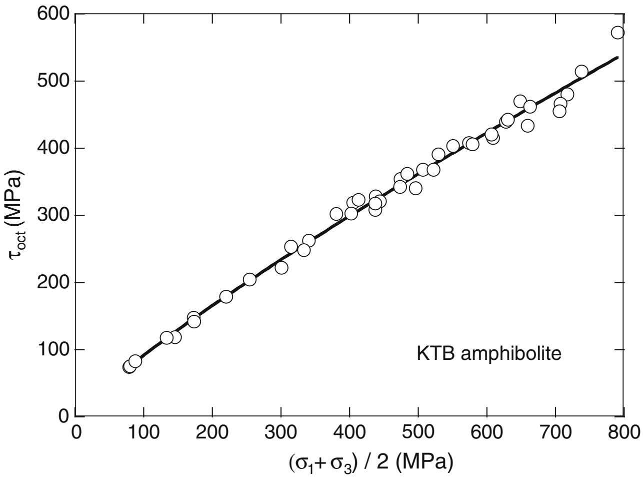

Figure 12
A true triaxial strength criterion for KTB amphibolite, based on all the experimental results shown in Figure 11, in terms of τ_{oct} versus $(\sigma_1 + \sigma_3)/2$ (after CHANG and HAIMSON, 2000).

reversing itself. Dilatancy has been correlated to internal microcracking responsible for enlarging specimen volume and for leading eventually to brittle fracture (BRACE *et al.*, 1966). We determined that dilatancy is more pronounced at low σ_2 magnitudes but diminishes at higher σ_2 levels, supporting results obtained by TAKAHASHI and KOIDE (1989) in Shirahama sandstone. The onset of dilatancy for a given σ_3 generally increases with the magnitude of σ_2 (Fig. 13) similar to observations by MOGI (1971) in Mizuho trachyte. These findings are significant in that previous conventional triaxial tests could only identify a unique dilatancy onset for a certain σ_3 in a given rock. The other significance is that higher intermediate principal stress magnitudes appear to extend the elastic range of the stress-strain behavior for a given σ_3, and by doing so it retards the onset of the failure process. The micromechanics leading to brittle fracture under true triaxial stress conditions begins at dilatancy onset, when microcracks, subparallel to the major principal stress σ_1, develop. As σ_1 increases microcracks grow and localize creating a shear-band dipping in the σ_3 direction. Upon brittle fracture the shear band fails, forming a fault (Fig. 14).

Using the same true triaxial system, we also conducted an extensive series of tests in Westerly granite, which had been thoroughly studied in rock mechanics laboratories in the U.S., and which is well known for its homogeneity and isotropy. The results were remarkably similar to those obtained in KTB amphibolite, both with respect to the effect of σ_2 on brittle fracture and the true triaxial strength criterion (Figs. 15 and 16), as well as dilatancy onset and the micromechanics of failure. From this analogous behavior between Westerly granite and KTB amphibolite we inferred that it represents typical true triaxial mechanical characteristics of fine- to medium-grained crystalline rocks.

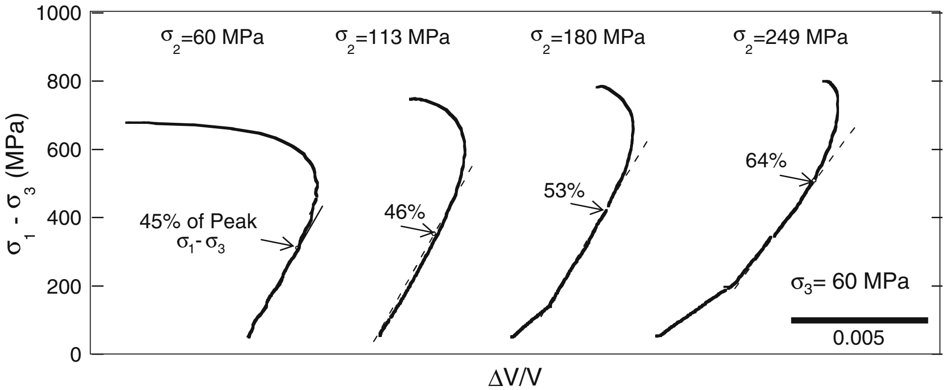

Figure 13
Differential stress ($\sigma_1 - \sigma_3$) versus volumetric strain (ΔV/V). showing that for a constant σ_3 (60 MPa) the onset of dilatancy rises with the increase in σ_2.

