Intuitionistic Fuzzy Graphs

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Summary. A new definition for intuitionistic fuzzy graph is given. Some properties of intuitionistic fuzzy graphs are considered and the authors introduced the notions of various concepts. These concepts are analyzed through suitable illustrations.

Key words: Intuitionistic fuzzy graph, Semi-μ strong path, Semi-γ strong path, Bridge, Composition.

1 Introduction

Fuzzy set [4] has emerged as a potential area of interdisciplinary research and fuzzy graph theory is of recent interest. The concept of a fuzzy relation was defined by Zadeh [9] and it has found applications in the analysis of cluster patterns [3]. Rosenfeld [6] considered fuzzy relations on fuzzy sets and developed the structure of fuzzy graphs, obtaining analogs of several graph theoretical concepts. Then Bhattacharya [2] introduced some remarks on fuzzy graphs. Later, complement of fuzzy graphs and some operations on fuzzy graphs are introduced by Mordeson and Peng [5]. Further, Sunitha and Vijayakumar [7] defined the complement of a fuzzy graph in a different way and studied some operations on it. Yeh and Banh [8] have also introduced various connectedness concepts in fuzzy graphs. After the pioneering work of Rosenfeld [6], Yeh and Banh [8] in 1975, when some basic fuzzy graph theoretic concepts and applications have been indicated.

Atanassov [1] introduced the concept of intuitionistic fuzzy (IF) relations and intuitionistic fuzzy graphs (IFGs). Research on the theory of intuitionistic fuzzy sets (IFSs) has been witnessing an exponential growth in Mathematics and its applications. This ranges from traditional Mathematics to Information Sciences.

This leads to consider IFGs and their applications. In this paper, we introduced IFG and analyzed its components. It is further proposed by the authors that these concepts can be extended to other types of IFSs and analyzing various components.

2 Preliminaries

Definition 1. *An IFG is of the form* $G = \langle V, E\rangle$ *where*

(i) $V = \{v_1, v_2, ..., v_n\}$ *such that* $\mu_1 : V \longrightarrow [0,1]$ *and* $\gamma_1 : V \longrightarrow [0,1]$ *denote the degree of membership and nonmembership of the element* $v_i \in V$*, respectively, and*

$$0 \le \mu_1(v_i) + \gamma_1(v_i) \le 1, \qquad \ldots\ldots (1)$$

for every $v_i \in V$*,* $(i = 1, 2, \ldots n)$*,*

(ii) $E \subseteq V \times V$ *where* $\mu_2 : V \times V \longrightarrow [0,1]$ *and* $\gamma_2 : V \times V \longrightarrow [0,1]$ *are such that*

$$\mu_2(v_i, v_j) \le \min[\mu_1(v_i), \mu_1(v_j)], \qquad \ldots\ldots (2)$$

$$\gamma_2(v_i, v_j) \le \max[\gamma_1(v_i), \gamma_1(v_j)] \qquad \ldots\ldots (3)$$

$$\textit{and} \quad 0 \le \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \le 1 \qquad \ldots\ldots (4)$$

for every $(v_i, v_j) \in E$*,* $(i, j = 1, 2, \ldots n)$*.*

Notations

The triple $\langle v_i, \mu_{1i}, \gamma_{1i}\rangle$ denotes the degree of membership and nonmembership of the vertex v_i. The triple $\langle e_{ij}, \mu_{2ij}, \gamma_{2ij}\rangle$ denotes the degree of membership and nonmembership of the edge relation $e_{ij} = (v_i, v_j)$ on V.

Note 1.

(i) When $\mu_{2ij} = \gamma_{2ij} = 0$, for some i and j, then there is no edge between v_i and v_j.

(ii) When either one of the following is true, then there is an edge relation between v_i and v_j.
   - $\mu_{2ij} > 0$ or $\gamma_{2ij} > 0$.
   - $\mu_{2ij} = 0$ or $\gamma_{2ij} > 0$.
   - $\mu_{2ij} > 0$ or $\gamma_{2ij} = 0$.

(iii) If one of the inequalities (1) or (2) or (3) or (4) is not satisfied, then G is not an IFG.

