13

Time-Series Models

13.1 Introduction

Appendix C recapitulates the most commonly assumed linear stochastic processes. In this chapter the sampling aspects of time-series modelling are dealt with. Section 13.2 focuses on the sampling aspects of estimating process parameters which characterize the dynamic behaviour of the process. In Sect. 13.3 the estimation of model means, such as annual model means, is elaborated upon. Finally, Sect. 13.4 discusses the sampling aspects in the perspective of detecting trends in time series.

13.2 Estimation of Process Parameters

This section focuses on the development of monitoring strategies to obtain parameters which give a *general* description of the temporal behaviour of a physical phenomenon, i.e., status monitoring (Sect. 1.1). Statistical inference is made by assuming that the temporal behaviour results from a stochastic process which can be specified by a time-series model. Therefore, the main question to be answered in this section is: *How should samples be distributed in time, in order to derive time-series models that describe the dynamic behaviour of a physical phenomenon adequately?* The answer to this question starts with making assumptions about the stochastic processes, the most applied of which are recapitulated in Appendix C. As is mentioned in Sect. C.1, stochastic processes are data-based. The discrete-time stochastic processes described in Sects. C.2 to C.4 reflect the dynamic behaviour of physical phenomena, given the sampling frequency and the length of the monitoring period. Decisions must be taken on the sampling frequency and the length of the monitoring period. These decisions are discussed in this section.

We restrict ourselves here to discrete-time stochastic processes with discrete, constant time-intervals . Statistical inference for this type of processes is possible by using relatively simple mathematical methods, as compared with continuous-time processes. Box and Jenkins (1976) give a procedure of model identification, fitting (calibration) and diagnostic checking (verification). In Fig. 13.1 this procedure is summarized and extended for decisions on the length of the monitoring period and the sampling frequency.

Important tools in both model identification and diagnostic checking are the sample autocorrelation function (sample ACF), the sample partial autocorrelation function (sample PACF) and the residual cross-correlation function (residual CCF), see Appendix C. As an alternative to the identification procedure, automatic model selection can be applied, using a selection criterion (e.g., Akaike's Information Criterion, AIC, or Bayes Information Criterion, BIC). The procedures of either model identification or automatic model selection, fitting and diagnostic checking result in a model that describes the *data* adequately. The next step is to analyze the extent to which *the underlying physical processes* are described adequately by the model. This can be done by physical interpretation of the modelling results. Besides this, whenever possible validation is advised, which means that the model performance is tested using independent validation data. Both physical interpretation and validation results may not only give rise to further model improvements, but also to extension of the monitoring period, and adjustment of the sampling frequency. This is illustrated by the following two situations, which are given without aiming to be complete:

1. A large value of the autoregressive parameter of a first-order autoregressive model (Sect. C.2.1) is found. The validation results show large systematic errors. In this case the monitoring period may not fully cover the correlation length, i.e., the time lag at which the autocorrelation is (approximately) zero. In the case of a dynamic relationship between two variables, the monitoring period may not cover the response time. The monitoring should be continued at least until the correlation length or the response time is completely covered;
2. Although autoregressive relationships were expected on the basis of physical insights, no significant autoregressive relationships were found. The validation results show large random errors. In this case the sampling interval may be larger than the correlation length or, in the case of a dynamic relationship between two variables, than the response time.

13.3 Sampling Frequency for Estimation of Model Means

As is mentioned in Sect. 1.1 in status monitoring the status of a system is characterized and followed in time. A general statistic of a system is the mean, for instance the annual mean. If monitoring is restricted to systematic sampling