Unjacketed Amphibolite and the State of in situ Stress at KTB

A second series of true triaxial tests was conducted on KTB amphibolite, specifically for the purpose of estimating *in situ* stress magnitudes from breakout dimensions logged in the ultra deep hole. These tests were intended to simulate field conditions, in which the least principal stress at the borehole wall equals the radial pressure applied by the borehole fluid to the exposed rock, and where owing to the extremely low porosity and permeability of the KTB amphibolite, the pore pressure in the rock is assumed to be negligible (Brudy *et al.*, 1997). Thus, in this series of tests samples were initially dry, but the confining-pressure fluid (kerosene), which applies σ_3, was in direct contact with two opposing specimen faces that were left unjacketed. The results of this series of tests were substantially different from those obtained in the jacketed specimens (Haimson and Chang, 2002).

Since KTB amphibolite is a nearly impermeable, practically no confining pressure fluid penetrated the rock prior to the opening of microcracks. However, fluid infiltration upon dilatancy onset completely changed the failure mode. Brittle fracture occurred at or soon after dilatancy onset, roughly at 50–60% of σ_1 at failure under jacketed conditions, and resulted from the development of densely spaced through-going extensile cracks adjacent and subparallel to one of the unjacketed faces (Fig. 17). Confining fluid apparently intruded newly opened microcracks upon dilatancy onset, and facilitated their extension into long tensile fractures subparallel to σ_1–σ_2 plane. For any given least principal stress, the compressive strength typically increased with the rise in the intermediate principal stress, but overall it maintained its much lower magnitude than that of jacketed dry samples of the same rock (Fig. 18). The true triaxial strength criterion of amphibolite under borehole wall conditions is not well represented by the criterion

Figure 14
SEM micrograph of a failed KTB amphibolite, showing the microcrack localization that preceded the fault, which is steeply inclined and dips in the σ_3 (acting laterally in this image) direction. Scattered grain debris within the fault is inferred as a result of shear slip.

given in equation (12), but can be expressed as a linear relationship between the octahedral shear stress and the octahedral normal stress at failure (equation 8; Fig. 19). The discrepancy can be explained based on the fact that failure occurs immediately upon dilatancy onset, which is approximately equal to the yield point in this brittle rock and which occurs throughout the specimen and not just along the fracture plane (MOGI, 1971).

Using this criterion together with all the other known data from the KTB hole (BRUDY *et al.*, 1997), we computed the estimated magnitude of the maximum horizontal *in situ* stress there (HAIMSON and CHANG, 2002). Since the dimensions of the laboratory specimens were close to those of the breakouts in the KTB hole, scale effect was considered negligible. Our results (Fig. 20) show that σ_H increases steadily

