Pure appl. geophys. 163 (2006) 1091–1100
0033–4553/06/061091–10
DOI 10.1007/s00024-006-0056-8

Pure and Applied Geophysics

A Method for Testing Dynamic Tensile Strength and Elastic Modulus of Rock Materials Using SHPB

Q. Z. Wang,[1] W. Li,[1] and X. L. Song[1]

Abstract—An experimental procedure for testing dynamic tensile strength and elastic modulus of rock materials at high strain rate loading is presented in this paper. In our test the split Hopkinson pressure bar (SHPB) was used to diametrally impact the Brazilian disc (BD) and flattened Brazilian disc (FBD) specimens of marble. A tensile strain rate of about 45 1/s was achieved at the center of the specimen. In order to improve the accuracy of the analysis, the initiation time difference between the strain waves acting on the two flat ends of the FBD specimen was treated properly. Typical failure modes corresponding to different loading conditions were observed. It was verified with a finite-element simulation that the equilibrium condition was established in the specimen before its failure. This numerical simulation validates the experimental procedure and also proves the suitability of formulation for the basic equations.

Key words: Flattened Brazilian disc, dynamic splitting test, dynamic tensile strength, dynamic elastic modulus, split Hopkinson pressure bar (SHPB), rock.

1. Introduction

Drilling, excavation, blasting and crashing of rocks are often encountered in civil and mining engineering, where dynamic tensile strength and elastic modulus of rock materials are needed, however, the research on this topic is relatively limited. Dynamic tension test at high strain rate is very difficult to conduct for rocks, since "in most cases, existing experimental techniques will not allow for reaching the maximum strain rate of $\sim$ 30 1/s" (Brara and Klepaczko 2004). The international Society for Rock Mechanics (ISRM) issued the suggested method for determining tensile strength of rock materials by using the Brazilian disc (BD) specimen (ISRM, 1978). This splitting test method has been widely used in static indirect tension for rocks, concrete and many other materials. Wang and Xing (1999) proposed introducing two parallel flat ends to the disc specimen for loading, in this way a BD becomes the flattened Brazilian disc (FBD). The FBD is favorable for reducing stress concentration effect, thus ensuring crack initiation from the center of the disc instead of from the loading

[1]Department of Civil Engineering and Applied Mechanics, Sichuan University, Chengdu, Sichuan 610065, China. E-mail: qzwang2004@163.com

point. RODRIGUEZ *et al.* (1994) extended the static Brazilian test into dynamic testing of ceramics using the split Hopkinson pressure bar (SHPB) setup.

In this paper, dynamic splitting test for dynamic tensile strength and elastic modulus of marble using BD and FBD specimens is reported, also presented are basic equations for analysis and numerical simulation. The dynamic load can be calculated with the dynamic strain recordings on the incident bar and transmission bar. RODRIGUEZ *et al.* (1994) pointed out: "A complete assessment of the splitting tests as a good method of determining the dynamic tensile strength should consider at least three critical aspects: the elastic behavior assumption, the time evolution of the stress distribution in the specimen and the failure pattern." These aspects are observed in our test and analysis, special attention is paid to the nonuniformity of stress waves in the specimen. A finite element simulation for the dynamic stress evolution in the specimen was performed to testify the validity of the experimental procedure.

2. Experimental Procedure

2.1 Specimens, SHPB Setup and Basic Formulas

The SHPB setup with the BD or FBD specimen is shown in Figure 1, the specimen is subjected to diametral impact. The diameter of the bars is 100 mm. In order to reduce the high frequency fluctuation, a soft media acting as wave shaper is glued to the left end of the incident bar (Fig. 1), the wave shaper filters out the high-frequency oscillation and makes the rising front of the pulse exerted on the specimen less steep, which facilitates to satisfying the equilibrium condition before specimen failure occurs. The stress wave in the specimen could be uniform after some time, as demonstrated by both the experimental recordings and the numerical simulation.

According to the theory of elasticity (TIMOSHENKO and GOODIER *et al.*, 1970), under static diametral compression of BD, the principal stress conditions on the loading diameter (Fig.2, excluding the near region of load application) are as follows:

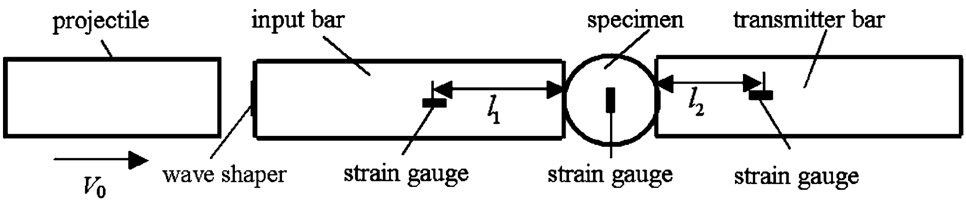


Figure 1
The SHPB with specimen under diametral compression.

