Interval Evaluations in DEA and AHP

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Summary. Even if the given data are crisp, there exists uncertainty in decision making process and inconsistency based on human judgements. The purpose of this paper is to obtain the evaluations which reflect such an uncertainty and inconsistency of the given information. Based on the idea that intervals are more suitable than crisp values to represent evaluations in uncertain situations, we introduce this interval analysis concept into two well-known decision making models, DEA and AHP. In the conventional DEA, the relative efficiency values are measured and in the proposed interval DEA, the efficiency values are defined as intervals considering various viewpoints of evaluations. In the conventional AHP, the priority weights of alternatives are obtained and in the proposed interval AHP, the priority weights are also defined as intervals reflecting the inconsistency among the given judgements.

Key words: Decision making, Uncertain information, Efficiency interval, Interval priority weight

1 Introduction

In the decision making problem involving human judgements, usually the information is uncertain, even if the data are given as crisp values. Through most of the conventional decision making models, the results such as evaluations from the given data are obtained as crisp values. However, there exists uncertainty in the decision making process involved in different viewpoints, human intuitive judgements and fuzzy environments. It seems to be suitable to obtain the evaluations as intervals in order to reflect various uncertainty in the given data and evaluating process. In this viewpoint, the concept of interval analysis is introduced into DEA(Data Envelopment Analysis) and AHP(Analytic Hierarchy Process), which are well-known evaluation models. DEA is relative evaluation model to measure the efficiency of DMUs (Decision Making Units) with common input and output terms. In Section 2, we propose

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Interval DEA, where the efficiency value calculated from various viewpoints for each DMU are considered and efficiency intervals are obtained. AHP is the useful method to obtain the priority weight of each item in multiple criteria decision making problems. In Section 3, we propose Interval AHP, where inconsistency based on human intuition in the given pairwise comparisons are considered and interval weights are obtained. In this paper, evaluations as results with crisp data through models are obtained as intervals reflecting the uncertainty of the given data and the decision making process. Interval evaluations are more useful information for decision making and helpful for decision makers than crisp evaluations, since the former can consider various viewpoints and inconsistency in the given data.

2 Interval DEA

2.1 Efficiency Value by Conventional DEA

DEA (Data Envelopment Analysis) is a non-parametric technique for measuring the efficiency of DMUs (Decision Making Units) with common input and output terms [1, 2]. In DEA, the efficiency for DMU_o which is the analyzed object is evaluated by the following basic fractional model.

$$\begin{aligned} \theta_o^{E*} &= \max_{\boldsymbol{u},\boldsymbol{v}} \frac{\boldsymbol{u}^t \boldsymbol{y}_o}{\boldsymbol{v}^t \boldsymbol{x}_o} \\ \text{s.t. } & \frac{\boldsymbol{u}^t \boldsymbol{y}_j}{\boldsymbol{v}^t \boldsymbol{x}_j} \leq 1 \quad \forall j \\ & \boldsymbol{u} \geq \boldsymbol{0} \\ & \boldsymbol{v} \geq \boldsymbol{0} \end{aligned} \tag{1}$$

where the decision variables are the weight vectors $\boldsymbol{u}$ and $\boldsymbol{v}$, $\boldsymbol{x}_j \geq \boldsymbol{0}$ and $\boldsymbol{y}_j \geq \boldsymbol{0}$ are the given input and output vectors for DMU_j and the numbers of inputs, outputs and DMUs are m, k and n, respectively.

The efficiency is obtained by maximizing the ratio of weighted sum of outputs to that of inputs for DMU_o under the condition that the ratios for all DMUs are less than or equal to one. To deal with many inputs and outputs, the weighted sum of inputs and that of outputs are considered as a hypothetical input and a hypothetical output, respectively. The maximum ratio of this output to this input is assumed as the efficiency which is calculated from the optimistic viewpoint for each DMU. The efficiency for DMU_o is evaluated relatively by the other DMUs.