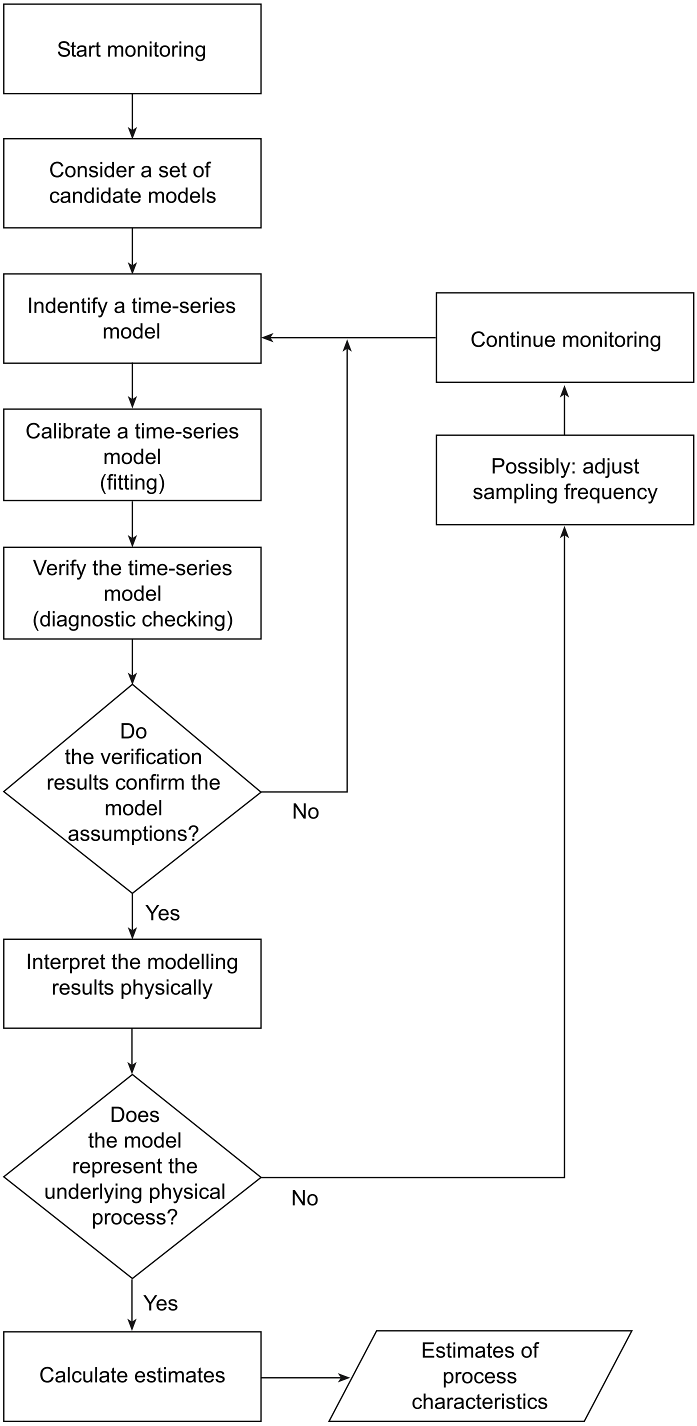

Fig. 13.1. Flow chart for the procedure of time-series model identification, fitting and diagnostic checking, extended with sampling aspects

in time, a sampling frequency needs to be chosen which enables estimates of means which are sufficiently accurate given the purposes of monitoring.

An observed series $\{z_i\}, i = 1 \ldots n$ is considered to be a realization of a second-order stationary stochastic process with a random fluctuation ϵ_i with variance σ_ϵ^2 around a deterministic mean μ:

$$Z_i = \mu + \epsilon_i \ . \tag{13.1}$$

Note that μ is a model parameter, and not the average of z over the universe of interest. Sanders and Adrian (1978) presented a sampling frequency criterion based on the relationship between sampling frequency and the magnitude of half the confidence interval of the model mean. Their approach will be followed here.

Suppose that n second-order stationary, independent and identically distributed observations on z are available. To obey the stationarity assumption, it may be necessary to remove seasonal nonstationarity first, for instance by fitting a deterministic annual cycle to the data and to use the residuals in further analysis. The variance σ_ϵ^2 can be estimated by:

$$\widehat{\sigma_\epsilon^2} = \frac{1}{n-1} \sum_{i=1}^{n} (z_i - \bar{z})^2 \ , \tag{13.2}$$

where $\bar{z}$ is the calculated mean of $z_i, \ i = 1, \ldots, m$. Confidence intervals for estimates of μ are estimated using the Student's t statistic:

$$t = \frac{\bar{z} - \mu}{\sigma_\epsilon / \sqrt{n}} \ . \tag{13.3}$$

The probability that t is within the confidence interval is given by the confidence level $1 - \alpha$:

$$\Pr \left(t_{\alpha/2} < \frac{\bar{z} - \mu}{\sigma_\epsilon / \sqrt{n}} \right) \ , \tag{13.4}$$

where $t_{\alpha/2}$ and $t_{1-\alpha/2}$ are constants from the Student's t distribution for a corresponding number of observations and confidence level. Since $t_{1-\alpha/2} = -t_{\alpha/2}$, the confidence interval of the model mean μ is given by

$$\bar{z} - \frac{t_{\alpha/2}\, \sigma_\epsilon}{\sqrt{n}} < \mu < \bar{z} + \frac{t_{\alpha/2}\, \sigma_\epsilon}{\sqrt{n}} \ . \tag{13.5}$$

If σ_ϵ has been estimated from prior information, and a decision on the confidence level $1 - \alpha$ has been taken, the width of the confidence interval can be plotted against values of n. If annual means are considered, n is the yearly number of observations.

Until now it was assumed that the values of z are mutually independent. However, the presence of autocorrelation is often indicated in time series of environmental variables. Dependency can be dealt with in two ways: (a) it is avoided, or (b) it is accounted for.

Avoiding dependency

To avoid dependency, the interval length should be larger than the length of serial correlation. Thus, prior information is needed on the length of serial correlation, but this is not always available. Alternatively, one could start to measure with a high frequency to estimate the correlation length, and then to continue at intervals longer than the estimated correlation length. Another possibility is to take average values over sufficiently long periods. For instance, if daily values appear to be correlated, one could calculate a series of monthly averages which may possibly be uncorrelated.

The correlation length can be estimated from observed time series using (C.8), provided that the interval length is shorter than the correlation length and the series amply covers the correlation length. Once a sampling frequency is found which provides serially uncorrelated observations on a target variable z_t, the maximum sampling frequency is known. Equation (13.5) is applied to investigate lower frequencies.