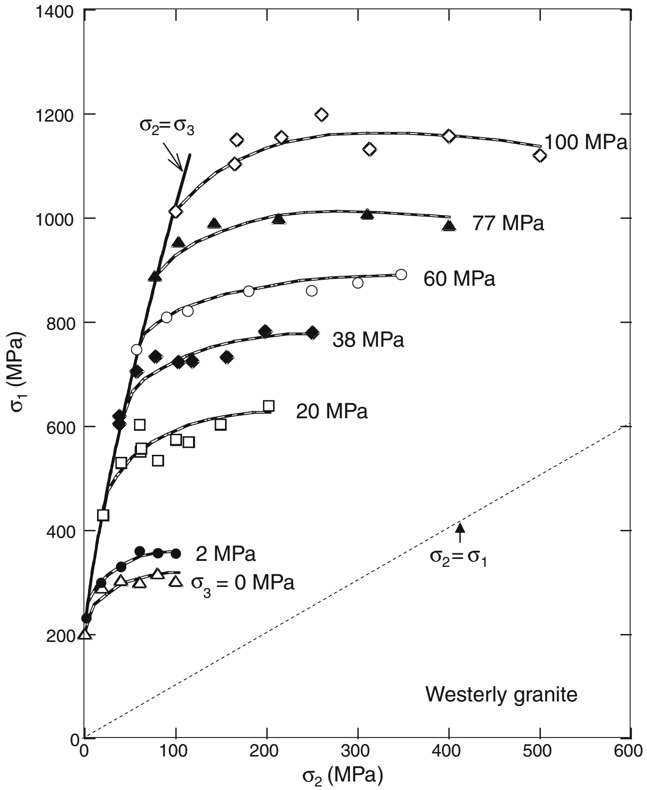

Figure 15
True triaxial test results in Westerly granite, plotted as the peak σ_1 versus σ_2 for several constant σ_3 magnitudes (after HAIMSON and CHANG, 2000).

with depth, within a relatively narrow band of uncertainty, confirming previous assessments of a strike-slip stress regime by BRUDY *et al.* (1997).

Long Valley Hornfels and Metapelite

More recent experiments have shown that the true triaxial mechanical behavior described above cannot be generalized to all brittle rocks. True triaxial strength experiments in two ultra fine-grained brittle rocks, hornfels and metapelite, which together are the major constituent of the Long Valley (California, U.S.A.) caldera basement in the 2000–3000 m depth range, exhibit a behavior unlike that previously observed in other crystalline and clastic rocks under similar testing conditions (CHANG and HAIMSON, 2005). For a given magnitude of σ_3, compressive strength σ_1 does not vary significantly in either hornfels or metapelite, regardless of the applied σ_2, suggesting little or no intermediate principal stress effect on brittle fracture (Fig. 21).

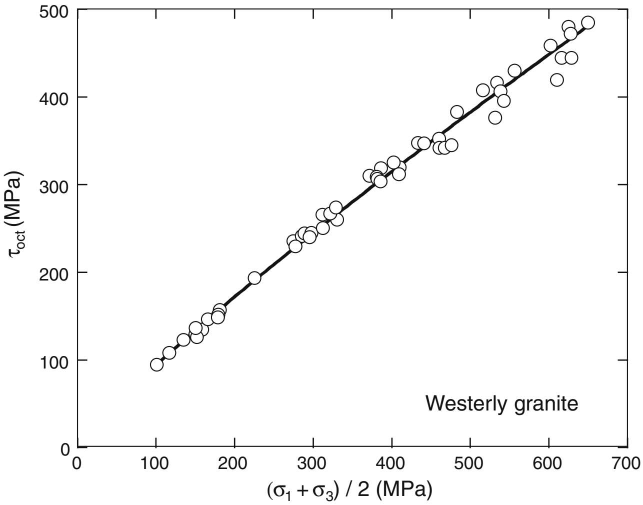

Figure 16
True triaxial strength criterion for Westerly granite, resulting from plotting all the experimental results shown in Figure 15 as τ_{oct} versus $(\sigma_1 + \sigma_3)/2$ (after HAIMSON and CHANG, 2000).

Measured σ_1 versus volumetric strain is linear almost to the point of rock failure, suggesting the absence of dilatancy. This unique mode of compressive failure that is not preceded by dilatancy was not previously detected in true triaxial tests. SEM inspection of failed specimens corroborates the observed nondilatant deformation by failing to reveal microcrack development prior to the emergence of the through-going steeply dipping shear failure plane (Fig. 22). This recent study implies that the commonly accepted mechanism of growth, localization, and coalescence of stress-induced extensile microcracks preceding brittle failure is not universally applicable to all hard rocks. It also indirectly confirms that the onset of microcracking is responsible for both dilatancy and the strengthening effect of the intermediate principal stress.