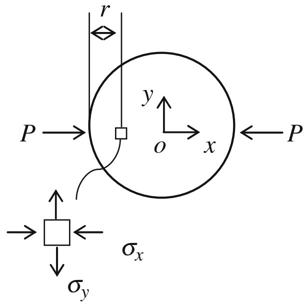


Figure 2
BD with point load.

$$\sigma_x = \frac{2P}{\pi DB}\frac{D^2}{r(D-r)}, \tag{1}$$

$$\sigma_y = -\frac{2P}{\pi DB}. \tag{2}$$

Compression is considered positive in this paper, so the minus sign belongs to tensile stress and tensile strain. D is the diameter, and B is the thickness of the specimen, r is the distance measured from the loading point. The tensile strength measured with the BD specimen can be calculated from eq. (2) using the maximum load recorded in the test.

BD is adapted into FBD as shown in Figure 3 (WANG and XING 1999), thus the original stress concentration on the loading point can be reduced, also crack initiation at the center of the disc can be more likely guaranteed. However, the stress distribution on the loading diameter of FBD cannot be described with eq. (1) and eq. (2), so that WANG *et al.* (2004) did the calibration for FBD with the loading angle $2\alpha = 20°$ and obtained the formulas for σ_x and σ_y on the center of FBD, respectively. Furthermore the formula for the tensile strength σ_t based on the Griffith strength criterion was derived. These three formulas are listed below:

$$\sigma_x = 2.973\frac{2P}{\pi DB}, \quad \sigma_y = -0.964\frac{2P}{\pi DB} \tag{3}$$

$$\sigma_t = 0.95\frac{2P}{\pi DB}. \tag{4}$$

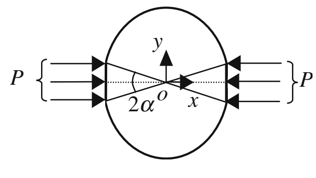


Figure 3
FBD with distributed load.

For the dynamic splitting test using SHPB, the following formula for calculating the load is based on recorded dynamic strains on the bars. It is assumed that the equilibrium condition is satisfied before the specimen breaks, so that $\varepsilon_i(t) + \varepsilon_r(t) = \varepsilon_t(t)$. The average value of the two pressures P_1 and P_2, where P_1 corresponds to $\varepsilon_i(t) + \varepsilon_r(t)$, and P_2 to $\varepsilon_t(t)$, exerted on the two sides of the specimen, is taken as the load (P) applied to the specimen, thus the average load is as follows (TEDESCO *et al.*, 1989; HUGHES *et al.*, 1993):

$$P = \frac{P_1 + P_2}{2} = \frac{E_0 A_0}{2}(\varepsilon_i(t) + \varepsilon_r(t) + \varepsilon_t^*(t)), \tag{5}$$

where A_0 and E_0 are the cross-sectional area and elastic modulus of the bar, respectively, $\varepsilon_i(t)$, $\varepsilon_r(t)$, $\varepsilon_t^*(t)$ are the incident strain, reflected strain and adapted transmitted strain on the bars, respectively. $\varepsilon_t^*(t)$ is adapted from the $\varepsilon_t(t)$ in order to reduce the effect of nonuniformity during loading. The adaptation is a shift of the transmitted strain-wave form along the time axis for reasonable superposition (ZHOU *et al.*, 1992). As the initiation time for the transmitted wave on the right end of the specimen is different from that of the incident wave and reflected wave on the left end of the specimen, it takes a time interval of τ_0 for the elastic wave to travel through the specimen, so that we have $\varepsilon_t^*(t) = \varepsilon_t(t + \tau_0)$.