Accounting for dependency

A first way to account for serial correlation in estimating model means μ is by using the relationship between the actual number of observations and the equivalent number of independent observations for an autocorrelated time series, given by Bayley and Hammersley (1946):

$$V(\hat{\mu}) = \frac{\sigma_\epsilon^2}{n_\mathrm{b}^*} , \tag{13.6}$$

where n_b^* is the equivalent number of independent observations, $\hat{\mu}$ is the estimated mean of a process $\{Z_t\}$ and σ^2 is the variance for the random fluctuation ϵ_i in (13.1). For second-order stationary stochastic processes with n observations, n_b^* can be calculated by

$$\frac{1}{n_\mathrm{b}^*} = \frac{1}{n} + \frac{2}{n^2} \sum_{j=1}^{n-1} (n-j)\, \rho_{j\Delta t} , \tag{13.7}$$

where Δt is the observation interval and $\rho(j\Delta t)$ is the correlation coefficient for lag $j\Delta t$. For first-order autoregressive processes (AR(1), see Sect. C.2.1), (13.7) reduces to

$$\frac{1}{n_\mathrm{b}^*} = \frac{1}{n} + \frac{2}{n^2} \cdot \frac{\rho^{(n+1)\,\Delta t} - n\,\rho^{2\,\Delta t} + (n-1)\,\rho^{\Delta t}}{(\rho^{\Delta t} - 1)^2} , \tag{13.8}$$

(Matalas and Langbein, 1962), where ρ is the lag 1 correlation coefficient for a selected base lag period. If the base lag period equals the observation interval the AR(1) process is given by

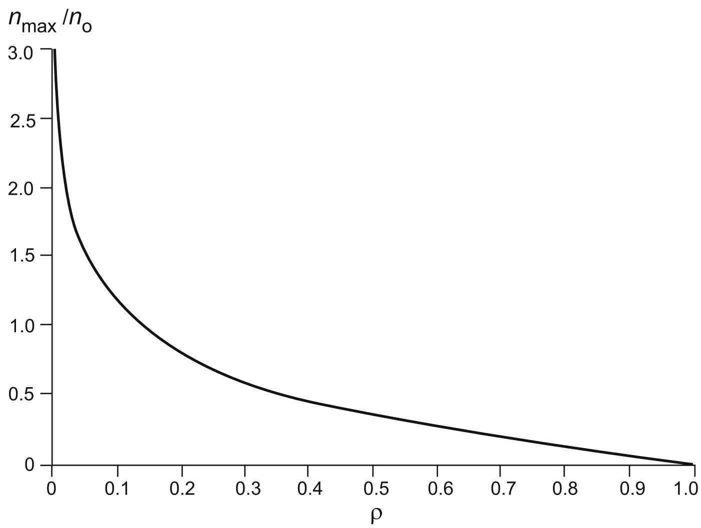

Fig. 13.2. Maximum equivalent number of independent observations as a function of the daily lag 1 correlation coefficient ρ for a Markov process; $n_o = 365$ days. (Reproduced from Lettenmaier (1976, Fig. 6) with permission of American Geophysical Union.)

$$Z_i - \mu = \phi\,(Z_{i-1} - \mu) + \epsilon_i \ , \tag{13.9}$$

and $\rho^{\Delta t}$ in (13.8) can be replaced by ϕ. The width of the confidence interval can be estimated by (13.5), replacing n by n_b^*.

Within a specified period n_o a maximum number of equivalent independent observations may be collected:

$$n_{max} = \frac{n_o}{2} \cdot \frac{(\ln \rho)^2}{\rho^{n_o} - n_o \ln \rho - 1} \ , \tag{13.10}$$

(Lettenmaier, 1976), where n_o is the specified period, for example 365 days, and ρ is the daily lag 1 correlation coefficient. Thus, if the number of observations n goes to infinity within the period n_o, the equivalent number of independent observations will not exceed a certain n_{max}.

In Fig. 13.2 n_{max} is given as a function of the lag 1 correlation coefficient ρ. Figure 13.2 shows that n_{max} decreases with an increasing lag 1 correlation coefficient. If the number of actual samples were infinite and ρ were 0.14, then from (13.10) and Fig. 13.2 it follows that the equivalent number of independent observations equals 365.

Figure 13.3 gives the ratio n_b^*/n_{max} as a function of the normalized sampling frequency n/n_o, for various values of the daily lag 1 correlation coefficient ρ. It can be seen that n_{max} is approached quite rapidly for large values of ρ. If observations are taken weekly ($n/n_o = 0.14$) and $\rho = 0.85$, then from Fig. 13.3