This fractional programming problem is replaced with the following linear programming (LP) problem, which is the basic DEA model called CCR (Charnes Cooper Rhodes) model, by fixing the denominator of the objective function to one.

$$\begin{aligned} \theta_o^{E*} &= \max_{\boldsymbol{u},\boldsymbol{v}} \boldsymbol{u}^t \boldsymbol{y}_o \\ \text{s.t. } & \boldsymbol{v}^t \boldsymbol{x}_o = 1 \\ & \boldsymbol{u}^t \boldsymbol{y}_j - \boldsymbol{v}^t \boldsymbol{x}_j \leq 0 \quad \forall j \\ & \boldsymbol{u} \geq \boldsymbol{0} \\ & \boldsymbol{v} \geq \boldsymbol{0} \end{aligned} \tag{2}$$

θ_o^{E*} is obtained with the superior inputs and outputs of DMU_o by maximizing the objective function in (2) with respect to the weight variables. Therefore, it can be said that it is the evaluation from the optimistic viewpoint for DMU_o.

When the optimal value of objective function is equal to one, DMU_o is rated as efficient and otherwise it is not rated as efficient. Precisely speaking, the word "efficient" which we use in this paper is called "weak efficient". In this model the production possibility set is assumed as follows.

$$P = \{(\boldsymbol{x}, \boldsymbol{y}) | \boldsymbol{x} \geq X\boldsymbol{\lambda}, \boldsymbol{y} \leq Y\boldsymbol{\lambda}, \boldsymbol{\lambda} \geq \boldsymbol{0}\} \tag{3}$$

where $X \in \Re^{m \times n}$ is an input matrix consisting of all input vectors, $Y \in \Re^{k \times n}$ is an output matrix consisting of all output vectors. (3) means that the more inputs, smaller outputs or both than those of given data can be productive.

On the other hand, the inefficiency measure has defined by using inverse relation to the ratio defined in DEA in [3], that is the ratio of weighted sum of inputs to that of outputs. Thus, the inefficiency model is called "Inverted DEA". However the ratios considered in DEA and Inverted DEA are different each other so that there is no mathematical relation between the efficiency by DEA and the inefficiency by Inverted DEA. In the literature [4], the maximum and minimum efficiency values for a new DMU have been proposed using the benchmarks obtained by DEA. This approach is called DEA-based benchmarking model and it is an effective measure as an interval for a new DMU, considering a set of the benchmark frontier by DEA. In this paper, the proposed minimum efficiency for each DMU has been defined by using all the given DMUs. We propose Interval DEA [5], where the efficiency intervals are obtained so as to reflect uncertainty in evaluating viewpoints. The following points should be noted: 1) the proposed approach has the same mathematical structures for the maximum and minimum efficiency values, and 2) efficiency interval is obtained by all the given DMUs. These are different from Inverted DEA [3] and DEA-based benchmarking model [4].

2.2 Efficiency Interval

The relative efficiency can be obtained from various viewpoints. In this section, we propose Interval DEA model to obtain the efficiency interval [5]. The efficiency interval is denoted as its upper and lower bounds. Then, they are obtained by solving two optimization problems such that the relative ratio of the analyzed DMU to the others is maximized and minimized with respect to input and output weights, respectively. In both models the same ratios are considered to be maximized and minimized respectively. The upper and lower bounds of efficiency interval denote the evaluations from the optimistic and pessimistic viewpoints, respectively.

Since the conventional DEA can be regarded as the evaluation from the optimistic viewpoint, the upper bound of efficiency interval for DMU_o can be obtained by the conventional CCR model in [1, 8]. Considering the original CCR model formulated as a fractional programming problem (1), the problem to obtain the upper bound of efficiency interval is formulated as follows.

$$\begin{aligned} \theta_o^{E*} &= \max_{\boldsymbol{u},\boldsymbol{v}} \frac{\dfrac{\boldsymbol{u}^t\boldsymbol{y}_o}{\boldsymbol{v}^t\boldsymbol{x}_o}}{\max_j \dfrac{\boldsymbol{u}^t\boldsymbol{y}_j}{\boldsymbol{v}^t\boldsymbol{x}_j}} \\ \text{s.t. } & \boldsymbol{u} \geq \boldsymbol{0} \\ & \boldsymbol{v} \geq \boldsymbol{0} \end{aligned} \tag{4}$$

It should be noted that the denominator in (4) plays an important role of normalizing efficiency value. The ratio of the weighted sum of outputs to that of inputs for DMU_o is compared to the maximum ratio of all DMUs. In (1), the ratios of the weighted sum of outputs to that of inputs for all DMUs are constrained to be less than one for normalization. Furthermore, formulating the upper bound of efficiency interval as (4) is very useful for defining the lower bound of efficiency interval.

