Linguistic Summarization of Some Static and Dynamic Features of Consensus Reaching

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Summary. Consensus reaching has been widely recognized as an important component of the decision-making process. In previous works Fedrizzi and Kacprzyk introduced a new concept of consensus referring to the idea of fuzzy majority and based on the Zadeh's calculus of linguistically quantified propositions. Basically, the (degree of) consensus was meant as the degree to which Q_1 (e.g., most) of the I (e.g., important) individuals agree as to Q_2 (e.g., almost all) of B (e.g., relevant) options. The approach was extended in further works by Fedrizzi, Kacprzyk, Nurmi, and Zadrożny. Recently Kacprzyk and Zadrożny proposed to apply linguistic summaries in the sense of Yager to support the consensus reaching process. For instance, "most individuals definitely preferring option o_1 over option o_2 also definitely prefer option o_5 to option o_7," "almost all options dominating option o_3 in the opinion of expert e_2 also dominate option o_6 in the opinion of expert e_4," etc. In the present paper, we extend this idea and propose to take into account dynamic features of the consensus reaching process while constructing the linguistic summaries. Basically, linguistic summaries are meant as a concise description of the current status in the group of individuals in terms of their preferences. These descriptions may concern particular individuals, the whole group, or particular options. Moreover we propose here to take into account also how the preferences are evolving over time. For instance, "individual e_2 is very flexible with respect to his or her preferences between options o_3 and o_5," etc. Such an information might be even more useful for the running, moderating, etc. of a consensus reaching process than a static description.

Key words: Consensus reaching, Fuzzy majority, Fuzzy preferences, Fuzzy quantifiers, Linguistic data summary, Linguistic summarization of times series.

1 Introduction

This paper deals with an aspect of consensus reaching processes in a fuzzy environment, i.e., under fuzzy preferences and a fuzzy majority. We assume that there is a set of individuals (experts, decision-makers, ...) and a set of options (alternatives, variants, decisions, issues, ...). The individuals provide their testimonies concerning alternatives in question which are assumed to be

fuzzy preference relations. Normally, the individuals initially disagree in their testimonies, i.e., they are far from "consensus." Then, assuming that the individuals are seriously committed to reaching consensus, they are expected to update step-by-step their testimonies via an exchange of information, rational argument (e.g., by a moderator), etc., and hopefully to finally attain a "consensus."

Traditionally, consensus is meant as a full and unanimous agreement, i.e., that the testimonies of all the individuals should be the same at consensus. Unfortunately, this is utopian in practice. This has implied a need of some reconsideration of the very essence of "consensus" exemplified by a citation from Lower and Laddaga [31]: "... It can correctly be said that there is a consensus among biologists that Darwinian natural selection is an important cause of evolution though there is currently no consensus concerning Gould's hypothesis of speciation. This means that there is a widespread agreement among biologists concerning the first matter but disagreement concerning the second ..." (cf. also [30]). The above given quotation suggests that a *fuzzy majority* is appropriate, and that it makes sense to speak about a consensus to a degree.

A "soft" degree of consensus meant as the degree to which, say, "*most* of the *relevant* individuals agree as to *almost all* of the *relevant* issues (aspects, etc.) was proposed by Kacprzyk [9], Kacprzyk and Fedrizzi [10–15], and Fedrizzi and Kacprzyk [2,3] (see also Kacprzyk et al. [16–18]), and then by Kacprzyk and Zadrożny [22,24,25,27]. Fuzzy logic with linguistic quantifiers (cf. [35]) was employed.

Consensus is normally reached via a consensus reaching process run by a moderator (cf. [5,29]). Thus some tools may be helpful, notably linguistic summaries as proposed by Kacprzyk and Zadrożny (c.f., e.g., [28]).

In this paper we will add some other analytic tools, notably some linguistic assessment of how opinions of the individuals evolve over time.

In Sect. 2 we will show how to derive soft degrees of consensus under fuzzy preferences and a fuzzy majority, in Sect. 3 the idea and basic aspects of linguistic data summaries are presented, and some methods of their derivation are presented; moreover, it is shown how they can be used to support the running of a consensus reaching process. In Sect. 4 dynamic aspects of consensus reaching process are dealt with.

2 Degrees of Consensus under Fuzzy Preferences and a Fuzzy Majority

We operate in the following basic setting. We have a set of $n \geq 2$ options (alternatives, variants, issues, ...), $O = \{o_1, \ldots, o_n\}$, and a set of $m \geq 2$ individuals, $E = \{e_1, \ldots, e_m\}$. Moreover B is a fuzzy set of *relevant* options and I is a fuzzy set of *important* individuals. Each individual $e_k \in E$ provides his or her testimony as to the options in O, assumed to be an individual fuzzy preference relation in $O \times O$.