For the determination of the dynamic elastic modulus, noting that the stress condition in the specimen is two-dimensional, and compressive normal stress is taken as positive, then we have (SU and WANG, 2004):

$$E = \frac{\sigma_y(t) - \mu\sigma_x(t)}{\varepsilon_y(t)}. \tag{6}$$

Thus, from eqs. (3) and (6) and taking Poisson's ratio μ as 0.3, it is easy to see that the elastic modulus can be calculated using the average load $P(t)$ from eq. (5) and the tensile strain $\varepsilon_y(t)$ on the center of the specimen as follows

$$E(t) = -1.856\frac{2P(t)}{\pi D B \varepsilon_y(t)}. \tag{7}$$

2.2 Experimental Records

A white marble taken from Ya'an, Sichuan province of China was tested; its Poisson's ratio is 0.3, Young's modulus is 16 GPa, and density is 2527 kg/m^3. 15 BDs and 5 FBDs with $2\alpha = 20°$ (see Fig. 3) were prepared. The disc diameter is 75 mm, and the thickness is 30 mm. The diameter of SHPB is 100 mm, the incident bar length 450 cm, the output bar length 250 cm, their elastic modulus 210 GPa, Poisson's ratio 0.25~0.30, the density 7850 kg/m^3, and the elastic wave speed is 5172 m/s. The length of the projectile is 500 mm. The wave shape of the dynamic load can be adjusted by changing the speed of the projectile punching the incident

Table 1

Specimen specifications, loading conditions and wave forms

Spec. No.	Diam. /mm	Thickness /mm	Flat ends	Gas gun pressure /MPa	Projectile speed /m/s	Incident rising time/μs	Incident height /$\mu\varepsilon$	Transmitted height /$\mu\varepsilon$
1	74.53	30.52	no	0.125	2.01	184	312.10	39.05
2	75.40	30.35	no	0.125	2.07	188	321.80	36.60
3	75.65	29.85	yes	0.4	6.83	84	1302.20	62.15
4	75.42	31.30	yes	1.0	12.47	88	2009.50	147.25
5	75.52	29.70	yes	0.2	3.75	372	469.85	56.05

bar and also by modifying the configuration of the projectile. Strain gauges were stuck at the center of two sides of the disc respectively, and the average of the two measured values was taken for analysis. Also strain gauges were put at $l_1 = 149.5$ cm of the incident bar, and $l_2 = 20$ cm of the transmission bar, as shown in Figure 1, the average values at the two lateral sides were taken for the needed strain in order to avoid the bending effect of the bars. Typical data of our test are presented in Table 1, where the first three columns are specifications for the five representative specimens, the following two columns are loading conditions, and the last three columns correspond to characteristics for wave forms.

3. Analysis of Experimental results

The corresponding failure mode of the specimen is shown in Figure 4. The impact speed for specimen No. 4 is so high that it broke into three pieces. The relatively ideal failure mode (two equal halves divided along a loading diameter) belongs to No. 3 and No. 5, they are all FBDs. Taking specimen No. 5 for example, its incident, reflected and transmitted wave forms are shown in Figure 5. The signal recorded on the transmitted bar is small and is also interfered by the noise as shown in Figure 5(b). From Figures 5(a) and 5(b) it can be seen that the transmitted strain signal is only about 1/10 of that on the incident bar.

Figure 6(a) shows the tensile strain record on the center of specimen No. 5. The lower horizontal line was caused by the signal exceeding the range of the instrument, as the maximum strain is too large, however we did not use this maximum strain, we chose the steepest point on the curve as the critical point using a differentiation operation on the curve, done with the software Origin. Figure 6(b) is a comparison of the strain signal on the two ends of the specimen, which shows that the signal on the right end lags behind that on the left end with a time lag of 97.5 μs. The two waves are not of the same shape, implying that nonuniformity exists both in time and space, which should be handled properly (ZHOU *et al.*, 1992). The adaptation of the transmitted strain wave in eq. (5) is based on this consideration.