The mechanism that culminates in compressive failure of the hornfels and metapelite is not clearly understood. One attractive speculation is that minute shear microcracks, which have been found to induce brittle fracture in a serpentinite and a syenite without discernible volumetric increase (ESCARTIN *et al.*, 1997; KATZ and RECHES, 2000) are responsible for the nondilatant deformation and σ_2-independent compressive strength of the Long Valley rocks. However, no such discontinuities were detected in our microscopic study.

Summary, Conclusions and Future Work

In this paper I attempted to demonstrate the ever increasing evidence accumulated over the last century of the effect of the intermediate principal stress on brittle

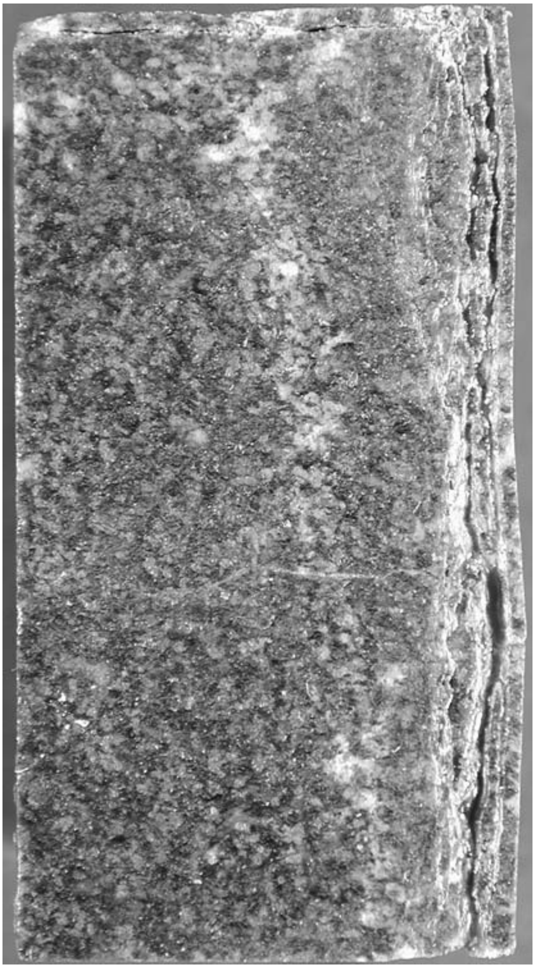

Figure 17
Cross section of failed unjacketed KTB amphibolite specimen along a $\sigma_1 - \sigma_3$ plane, showing a cluster of through-going extensile cracks parallel to the unjacketed face subjected to σ_3. In this figure σ_1 acts vertically.

fracture and failure criteria of rock. The Coulomb, Mohr, and Griffith criteria have been universally used to depict brittle failure based only on knowledge of the least and largest principal stresses. This approach enables the use of uncomplicated conventional triaxial cells to yield expressions for failure criteria. However, the intermediate principal stress effect on strength and deformability has proven over and over to be substantial. Strength differences of 10 to 50% have been observed in most rocks tested thus far, with the exception of the ultra fine-grained Long Valley hornfels and metapelite. In addition, the angle of fracture orientation with respect to