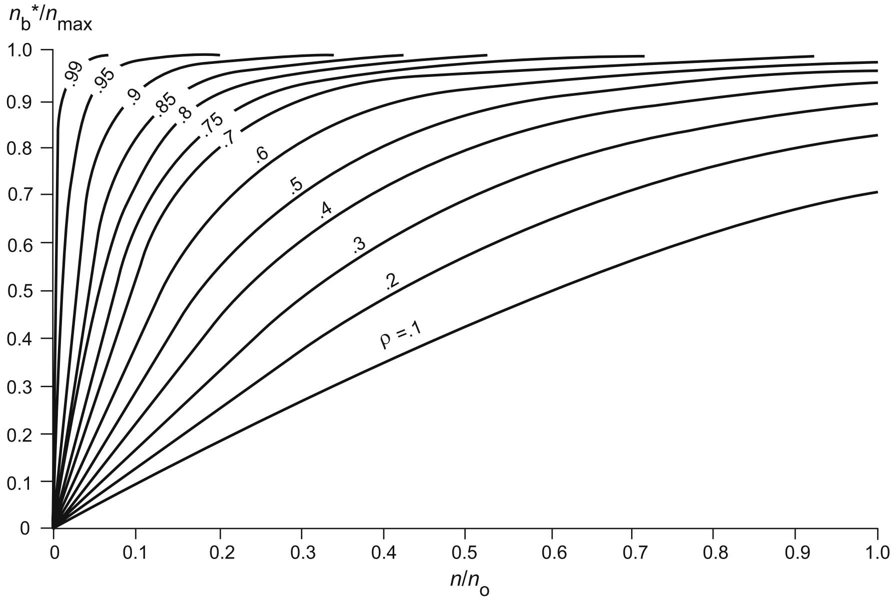

Fig. 13.3. Ratio of equivalent and maximum equivalent number of independent observations, as a function of the normalized sampling frequency; $n/n_{\mathrm{o}} = 1$ corresponds to daily sampling. (Reproduced from Lettenmaier (1976, Fig. 7) with permission of American Geophysical Union.)

follows that $n_{\mathrm{b}}^*/n_{\mathrm{max}}$ equals 0.9. Thus, 90 % of the information that may be collected in a period of 365 days is already provided by weekly sampling.

13.4 Sampling Frequency for Detecting Trends

In the previous section second-order stationarity of Z in (13.1) was assumed, which implies a time invariant model mean μ. This section focuses on the detection of temporal changes in the model mean μ. In Sect. 13.4.1 sampling aspects of tests for step trends and linear trends are discussed. Section 13.4.2 deals with sampling aspects of intervention analysis. Note that the methods described here are based on model means, in contrast to the method for trend testing described in Sect. 11.2 which is based on temporal means.

13.4.1 Tests for Step Trends and Linear Trends

If temporal sampling is restricted to systematic sampling, then the choice of an appropriate sampling design reduces to choosing an appropriate sampling frequency. Analogous to Sect. 13.3, serial correlation in regularly spaced series is either prevented for or accounted for.

Avoiding dependency

In Sect. 13.3 it was discussed how the sampling frequency can be found below which observations are practically serially uncorrelated. Once a sampling frequency is found which provides serially uncorrelated observations on a target variable Z, the minimum sample size and thus the minimum length of the monitoring period must be decided on. Lettenmaier (1976) gives criteria for the minimum sample size if trend detection is the purpose of the monitoring. The trend can concern either a step trend or a linear trend. Lettenmaier (1976) considers trend detection as a testing problem, with H_0: a trend is not present in the underlying process and H_1: a trend is present. Following Lettenmaier (1976), it is explained below how the minimum length of series can be determined for testing on step trends and linear trends, respectively.

Sampling Frequency for Detecting Step Trends

A step trend is defined here as a sudden change in the mean level of a process. Suppose that this sudden change occurs halfway a series with an even number of n measurements. Furthermore, let μ_1 and μ_2 be the mean levels of the series before and after the abrupt change. The underlying process is now defined as

$$Z_i = \begin{cases} \mu_1 + \epsilon_i & \text{if } i \leq n/2 \\ \mu_2 + \epsilon_i & \text{if } i > n/2 \;, \end{cases} \tag{13.11}$$

where $i = 1, \ldots, n$ indicates the ith element of an equidistant series of length n. In (13.11) the ϵ's are independent identically distributed random variables with zero mean and variance σ_ϵ^2.

The test chooses between

$$\begin{aligned} \mathrm{H}_0 &: \mu_1 = \mu_2 \\ \mathrm{H}_1 &: \mu_1 \neq \mu 2 \;. \end{aligned}$$

The test statistic is

$$T = \frac{|\bar{z}_1 - \bar{z}_2|\sqrt{n}}{2\,\hat{\sigma}_\epsilon} - t_{1-\alpha/2,\,\nu} \;, \tag{13.12}$$

with

$$\bar{z}_1 = \frac{1}{n/2}\sum_{i=1}^{n/2} z_i, \quad \bar{z}_2 = \frac{1}{n/2}\sum_{i=n/2+1}^{n} z_i \;, \tag{13.13}$$

being estimators for μ_1 and μ_2, respectively, $t_{1-\alpha/2,\nu}$ is the quantile of the Student's t distribution at confidence level $1-\alpha/2$ and for $\nu = n-2$ degrees of freedom, and with $\hat{\sigma}_\epsilon$ being the sample standard deviation,

$$\hat{\sigma}_\epsilon^2 = \frac{1}{n-2}\left(\sum_{i=1}^{n/2}(z_i - \bar{z}_1)^2 + \sum_{i=n/2+1}^{n}(z_i - \bar{z}_2)^2\right) \;. \tag{13.14}$$

If $T \leq 0$ then H_0 is accepted, otherwise H_0 is rejected.

Now, for the purpose of sample design, the absolute value of the true difference between μ_1 and μ_2 is assumed to be known: $T_\mathrm{r} = |\mu_1 - \mu_2|$, as well as the variance σ_ϵ^2. Then, the following population statistic can be formed:

$$N_T = \frac{\sqrt{n}}{2\,\sigma_\epsilon} T_\mathrm{r} \,. \tag{13.15}$$

The power of the test is now given by

$$1 - \beta = F(N_T - t_{1-\alpha/2,\,\nu}) \,, \tag{13.16}$$

where F is the cumulative distribution of a standard Student's t distribution with $\nu = n - 2$ degrees of freedom.

