Data and Model-Driven Selection using Color Regions *

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Abstract. A key problem in model-based object recognition is selection, namely, the problem of determining which regions in an image are likely to come from a single object. In this paper we present an approach that uses color as a cue to perform selection either based solely on image-data (data-driven), or based on the knowledge of the color description of the model (model-driven). It presents a method of color specification by color categories which are used to design a fast segmentation algorithm to extract perceptual color regions. Data driven selection is then achieved by selecting salient color regions while model-driven selection is achieved by locating instances of the model in the image using the color region description of the model. The approach presented here tolerates some of the problems of occlusion, pose and illumination changes that make a model instance in an image appear different from its original description.

1 Introduction

A key problem in object recognition is selection, namely, the problem of isolating regions in an image that are likely to come from a single object. This isolation can be either based solely on image data (data-driven) or can incorporate the knowledge of the model (task-driven or model-driven). It has been shown that the search in the matching stage of recognition can be considerably reduced if recognition systems were equipped with a selection mechanism thus allowing the search to be focused on those matches that are more likely to lead to a correct solution [3]. Even though selection can be of help in recognition, it has largely remained unsolved. The lack of knowledge of illumination conditions and surface geometries of objects in the scene, and the problems of occlusion, shadowing, specularities, and interreflections in the image make it difficult to interpret groups of data features as belonging to a single object. Previous approaches to selection have focused on the problem of data-driven selection by grouping data features such as edges and lines based on constraints such as parallelism, or collinearity, distance and orientation, etc.[4][3]. But ensuring the reliability of such grouping has been found to be difficult, thus restricting their effectiveness in reducing the search complexity in recognition.

In this paper we present a way of performing data and model-driven selection by extracting color regions from an image. A color region almost always comes entirely from a single object, giving, therefore, more reliable groups than existing grouping methods and this can be useful for data-driven selection. Because objects tend to show color constancy under most illumination conditions, color when specified appropriately, can be a stable cue for most appearances of objects in scenes, thus making it also suitable for model-driven selection.

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2 Color Specification for Selection

Existing approaches to color have either tried to recover the surface color, i.e. the surface reflectance function, [6] [7] or the image color, i.e., the color of the objects as they appear under the present illumination conditions [5]. The recovery of surface color is known to be an underconstrained problem and the solutions usually make some assumptions either about the colored surface or the illumination conditions [6][7]. The image color, on the other hand, is a very unstable description, changing easily with illumination conditions. For the purposes of selection, therefore, we propose that the color of a region be specified by its perceived color. Using the perceptual color, two adjacent color regions would be distinguished if their perceived colors were different, and this is sufficient for data-driven selection. Because objects tend to obey color constancy under most changes in illumination, their perceived color remains more or less the same thus making it sufficient also for model-driven selection.

We now present a way of specifying the perceptual color of image regions. The color of pixels constituting color regions can be described by a triplet <R,G,B> (called specific color henceforth), representing the components of image intensity at that point along three wavelengths (usually red, green and blue as dominant wavelengths to correspond to the filters used in the color cameras). When all possible triples are mapped into a 3-dimensional color space with axes standing for the pure red, green and blue respectively, we get a color space that represents the entire spectrum of computer recordable colors. Such a color space, must therefore, be partitionable into subspaces where the color remains perceptually the same, and is distinctly different from that of neighboring subspaces. Such subspaces can be called perceptual color categories. Now the color of each pixel maps to a point in this color space, and hence will fall into one of the categories. The perceptual color of the pixel can, therefore, be specified by this color category. To get the perceived colors of regions, we note that although the individual pixels of an image color region may show considerable variation in their specific colors, the overall color of the region is fairly well-determined by the color of the majority of pixels (called dominant color henceforth). Therefore, the perceived color of a region can be specified by the color category corresponding to the dominant color in the region.

The category-based specification of perceptual color (of pixels or regions) remains fairly stable under changes in illumination conditions and as we show next, can be used to give a reliable segmentation of the scene. In addition, since the perceptual categories depend on the color space and are independent of the image, they can be found in advance and stored. Finally, a category-based description is in keeping with the idea of perceptual categorization that has been explored extensively through psychophysical studies [8].