An *individual fuzzy preference relation* of individual e_k, $R^k = [r_{ij}^k]$, is given as

$$\mu_{R^k}(o_i, o_j) = \begin{cases} 1 & \text{if } o_i \text{ is definitely preferred to } o_j, \\ c \in (0.5, 1) & \text{if } o_i \text{ is slightly preferred to } o_j, \\ 0.5 & \text{in the case of indifference,} \\ d \in (0, 0.5) & \text{if } o_j \text{ is slightly preferred to } o_i, \\ 0 & \text{if } o_j \text{ is definitely preferred to } o_i. \end{cases} \tag{1}$$

The "soft" degree of consensus is derived in three steps:

1. for each pair of individuals we derive a degree of agreement as to their preferences between *all* the pairs of options,
2. we aggregate these degrees to obtain a degree of agreement of each pair of individuals as to their preferences between Q_1 (a linguistic quantifier as, e.g., "most," "almost all," "much more than 50%," ...) pairs of *relevant* options B, and
3. we aggregate these degrees to obtain a degree of agreement of Q_2 (a linguistic quantifier similar to Q_1) pairs of *important* individuals I as to their preferences between Q_1 pairs of *relevant* options B, and this is meant to be the *degree of consensus* sought.

The point of departure is clearly a degree to which each pair of individuals agrees as to the preference between a particular pair of options. In the most basic case, we may define the degree of strict agreement between individuals e_k and e_l as to their preferences between options o_i and o_j

$$v_{ij}(k, l) = \begin{cases} 1 \text{ if } r_{ij}^k = r_{ij}^l, \\ 0 \text{ otherwise,} \end{cases} \tag{2}$$

where here and later on in this section, if not otherwise specified, $k = 1, \ldots, m-1$; $l = k+1, \ldots, m$; $i = 1, \ldots, n-1$; $j = i+1, \ldots, n$.

The relevance of a pair of options, $(o_i, o_j) \in O \times O$, may be defined, say, as $b_{ij}^B = \frac{1}{2}[\mu_B(o_i) + \mu_B(o_j)]$, which is clearly the most straightforward choice; evidently, $b_{ij}^B = b_{ji}^B$, and b_{ii}^B do not matter; for each i, j.

The importance, $b_{k,l}^I$, of a pair of individuals, (e_k, e_l), may be defined as $b_{k,l}^I = \frac{1}{2}[\mu_I(e_k) + \mu_I(e_l)]$.

The degree of agreement between individuals e_k and e_l as to their preferences between *all* the pairs of *relevant* options is

$$v_B(k, l) = \frac{\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} [v_{ij}(k, l) \wedge b_{ij}^B]}{\sum_{i=1}^{n-1} \sum_{j=i+1}^{n} b_{ij}^B}. \tag{3}$$

The degree of agreement between individuals e_k and e_l as to their preferences between Q_1 *relevant* pairs of options is

$$v_{Q_1}^B(k, l) = \mu_{Q_1}[v_B(k, l)]. \tag{4}$$

In turn, the degree of agreement of *all* the pairs of *important* individuals as to their preferences between Q_1 pairs of *relevant* options is

$$v_{Q_1}^{I,B} = \frac{2}{m(m-1)} \frac{\sum_{k=1}^{m-1} \sum_{l=k+1}^{m} [v_{Q_1}^{B}(k,l) \wedge b_{k,l}^{I}]}{\sum_{k=1}^{m-1} \sum_{l=k+1}^{m} b_{k,l}^{I}}. \tag{5}$$

Finally, the degree of agreement of Q_2 pairs of *important* individuals as to their preferences between Q_1 pairs of *relevant* options, called the *degree of* $Q1/Q2/I/B$*-consensus*, is

$$con(Q_1, Q_2, I, B) = \mu_{Q_2}(v_{Q_1}^{I,B}). \tag{6}$$

For some extensions, see, e.g. [6–8].

It is worth noticing that the required consensus does not have to concern directly the preference relations expressed by the individuals. Usually the ultimate goal of the decision-making session is the *choice* of the *best* option(s). Obviously an agreement between the individuals on the level of their preference relations makes this choice easier [32]. However if the individuals do not agree with respect to *all* or *most* pairs of options they still may be fairly in agreement as to which option is the best. Thus it may be advantageous to consider the agreement also on the level of the *choice sets* of options, i.e., the sets of options that should be selected as the best taking into account each individual preference relation separately. Such a multilevel measuring of the consensus was proposed by Kacprzyk and Zadrożny in [23, 24, 26, 27], see also [22] for a discussion of *linguistic choice rules*.