When the denominator of the objective function is fixed to one, (4) can be reduced to the following problem.

$$\begin{aligned} \theta_o^{E*} &= \max_{\boldsymbol{u},\boldsymbol{v}} \frac{\boldsymbol{u}^t\boldsymbol{y}_o}{\boldsymbol{v}^t\boldsymbol{x}_o} \\ \text{s.t. } & \max_j \frac{\boldsymbol{u}^t\boldsymbol{y}_j}{\boldsymbol{v}^t\boldsymbol{x}_j} = 1 \\ & \boldsymbol{u} \geq \boldsymbol{0} \\ & \boldsymbol{v} \geq \boldsymbol{0} \end{aligned} \tag{5}$$

Comparing with (5) and (1), the conditions of (5) is stricter than that of (1). However, the optimization problem (5) is equal to (2), which is the original CCR model described as LP problem (see [5]).

On the other hand, by minimizing the objective function in (4) with respect to the weight variables, the lower bound of efficiency interval is obtained by the following problem.

$$\begin{aligned} \theta^E_{o\,*} &= \min_{\boldsymbol{u},\boldsymbol{v}} \frac{\dfrac{\boldsymbol{u}^t\boldsymbol{y}_o}{\boldsymbol{v}^t\boldsymbol{x}_o}}{\max_j \dfrac{\boldsymbol{u}^t\boldsymbol{y}_j}{\boldsymbol{v}^t\boldsymbol{x}_j}} \\ \text{s.t. } & \boldsymbol{u} \geq \boldsymbol{0} \\ & \boldsymbol{v} \geq \boldsymbol{0} \end{aligned} \tag{6}$$

$\theta^E_{o\,*}$ is obtained with inferior inputs and outputs of DMU_o. Therefore, it can be said that it is the evaluation from the pessimistic viewpoint considering all DMUs. The optimization problem (6) can be reduced to the following problem (see [5]).

$$\theta^E_{o\,*} = \min_{p,r} \frac{\dfrac{y_{op}}{x_{or}}}{\max\limits_j \dfrac{y_{jp}}{x_{jr}}} \tag{7}$$

where the rth element of input weight vector $\boldsymbol{v}$ and the pth element of output vector $\boldsymbol{u}$ are one and the other elements are all zero in (6). Only the rth input and pth output are used to determine the lower bound of efficiency interval. These are inferior input and output of DMU_o relatively to the others.

The efficiency interval denoted as $[\theta^E_{o*}, \theta^{E*}_o]$ illustrates all the possible evaluations for DMU_o from various viewpoints. Thus, Interval DEA gives a decision maker all the possible efficiency values that reflect different perspectives. The efficiency intervals are important and useful information to a decision maker in a sense of perspectives.

In order to improve the efficiency, in the conventional DEA the efficiency value is obtained as a real value and the inputs and outputs are adjusted to make the efficiency value be one. Several approaches to improvement for the conventional efficiency value by adjusting inputs and outputs have been proposed in [4, 6, 7, 8, 9] On the contrary, in Interval DEA the efficiency is obtained as an interval. The given inputs and outputs are adjusted so that the efficiency interval with the adjusted ones become larger than one before the improvement. The approach to improve the efficiency interval can be described in [10]. It is done with the following way: the upper bound of efficiency interval becomes one and the lower one becomes as large as possible. It can be said that the superior inputs and outputs are shown as a target for each improved DMU.

2.3 Numerical Example

We calculate the efficiency interval by using one-input and two-output data in Table 1. Efficiency intervals determined by (2) and (7) are shown in Table 1 and Fig. 1. The conventional efficiency value and the upper bound of efficiency intervals are the same, since both of them are obtained from the optimistic viewpoint by (2).