To find the perceptual color categories, we performed some rather informal but extensive psychophysical experiments that systematically examined a color space and recorded the places where qualitative color changes occur, thus determining the number of distinct color categories that can be perceived. The details of these experiments will be skipped here except to mention the following. The entire spectrum of computer recordable colors (2²⁴ colors) was quantized into 7200 bins corresponding to a 5 degree resolution in hue, and 10 levels of quantization of saturation and intensity values and the color in each such bin was then observed to generate the categories. From our studies, we found about 220 different color categories were sufficient to describe the color space. The color category information was then summarized in a color-look-up table. Similarly, the categories that can be grouped to give an even rougher description of a particular hue were found and stored in a category-look-up table to be indexed using the color categories given by the color-look-up table.

3 Color Region Segmentation

The previous section described how to specify the color of regions, after they have been isolated. But the more crucial problem is to identify these regions. If each surface in the scene were a mondrian, then all its pixels would belong to single color category, so that by grouping spatially close pixels belonging to a category, the desired segmentation of the image can be obtained. But even for real surfaces, an analysis assuming a single light source and the separability of surface reflectance has shown that the color variations over a surface are mostly in intensity [2]. In practice, even when these assumptions are not satisfied, the general observation is that the intensity and purity of colors get affected but the hue still remains fairly constant. In terms of categories, this means that different pixels in a surface belong to compatible categories, i.e. have the same overall hue but vary in intensity and saturation. Conversely, if we group pixels belonging to a single category, then each physical surface is spanned by multiple overlapping regions belonging to such compatible color categories. These were the categories that were grouped in the category-look-up-table mentioned earlier.

The algorithm for color image segmentation performs the following steps. (1) First, it maps all pixels to their categories in color space. (2) It then groups pixels belonging to the same category, (3) and finally merges overlapping regions in the image that are of compatible color categories. The grouping is done by dividing the image into small-sized bins and running a connected component algorithm to assemble the groups in linear time. Similarly, the overlapping regions of compatible color categories are found and merged by using the bin-wise representation of the image, also in linear time.

Figure 1 demonstrates the color region segmentation algorithm. Figure 1a shows a 256 x 256 pixel-size image of a color pattern on a plastic bag. The result of step-2 of the algorithm is shown in Figure 1b, and there it can be seen that the glossy portions on the big blue Y and the red S cause overlapping color regions. These are merged in step 3 and the result is shown in Figure 1c. Similarly, Figure 2 shows another example of color region segmentation using the algorithm on an image of a realistic indoor scene.

4 Color-based Data-driven Selection

We now present an approach to data-driven selection using color regions. The segmentation algorithm described above gives a large number of color regions, some of which may span more than one object, while others may come from the scene clutter rather then objects of interest in the scene. It would be useful for the purposes of recognition, therefore, to order and consider only some salient color regions. This is based on the observation that an object stands out in a scene because of some salient features (such as, say, color) that are usually localized to some portion of the object. Therefore isolating salient regions is more likely to point to a single object and hence to a more reliable grouping strategy. The next section describes how such salient color regions can be found.

4.1 Finding Salient Color Regions in Images

In finding salient color regions, we focus on the sensory components of their distinctiveness and propose that the saliency be a linear combination of two components, namely, self-saliency and relative saliency. Self-saliency determines how conspicuous a region is on its own and measures some intrinsic properties of the region, while relative saliency measures how distinctive the region appears when there are regions of competing distinctiveness in the neighborhood. To determine these components some region features were selected and weighting functions were designed to appropriately reflect sensory judgments of saliency. Specifically, the color of a region and its size were used as features for determining self-saliency and were measured as follows. The color was given

by (s(R),v(R)), where s(R) = saturation or purity of the color of region R, and v(R) = brightness, and $0 \le s(R),v(R) \le 1.0$. And the size is simply the normalized size given by r(R) = Size(R)/Image-size. Similarly, the color and size contrast were chosen as features for determining relative saliency. The color contrast measure chosen enhances a region R's contrast if it is surrounded by a region T of different hue and is given by c(R,T) below:

 $c(R,T) = \begin{cases} k_1 d(C_R, C_T) & \text{if R and T are of same hue} \\ k_2 + k_1 d(C_R, C_T) & \text{otherwise} \end{cases}$ (1)

where $k_1 = \frac{0.5}{\sqrt{2}}$ and $k_2 = 0.5$, so that $0 \le c(R,T) \le 1.0$, and $d(C_R,C_T)$ is the cie-distance between the two regions R and T with specific colors as $C_R = (r_0, g_0, b_0)^T$ and $C_T = (r, g, b)^T$ and is given by $d(C_R, C_T) = \sqrt{(\frac{r_0}{r_0 + g_0 + b_0} - \frac{r}{r + g + b})^2 + (\frac{g_0}{r_0 + g_0 + b_0} - \frac{g}{r + g + b})^2}$. The size contrast is simply the relative size and is given by $d(R, T) = min\left(\frac{size(R)}{size(T)}, \frac{size(T)}{size(R)}\right)$. In both cases the neighboring region T is the rival neighbor that ranks highest when all neighbors are sorted first by size, then by extent of surround, and finally by contrast (size or color contrast as the case may be), and will be left implicit here.

The weighting functions for these features were chosen both from the point of data-driven selection and the extent to which they reflect our sensory judgments. Thus for example, the functions for weighting intrinsic color and color contrast, $f_1(s(R))$ and $f_2(v(R))$ and $f_3(c(R))$ were chosen to be linear $(f_1(s(R)) = 0.5s(R))$, and $f_2(v(R)) = 0.5v(R)$, and $f_4(c(R)) = c(R)$ respectively) to emphasize brighter and purer colors and higher contrast respectively. The size of a region is given a non-linear weight to deemphasize both very small and very large regions. Very small regions are usually spurious while very large regions tend to span more than one object, making both unsuitable for selection. The corresponding weighting function $f_3(r(R))$ was found by performing some informal psychophysical experiments and is given by

$$f_3(n) = \begin{cases} -\frac{\ln(1-n)}{c_1} & 0 \le n \le t_1\\ 1 - e^{-c_2 n} & t_1 < n \le t_2\\ s_2 - c_3 \ln(1-n+t_2) & t_2 < n \le t_3\\ s_3 e^{-c_4(n-t_3)} & t_3 < n \le t_4\\ 0 & t_4 < n < 1.0 \end{cases}$$
 (2)

where $t_1 = 0.1$, $t_2 = 0.4$, $t_3 = 0.5$, $t_4 = 0.75$, $s_1 = 0.8$, $s_2 = 1.0$, $s_3 = 0.7$, $s_4 = 10^{-3}$ and $c_1 = -\frac{\ln(1-t_1)}{s_1}$, $c_2 = -\frac{\ln(1-s_1)}{t_1}$, $c_3 = -\frac{(s_2-s_3)}{\ln(1+t_2-t_3)}$, $c_4 = -\frac{\ln\frac{t_4}{t_3}}{(t_4-t_3)}$ and n = size of region R = r(R). A function $f_5(t(R)) = 1 - e^{-12t(R)}$ for relative size was similarly designed. The color saliency of region R was obtained by combining all these features as

Color-saliency(R) =
$$f_1(s(R)) + f_2(v(R)) + f_3(r(R)) + f_4(c(R)) + f_5(t(R))$$
 (3)

Figure 1d-1f and 2c-2f show the four most distinctive regions found by applying the color-saliency measure to all the color regions extracted from the scene shown in Figure 1a and 2a respectively. In the experiments done so far, the color-saliency measure was found to select fairly large bright-colored regions that showed good contrast with their neighbors, and appeared perceptually significant.

4.2 Use of Salient Color-based Selection in Recognition

Data-driven selection based on salient color regions is primarily useful when the object of interest has at least one of its regions appearing salient in the given scene, since the search for data features that match model features can be restricted to these regions. Selecting salient regions gives a small number of large-sized groups which were shown to be very useful for indexing into the library of models [1]. But to recognize a single object, it is desirable to have small-sized groups. For this, existing grouping techniques can be applied to the data features found within the color regions to obtain reliable small-sized groups.