Example 1. Consider $G = \langle V, E\rangle$ where $V = \{v_1, v_2, v_3, v_4, v_5\}$. (refer Fig. 1)

Example 2. Consider $G = \langle V, E\rangle$ where $V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$. (refer Fig. 2)

Definition 2. *An IFG* $H = \langle V', E'\rangle$ *is said to be an IF subgraph (IFSG) of the IFG,* $G = \langle V, E\rangle$ *if* $V' \subseteq V$ *and* $E' \subseteq E$.

In other words, if $\mu'_{1i} \le \mu_{1i}$ *;* $\gamma'_{1i} \ge \gamma_{1i}$ *and* $\mu_{2ij}{}' \le \mu_{2ij}$ *;* $\gamma_{2ij}{}' \ge \gamma_{2ij}$ *for every* $i, j = 1, 2, \ldots, n$.

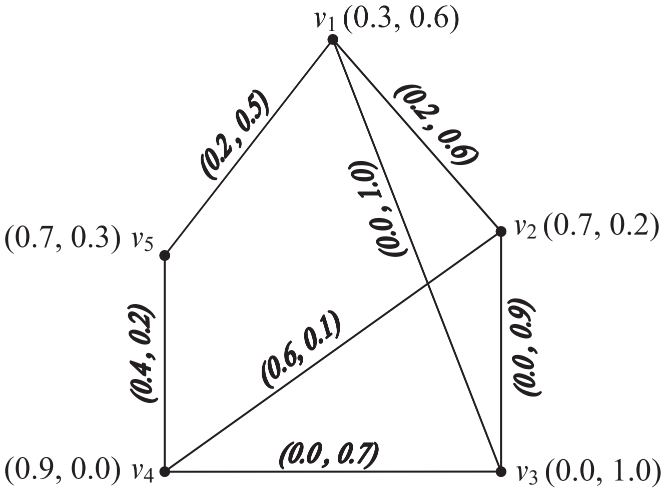

Fig. 1. Intuitionistic fuzzy graph

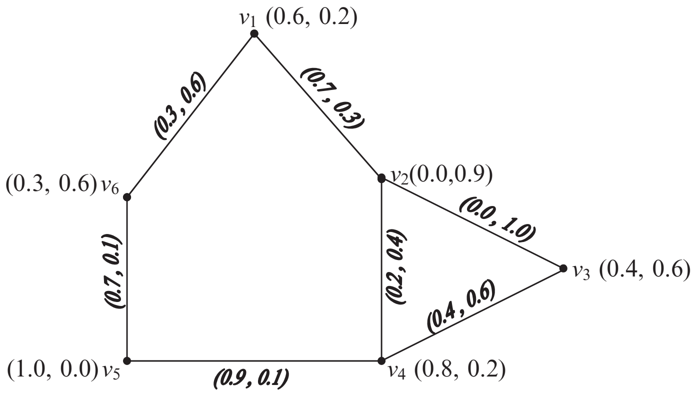

Fig. 2. G is not an intuitionistic fuzzy graph

Definition 3. *An IFG* , $\mathrm{G} = \langle \mathrm{V}, \mathrm{E} \rangle$ *is said to be a semi-*μ *strong IFG if*

$$\mu_{2ij} = \min\ (\mu_{1i}, \mu_{1j}),\ \ for\ every\ (v_i, v_j) \in \mathrm{E}.$$

Definition 4. *An IFG* , $\mathrm{G} = \langle \mathrm{V}, \mathrm{E} \rangle$ *is said to be a semi-*γ *strong IFG if*

$$\gamma_{2ij} = \max\ (\gamma_{1i}, \gamma_{1j}),\ \ for\ every\ (v_i, v_j) \in \mathrm{E}.$$

Definition 5. *An IFG,* $\mathrm{G} = \langle \mathrm{V}, \mathrm{E} \rangle$ *is said to be a strong IFG if*

$$\mu_{2ij} = \min (\mu_{1i}, \mu_{1j})\ \ and\ \gamma_{2ij} = \max (\gamma_{1i}, \gamma_{1j})\ \ for\ all\ \ (v_i, v_j) \in \mathrm{E}.$$

Example 3. Let $\mathrm{V} = \{v_1, v_2, v_3, v_4, v_5\}$. (refer Fig. 3)