If prior information on the variance σ_ϵ^2 is available, a guess can be made of the minimum length of the series needed for the detection of a step trend which occurs halfway this series, for a given α and β. Figure 13.4 shows a diagram for the relationship between the normalized magnitude of the step trend ($T_\mathrm{r}/\sigma_\epsilon$) and the minimum length of the series needed for given values of β and with $\alpha = 0.05$. The step trend is assumed to occur halfway the series, and $n = n_1 + n_2$, where $n_1 = n_2$ are the numbers of observations before and after the step change, respectively. The effect on the sample size of a decision for a lower confidence level $1 - \alpha$ is illustrated by Fig. 13.5, where $\alpha = 0.10$.

Minimum Sample Size for Detecting Linear Trends

According to Lettenmaier (1976, p. 1038), a linear trend is parameterized as

$$Z_i = \epsilon_i + i\,\tau + \gamma \,, \tag{13.17}$$

where ϵ_i is a series of random disturbances from a normal distribution with mean zero and variance σ_ϵ^2, τ is the trend magnitude, and γ is the process base level. The parameter τ is estimated by

$$\hat{\tau} = \frac{\sum\limits_{i=1}^{n} i'\, z_i'}{\sum\limits_{i=1}^{n} i'^2} \,, \tag{13.18}$$

where

$$i' = i - \frac{n+1}{2}, \qquad z_i' = z_i - \frac{1}{n}\sum_{i=1}^{n} z_i \,. \tag{13.19}$$

The estimate $\hat{\tau}$ has variance

$$V(\hat{\tau}) = \frac{\sigma_\epsilon^2}{\sum\limits_{i=1}^{n} i'^2} \,. \tag{13.20}$$

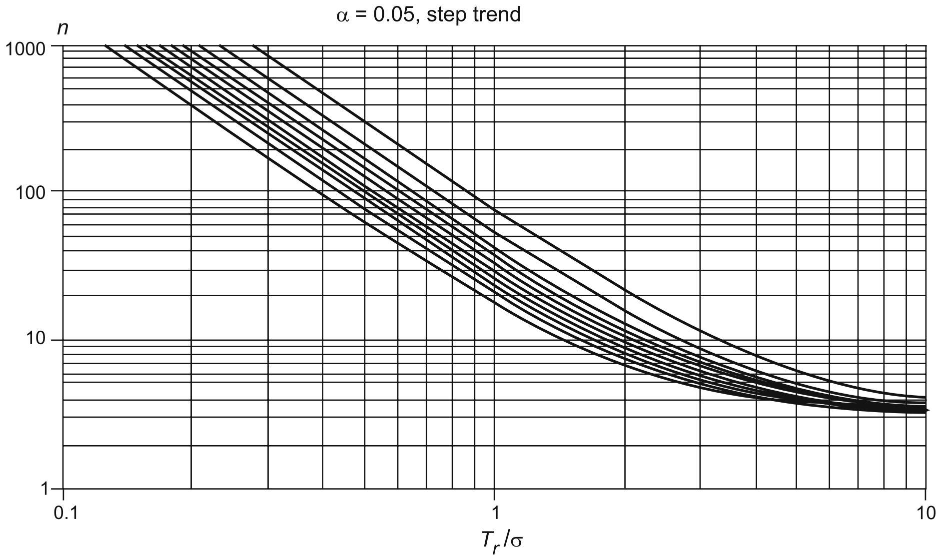

Fig. 13.4. Sample size against normalized step trend for β = 0.5, 0.4, 0.35, 0.3, 0.25, 0.2, 0.15, 0.1, 0.05, 0.01 (from left to right) and $\alpha = 0.05$. T_r: the magnitude of the step trend, σ: the standard deviation, $n = n_1 + n_2$, with $n_1 = n_2$: the number of equidistant observations before and after the step change, respectively.

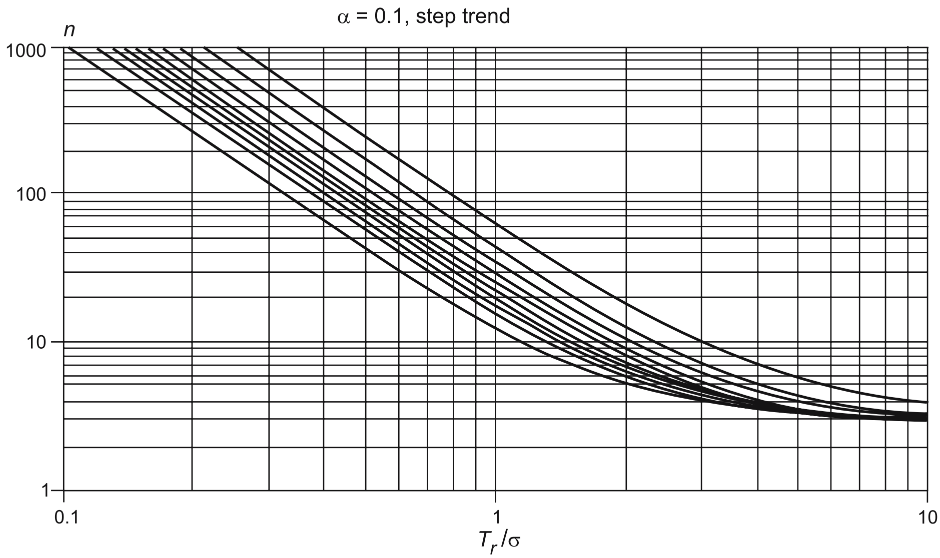

Fig. 13.5. Sample size against normalized step trend for given values of β, and $\alpha = 0.1$. See Fig. 13.4