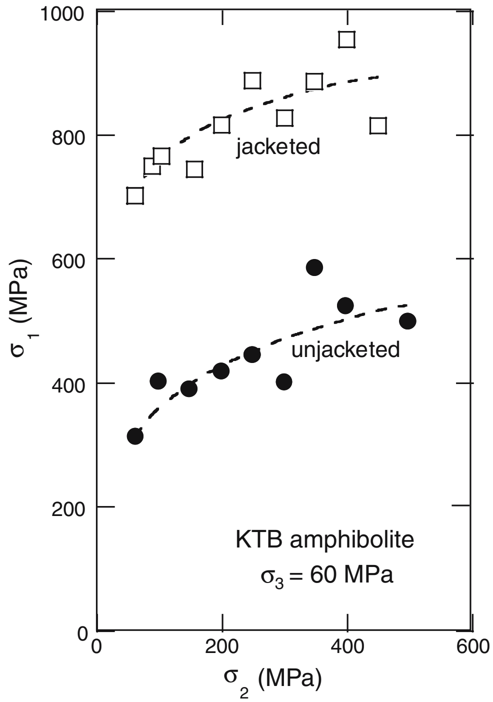

Figure 18
Compressive strength of unjacketed KTB amphibolite (blocked circles) as a function of σ_2 for a constant σ_3 (60 MPa). For comparison the strength of the same rock and for the same σ_3 under jacketed conditions is also plotted (open squares). The increase in strength with σ_2 is similar, but the unjacketed rock is about half as strong as the jacketed.

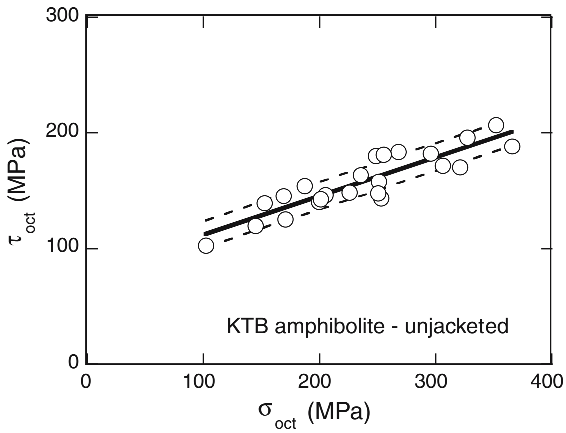

Figure 19
True triaxial strength criterion of unjacketed KTB amphibolite is best described as a linear relationship between the octahedral shear stress τ_{oct} and the octahedral normal stress σ_{oct}.

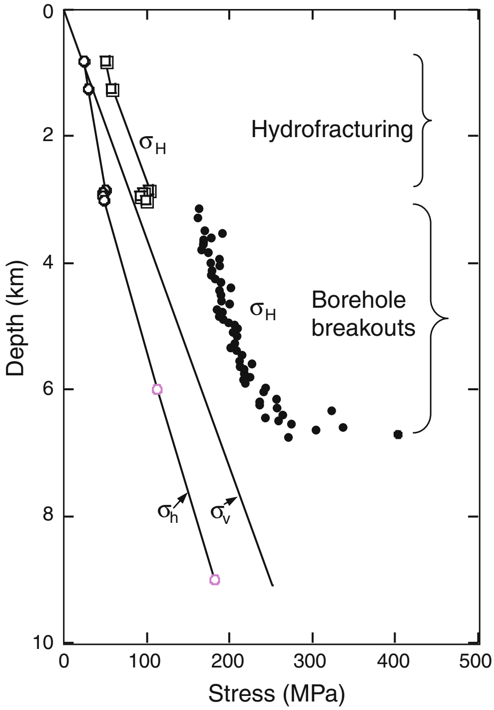

Figure 20
The estimated state of stress around the KTB ultra deep hole. In the 3 to 7 km range hydraulic fracturing yielded only an estimate of the least horizontal stress σ_h (see open circle at 6.0 km depth). The vertical stress σ_v was assumed to equal the weight of the overlying rock (BRUDY *et al.*, 1997). The major horizontal principal stress σ_H was computed from knowledge of the logged breakout dimensions and the true triaxial strength criterion of unjacketed amphibolite (HAIMSON and CHANG, 2002).

the directions of the principal stresses has been found to depend not only on σ_3, but also on σ_2. Also, the onset of dilatancy was shown to increase sharply with the rise in σ_2 in tests run under a constant σ_3.