Although the upper bounds of efficiency intervals for A and J are equal to 1, their lower bounds are small. Their ranges of efficiency intervals are large, therefore, they are called as peculiar. Peculiar DMUs have some inferior data so that the interval ranges are large, while the upper bounds are one.

The interval order relation is defined as follows in [11].

Definition 1. *Interval order relation*
$A = [\underline{a}, \overline{a}] \succ B = [\underline{b}, \overline{b}]$ *holds if and only if* $\underline{b} \leq \underline{a}$ *and* $\overline{b} \leq \overline{a}$.

Using Definition 1, the relations between DMUs by the efficiency intervals are illustrated in Fig. 2. By the obtained efficiency intervals, E and G do not have any DMUs whose efficiency intervals are greater than those of them. Then they are picked out as non-dominated DMUs and rated as efficient in Interval DEA. Peculiar DMUs such as A and J are not rated as efficient. Considering all the possible viewpoints of evaluations by Interval DEA, the partial order relation of DMUs is obtained. Efficiency intervals reflect uncertainty on perspectives of evaluations so that they are similar to our natural evaluation and give more useful information than crisp efficiency values do.

Table 1. Given crisp data and efficiency intervals

DMU	input	output1	output2	efficiency interval
A	1	1	8	[0.143,1.000]
B	1	2	3	[0.286,0.522]
C	1	2	6	[0.286,0.824]
D	1	3	3	[0.375,0.652]
E	1	3	7	[0.428,1.000]
F	1	4	2	[0.250,0.696]
G	1	4	5	[0.571,0.957]
H	1	5	2	[0.250,0.826]
I	1	6	2	[0.250,0.957]
J	1	7	1	[0.125,1.000]

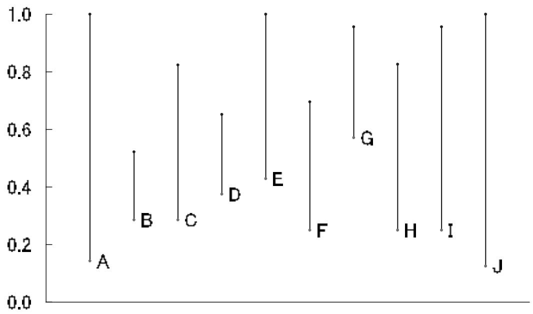

Fig. 1. Efficiency intervals

3 Interval AHP

3.1 Crisp Weights by Conventional AHP

AHP (Analytic Hierarchy Process) is useful in multi-criteria decision making problems. AHP is a method to deal with the priority weights with respect

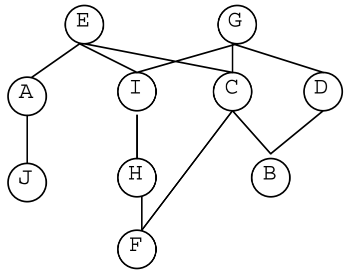

Fig. 2. Partial order of DMUs by efficiency intervals

to many items and proposed to determine the priority weight of each item [12]. When there are n items, a decision maker compares a pair of items for all possible pairs then we can obtain a comparison matrix A as follows. The elements of the matrix called pairwise comparisons are relative measurements and given by a decision maker.

$$A = [a_{ij}] = \begin{pmatrix} 1 & \cdots & a_{1n} \\ \vdots & a_{ij} & \vdots \\ a_{n1} & \cdots & 1 \end{pmatrix}$$

where a_{ij} shows the priority ratio of item i comparing to item j.

The elements of pairwise comparison matrix satisfy the following relations. The decision maker gives $n(n-1)/2$ pairwise comparisons in case of n items.

$$\begin{array}{ll} \text{Diagonal elements} & a_{ii} = 1 \\ \text{Reciprocal elements} & a_{ij} = 1/a_{ji} \end{array} \tag{8}$$

From the given comparison matrix, the priority weights w_i^* are obtained by the well-known eigenvector method. The eigenvector problem is as follows.

$$A\boldsymbol{w} = \lambda \boldsymbol{w} \tag{9}$$

where λ is an eigenvalue and $\boldsymbol{w}$ is a corresponding eigenvector. By (9), the eigenvector $\boldsymbol{w}^* = (w_1^*, \ldots, w_n^*)^t$ corresponding to the principal eigenvalue λ_{max} is obtained as the weight vector. It is noted that the sum of the obtained weights w_i^* is normalized to be one; $\sum_i w_i^* = 1$. The obtained weights from the given comparison matrix can reflect his/her attitude in the actual decision problem.