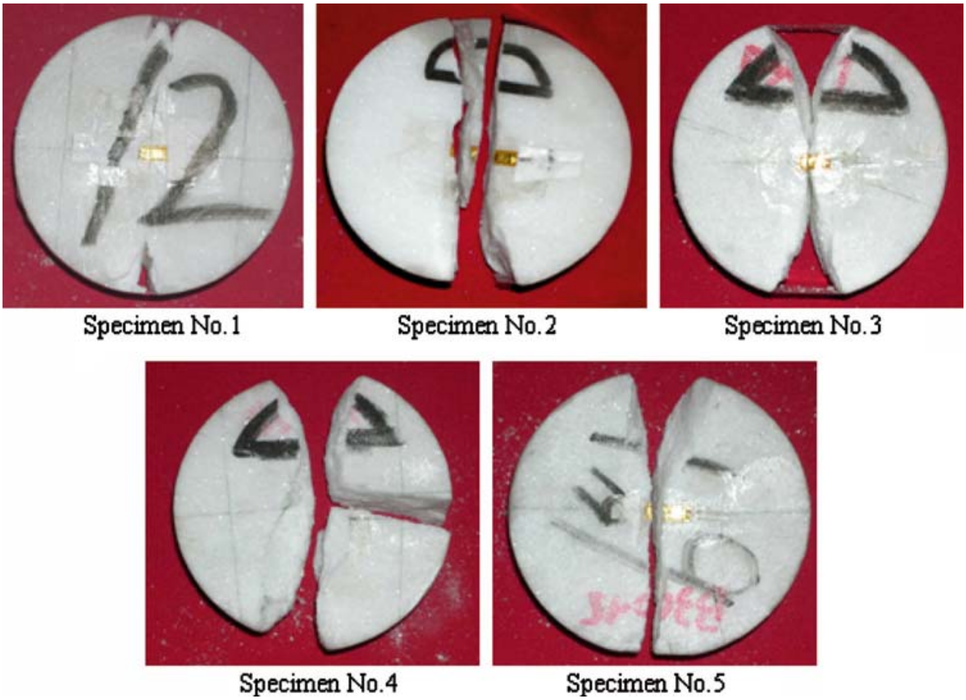


Figure 4
Failure modes of five representative specimens.

SU and WANG (2004) used an Instron 1342 type servohydraulic testing machine to test marble with a strain rate of 0.11 1/s. They derived a tensile strength of 4.47 MPa and an elastic modulus of 20.9 GPa. Now using SHPB, the strain rate for specimen No. 5 is 44.86 1/s, which can be estimated with Figure 6 using the strain difference divided by the corresponding time interval. At such a high strain rate, the tensile strength is 20.55 MPa and the maximum elastic modulus is 114.2 Gpa. These quantities are 4.60 times and 5.46 times as large as their static values, respectively.

4. Finite Element Simulation

To prove the validity of the present experiment, the stress history in the specimen was analyzed using the finite element software ANSYS. The mesh consists of 519 plane 8-node elements and 1648 nodes in total. A ramp load with a peak value of 99.23 kN and a duration of 288 μs, was applied. These data were taken from experiments. Figures 7(b) to (f) shows the time evolution of the dynamic tensile stress σ_y in the specimen. The isolines of the tensile stress in FBD under dynamic split

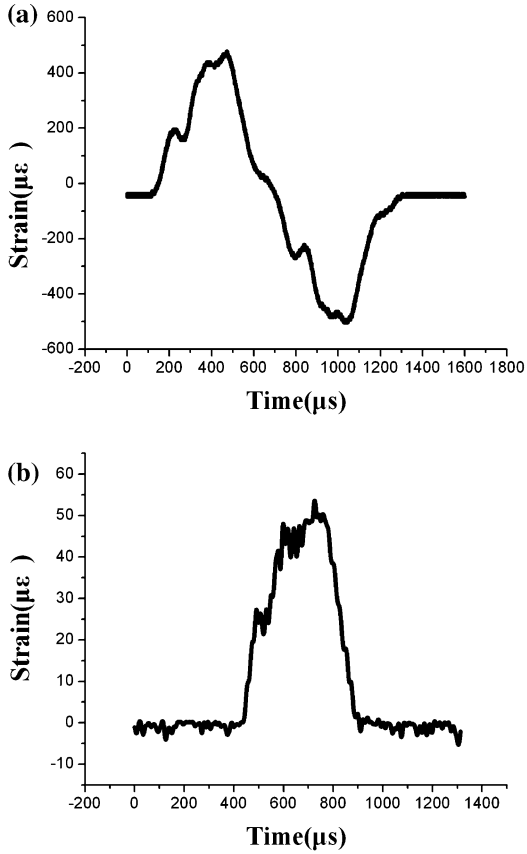


Figure 5
Wave forms on bars for testing specimen No. 5. (a) Incident and reflected waves. (b) Transmitted wave.

loading are given. Shown in Figure 7(a) is the counterpart static loading case, which is given as a comparison base. Figure 7(d) basically resembles Figure 7(a), and this identity proves that under dynamic loading and after a time interval of 75 μs the equilibrium is reached, which occurs before the specimen breaks. Figures 7(d), (e) and (f) show that after a time interval of 75 μs, σ_y distribution in the specimen is uniform, thereafter it does not change with time. For the corresponding BD specimen, about 150 μs is needed to arrive at such uniformity.

