Max-Product Fuzzy Relational Equations as Inference Engine for Prediction of Textile Yarn Properties

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Summary. This work presents first practical implementation of a new algorithm for solving max-product fuzzy relational equations as inference engine. The original, analytical provided procedure computes the greatest solution and the set of all minimal solutions, in case of consistency. In case of inconsistency, which presents not adequate knowledge base or not adequate case for solution, the equations, that correspond to the unsatisfied rules, are obtained. The algorithm is implemented for solving max-product fuzzy linear system for predicting properties of textile yarns, but these systems as inference engine are applicable in wide range of areas. Several methodology problems of the practical implementation like the type of membership functions, relation coefficients, dealing with multiple interactions are presented.

Key words: Inference engine, Inverse problem resolution, Max-prod composition, Textile yarn.

1 Introduction

There are several approaches for building the inference engine. Those of them, which follow directly programmed "if-then" rules are suitable for the diagnosis problems, but not for engineering applications for prediction of certain product properties as function of the technological process. Most popular are the neuronal networks, often combined with fuzzy input and/or output. They are very powerful because of their multilayer nonlinear approximation nature, but they do not present clearly and user-friendly the knowledge base. The most important disadvantage for the current case is the impossibility to work in inverse direction, for backward reasoning. The fuzzy linear equations present clear definitions of the relations between output and input for a given system. Due to further development of the theory, they can be successfully used in both the directions: for forward reasoning – for calculating the outputs when the inputs and the relation matrix are given, and for backward reasoning – to calculate which input has to be provided in order to receive certain output.

There are several applications of fuzzy relational equations in the textile engineering. Some of them use fuzzy max–min linear systems for diagnostics [1–4]. The max–min composition is suitable for a specific kind of reasoning, if the main task is to establish whether some events are present or not, taking into account the fuzziness of the used data. For predicting the properties of the materials or for similar engineering applications, often proportionality between the variables has to be present. In this case max-product law of composition provides a suitable mathematical description of the relations among the physical parameters. The direct problem (calculation of the max-product composition between the input vector X and the weight matrix A) is trivial, but for solving inverse problem there are still open fields for researching. Here we present first practical implementations of the algorithm presented in [5]. It uses algebraic-logical approach by using objects for representing the way of thinking of the man by solving fuzzy equations and is based on universal algorithm [6], developed as an extension of the theory and software in [7]. In Sect. 2 is explained basically how to use the fuzzy linear systems as inference engine. After that, in Sect. 3 are described some methodology problems, which we had to solve during the practical implementation of the fuzzy linear system as inference engine. At the end is presented a short example.

2 Max-Prod Fuzzy Linear Equations as Inference Engine

2.1 Mathematical Model

The properties of the textile yarns depend on a large number of parameters. Detailed experimental investigations over influence of some preparation processes over the parameters of the fibers sliver [8], some machine construction and adjustments [9,10], as well as the complex numerical and experimental investigations of the drafting process [11] prove, that predicting the yarn properties requires a complex mathematical model.

Formalized description of the process of prediction of yarn properties is presented in Fig. 1, where are mentioned only the most important input and output parameters. Let us present the process of prediction as a general system with N inputs x_i and M outputs b_j, where $i = 1 \cdots N$ and $j = 1 \cdots M$. In the yarn production is important to have estimation of the maximum possible strength of the yarn for a certain material. At the same time, the yarn irregularity CV, as well as the yarn hairiness have to remain in certain limits. All these requirements are output parameters. They depend on a set of input parameters, like machine adjustment, working speed, material preparation, temperature and humidity in the rooms etc. The relations between all inputs and outputs are usually nonlinear and they include complex interactions among several single inputs. One full experimental investigation of these relations by using design of experiments requires a large number of tests, which is not usually possible in industrial conditions and is time and resources consuming. On the other side, the complexity and multiscaling of the real problems particularly for spinning and for other textile processes, complicates building of phenomenological models, which represent the physics of all interactions. Our