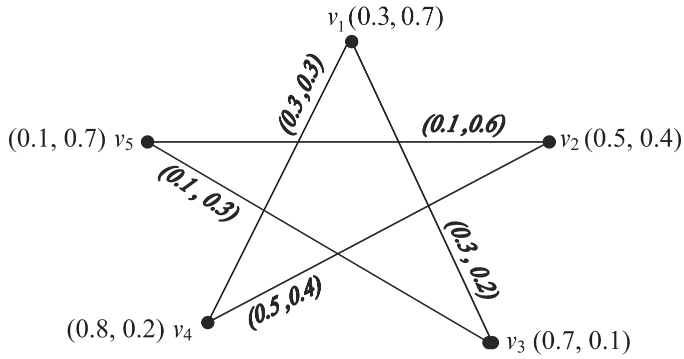

Fig. 3. Semi-μ strong IFG

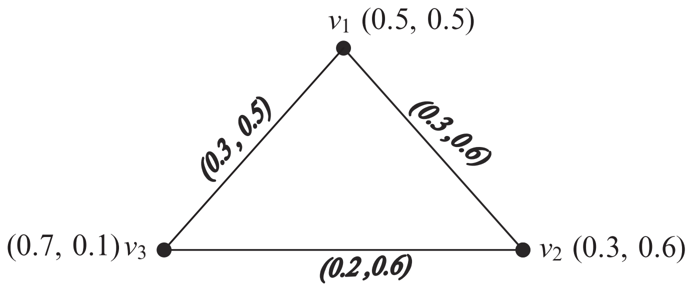

Fig. 4. Semi-γ strong IFG

Example 4. Let V $= \{v_1, v_2, v_3\}$. (refer Fig. 4)

Definition 6. *A path P in an IFG is a sequence of distinct vertices* v_1, $v_2 \ldots v_n$ *such that either one of the following conditions is satisfied:*

(a) $\mu_{2ij} > 0$ *and* $\gamma_{2ij} = 0$ *for some* i *and* j,
(b) $\mu_{2ij} = 0$ *and* $\gamma_{2ij} > 0$ *for some* i *and* j,
(c) $\mu_{2ij} > 0$ *and* $\gamma_{2ij} > 0$ *for some* i *and* j $(i, j = 1, 2, \ldots \mathrm{n})$.

Example 5. Let V $= \{v_1, v_2, v_3, v_4, v_5\}$. (refer Fig. 5)
Here $v_1v_4v_3v_2$ is a path.

Definition 7. *The length of a path* P $= v_1v_2 \ldots v_{n+1}$ $(n > 0)$ *is* n.

Definition 8. *A path* P $= v_1v_2 \ldots v_{n+1}$ *is called a cycle if* $v_1 = v_{n+1}$, *and* $n \geq 3$.

Definition 9. *Two vertices that are joined by a path are said to be connected.*

Definition 10. *The* μ*-strength of a path* P $= v_1v_2 \ldots v_n$ *is defined as*

$$\min_{i,\mathrm{j}} \{\mu_{2ij}\} \qquad \ldots\ldots \quad (5)$$

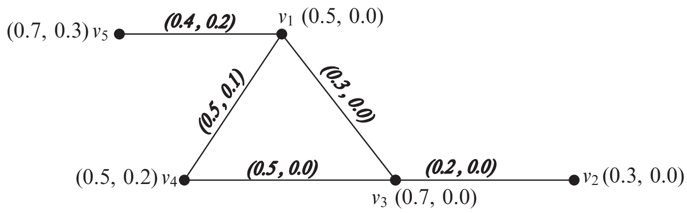

Fig. 5. A path in an IFG

and is denoted by S_μ.

The γ*-strength of a path* $\mathrm{P} = v_1 v_2 \dots v_n$ *is defined as*

$$\max_{i,j} \{\gamma_{2ij}\} \qquad \dots\dots \ (6)$$

and is denoted as S_γ.

Note 2.

If an edge possesses both the values (5) and (6), then it is the *strength* of the path P and is denoted by $\mathrm{S_P}$.