Employing Coulomb, Mohr or other criteria derived from them (such as the 'Mohr-Coulomb' criterion) for assessing rock strength yields only the lower limit of this parameter, appropriate when σ_2 is equal or only slightly larger than σ_3. For larger intermediate principal stresses a true triaxial strength criterion is recommended, based on true triaxial tests conducted on rectangular prismatic specimens. Several examples are scattered in the literature in which it is reported that using the conventional strength criterion (equation 3) in some specific applications leads to nonsensical results. VERNIK and ZOBACK (1992) found that use of the Mohr-Coulomb criterion in relating borehole breakout dimensions to the prevailing *in situ* stress conditions in crystalline rocks did not provide realistic results. They suggested the use of a more general criterion that accounts for the effect of the intermediate

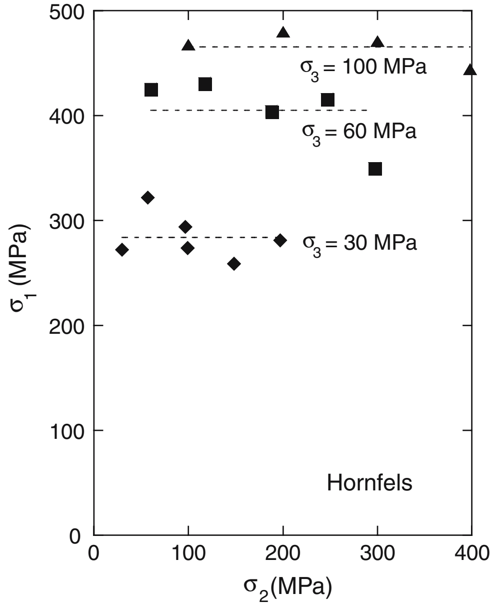

Figure 21
True triaxial strength σ_1 as a function of σ_2 for three separate magnitudes of σ_3 in Long Valley Caldera hornfels extracted from a depth of 2,250 m. The relationship is best represented by a constant (CHANG and HAIMSON, 2005).

principal stress on strength. EWY (1998) reported that for the purpose of calculating the critical mud weight necessary to maintain wellbore stability, the Mohr-Coulomb criterion is too conservative because it neglects the strengthening effect of the intermediate principal stress. Ewy considered using the DRUCKER and PRAGER (1952) criterion or a modified Lade criterion (LADE and DUNCAN, 1973) to take into account the σ_2 effect.

The importance of incorporating all principal stresses in determining experimentally the mechanical behavior of rocks has contributed to our present involvement in conducting true triaxial tests on extracted core from two major site investigations, the San Andreas Fault Observatory as Depth (SAFOD), California, U.S.A., and the Taiwan Chelungpu-fault Drilling Project (TCDP), both programs being under the auspices of the International Continental Scientific Drilling Program (ICDP).

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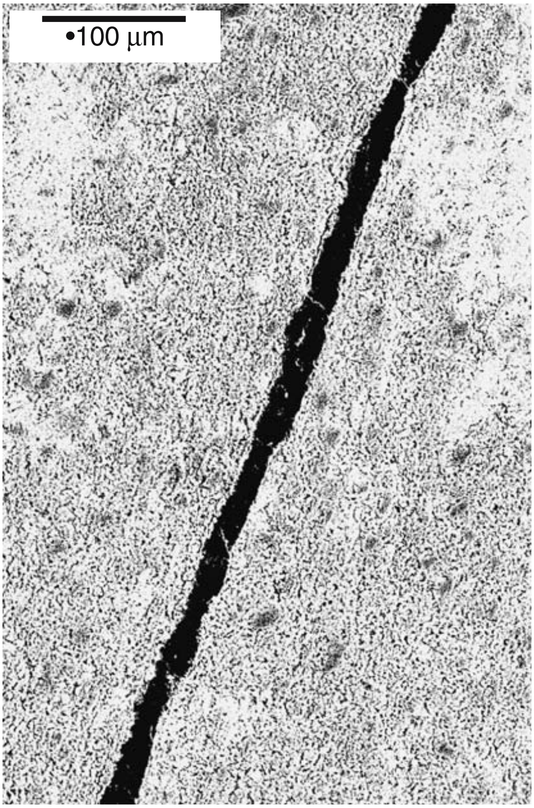

Figure 22
SEM micrograph of a cross section of a failed Long Valley hornfels specimen along a $\sigma_1 - \sigma_3$ plane, showing a steeply inclined shear fault, but lacking any visible microcracks. (Compare with the KTB amphibolite, Fig. 14.) The lack of microcracks is inferred as the reason for the nondilatancy in hornfels. Long Valley metapelite failure mechanism is identical to that of the hornfels (CHANG and HAIMSON, 2005).