The weights obtained by the conventional AHP lead to a linear order of items. Uncertainty of an order of items in AHP is discussed in [13]. However, there exists a problem that pairwise comparisons might be inconsistent with each other because they are based on human intuition. The approach for dealing with interval comparisons has been proposed in [14]. It is easier for a decision maker to give interval comparisons than crisp ones. This approach is rather complex comparing to our approach shown in this paper in view of solving problems on all vertices for obtaining interval weights. In the similar setting to [14], the approaches for dealing with decision maker's preference statements instead of pairwise comparisons have been described in [15]. This seems to be very practical, but obtaining the upper and lower bounds of interval weights has been proposed without defining the interval weights. We propose Interval AHP where interval weights are obtained so as to reflect inconsistency among the given crisp comparisons.

3.2 Interval Priority Weight

It is assumed that the estimated weights are intervals to reflect inconsistency of pairwise comparisons. Since the decision maker's judgements are usually inconsistent [16, 17]. We obtain the interval weights so as to include all the given pairwise comparisons and minimize the widths. We formulate the approach for obtaining interval weights as a LP problem, instead of the eigenvector problem in the conventional AHP. This concept is similar to interval regression analysis [18]. The width of the obtained interval weight represents inconsistency of the pairwise comparisons. A decision maker always gives inconsistent information since his/her judgements on each item's weight are uncertain. Then, such inconsistency in the item's weight can be denoted as the widths of interval weights. The given pairwise comparison a_{ij} is approximated by the ratio of priority weights, w_i and w_j, symbolically written as follows.

$$a_{ij} \approx w_i / w_j$$

It is noted that the consistent comparison matrix satisfy the following relations.

$$a_{ij} = a_{ik} a_{kj} \quad \forall (i, j, k) \tag{10}$$

In usual cases such that comparisons are based on the decision maker's intuitive judgements, the relation (10) is not satisfied. Therefore, there is some inconsistency in the given matrix.

Assuming the priority weight as an interval W_i, the interval priority weights are denoted as $W_i = [\underline{w}_i, \overline{w}_i]$. Then, the approximated pairwise comparison with the interval weights is defined as the following interval.

$$\frac{W_i}{W_j} = \left[\frac{\underline{w}_i}{\overline{w}_j}, \frac{\overline{w}_i}{\underline{w}_j} \right]$$

where the upper and lower bounds of the approximated comparison are defined as the maximum range considering all the possible values.

Interval Weights Normalization

While the sum of weights obtained by AHP is normalized to be one, interval probability proposed in [19] can be regarded as a normalization of interval weights. The normalization for interval weights is defined as follows.

Definition 2. *Interval normalization*
Interval weights $(W_1, ..., W_n)$ *are called interval probability if and only if*

$$\begin{array}{l} \sum_{i \neq j} \overline{w}_i + \underline{w}_j \geq 1 \quad \forall j \\ \sum_{i \neq j} \underline{w}_i + \overline{w}_j \leq 1 \quad \forall j \end{array} \tag{11}$$

where $W_i = [\underline{w}_i, \overline{w}_i]$.

It can be said that the conventional normalization is extended to the interval normalization by using the above conditions. In order to explain interval normalization, we use the following example intervals which do not satisfy the conditions (11),

$$W_1 = [0.3, 0.6], \quad W_2 = [0.2, 0.4], \quad W_3 = [0.1, 0.2].$$

Assuming the value $w_1^* = 0.3$ in W_1, there do not exist the values, w_2^* and w_3^*, in W_2 and W_3 whose sum is one, $w_1^* + w_2^* + w_3^* = 1$. Transforming W_1 into $W_1' = [0.5, 0.6]$, these intervals satisfy the conditions for interval normalization (11) and the sum of values in the intervals can be one. Definition 2 is effective to reduce redundancy under the condition that the sum of crisp weights in the interval weights is equal to one.