3 A Consensus Reaching Process and Linguistic Data Summarization

We assume the following setting of the consensus reaching process (cf. [4,5,29, 36]). We have a set of individuals and a distinguished person, a *moderator* who is responsible for running the consensus reaching session. The individual fuzzy preference relations may initially differ to a large extent, i.e., the group may be far from consensus. A moderator stimulates an exchange of information, rational argument, discussion, creative thinking, clarification of positions, etc. If the individuals are rationally committed to consensus, a change of testimonies usually occurs, and they get closer to consensus. It is assumed that some individuals, even if they are still convinced they are right with their original preferences, they can accept a consensual preferences established by the group provided their arguments has been heard and discussed. Thus, their acceptance of consensus may be effectively treated as a change of their preferences. This is repeated until the group gets sufficiently close to consensus.

Among some means for supporting consensus reaching, linguistic summaries of what happens to the preferences, relations between options, etc. may be useful.

A linguistic summary is meant as a natural language like sentence that subsumes the very essence (from a certain point of view) of a set of data. This set is assumed to be numeric and is usually large, not comprehensible in its original form by the human being. The original Yager's approach to the linguistic summaries (cf. Yager [33], Kacprzyk and Yager [20], Kacprzyk et al. [21], and Kacprzyk and Zadrożny [28]) may be expressed as follows:

- $Y = \{y_1, \ldots, y_n\}$ is a set of objects
- $A = \{A_1, \ldots, A_m\}$ is a set of attributes characterizing objects from Y, $A_j(y_i)$ denotes a value of attribute A_j for object y_i

A linguistic summary of set Y consists of:

- A summarizer S, i.e., an attribute together with a linguistic term (label) defined on the domain of attribute A_j
- A quantity in agreement Q, i.e., a linguistic quantifier (e.g., most)
- Truth (validity) T of the summary, i.e., a number from the interval $[0, 1]$ assessing the truth (validity) of the summary (e.g., 0.7); usually, only summaries with a high value of T are interesting
- Optionally, a qualifier P, i.e., another attribute together with a linguistic term (label) defined on the domain of attribute A_k determining a (fuzzy) subset of Y

Note that for brevity we will often identify summarizers and qualifiers with the linguistic terms they contain.

Basically, the core of a linguistic summary is a *linguistically quantified proposition* in the sense of Zadeh [35]. A linguistically quantified proposition, of type I may be written as:

$$Qy\text{s are } S \tag{7}$$

and the one of type II may be written as

$$QPy\text{s are } S. \tag{8}$$

Then, the component of a linguistic summary, T, i.e., its truth (validity), directly corresponds to the truth value of (7) or (8). This may be calculated by using either original Zadeh's calculus of linguistically quantified statements (cf. [35]), or other interpretations of linguistic quantifiers, including Yager's OWA operators [34].

Using Zadeh's [35] fuzzy logic-based calculus of linguistically quantified propositions, a (proportional, nondecreasing) linguistic quantifier Q is assumed to be a fuzzy set in the interval $[0, 1]$ as, e.g.,

$$\mu_Q(x) = \begin{cases} 1 & \text{for } x \leq 0.8, \\ 2x - 0.6 & \text{for } 0.3 < x < 0.8, \\ 0 & \text{for } x \geq 0.3. \end{cases} \tag{9}$$

Then, the truth values (from $[0,1]$) of (7) and (8) are calculated, respectively, as

$$\text{truth}(Qy\text{s are } S) = \mu_Q\left(\frac{1}{n}\sum_{i=1}^{n}\mu_S(y_i)\right), \tag{10}$$

$$\text{truth}(QPy\text{s are } S) = \mu_Q\left(\frac{\sum_{i=1}^{n}(\mu_P(y_i)\wedge\mu_S(y_i))}{\sum_{i=1}^{n}\mu_P(y_i)}\right), \tag{11}$$

where "$\wedge$" is the minimum operation, i.e., $a \wedge b = \min(a,b)$, which can be replaced by, e.g., a t-norm.

Linguistic summaries may, in a convenient way, describe the current state of agreement in the group and serve as guiding indicators for a further discussion, if needed. They may point out how far the group is from consensus, what are main obstacles in reaching consensus, which preference matrix may be a candidate for a consensual one, etc. As shown by Kacprzyk and Zadrożny [27] the very definition of consensus (6) may be interpreted as a kind of a linguistic summary.