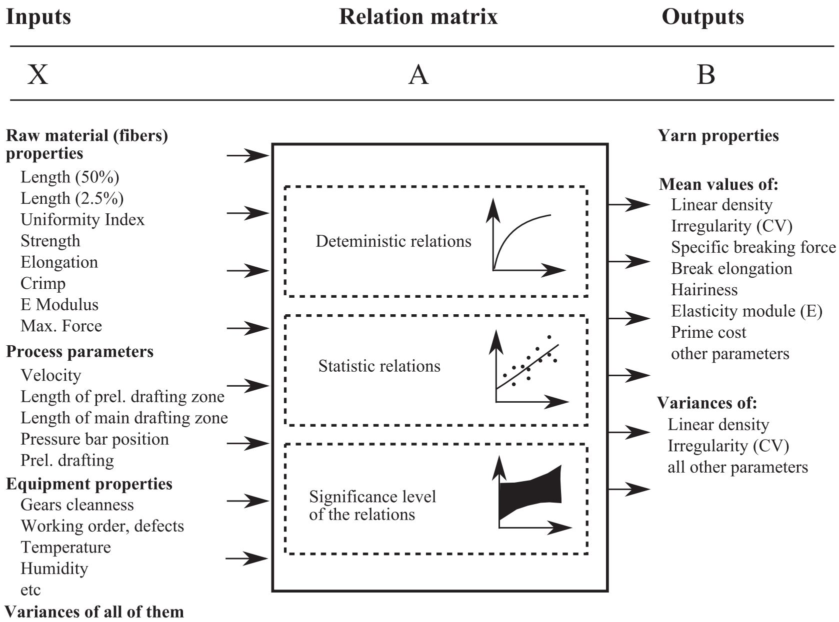

Fig. 1. Inputs, relation matrix and outputs when building a system for prediction of the properties of textile yarns

goal is to create a simple, fast and user friendly model, which consists of the main important relations between single inputs and outputs, and allows both forward and backward reasoning. The fuzzy linear system of equations fulfill these requirements, as for the case most appropriate is the use of *max-product* composition.

Let the relation between all inputs $X = x_i$ and the output b_j is presented with the equation

$$(a_{j1}. x_1) \vee \cdots \vee (a_{j\,n-1}. x_{n-1}) \vee (a_{j\,n}. x_n) = b_j, \qquad (1)$$

where $\vee$ denotes *max* operator and . – multiplication. The complete system for all outputs is

$$\left| \begin{array}{ccccc} (a_{11}. x_1) & \vee \cdots \vee & (a_{1\,n}. x_n) & = & b_1 \\ \cdots & \cdots & \cdots & & \cdots\cdots \\ (a_{m\,1}. x_1) & \vee \cdots \vee & (a_{m\,n}. x_n) & = & b_m \end{array} \right. , \qquad (2)$$

written in the following equivalent matrix form

$$A \odot X = B,$$

where $A = (a_{ij})_{m \times n}$ stands for the matrix of coefficients, $X = (x_j)_{n \times 1}$ stands for the matrix of unknowns, $B = (b_i)_{m \times 1}$ is the right side of the system. For each $i, 1 \leq i \leq m$ and for each $j, 1 \leq j \leq n$, we have a_{ij}, b_i, $x_j \in [0, 1]$ and the max-prod composition is written as $\odot$.

The coefficients a_{ij} matematically represent the influence (weight) of the input x_i over the output b_j. In industry, the experts are looking for the best performance, quality or seek the reasons for the worst cases. The best and worst cases of one output b_j correspond to its maximal and minimal value. Looking for minimum can be inverted to looking for maximum [7], thus we will work furtherly only with the maximal value.