Definition 11. *For any t,* $0 \leq t \leq 1$*, the set of triples* $\langle \mathrm{V_t}, \mu_{1\mathrm{t}}, \gamma_{1\mathrm{t}} \rangle$*, where*

$$\mu_{1t} = \{v_i \in \mathrm{V} : \mu_{1i} \geq t\} \qquad \dots\dots (7)$$

$$\text{or} \qquad \gamma_{1t} = \{v_i \in \mathrm{V} : \gamma_{1i} \leq t\} \qquad \dots\dots (8)$$

for some $i = 1, 2, \dots n$*, is a subset of* V
and the set of triples $\langle \mathrm{E_t}, \mu_{2\mathrm{t}}, \gamma_{2\mathrm{t}} \rangle$*, where*

$$\mu_{2t} = \{(v_i, v_j) \in \mathrm{V} \times \mathrm{V} : \mu_{2ij} \geq t\} \qquad \dots\dots (9)$$

$$\text{or} \qquad \gamma_{2t} = \{(v_i, v_j) \in \mathrm{V} \times \mathrm{V} : \gamma_{2ij} \leq t\} \qquad \dots\dots \ (10)$$

for some $i, j = 1, 2, \dots n$*, is a subset of* E.

Example 6. Let $\mathrm{V} = \{v_1, v_2, v_3, v_4, v_5\}$. (refer Fig. 6)
Here, $\mathrm{V}_{0.6} = \{v_1, v_2, v_4, v_5\}$,
$\mathrm{E}_{0.6} = \{v_1 v_2, v_2 v_5, v_4 v_5, v_5 v_1\}$.

3 Properties

Theorem 1. *If* $0 \leq \mathrm{x} \leq \mathrm{y} \leq 1$*, then* $(\mathrm{V_x}, \mathrm{E_x})$ *is a subgraph of* $(\mathrm{V_y}, \mathrm{E_y})$.

Proof. Let $\mathrm{G} = \langle \mathrm{V_y}, \mathrm{E_y} \rangle$ and $\mathrm{H} = \langle \mathrm{V_x}, \mathrm{E_x} \rangle$.

To prove H is a subgraph of G, it is enough to prove that $\mathrm{V_x} \subseteq \mathrm{V_y}$ and $\mathrm{E_x} \subseteq \mathrm{E_y}$.

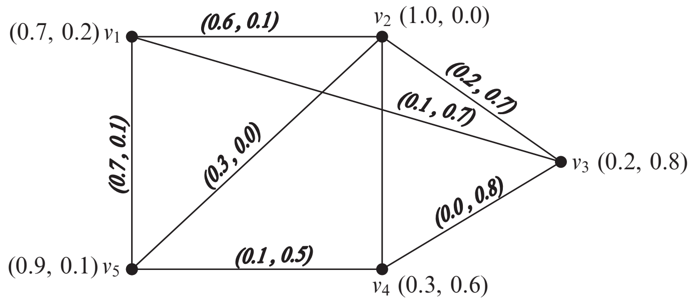

Fig. 6. $V_{0.6}$ and $E_{0.6}$

Let $v_i \in V_x$. Therefore, $\gamma_{1i} \leq x$

$$\leq y, \text{ since } x \leq y.$$
$$\Rightarrow v_i \in V_y. \text{ Hence } V_x \subseteq V_y.$$

Let $(v_i, v_j) \in E_x$. Therefore, $\gamma_{2ij} \leq x$

$$\leq y, \text{ since } x \leq y.$$

Thus, we have $(v_i, v_j) \in E_y$. Hence, $E_x \subseteq E_y$.

Hence, (V_x, E_x) is a subgraph of (V_y, E_y). □

Theorem 2. *If* $H = \langle V', E' \rangle$ *is an IF subgraph of* $G = \langle V, E \rangle$*, then for any* $0 \leq x \leq 1$, $\langle V'_x, E'_x \rangle$ *is an IF subgraph of* $\langle V_x, E_x \rangle$.

Proof. Given $V' \subseteq V$ and $E' \subseteq E$.

To prove $V'_x \subseteq V_x$; $E'_x \subseteq E_x$, it is enough to prove (7)–(10) for μ or γ.

Let $v_i \in V'_x$
$\Rightarrow \mu'_{1i} \geq x$
$\Rightarrow \mu_{1i} \geq x$, since $\mu'_1 \leq \mu_1$
$\Rightarrow v_i \in V_x$
$\Rightarrow V'_x \subseteq V_x$

Let $(v_i, v_j) \in E'_x$

Therefore, $\mu'_{2ij} \geq x$

$\Rightarrow \mu_{2ij} \geq x$, since $\mu'_2 \leq \mu_2$
$\Rightarrow (v_i, v_j) \in E_x$

Hence, $E'_x \subseteq E_x$.