In [27] other types of linguistic summaries have also been proposed. The summarizers S and qualifiers P refer to features of either individuals or options and linguistic terms expressing degrees of preferences, importance of individuals, and relevance of options.

First, the summarized objects Y may be identified with the individuals E, and their attributes A are preference degrees for particular pairs of options as well as their importance degrees. Then, the summaries of the following type may be useful and helpful for running a consensus reaching session:

Most individuals *definitely* prefer o_1 to o_2, *moderately* prefer o_3 to $o_4, \ldots$

Most individuals *definitely* preferring o_1 to o_3 also *definitely* prefer o_2 to o_4.

The summaries concerning the *choice sets* of particular individuals may be exemplified by:

Most individuals choose options $o_1, o_3, \ldots$

Most individuals reject options $o_1, o_4, \ldots$

The summaries of the latter type may help exclude some options from a further consideration and thus better focus the discussion.

Second, the summarized objects Y may be identified with the options O, and their attributes A are preference degrees over other options as expressed by particular individuals as well as their relevance degrees. Then, for instance, the following summaries may be useful:

Most options are dominated by option o_2 in opinion of individual e_3.

Most options are dominated by option o_2 in opinion of individual $e_2, e_4, \ldots$

Most options are preferred to option o_1 in opinion of individual e_3 also preferred to option o_2 in opinion of individual e_4.

Thus, summarized are here the preference matrices of the individuals at a given point in time, i.e., at a given iteration (stage) of the discussion. Quite clearly, preferences evolve over time, and an interesting problem is to how to linguistically assess those changes. This is dealt with in the next section.

4 A Dynamic View of Consensus Reaching

Let us now consider some linguistic summaries that take into account dynamic, temporal aspects of the consensus reaching process, and notably express how the preferences evolve over time. These may be exemplified by a summary

$$\text{Individual } e_k \text{ is } \textit{very flexible} \text{ with respect to his/her preferences between options } o_i \text{ and } o_j. \tag{12}$$

In such a summary we do not capture an "up" and "down" direction of change, and what really matters is an absolute change of a preference degree r_{ij}^k. Thus the time series data considered here represent the cumulative changes up to certain point of time (i.e., iteration of discussion) of an expert's preferences regarding a pair of options. This may be expressed as the sum of absolute values of all changes that have occurred until a given point of time t_s, i.e.

$$\text{change}(t_s) = \sum_{q=1}^{s} \left| r_{ij}^k(t_q) - r_{ij}^k(t_{q-1}) \right|, \tag{13}$$

where $r_{ij}^k(t)$ denotes the r_{ij}^k element of the preference matrix of the kth expert at time (iteration) t. Clearly, the maximal value of change in a single step in comparison to the previous one equals 1.

As the consensus reaching sessions, and thus a "time series of preference changes," are usually very short, then we assume that only one trend, concerning the whole time span is observed. Such a trend is meant to describe a *flexibility* of an expert's opinion: an expert is identified as either flexible or inflexible (stubborn). To extract it we use a simple least squares approximation (LSA) of function (13) by a linear function.

In the summaries we use the slope of the line obtained via LSA to characterize the trend. However it might be impractical to use the actual precise value of the slope. Instead we use a *fuzzy granulation* in order to meet the users' needs and a task specificity. The user may construct a scale of *linguistic terms* corresponding to various slopes of the line identifying the trend as, e.g.,:

- very flexible,
- flexible,
- moderately flexible,

– inflexible, and
– very inflexible (stubborn).

Figure 1 illustrates the lines corresponding to the particular linguistic terms. In fact, each term represents a fuzzy granule of directions.

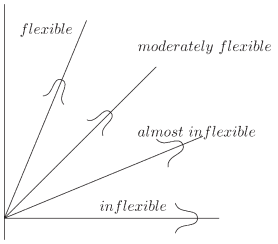

Fig. 1. A visual representation of slope granules defining the dynamics of change

The truth value of the summary (12) is computed via the following three steps:

1. Compute the values of a cumulative change for all time points until the current one – we treat them as a time series
2. Find the slope of the LSA line (α)
3. The truth value of the summary is equal to $\mu_S(\alpha)$, where S is the fuzzy set representing the summarizer of the summary; in case of (12) it is the fuzzy set representing the linguistic term *very flexible*

5 Concluding Remarks

We have proposed to use linguistic summaries to evaluate trends of how the individuals' preferences evolve over time. Starting from a "soft" definition of a consensus degree, we have shown that these summaries provide much of information that can help run a consensus reaching session aimed at reaching a possibly good agreement among a group of individuals concerning their preferences.

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