2.2 Forward and Backward Schemes of Reasoning

We suppose that the relations' matrix A and one vector X with input parameters, for example raw material, equipment and process data are given. The estimation of the output parameters requires only computing of the composition of the left side of (1). This corresponds to the forward scheme of reasoning, or to the so called direct problem. This way of calculation is fast, because is connected to one matrix composition. It is useful for predicting of the properties of the yarn, when the different materials, process parameters or machine equipment are used. One can define goal function for some elements of the vector B and to start optimization problem, looking for the most suitable inputs X. Such optimization is often not effective, because the system (1) can have a large number of solutions and the standard optimization algorithms will find only a local solution. More effective is the optimization, when is used backward scheme of reasoning.

In this case, used also for diagnosis problems, we have to solve the *inverse* problem, finding all solutions of (1) for given outputs B and relations A. If the system (1) has solutions, it has one greatest and one or lots of lower solutions. The lower solutions can be interpretated as the cheapest and the worst material, which can be used for production of the yarn with the required in B quality and properties. The greatest solution gives the best (and expensive) material, which still will lead to producing the yarn with the same properties. The interval solutions of the system build the range of variations of the input parameters, where the output will remain unchanged. The interval solutions are of great importance for the application engineers, as they show which input parameters can be changed without loss of quality.

The application of the backward scheme of reasoning for optimization is a little bit different from the optimization with forward scheme. Here the required (maximum) values in B have to be given initially. Then, solving the system (1), all interval solutions can be calculated. For the experts remains the task to select this or these from the solutions X, which are more effective.

2.3 Solution Notes

If $A \odot X = B$ is consistent, it has unique greatest solution $X_{\text{gr}} = A^t \diamond B$ [5,13–15].

The $\diamond$-*product* of matrices A and B is in general defined as matrix $C = (c_{ij})_{m \times n} = A \diamond B$, if

$$c_{ij} = \min_{k=1}^{p}(a_{ik} \diamond b_{kj}), \text{when } 1 \leq i \leq m, 1 \leq j \leq n.$$

In our case B is a vector and C becomes vector too, and the A^t denotes transpose matrix of the A, where $A^t = (a^t_{ij}) = a_{ji}$.

A program implementation of exact method and an algorithm for solving the system $A \odot X = B$ for the unknown X is explained in [5, 7]. As much as possible improvements over the straightforward exhaustive depth search of this NP-hard problem are obtained. Rather than work with the system $A \odot X = B$, is used a matrix, whose elements capture all the properties of the equations. In depth first search, it is proposed how to drop branches that do not lead to minimal solutions. A sequence of simplification rules is defined, which brings the matrix into a new form. Once in this form, dominance is applied to remove redundancy. In this manner the time complexity of an exhaustive search is reduced merely by making a more clever choice of the objects over which the search is performed. This provides an easy finding of the complete solution of the original system.

3 Implementation Methodology

3.1 Membership Functions

The selection of the membership functions depends on the specificity of the problem.

For instance the yarn strength depends on the yarn twist nearly quadratically, as the type of the real function is presented in Fig. 2. In this case, we split the single input variable *twist* into new four input variables, which can be named like "very low","low","normal", and "high" twist, and which have also a unique range. Furtherly, we input four output variables for yarn strength, no matter that, for the forward reasoning this is not obligatory. These four output variables in this case are required, because some of the physical value of the yarn strength can be obtained for **two** different values of the variable "twist" – one before the maximum and one after the maximum. With the additional new output variables can be exactly specified, if the value is "high strength, but under the critical twist" or "high strength, but above the critical twist."