Assuming normality of $\hat{\tau}$, a test statistic T is given by

$$T = |\hat{\tau}| - \frac{t_{1-\alpha/2,\,\nu} \cdot \hat{\sigma}_\epsilon}{\sqrt{\sum_{i=1}^{n} i'^2}} , \tag{13.21}$$

with $\nu = n - 2$ degrees of freedom. The sample estimate $\hat{\sigma}_\epsilon$ is calculated by

$$\hat{\sigma}_\epsilon = \frac{1}{n-2} \sum_{i=1}^{n} (z_i - i\,\hat{\tau} - \hat{\gamma})^2 , \tag{13.22}$$

and the sample estimate $\hat{\gamma}$ is calculated by

$$\hat{\gamma} = \frac{1}{n} \sum_{i=1}^{n} z_i - \hat{\tau} \frac{n+1}{2} . \tag{13.23}$$

The test statistic T in (13.21) can be normalized as follows:

$$T' = \frac{|\hat{\tau}| \left(\sum_{i=1}^{n} i^2\right)^{1/2}}{\hat{\sigma}_\epsilon} - t_{1-\alpha/2,\,\nu} . \tag{13.24}$$

Given the identity

$$\sum_{i=1}^{n} i^2 = \frac{1}{6} n\,(n+1)\,(2n+1) , \tag{13.25}$$

Lettenmaier (1976) derives the following dimensionless statistic for the existence of a linear trend, assuming that the population trend magnitude τ is known:

$$N_T' = \frac{\{n\,(n+1)\,(n-1)\}^{1/2}\,\tau}{\sqrt{12}\sigma_\epsilon} . \tag{13.26}$$

If $n\,\tau$ is substituted by T_r', then N_T' becomes

$$N_T' = \frac{\{n\,(n+1)\,(n-1)\}^{1/2}\,T_\mathrm{r}'}{n\sqrt{12}\sigma_\epsilon} . \tag{13.27}$$

The power of the test for linear trend can be calculated by (13.16), with N_T replaced by N_T'. Figures 13.6 and 13.7 give minimum series lengths needed for given values of β and for $\alpha = 0.05$ and 0.1, respectively, to detect linear trends with normalized magnitudes T'/σ_ϵ.

Lettenmaier (1976) gives power curves for nonparametric trend tests. These tests are appropriate if the assumptions of parametric tests are violated. Lettenmaier (1976) shows that Mann–Witney's test and Spearman's rho test are adequate for testing against a step trend and a linear trend, respectively.

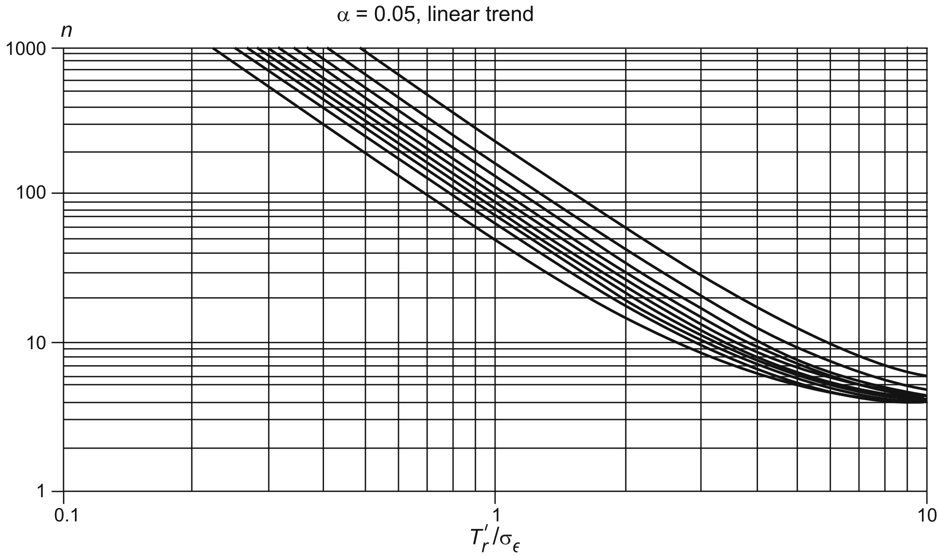

Fig. 13.6. Sample size against normalized linear trend for given values of β (See Fig. 13.4), and $\alpha = 0.05$. T'_{r}: magnitude of the linear trend, σ_ϵ: the residual standard deviation of the linear trend model, n: the number of equidistant observations