5. Conclusion

1. An experimental procedure for measuring dynamic tensile strength σ_t and elastic modulus E using SHPB and BD and FBD specimens is proposed, the basic

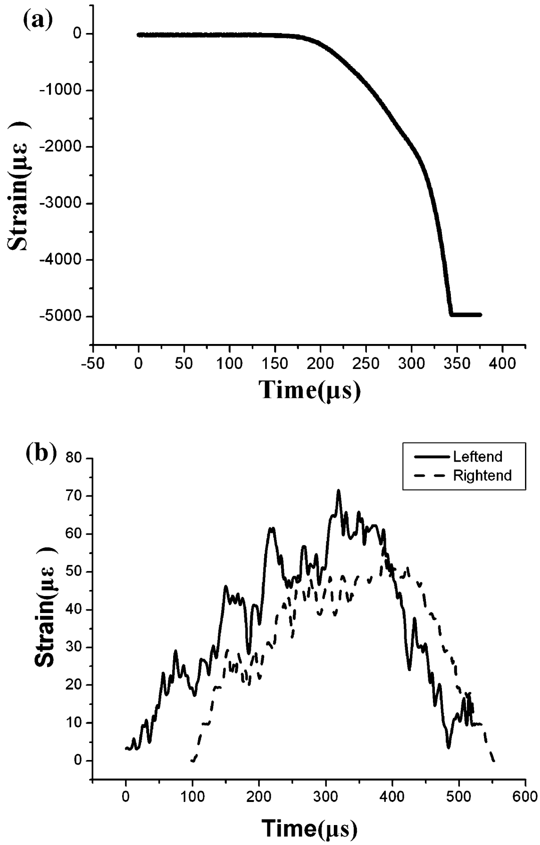
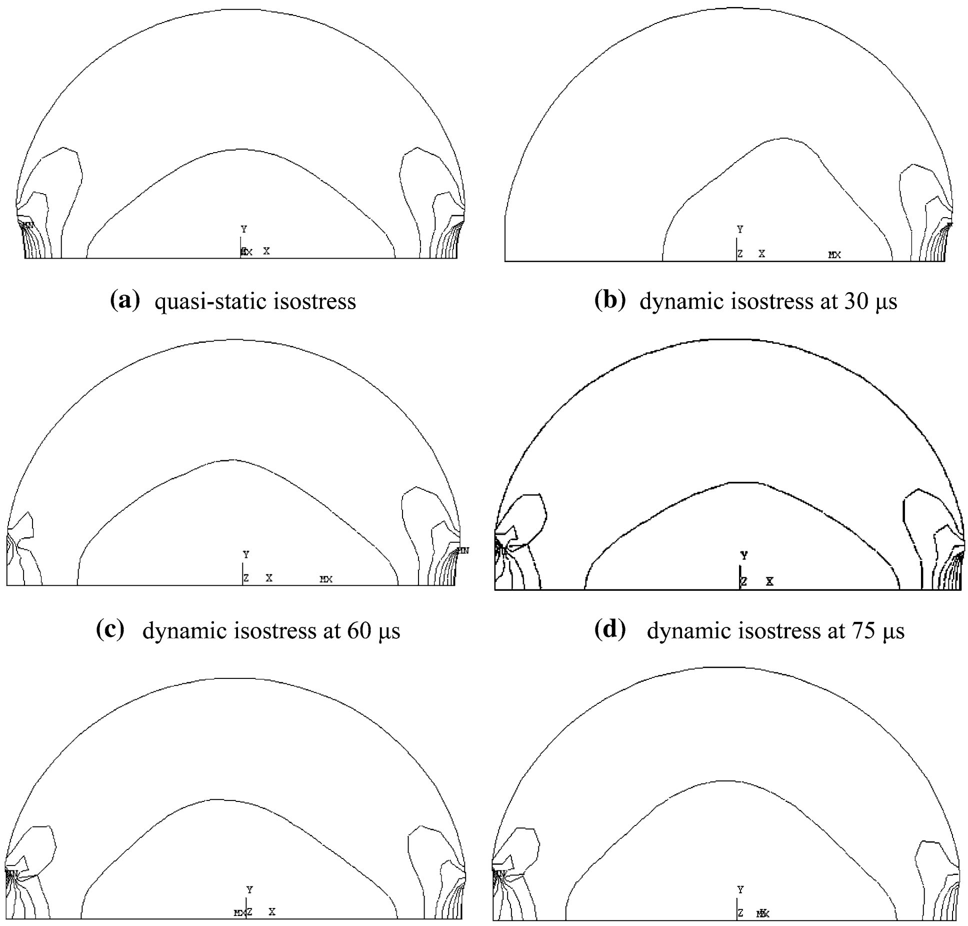