EAR-0346141. C. Chang carried out all the reported University of Wisconsin experimental work. I. Song and R. Sandberg assisted in the design of the University of Wisconsin true triaxial cell.

REFERENCES

ATKINSON, R.H., and KO, H.Y. (1973), *A fluid cushion, multiaxial cell for testing cubical rock specimens*, Intl. J. Rock Mech. and Mining Sci. *10,* 351–361.

BÖKER, R. (1915), *Die Mechanik der bleibenden Formanderung in kristallinisch aufgebauten Körpern*, Verhandl. Deut. Ingr. Mitt. Forsch. *175*, 1–51.

BRACE, W.F., PAULING, B.W., and SCHOLZ, C.H. (1966), *Dilatancy in the fracture of crystalline rocks*, J. Geophys. Res. *71*, 3939–3953.

BRESLER, B. and PISTER, K.S. (1957), *failure of plane concrete under combined stresses*, Trans. Am. Soc. Civ. Engrs. *122*, 1049–1068.

BRUDY, M., ZOBACK, M.D., FUCHS, K., RUMMEL, F., and BAUMGÄRTNER, J. (1997), *Estimation of the complete stress tensor to 8 km depth in the KTB scientific drill holes: implications for crustal strength*, J. Geophys. Res. *102*, 18,453–18,475.

CHANG, C. and HAIMSON, B.C. (2000), T*rue triaxial strength and deformability of the KTB deep hole amphibolite*, J. Geophys. Res. *105*, 18,999–19,014.

CHANG, C. and HAIMSON, B.C. (2005), N*ondilatant deformation and failure mechanism in two long valley caldera rocks under true triaxial compression*, Intl. J. Rock Mech. and Mining Sci. *42*, 402–414.

CRAWFORD, B.R., SMART, B.G.D., MAIN, I.G., and LIAKOPOULOU-MORRIS, F. (1995), S*trength characteristics and shear acoustic anisotropy of rock core subjected to true triaxial compression*. Int. J. Rock Mech. and Min. Sci. *32*, 189–200.

DESAI, C.S., JANARDAHONAN, R., and STURE, S. (1982), *High capacity multiaxial testing device*, Geotech. Testing J. *5*, 26–33.

DRUCKER, D.C., and PRAGER, W. (1952), *Soil mechanics and plastic analysis or limit design,* Quart. Appl. Math. *10*, 157–165.

EMMERMANN, R., and LAUTERJUNG, J. (1997), *The german continental deep drilling program KTB: Overview and major results*, J. Geophys. Res. *102*, 18,179–18,201.

ESCARTIN, J., HIRTH, G., and EVANS, B. (1997), *Nondilatant brittle deformation of serpentinites: Implications for Mohr-Coulomb theory and the strength of faults*. J. Geophys. Res. *102*, 2897–2913.

EWY, R.T., *Wellbore stability predictions using a modified lade criterion*. In *Rock Mechanics in Petroleum Engineering*, vol. 1, Proc. Eurock 98 (Society of Petroleum Engineers, 1998), pp. 247–254..

FREUDENTHAL, A., *The inelastic behavior and failure of concrete*. In Proc. I (ASME, New York, 1951), pp. 641–646.

HAIMSON, B., and CHANG, C. (2000), *A new true triaxial cell for testing mechanical properties of rock, and its use to determine rock strength and deformability of westerly granite,* Int. J. Rock Mech. Min. Sci. *37*, 285–296.