Therefore, $\langle V'_x, E'_x \rangle$ is an IF subgraph of $\langle V_x, E_x \rangle$. □

Definition 12. *Let* $\langle \mathrm{e}_{ij},\ \mu_{2ij},\ \gamma_{2ij}\rangle$ *and* $\langle \mathrm{e}_{jk},\ \mu_{2jk},\ \gamma_{2jk}\rangle$ *be two edge relations on* V. *The composition of these two edge relations is an IFS, denoted by* $\mathrm{e}_{ij} \bullet \mathrm{e}_{jk}$, *is of the form* $\langle \mathrm{e}_{ik},\ \mu_{2ik},\ \gamma_{2ik}\rangle$ *where*

$$\mu_{2ik} = \max\{\min_j[\mu_{2ij}, \mu_{2jk}]\} \ \textit{and}$$

$$\gamma_{2ik} = \min\{\max_j[\gamma_{2ij}, \gamma_{2jk}]\},\ \textit{for all}\ v_i, v_k \in \mathrm{V}.$$

Definition 13. *Let* $\langle \mathrm{e}_{ij}, \mu_{2ij}, \gamma_{2ij}\rangle$ *be an edge relation on* V. *Then it is said to be*

(i) *reflexive if* $\langle \mathrm{e}_{ii}, \mu_{2ii}, \gamma_{2ii}\rangle = \langle v_i, \mu_{1i}, \gamma_{1i}\rangle$ *for all* $v_i \in \mathrm{V}$.
(ii) *symmetric if* $\langle \mathrm{e}_{ij}, \mu_{2ij}, \gamma_{2ij}\rangle = \langle \mathrm{e}_{ji}, \mu_{2ji}, \gamma_{2ji}\rangle$, *for all* $v_i, v_j \in \mathrm{V}$.
(iii) *transitive if the edge relations* (v_i, v_j) *and* (v_j, v_k) *imply the edge relation* (v_i, v_k).

Definition 14. *The powers of edge relation* e_{ij} *are defined as*

$$\mathrm{e}^1_{ij} = \mathrm{e}_{ij} = \langle \mathrm{e}_{ij}, \mu_{2ij}, \gamma_{2ij}\rangle$$
$$\mathrm{e}^2_{ij} = \mathrm{e}_{ij} \bullet \mathrm{e}_{ij} = \left\langle \mathrm{e}_{ij}, \mu^2_{2ij}, \gamma^2_{2ij}\right\rangle$$
$$\mathrm{e}^3_{ij} = \mathrm{e}_{ij} \bullet \mathrm{e}_{ij} \bullet \mathrm{e}_{ij} = \left\langle \mathrm{e}_{ij}, \mu^3_{2ij}, \gamma^3_{2ij}\right\rangle \ \textit{and so on.}$$

Also,

$$\mathrm{e}^\infty_{ij} = \left\langle \mathrm{e}_{ij}, \mu^\infty_{2ij}, \gamma^\infty_{2ij}\right\rangle$$

where $\mu^\infty_{2ij} = \max\limits_{k=1,2,\dots n}\{\mu^k_{2ij}\}$ *and* $\gamma^\infty_{2ij} = \min\limits_{k=1,2,\dots n}\{\gamma^k_{2ij}\}$ *are the* μ*-strength and* γ*-strength of connectedness between any two vertices* v_i *and* v_j.

Also,

$$e^0_{ij} = \begin{cases} 0, & \text{if } v_i \neq v_j, \\ \langle v_i, \mu_{1\mathrm{i}}, \gamma_{1\mathrm{i}}\rangle, & \text{if } v_i = v_j. \end{cases}$$

Theorem 3. *If* $\mathrm{H} = \langle V', \mathrm{E}'\rangle$ *is an IF subgraph of* $\mathrm{G} = \langle \mathrm{V}, \mathrm{E}\rangle$, *then for some* $(v_i, v_j) \in \mathrm{E}$, $\mu'^\infty_{2ij} \leq \mu^\infty_{2ij}$ *and* $\gamma'^\infty_{2ij} \geq \gamma^\infty_{2ij}$.