Approximation of Crisp Pairwise Comparison Matrix

The model to obtain the interval weights is determined so as to include the given interval comparisons [16]. The obtained interval weights satisfy the following inclusion relations.

$$a_{ij} \in \frac{W_i}{W_j} = \left[\frac{\underline{w}_i}{\overline{w}_j}, \frac{\overline{w}_i}{\underline{w}_j} \right] \quad \forall (i, j)$$

It is denoted as the following two inequalities.

$$\frac{\underline{w}_i}{\overline{w}_j} \leq a_{ij} \leq \frac{\overline{w}_i}{\underline{w}_j} \Leftrightarrow \begin{cases} \underline{w}_i \leq a_{ij} \overline{w}_j & \forall (i, j) \\ \overline{w}_i \geq a_{ij} \underline{w}_j & \forall (i, j) \end{cases} \tag{12}$$

The interval weights include the given inconsistent comparisons. In order to obtain the least interval weights, the width of each weight must be minimized. The problem for obtaining interval weights is formulated as the following LP problem.

$$
\begin{array}{ll}
\min & \sum_i (\overline{w}_i - \underline{w}_i) \\
\text{s.t.} & \underline{w}_i \leq a_{ij}\overline{w}_j \quad \forall (i,j) \\
& \overline{w}_i \geq a_{ij}\underline{w}_j \quad \forall (i,j) \\
& \sum_{i \neq j} \overline{w}_i + \underline{w}_j \geq 1 \quad \forall j \\
& \sum_{i \neq j} \underline{w}_i + \overline{w}_j \leq 1 \quad \forall j \\
& \overline{w}_i \geq \underline{w}_i \geq \varepsilon \quad \forall i
\end{array}
\tag{13}
$$

where ε is a small positive value and the first two and the next two conditions show the inclusion relations (12) and interval normalization (11), respectively.

The width of the interval weight represents uncertainty of each weight and the least uncertain weights are obtained by this model (13). When a decision maker has some information over uncertainties of the items' priority weights, he/she gives them as the uncertainty weights $p_i \forall i$. Then, the weighted sum of widths $\sum_i p_i(\overline{w}_i - \underline{w}_i)$ can be minimized. However, it is not easy for a decision maker to give the weight of each width. Simply the sum of widths of all weights is minimized as in (13) without information over uncertainties of items.

Since the proposed Interval AHP is the ratio model, its concept is similar to interval regression analysis in view of the least approximation. The proposed model (13) is formulated as LP problem, the following inequality should be satisfied.

$$
\frac{n(n-1)}{2} \geq 2n \tag{14}
$$

where n is the number of items. (14) requires that the number of given comparison data should be larger than that of decision variables.

If (14) is satisfied, whatever the given comparison matrix is, there exist an optimal solution that minimizes the objective function in (13). If the optimal value of the objective function is equal to zero; $\sum_i(\overline{w}_i^* - \underline{w}_i^*) = 0$, it can be said that the given comparison matrix is perfectly consistent. The weights are obtained as crisp values and they are the same as those by the conventional eigenvector method (9). In the conventional AHP, the consistency index is defined considering the eigenvector corresponding to the principal eigenvalue of the given matrix. If it is equal to 0, the elements of the matrix satisfy the relations (10), that is, the matrix is perfectly consistent. Experimentally it can seem to be consistent in case where the index is less than 0.1. In the proposed LP method, the consistency of the matrix is represented as the optimal value of the objective function that is the sum of widths of the obtained interval weights. The optimal value becomes small for consistent matrix.

The decision problem in AHP is structured hierarchically as criteria $(C_1, ..., C_k)$ and alternatives $(A_1, ..., A_n)$ as in Fig.3. In Fig. 3 the criteria are at one layer, however, it is possible to construct several layers of criteria.

The criterion weights and alternative scores with respect to the criteria are obtained from the corresponding pairwise comparison matrices. Concerning all criteria, the overall priority of each alternative is obtained as the sum of

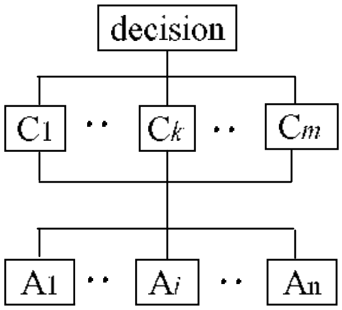

Fig. 3. Structure of decision problem in AHP

the products of criterion weights and corresponding alternative scores. By the proposed model, the criterion weights and alternative scores are obtained as intervals. Then, by interval arithmetic the overall priority is also obtained as an interval.