HAIMSON, B., and CHANG, C. (2002), *True triaxial strength of the KTB amphibolite under borehole wall conditions and its use to estimate the maximum horizontal in situ stress*, J. Geophys. Res. *107(B10)*, ETG 15–1 to 14.

HANDIN, J., HEARD, H.C., and. MAGOUIRK, J.N. (1967), *Effect of the intermediate principal stress on the failure of limestone, dolomite, and glass at different temperature and strain rate*, J. Geophys. Res. *72*, 611–640.

JAEGER, J.C., and COOK, N.G.W., *Fundamentals of Rock Mechanics*, 3rd ed., (Chapman and Hall, London 1979) 593 pp.

JAEGER, J.C., and HOSKINS, E.R. (1966), *Rock failure under the combined Brazilian test*, J. Geophys. Res. *71*, 2651–2659.

KATZ, O. and RECHES, Z. (2000), *micro- and macro-structural analysis of small faults in a quartz-syenite intrusion: Faulting of a brittle rock without microcracking?* EOS Transactions, AGU *81*, F1121.

KO, H.Y. and SCOTT, R. F. (1967), *A new soil testing apparatus*, Geotechnique *17*, 40–57.

LADE, P.V. and DUNCAN, J.M. (1973), *Cubical triaxial tests on cohesionless soil,* J. Soil Mech. and Foundation Div., ASCE *99*, 793–812.

MICHELIS, P. (1985), *A true triaxial cell for low and high-pressure experiments*. Int. J. Rock Mech. Min. Sci. *22*, 183–188.

MOGI, K. (1966), *Some precise measurements of fracture strength of rocks under uniform compressive strength*, Rock Mech. Engin. Geology *4*, 51–55.

MOGI, K. (1967), *Effect of the intermediate principal stress on rock failure*, J. Geopys. Res. *72*, 5117–5131.

MOGI, K. (1971). *Fracture and flow of rocks under high triaxial compression*, J. Geophys. Res. *76*, 1255–1269.

MURRELL, S.A.F., *A criterion for brittle fracture of rocks and concrete under triaxial stress, and the effect of pore pressure on the criterion*. In Proc. *Fifth Symp. on. Rock Mechanics*, (Pergamon Press, 1963), pp. 563–577.

NADAI, A., *Theory of Flow and Fracture of Solids*, vol. 1 (McGraw-Hill, New York, 1950).

SMART, B.G.D. (1995), *A true triaxial cell for testing cylindrical rock specimens,* Int. J. Rock Mech. and Min. Sci. *32*, 269–275.

STURE, S., and DESAI, C.S. (1979), *Fluid cushion truly triaxial or multiaxial testing device*, Geotech. Testing J. *2*, 20–33.

TAKAHASHI, M., and KOIDE, H., *Effect of the intermediate principal stress on strength and deformation behavior of sedimentary rocks at the depth shallower than 2000 m,* In *Rock at Great Depth* (eds. V. Maury and D. Fourmaintraux) (Balkema, Rotterdam, 1989), pp. 19–26.

VERNIK, L., and ZOBACK, M.D. (1992), *Estimation of maximum horizontal principal stress magnitude from stress-induced well bore breakouts in the cajon pass scientific research borehole*, J. Geophys. Res. *97*, 5109–5119.

VON KARMAN, T. (1911), *Festigkeitsversuche unter all seitigem Druck*, Z. Verein Deut. Ingr. *55*, 1749–1759.

WAWERSIK, W.R., CARLSON, L.W., HOLCOMB, D.J., and WILLIAMS, R.J. (1997), *New method for true-triaxial rock testing*, Int. J. Rock Mech. and Min. Sci. *34*, Paper no. 330.

WIEBOLS, G.A. and COOK, N.G.W. (1968), *An energy criterion for the strength of rock in polyaxial compression*, Int. J. Rock Mech. Min. Sci. *5*, 529–549.

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