To estimate the search reduction that can be achieved with such a selection mechanism, let (M,N) = total number of features (such as edges, lines, etc.) in the model and image respectively. Let $(M_R, N_R) = \text{total number of color regions in the model and image}$ respectively. Let N_S = number of salient regions that are retained in an image. Let g = average size of a group of data features, within a model or image. Let (G_M, G_N) = number of groups formed (using any existing grouping scheme) in the model and image respectively. Finally, let G_{Ni} be the number of groups in the salient image region i. Using the alignment method of recognition [3], at least three corresponding data features are needed to solve for the pose (appearance) of the model in the image. If no selection of the data features is done, then the brute-force search required to try all possible triples is $O(M^3N^3)$. If selection is done by only grouping methods (i.e., without color region selection), then the number of matches that need to be tried is $O(G_M G_N g^3 g^3)$ since only triples within groups need to be tried. When grouping is done within color regions, the groups obtained are even smaller in number and are more reliable, so that the overall effect is to reduce search (by as much as a factor of 107). When grouping is restricted to salient color regions, the number of matches further reduces to $O(\sum_{j=1}^{N_S} G_{Nj} G_M g^3 g^3)$.

To get an estimate of the number of matches and time taken for matching in real scenes when color-based selection is used, we recorded the number of color regions, and the number of data features within regions in some selected models and scenes (Figure 2 and 3 show typical examples of models and scenes tried). The regions were ordered using the color saliency measure and the four most salient regions were retained. Then search estimates were obtained using the above formulas, and assuming a grouping scheme that gives a number of groups within regions that is bounded by the number of features in a region (which is a good bound using simple grouping schemes such as grouping 'g' closely-spaced parallel lines in a region). The result of such studies is shown in Table I. As can be seen from this table, the number of matches is always smaller when salient color regions are used for selection.

5 Color-based Model-driven Selection

When the object of interest is not salient in color, saliency-based data-driven selection will no longer be useful. In such cases, the color description of the model can be used to perform selection. Previous approaches to using model color information to search for instances of objects have used histogram matching techniques [9] that cause a lot of false positive identifications since they do not explicitly address some of the problems such as pose changes, occlusions, or illumination conditions that make a model instance appear different from its original description. Our approach to color-based model-driven selection handles some of these problems by using a rich description of model color regions and a location strategy that exploits global relational information about the regions provided in this description. In addition, it provides correspondence between model and image regions, which can help reduce the search in recognition as matching can now be restricted to the corresponding regions. Since the model description affects the design of the location strategy, it is described first.

5.1 Model Description

The color region information in the model (an image or view of the model, that is) is represented as a region adjacency graph (RAG) $M_G = \langle V_m, E_m, C_m, R_m, S_m, B_{rm}, B_{sm} \rangle$, where $V_m = \text{color regions}$ in the model, $E_m = \text{adjacencies}$ between color regions, $C_m(u) = \text{color of region } u \in V_m$, $R_m(u,v) = \text{relative size of region } v$ w.r.t region u. $S_m(u) = \text{size of region } u$, and $B_{rm} = \text{a}$ bound on the relative size of regions given by R_m , and $R_{sm} = \text{a}$ bound on the absolute size of regions given by S_m .

This description exploits features of regions such as color and adjacency information that tend to remain more or less invariant in most scenes where the model appears. Also, the bounds B_{rm} and B_{sm} indicate the extent of pose changes and occlusions that a selection mechanism is expected to tolerate. The description therefore, is fairly rich and has some structural information about color regions that can be used to restrict the number of false positives, and some constraints on the relative and absolute size changes that can be used to restrict the number of false negatives made by the selection mechanism.

Finally, the color region information in the image is similarly organized as an image region adjacency graph as $I_G = \langle V_I, E_I, C_I, R_I, S_I \rangle$ where each term has a meaning analogous to $\langle V_m, E_m, C_m, R_m, S_m \rangle$ respectively.

5.2 Location Strategy

Given the image region adjacency graph I_G , the model object if present in the scene will form a subgraph in I_G . The location strategy, therefore, regards the problem of selection as the problem of searching for suitable subgraphs that satisfy the model description. Although the number of subgraphs is exponential, a set of unary and binary constraints supplied in the model description restrict the subgraphs to a small number of feasible subgraphs. The perceptual color of a region and its absolute size bound (B_{im}) were used as the unary constraints, while region adjacency and relative size were used as the binary constraints. Specifically, the lack of adjacency between two model regions was used to prune false matches to two adjacent image regions. The bound B_{rm} in the model was used to discard matches when the relative size exceeded this bound.