Proof. By given, $\mathrm{V}' \subseteq \mathrm{V}$ and $\mathrm{E}' \subseteq \mathrm{E}$.

$$\Rightarrow\ \mu'_{1i} \leq \mu_{1i}; \gamma'_{1i} \geq \gamma_{1i}, \text{for every } v_i \in \mathrm{V} \qquad \dots\dots(11)$$

and

$$\mu'_{2ij} \leq \mu_{2ij}; \qquad \dots\dots(12)$$
$$\gamma'_{2ij} \geq \gamma_{2ij} \qquad \dots\dots(13)$$

for every $v_i, v_j \in \mathrm{V}$.

Consider a path $v_1v_2\ldots v_n$ of H.
Here,

$$\mu_{2ij}^{'\infty} = \min_{k=1,2,\ldots n}\left\{(\mu'_{2ij})^k\right\} \qquad \ldots\ldots (14)$$

$$\gamma_{2ij}^{'\infty} = \max_{k=1,2,\ldots n}\left\{(\gamma'_{2ij})^k\right\} \qquad \ldots\ldots (15)$$

and

$$\mu_{2ij}^{\infty} = \min_{k=1,2,\ldots n}\left\{(\mu_{2ij})^k\right\} \qquad \ldots\ldots (16)$$

$$\gamma_{2ij}^{\infty} = \max_{k=1,2,\ldots n}\left\{(\gamma_{2ij})^k\right\} \qquad \ldots\ldots (17)$$

Therefore, we have

$$\begin{aligned}\mu_{2ij}^{'\infty} &= \min_{k=1,2,\ldots n}\left\{(\mu'_{2ij})^k\right\}\\ &\leq \min_{k=1,2,\ldots n}\left\{(\mu_{2ij})^k\right\},\ \text{by (12)}\\ &= \mu_{2ij}^{\infty}.\end{aligned}$$

Also,

$$\begin{aligned}\gamma_{2ij}^{'\infty} &= \max_{k=1,2,\ldots n}\left\{(\gamma'_{2ij})^k\right\}\\ &\geq \max_{k=1,2,\ldots n}\left\{(\gamma_{2ij})^k\right\}, \text{by (13)}\\ &= \gamma_{2ij}^{\infty}.\end{aligned}$$

Hence proved. □

Definition 15. *Let* $\mathrm{G} = \langle \mathrm{V}, \mathrm{E}\rangle$ *be an IFG. Let* v_i, v_j *be any two distinct vertices and* $\mathrm{H} = \langle \mathrm{V}', \mathrm{E}'\rangle$ *be an IF subgraph of* G *obtained by deleting the edge* (v_i, v_j).

That is, $\mathrm{H} = \langle \mathrm{V}', \mathrm{E}'\rangle$, *where*

$$\begin{aligned} &\mu'_{2ij} = 0 \text{ and } \gamma'_{2ij} = 0\\ \text{and} \quad &\mu'_2 = \mu_2\\ &\gamma'_2 = \gamma_2 \text{ for all other edges.}\end{aligned}$$

Now, (v_i, v_j) *is said to be a bridge in* G, *if either* $\mu_{2\mathrm{xy}}^{'\infty} < \mu_{2xy}^{\infty}$ *and* $\gamma_{2xy}^{'\infty} \geq \gamma_{2xy}^{\infty}$ *or* $\mu_{2\mathrm{xy}}^{'\infty} \leq \mu_{2xy}^{\infty}$ *and* $\gamma_{2xy}^{'\infty} > \gamma_{2xy}^{\infty}$, *for some* v_x, $v_y \in \mathrm{V}$.

In other words, deleting an edge (v_i, v_j) reduces the strength of connectedness between some pair of vertices (or) $(v_i,\ v_j)$ is a bridge if, there exists v_x, v_y such that, $(v_i,\ v_j)$ is an edge of every strongest path from v_x to v_y.

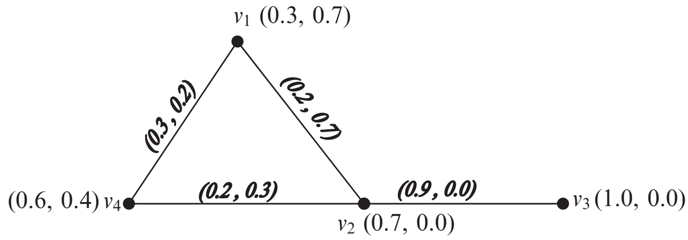

Fig. 7. (v_1, v_4) is a bridge

Example 7. Let $\mathrm{V} = \{v_1, v_2, v_3, v_4\}$.