3.3 Numerical Example

The following pairwise comparison matrix with five items is given by a decision maker. The decision maker gives 10 comparisons marked * and the other elements are filled by (8).

$$A = [a_{ij}] = \begin{pmatrix} 1 & 2^* & 3^* & 5^* & 7^* \\ 1/2 & 1 & 2^* & 2^* & 4^* \\ 1/3 & 1/2 & 1 & 1^* & 1^* \\ 1/5 & 1/2 & 1 & 1 & 1^* \\ 1/7 & 1/4 & 1 & 1 & 1 \end{pmatrix}$$

The crisp weights obtained by conventional eigenvector method (9) are shown in the right column of Table 2. The linear order relation of items is $1 > 2 > 3 > 4 > 5$. The elements of this comparison matrix satisfy $a_{ik} \geq a_{jk}$ for all k and such a comparison matrix is in row dominance relation. In case of row dominance relation, it is assured that the obtained weights by eigenvector method satisfy the order relation $w_i \geq w_j$. It can be seen from the results of this example.

The interval weights obtained by the proposed model (13) is shown in Table 2 and Fig. 4. With the obtained interval weights, by Definition 1, the order relation of items is $1 \succ 2 \succ 3 \succ 4 \succ 5$. Although the linear order relation is obtained in this example, it is noted that the partial order relation is often obtained because of interval weights. The obtained weights of items 1 and 2 are crisp values and items 3, 4 and 5 are intervals. From the given comparison matrix, it is estimated that item 1 is prior to item 2 and both of them are apparently prior to items 3, 4 and 5. However, the relations among items 3, 4 and 5 are not easily estimated, since the comparisons over them are contradicted each other. The obtained interval weights, W_3, W_4 and W_5, by the proposed Interval AHP reflect inconsistency among the given crisp comparisons. Since the decision maker gives comparisons of all pairs of items intuitively, it is natural to consider that the obtained weights are intervals reflecting the uncertainty.

Table 2. Interval weights with crisp comparison matrix

item	interval weights (13)	width	eigenvector (9)
1	0.453	0.000	0.464
2	0.226	0.000	0.241
3	[0.104, 0.151]	0.047	0.112
4	[0.091, 0.113]	0.023	0.100
5	[0.057, 0.104]	0.047	0.083

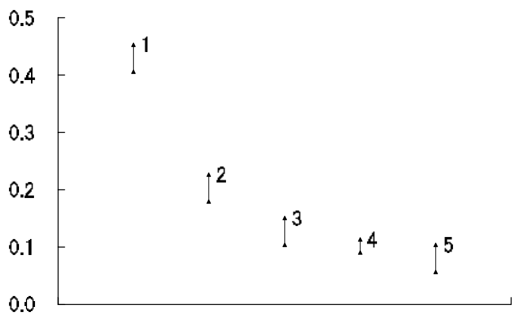

Fig. 4. Interval weights

4 Conclusion

The decision problems usually include uncertainty since humans are involved in the process. The given information and evaluations based on human intuitive judgements might be from various viewpoints and inconsistent. In order to deal with the uncertainty in the given information, interval evaluations have been introduced. Reflecting the uncertainty of the given data and evaluating process, the results obtained by the proposed interval evaluation models are intervals even in case of crisp given data. Interval evaluations are more suitable for our natural judgements than crisp evaluations.

By the proposed Interval DEA, the efficiency interval is obtained so as to include all the possible efficiency values from various viewpoints. By the proposed Interval AHP, the interval weights are obtained so as to include inconsistency among the given comparisons. The proposed interval evaluation models deal with the uncertainty by human judgements. In view of considering all the possibility of data and perspectives of evaluations, the proposed interval models are a kind of possibility analysis [18]. They can give useful and helpful information for decision makers, since various viewpoints and inconsistency in the given data are considered.

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