The location strategy searched among the feasible subgraphs for a subgraph (or subgraphs) that in some sense best matches the given model description. Such a subgraph $I_g = \langle V_g, E_g, C_g, R_g, S_g \rangle$ such that $||V_g|| \leq ||V_m||, ||E_g|| \leq ||E_m||$, has associated with it a node correspondence vector $\Upsilon = \{(u_m, u_g) | \forall u_m \in V_m, u_g \in V_g \cup \{\bot\}, \{\bot\} \text{ is a null match} \}$ and is chosen to be the one that minimizes the following measure:

$$SCORE(I_g) = \left(1 - \frac{||V_g||}{||V_m||}\right) + \frac{2\sum_{\forall (u_g, v_g) \in E_g, \Upsilon(u_m) = u_g, \Upsilon(v_m) = v_g} R_{mg}^2(u_m, v_m, u_g, v_g)}{||E_m||}.$$
(4)

where $R_{mg}(u_m, v_m, u_g, v_g)$ expresses the change in the relative size when adjacent model regions (u_m, v_m) are paired to corresponding image regions (u_g, v_g) and is given by $R_{mg}(u_m, v_m, u_g, v_g) = \frac{|R_m(u_m, v_m) - R_g(u_g, v_g)|}{\max(R_m(u_m, v_m), R_g(u_g, v_g))}$. SCORE (I_g) emphasizes rewards for making as many correspondences as possible as indicated by the first term, and penalties for a mismatch of the relative size, as indicated by the second term which accounts for occlusions and pose changes in a more refined way than the binary constraints alone. A branch and bound version of interpretation tree search [3] was then used to search for the best subgraph.

The result of using color-based model-driven selection is illustrated in Figure 3. Figure 3a and 3b show a model object, and its color description obtained by using the color-region segmentation algorithm of Section 3. Figure 3c shows a scene in which the model

object occurs. The scene shown has several other objects with one or more of the model colors. Also, the model appears in a different pose, being rotated to the left about the vertical axis. Figure 3d shows the result of applying the unary color constraints, and Figure 3e, the subsequent use of the absolute size constraint. Finally, the subgraph with the lowest value of SCORE is shown in Figure 3f. As can be seen from this figure, a region containing most of the model object has been identified even with an imperfect color image segmentation.

5.3 Search Reduction using Color-based Model-driven Selection

The color-based model-driven selection mechanism provides a correspondence of model regions to some image regions. The matching of model features to image features can be restricted to within corresponding regions, and when this is combined with grouping within regions as described in Section 4.2, the number of matches to be tried for recognition reduces further. To estimate the search reduction in this case, let N_l be the number of solution subgraphs given by the selection mechanism, and let I_k represent one such subgraph with the number of nodes = N_k . Let (G_{u_i}, G_{v_i}) = the number of groups in region u_i of the solution subgraph I_k , and region v_i of the model RAG that corresponds to u_i as implied by the correspondence vector Υ associated with I_k . Then assuming, as before, the average size of the group = g, the number of matches that need to be tried are $O(\sum_{k=1}^{N_l}\sum_{j=1}^{N_k}G_{u_j}G_{v_i}.g^3.g^3)$. By trying several models and images of scenes where they occurred, we recorded the average number of subgraphs generated by the model-driven selection mechanism. The search estimates were obtained using the above formula for model-driven selection with grouping, and the formulas for other methods mentioned in Section 4.2. The results are shown in Table II. The bound on the number of groups in a region was the same as used in Section 4.2. As can be seen from the table, the number of matches using correspondence between model and image color regions is always lower.

6. Conclusions

In this paper we have shown how color can be used as a cue to perform both data and model-driven selection. Unlike other approaches to color, we have used the intended task to constrain the kind of color information to be extracted from images. This led to a fast color image segmentation algorithm based on perceptual categorization of colors which later formed the basis of data and model-driven selection. Future work will be directed towards integrating the selection mechanism with a 3D from 2D recognition system to obtain statistics of false positives and negatives and the actual search reduction due to selection.