In Fig. 7, the strength of $\mathrm{P} = v_1v_4$ in G is (0.3, 0.2). Also, the strength of $\mathrm{P}' = v_1v_2v_4$ is (0.2, 0.3). Here, (v_1, v_4) is a bridge, because if we delete (v_1,v_4) from G, the strength of the connectedness between v_1 and v_4 in $\mathrm{G} - (v_1, v_4)$ is decreased.

Theorem 4. *Let* $\mathrm{G} = \langle \mathrm{V}, \mathrm{E} \rangle$ *be an IFG. For any two vertices* v_i, v_j *in* G, *the following conditions are equivalent:*

(i) (v_i, v_j) *is a bridge.*
(ii) $\mu_{2ij}^{'\infty} < \mu_{2ij}$ *and* $\gamma_{2ij}^{'\infty} > \gamma_{2ij}$.
(iii) (v_i, v_j) *is not an edge of any cycle.*

Proof. (ii) $\Rightarrow$ (i).

$$\text{Assume } \mu_{2ij}^{'\infty} < \mu_{2ij} \text{ and } \gamma_{2ij}^{'\infty} > \gamma_{2ij}.$$

To prove (v_i, v_j) is a bridge. If (v_i, v_j) is not a bridge, then

$$\mu_{2ij}^{'\infty} = \mu_{2ij}^{\infty} \geq \mu_{2ij}, \text{ and } \gamma_{2ij}^{'\infty} = \gamma_{2ij}^{\infty} \leq \gamma_{2ij}$$

which implies $\mu_{2ij}^{'\infty} \geq \mu_{2ij}$ and $\gamma_{2ij}^{'\infty} \leq \gamma_{2ij}$, a contradiction.

Hence, (v_i, v_j) is a bridge.

(i) $\Rightarrow$ (iii)

Assume (v_i, v_j) is a bridge. To prove (v_i, v_j) is not an edge of any cycle.

If (v_i, v_j) is an edge of a cycle, then any path involving the edge (v_i, v_j) can be converted into a path not involving (v_i, v_j) by using the rest of the cycle as a path from v_i to v_j. This implies (v_i, v_j) cannot be a bridge which is a contradiction to our assumption. Therefore, (v_i, v_j) is not an edge of any cycle.

(iii) $\Rightarrow$ (ii)

Assume (v_i, v_j) is not an edge of any cycle.

To prove $\mu_{2ij}^{'\infty} < \mu_{2ij}$ and $\gamma_{2ij}^{'\infty} > \gamma_{2ij}$.

Assume that $\mu_{2ij}^{'\infty} \geq \mu_{2ij}$ and $\gamma_{2ij}^{'\infty} \leq \gamma_{2ij}$. Then, there is a path from v_i to v_j not involving $(v_i,\, v_j)$ that has strength greater than or equal to μ_{2ij} and less than or equal to γ_{2ij} and this path together with $(v_i,\, v_j)$ forms a cycle which is a contradiction. Hence, $\mu_{2ij}^{'\infty} < \mu_{2ij}$ and $\gamma_{2ij}^{'\infty} > \gamma_{2ij}$. Therefore, the statements (i), (ii) and (iii) are equivalent. □

Theorem 5. *Let* $\mathrm{G} = \langle \mathrm{V}, \mathrm{E} \rangle$ *be an IFG with the set of vertices* V. *Then*

(i) If μ_{2ij} *and* γ_{2ij} *are constants for all* v_i , $v_j \in \mathrm{V}$, *then* G *has no bridge.*
(ii) If μ_{2ij} *and* γ_{2ij} *are not constants for all* $(v_i, v_j) \in \mathrm{E}$, *then* G *has at least one bridge.*

Proof. (i) Let μ_{2ij} and γ_{2ij} are constants for all v_i , $v_j \in \mathrm{V}$.

Let $\mu_{2ij} = \mathrm{c}_1$ and $\gamma_{2ij} = \mathrm{c}_2$ for all v_i , $v_j \in \mathrm{V}$, where $0 \leq \mathrm{c}_1 \leq 1$ and $0 \leq \mathrm{c}_2 \leq 1$.

In this IFG, since each edge has the same weight (the degree of membership and nonmembership values of an edge), deleting any edge does not reduce the strength of connectedness between any pair of vertices. Hence, G has no bridge.