The next not typically used is the type of certain membership functions. For the above mentioned case (Fig. 2) the physical input variable increases monotonously and there is no overlapping between the local areas. For such type of parameters we used "saw" – like membership functions for normalization of the input variables (Fig. 3, left). Of course a lot of parameters work

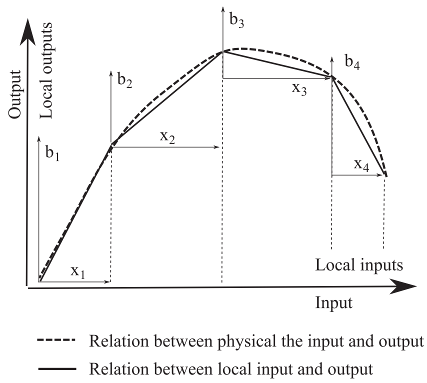

Fig. 2. Local approximation and setting up of new variables for the nonlinear relationships between input and output. The presented curve is typical for the relationship between yarn twist and yarn strength

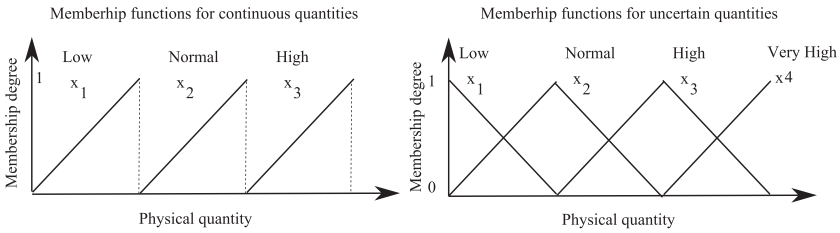

Fig. 3. Membership functions for continuous input variable without overlapping (*left*) and triangular membership functions for uncertain variables, like "state of the machine," connected with not exact definition of the state of working parts, gears, dirtiness level etc. (*right*)

well with the standard type functions (Fig. 3, right), for which more explanation can be found in almost all introductional literature about fuzzy logic, as for instance [12].

3.2 Coefficients of the Relation Matrix

The most important key point for expert system building is the selection of the proper structure of the relation matrix A. Its coefficients a_{ij} are obtained from experts, using mechanical models of the system for some of the relations or experimental results for the more complicated ones. They are divided into three groups, depending on the type of the relation between the input and output variables in the system (2):

- Physical (deterministic) relation between the inputs and outputs. Example: during the drafting process the fiber sliver becomes longer and finer. The drafting ratio I connects the input T_{inp} and output T_{out} fineness, as

$T_{out} = T_{in} \cdot I$. For this case, the corresponding coefficient in the system (1) has value $a_{ij} = I$, if the finenesses T_{out} and T_{in} are normalized.

– Stochastic correlation between the input and output parameters. Example: at higher working speed of the machine V the irregularity of the sliver CV becomes higher, too. This relation is not well (jet) deterministically described, but exists enough statistical data as a proof of its significance (Fig. 4, left). At the same time the speed of the machine does not influence significantly the mean fiber length $L_{50\%}$, and in this case we will set correspondent $a_{ij} = 0$.
– Relation between the variances of the input and output parameters. This can be the case not only for the stochastic relation between parameters, but also for the deterministic ones. The models usually do not describe all the influences, like the humidity and temperature of the room, some defects in the gears, sticking of the dust, which is usual for the textile production. We use the variances of all input and output variables, and input their relation in the corresponding coefficient a_{ij}. Here, in general can be assumed that $a_{ij} = 1 - r_{ij}^2$, where r_{ij} is the correlation coefficient of the regression equation for the connection between input x_i and output b_j. In this system all the input and output parameters are analyzed as pairs "parameter–variance": $x_i - x_{K+i}$, where K is the number of the independent input variables, $2K \leq N$. The use of the additional variables for the variances per input and output parameters makes the fuzzy linear system two times larger (actually four, but the half of the coefficients are zeros), but the variances are required for proper description of the processes. Example of confidential area of the influence between input "machine speed" and output "yarn mass irregularity" is presented on the Fig. 4 (right).