Figure 6
(a) ε_y strain at the specimen center. (b) Strains on the flat ends of the specimen.

formulas for σ_t and E are give in eqs. (4) and (7), respectively, while the average load is derived from dynamic strain recordings on the incident bar and transmission bar using eq. (5), the critical point is illustrated.

2. Dynamic strain recording and its analysis are key issues to the experimental procedure. The nonuniformity of strain on the two sides of the specimen, demonstrated in Figure 6(b), was considered for proper superposition of different stains. A method for the adaptation of the transmitted strain was used.
3. A tensile strain rate of almost 45 1/s was achieved at the center of the specimen as shown in Figure 6(a). At such high strain rate loading, the dynamic tensile strength and elastic modulus of a marble are several times higher than their static values.
4. It is proved that the equilibrium was established before the failure of the specimen occurred. This explains why the formulas for static loading can be used for



(a) quasi-static isostress

(b) dynamic isostress at 30 μs

(c) dynamic isostress at 60 μs

(d) dynamic isostress at 75 μs

(e) dynamic isostress at 90 μs

(f) dynamic isostress at 120 μs

Figure 7
Comparison of isostress contour for FBD under quasi-static and dynamic loading.

corresponding dynamic splitting test. The result of the numerical simulation shows that for reaching equilibrium in the specimen FBD needs about half the time of BD, hence FBD is superior to BD for dynamic splitting test.

Acknowledgement

This work was supported by the National Natural Science Foundation of China (Project No. 40172094). The authors are very grateful to Professor Ove Stephansson and another reviewer for their insightful comments.

References

BRARA, A. and KLEPACZKO, J.R. (2004), *Dynamic tensile behavior of concrete: Experiment and numerical analysis*, ACI Material J. *101*, 162–167.

GOMEZ, J.T., SHUKLA, A., and SHARMA, A. (2001), *Static and dynamic behavior of concrete and granite in tension with damage*, Theoretical and Appl. Fract. Mech. *36*, 37–49.

HUGHES, M.L., TEDESCO, J.W., and ROSS, C.A. (1993), *Numerical analysis of high strain rate splitting-tensile tests*, Computers and Structures *47*, 653–671.

ISRM (1978), *Suggested methods for determining tensile strength of rock materials*, Int. J. Rock Mech. Min. Sci. and Geomech. Abstr. *15*, 99–103.

RODRIGUEZ, J., NAVARRO, C., and SANCHEZ-GALVEZ, V. (1994), *Splitting tests: An alternative to determine the dynamic tensile strength of ceramic materials*, Journal de Physique IV, *4*, c8-101-c8-106.

SU, B.J. and WANG, Q.Z. (2004), *Experimental study of flattened Brazilian disc specimen under dynamic loading*, Journal of Yangtze River Academy (in Chinese) *21*, 22–24.

TEDESCO, J.W., ROSS, C.A., and BRUNAIR, R.M. (1989), *Numerical analysis of dynamic split cylinder test*, Computers and Structures *32*, 609–624.

TIMOSHENKO, S.P. and GOODIER, J.N., *Theory of elasticity* (McGraw-Hill, New York 1970).

WANG, Q.Z. and XING, L. (1999), *Determination of fracture toughness K_{IC} by using the flattened brazilian disk specimen for rocks*, Eng. Fracture Mech. *64*, 193–201.

WANG, Q.Z., JIA, X.M., KOU, S.Q., ZHANG, Z.X., and LINDQVIST, P.-A. (2004), *The flattened Brazilian disc specimen used for determining elastic modulus, tensile strength and fracture toughness of brittle rocks: Theoretical and numerical results*, Int. J. Rock Mech. Min. Sci. *41*, 245–253.

ZHOU, F.H., WANG, L.L., and HU, S.S. (1992), *On the effect of stress nonuniformness in polymer specimen of SHPB test*, Experim. Mech. (in Chinese) *7*, 23–29.

(Received February 17, 2005, revised September 20, 2005, accepted October 26, 2005)
Published Online First: May 12, 2006