(ii) Assume that μ_{2ij} and γ_{2ij} are not constants for all $(v_i, v_j) \in \mathrm{E}$.

Choose an edge $(v_x, v_y) \in \mathrm{E}$ such that

$$\begin{aligned} &\mu_{2\mathrm{xy}} = \max\{\mu_{2ij}\} \\ &\gamma_{2\mathrm{xy}} = \min\{\gamma_{2ij}\}, \text{ for all } v_i, v_j \in \mathrm{V}. \\ &\text{Therefore, } \mu_{2\mathrm{xy}} > 0 \text{ and } \gamma_{2\mathrm{xy}} < 1. \end{aligned}$$

There exists at least one edge $(v_\mathrm{s},\, v_\mathrm{t})$ distinct from $(v_\mathrm{x},\, v_\mathrm{y})$ such that

$$\mu_{2\mathrm{st}} < \mu_{2\mathrm{xy}} \text{ and } \gamma_{2\mathrm{st}} > \gamma_{2\mathrm{xy}}.$$

We claim that $(v_\mathrm{x},\, v_\mathrm{y})$ is a bridge of G. For, if we delete the edge $(v_\mathrm{x},\, v_\mathrm{y})$, then the strength of connectedness between v_x and v_y in the IF subgraph thus obtained is decreased. In other words, $\mu_{2\mathrm{xy}}^{\prime\infty} < \mu_{2\mathrm{xy}}$ and $\gamma_{2\mathrm{xy}}^{\prime\infty} > \gamma_{2\mathrm{xy}}$.

Therefore, by Theorem 4, $(v_\mathrm{x},\, v_\mathrm{y})$ is a bridge of G. □

Corollary 1. *In an IFG,* $\mathrm{G} = \langle \mathrm{V}, \mathrm{E} \rangle$ *for which* $\mu_2 : \mathrm{V} \times \mathrm{V} \longrightarrow [0, 1]$ *and* $\gamma_2 : \mathrm{V} \times \mathrm{V} \longrightarrow [0, 1]$ *are not constant mapping, an edge* (v_i, v_j) *for which* μ_{2ij} *is maximum and* γ_{2ij} *is minimum . Therefore it is a bridge of* G.

Definition 16. *A vertex* v_i *is said to be a cut-vertex in* G *if deleting a vertex* v_i *reduces the strength of connectedness between some pair of vertices or* v_i *is a cut vertex if and only if there exists* v_x, v_y *such that* v_i *is a vertex of every strongest path from* v_x *to* v_y.

In other words, $\mu_{2\mathrm{xy}}^{\prime\infty} \leq \mu_{2\mathrm{xy}}$ *and* $\gamma_{2\mathrm{xy}}^{\prime\infty} < \gamma_{2\mathrm{xy}}$ *(or)* $\mu_{2\mathrm{xy}}^{\prime\infty} < \mu_{2\mathrm{xy}}$ *and* $\gamma_{2\mathrm{xy}}^{\prime\infty} \leq \gamma_{2\mathrm{xy}}$ *for some* v_x, $v_y \in \mathrm{V}$.

Example 8. Let $\mathrm{V} = \{v_1, v_2, v_3, v_4, v_5\}$.

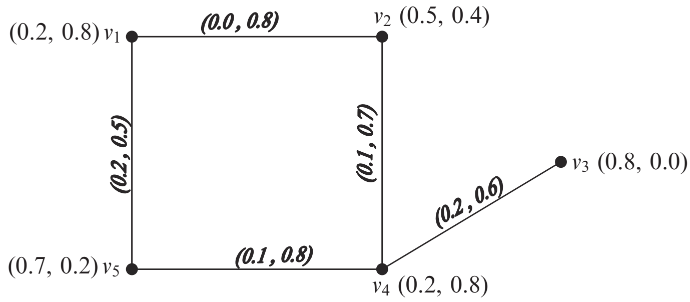

Fig. 8. v_1 is a cut-vertex

4 Conclusion

In this paper, the intuitionistic fuzzy extension of some known concepts of fuzzy graphs has been investigated. Much more work could be done to investigate the structure of IFG. It would be useful, since IFGs have applications in pattern clustering and network analysis which in turn would have applications in telecommunications. In this work, we have restricted our discussion to the first type IFS. It is also proposed to extend these concepts on the other extensions of IFSs.

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