3.3 Significant Multiple Interactions

In some cases the interactions between two or more input parameters are very important. They can not be properly modeled by the system (2) and for

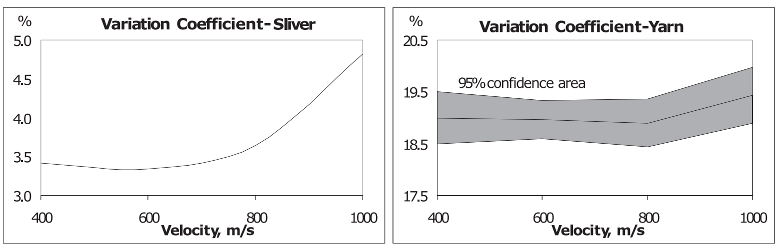

Fig. 4. Variation coefficient of the sliver (intermediate half-finished product), *left*, depending nonlinear on the machine velocity with very high degree of correlation. The same coefficient for the final product – yarn on the *right figure*, has the same trend, but with quite large confidence interval

this reason we used the idea for the multilayers from the neural networks. If the interaction between variables x_i and x_k is significant, we build new one, composite variable, $x_l = x_i \cdot x_k$. We use such composite variables during the solution of (1) formally as independent variables, but after that, during the decoding of the results, these variables require some additional operations and checks about possible logical contradictions.

4 Numerical Example

The relation matrix when working with industrial problems is usually bigger than 15×10, which is not convenient for printing. Because of this, the realization of forward and backward reasoning in the MATLAB environment are presented here as an example with highly reduced size. Let the relation matrix A is given as

$$A = \left(\begin{array}{ccc|ccc} 0.0 & 0.9 & 1.0 & 0.0 & 0.0 & 0.0 \\ 0.1 & 0.6 & 0.3 & 0.0 & 0.0 & 0.0 \\ 0.8 & 0.4 & 0.2 & 0.0 & 0.0 & 0.0 \\ \hline 0.0 & 0.0 & 0.0 & 0.4 & 0.8 & 0.1 \\ 0.0 & 0.0 & 0.0 & 0.2 & 0.1 & 1.0 \\ 0.0 & 0.0 & 0.0 & 0.4 & 0.1 & 0.8 \end{array}\right)$$

Here is demonstrated the block-architecture of this matrix, where the upper left block represents the coefficients of the relation between the input and output variables, the bottom right block – the coefficient for the variances between these variables. For forward scheme of reasoning, we need the outputs B, if the inputs X are given. For instance $X = \left(0.3\ 0.9\ 1.0\ 0.2\ 0.0\ 0.6\right)^t$. Here we have to compute the *max-prod* composition $B = A \odot X$, which in the MATLAB environment using the library, described in [5] is simple:

```
>> B=fuzzy_maxprod(A,X')

B =

    1.0000
    0.5400
    0.3600
    0.0800
    0.6000
    0.4800
```

Let us solve the inverse example, typical for the backward reasoning, asking – which inputs X have to be used, in order to receive the presented output B? The solver calculates the greatest and finds two lower solutions

```
>> s=solvedot(A,B)
 greatest solution - transposed
    0.4500    0.9000    1.0000    0.2000    0.1000    0.6000

 lower solutions - transposed
         0    0.9000    1.0000    0.2000         0    0.6000
         0    0.9000    1.0000         0    0.1000    0.6000
```

which builds two interval solutions of the problem

$$X_1 = \begin{pmatrix} [0,\ 0.45] \\ 0.9 \\ 1 \\ 0.2 \\ [0,\ 0.1] \\ 0.6 \end{pmatrix}, X_2 = \begin{pmatrix} [0,\ 0.45] \\ 0.9 \\ 1 \\ [0,\ 0.2] \\ 0.1 \\ 0.6 \end{pmatrix}.$$

This example demonstrates, that for certain relation matrix A there are three inputs variables, which value can be changed and despite of this to receive the same outputs B. The variable x_1 has to be between 0 and 0.45, which means, that it has no significant influence over the outputs in this case. The variables x_4 and x_5 are connected – one of them can vary in some limits if the other one is fixed. On the language of the application engineers this solution set means, that we can obtain the same yarn properties, by relative large variation of the input parameters x_1 and careful choice between the variation of one of the inputs x_4 or x_5.