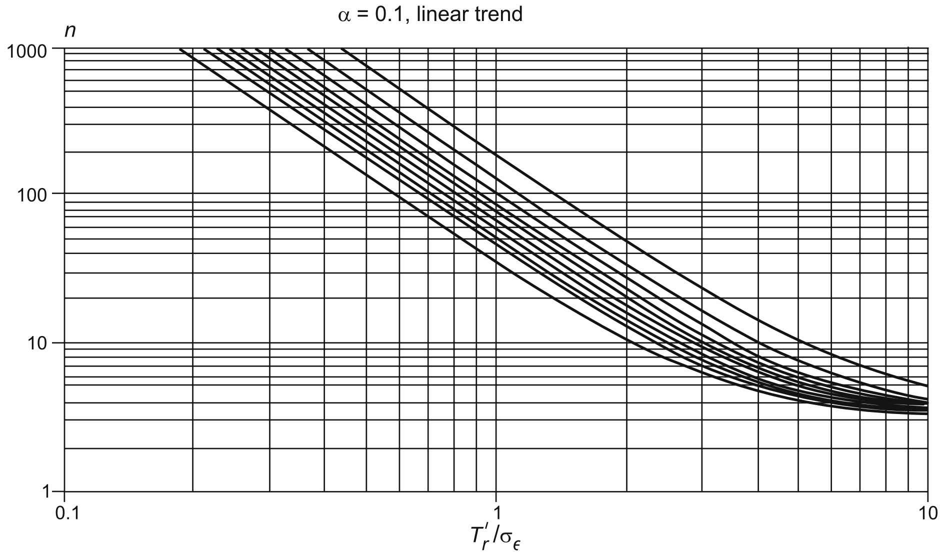

Fig. 13.7. Sample size against normalized linear trend for given values of β (See Fig. 13.4), and $\alpha = 0.1$. T'_{r}: magnitude of the linear trend, σ_ϵ: the residual standard deviation of the linear trend model, n: the number of equidistant observations

Accounting for dependency

The trend tests described above are based on the assumption that the observations are mutually independent. This is the case if the interval length is larger than the correlation length. However, in many cases the assumption of mutual independence is not very useful. The required minimum interval length implies extensive monitoring periods in order to obtain a sufficiently large number of observations to execute powerful tests. Furthermore, if serial correlation is removed by extending the interval length or by calculating averages over sufficiently long periods, information gets lost. Therefore, it may be attractive to use methods for trend detection that account for serial correlation.

Serial correlation can be accounted for in trend tests by using the relationship between the actual number of observations and the equivalent number of independent observations for an autocorrelated time series, given in (13.6) to (13.8). Tests for step trends or linear trends in autocorrelated series can be performed using the equations given before for independent series, replacing n by n^*_{b}.

Within a specified period n_{o} a maximum number of equivalent independent observations may be collected, see (13.10). This is important in deciding on the sampling frequency and the length of the observation period: the power of trend tests may increase more by observing longer than by observing more frequently. Lettenmaier (1976) made an extension of nonparametric trend tests to dependent time series, and gives diagrams of maximum power and power to maximum power ratio for Mann–Whitney's test against a step trend. Similar diagrams for parametric t-tests are given in Figs. 13.8 and 13.9. As compared to the diagrams for Mann–Whitney's test against step trend, the diagrams for t-tests given in Fig. 13.8 indicate that t-tests have smaller maximum power for given daily lag 1 correlation coefficient and trend to standard deviation ratio. For the application of nonparametric trend tests to hydrological time series we refer to Hirsch et al. (1982), van Belle and Hughes (1984), Hirsch and Slack (1984) and Yue et al. (2002).

Lettenmaier (1978) compared the statistical power of trend tests for uniformly collected time series (i.e., equidistant observation times) and for 'stratified' or unequally spaced time series, resulting from rotational monitoring designs (Sect. 14.1). In the latter case observations are taken, for instance, during one year in three, which may be more travel-economical than collecting equidistant time series in a monitoring network. It was concluded that equidistant time series are preferred over unequally spaced time series in trend detection. In the rotational design 2–3 times as many samples need to be taken to achieve the same power as in trend tests for equidistant time series.

13.4.2 Intervention Analysis

Methods accounting for serial correlation include the intervention models, described by Hipel et al. (1975) and Hipel and McLeod (1994). Intervention

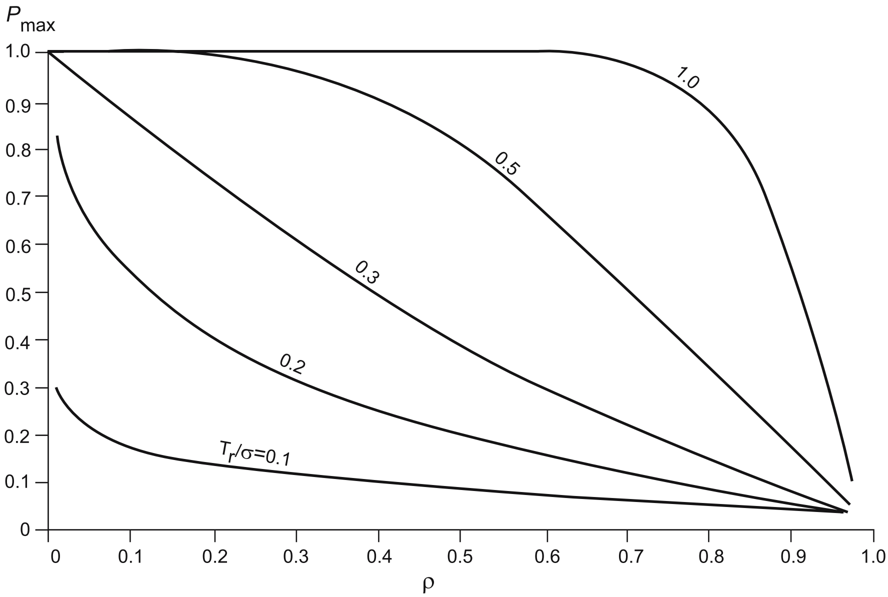

Fig. 13.8. Maximum power of a t-test against a step trend, as a function of the daily lag 1 correlation coefficient ρ and the trend to standard deviation ratio T_r/σ for a Markov process. Analogous to Lettenmaier (1976)