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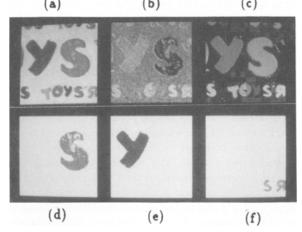


Fig. 1. Illustration of color region segmentation and color-saliency. (a) Input image consisting of regions of 3 different colors: red, green and blue against an almost white background. (b) Result of Step 2 of algorithm with regions colored differently from the original image. (c) Final segmentation of the image of Fig.3a. (d) — (f) The three most distinctive regions found using the color saliency measure.

Table 1. Search reduction using color-based data-driven selection. The last column shows the match time when color-based data-driven selection is combined with grouping. The color-based selection is done by choosing the four most salient regions. Here g=7, Time per match =1 microsecond, and the grouping method is as described in text.

					No sel	ection	Only gro	uping	Salient color + grouping		
S.No	M	N	M_R	N_R	Num.	Time	Num.	Time	Num.	Time	
				ł	matches		matches		matches		
1.	229	1170	1	18	1.92x1016	610yrs	6.52x108	11min	3.37×10^8	5min	
2.	507	2655	2	20	2.4x10 ¹⁸	77,341yrs	3.22x10°	54min	1.32x10°	22min	
3.	124	2655			3.57x1016					5min	
4.	507	2247	2	14	1.48x1018	46,884yrs	2.72x10°	46min	7.8x108	13min	

Table 2. Search reduction using color-based model-driven selection. The last column shows the match time when model-color-based selection is combined with grouping. Here g = 7, Time per match = 1 microsecond, and the grouping method is as described in text.

								No selection		Only grouping		Model-driven selection	
S.No	M	N	M_R	N_R	Objects	N_{I}	N.	Num.	Time	Num.	Time	Num.	Time
	L							matches		matches		matches	
1.	786	3268	5	30	20	1	(3)	1.69x10°	530000yrs	6.15x10 ⁹	103min	4.55×10^{7}	45sec
2.	83	3078	1	20	14	3	(1,1,1)	1.67x10 ¹⁶	528yrs	6.2x10 ⁸	11min	1.7x10 ⁸	3min
3.	507	2655	2	20	14	2	(2,1)	2.4x1018	77,341yrs	3.22x10°	54min	3.72×108	6min
4.	507	2247	2	14	6	1	(2)	1.48x10 ¹⁸	46,884yrs	2.72x109	46min	3.16x10 ⁸	5min

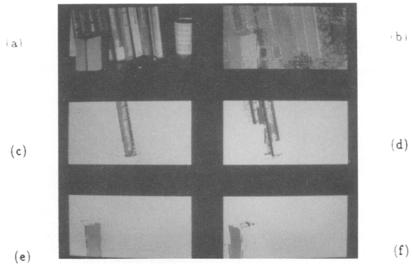


Fig. 2. Illustration of color region segmentation and color-saliency. (a) Input image depicting a scene of objects of different materials and having occlusions and inter-reflections. (b) Segmented image using the color region segmentation algorithm. (c)-(f) The four most distinctive regions detected using the color-saliency measure. The white portion in the red book appears so because of the white background.

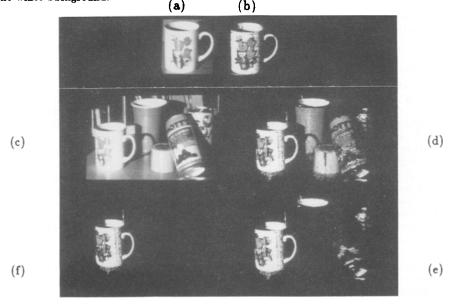


Fig. 3. Illustration of color-based model-driven selection. (a) The object serving as the model. (b) Its color description produced by the segmentation algorithm of Section 3. (c) A cluttered scene in which the object appears. (d) Regions selected based on unary color constraint. (e) Regions of (d) pruned after using the unary size constraint. (f) Regions corresponding to the best subgraph that matched the model specifications.