5 Discussion

In order to take into account the spread of the parameters, that is usually for the textile products, we use the variances for almost all input and output quantities as variables in the system, too. This increases the size of the system and worsens the clarity of the knowledge presentation. Better approach can be realization of the system by the means of logic [2, 7], where probably by using membership and nonmembership degrees, can be modeled the spread of the investigated variables as well. The use of intuitionistic approach formally would lead to saving the number of variables, but the amount of the data and calculations will be again almost identical with the presented here. The use of the intuitionistic approach still requires some additional development in the theory and software, which did not allow us to implement it.

6 Conclusions

The use of the *max-prod* fuzzy linear systems of equation as inference engine is explained. The practical implementation of these systems requires additional knowledge about the selection of the membership functions, presenting the highly nonlinear relations, spread of the variables, as well as the ways for filling the relation matrix. Interpretations of the mathematical model, from the point of view of the prediction of textile yarn properties are given, but the model is applicable in a wide range of areas.

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References

1. Lin J-J, Lin C-H, Tsai I-S (1995) Textile Research Journal 65(12):697–709
2. Kyosev Y (2003) Notes on Intuitionistic Fuzzy Sets, 9(4): 135–139
3. Kyosev Y (2003) Diagnostics of Sewing Process Using Fuzzy Linear Systems, in: 5-th International Conference Textile Science TEXSCI 2003, June 16–18, Liberec, CD-ROM Edition
4. Kyosev Y (2003) Einsatz der Fuzzy Logik zur Diagnostik der Streckprozesse und Streckwerksvorrichtung, in: 9. Chemnitzer Textilmaschinen-Tagung '03, 21–22 November 2003, Chemnitz, pp. 225–226
5. Peeva K, Kyosev Y(2006), Soft computing – A Fusion of Foundations, Methodologies and Applications, URL: http//dx.doi.org/10.1007/s00500006-0103-5
6. Peeva K (2006) Italian Journal of Pure and Applied Mathematics 19: 9–20
7. Peeva K, Kyosev Y (2004) Fuzzy Relational Calculus-Theory, Applications and Software (with CD-ROM), In: Advances in Fuzzy Systems – Applications and Theory, vol. 22, World Scientific, New Jersey
8. Angelova R, Ivanova T, Velickova G (2005) On the influence of the preprocessing in rotor spinning method: quality of the slivers, in: Kyosev Y, Angelova R, (ed.): Textile and clothing – research, production, education, Technical University of Sofia, Sofia, pp. 60–66
9. Reinbach I (2003) Analyse der Bandfhrung am Bandtrichter (AiF 13490 N), DWI Reports 2003/127, Paper: P-E2.pdf
10. Reinbach I, Gries T (2004) Einfluss des Vliesfhrers auf die Faserbandungleichmssigkeit (AiF 13490 N), DWI Reports 2004, PC3.pdf
11. Cherif C (2001) Analyse der Verzugsvorgnge auf Hochleistungsstrecken und deren Auswirkungen auf die nachfolgenden textilen Prozessstufen, RWTH Aachen, Habil. Thesis, Shaker
12. Klir G, Clair U, Bo Y (1997), Fuzzy Set Theory Foundations and Applications, Prentice Hall PRT

13. Loetamonphong J, Fang S (1999) IEEE Transactions on Fuzzy Systems 7(4): 441–445
14. Loetamonphong J, Fang S (2001) Fuzzy Sets and Systems 118(3): 509–517
15. Di Nola A, Lettieri A (1989) Relation Equations in Residuated Lattices, Rendiconti del Circolo Matematico di Palermo, s. II, XXXVIII, pp. 246–256