models form a special class of transfer function-noise models, see Sect. C.4. The intervention model for a step trend is given by

$$Z_t = I_t + N_t \; , \tag{13.28}$$

where $t = 1, \ldots, n$ indicates the t-th element of a series of length n, Z_t is the process of interest, I_t is the trend component and N_t is a noise component describing the part of Z_t that cannot be explained from the trend. The noise component is usually taken as an ARMA model, see (C.23). The trend component I_t is a transfer function with the following general form:

$$I_t = \delta_1 I_{t-1} + \delta_2 I_{t-2} + \cdots + \delta_r I_{t-r} + \omega_0 S_{t-b}^{(T)} - \omega_1 S_{t-1-b}^{(T)} - \cdots - \omega_m S_{t-m-b}^{(T)} \; , \tag{13.29}$$

where $\delta_1 \ldots \delta_r$ are autoregressive parameters up to order r, $\omega_0 \ldots \omega_m$ are moving average parameters up to order m, b is a pure delay parameter. Using the backward shift operator B, (13.29) can be written as

$$I_t = \frac{\omega(B)}{\delta(B)} B^b S_t^{(T)} \; , \tag{13.30}$$

with $B^k z_t = z_{t-k}$ and k is a positive integer.

$S_t^{(T)}$ is an input series indicating the step intervention:

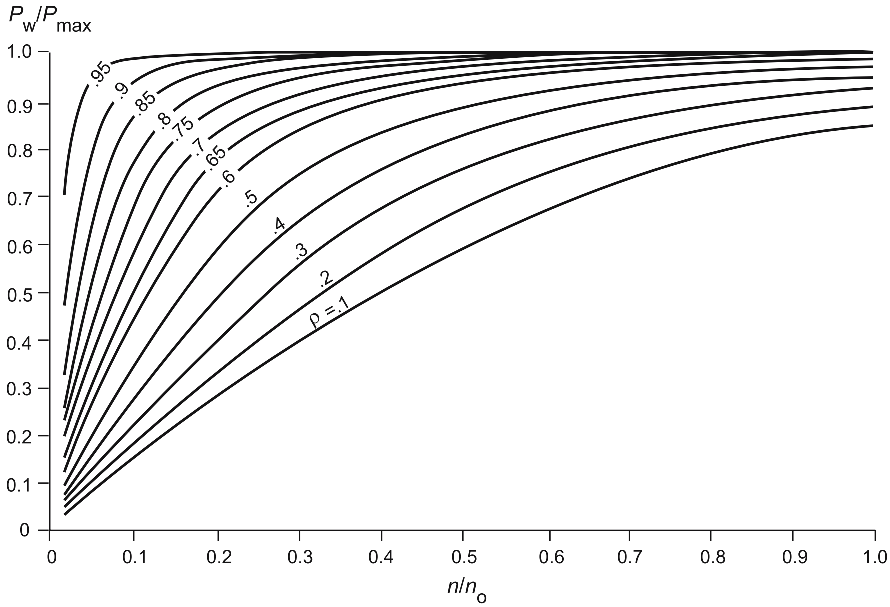

Fig. 13.9. Power to maximum power ratio as a function of relative sampling frequency for t-tests against a step trend in time series following a Markov process. ρ is the daily lag 1 correlation coefficient. $n_o = 365$. Analogous to Lettenmaier (1976).

$$\begin{aligned} S_t^{(T)} &= 0 \quad \text{if } t < T, \\ S_t^{(T)} &= 1 \quad \text{if } t \geq T \ . \end{aligned} \tag{13.31}$$

Step interventions influence processes in different ways, which can be expressed by different forms of the transfer function, see Fig. 13.10. As compared to the testing procedures described before, intervention modelling has the advantage that the effect of interventions can be separated from other independent influences. The model in (13.28) can be extended with other transfer components besides the intervention:

$$Z_t = I_t + X_{i,t} + N_t, \ \ \imath = 1, \ldots, m \ , \tag{13.32}$$

where $X_{i,t}$, $i = 1, \ldots, m$ are m transfer components of m independent inputs. Lettenmaier et al. (1978) discussed the sampling aspects of intervention analysis. They considered intervention analysis as a hypothesis test with H_0: no intervention has taken place, which means that $\omega B/\delta B = 0$ in (13.30). Based on knowledge of the covariance matrix of the model parameters, they constructed power functions for several specified intervention models. These models do not include models containing other inputs besides the intervention, as in (13.32). Nevertheless, the power curves presented by Lettenmaier et al. (1978) provide an indication of required sample sizes and the ratio of the number of observations collected before and after the intervention took place. In summary, their conclusions on the sampling aspects of intervention analysis are:

1. For the step decay model, the linear model and the impulse decay model (Figs. 13.10**c**, **d** and **e**, respectively), relatively small pre-intervention series lengths are required. For the step model (Fig. 13.10**a**, **b**) it is indicated that equal pre- and post-intervention series lengths are optimal;
2. For the impulse decay model, it is important that data are collected frequently during the period that the intervention response is non-constant;
3. It is indicated that the minimum detectable intervention effect depends on the complexity of the intervention model: more complex models require a larger number of observations. Let ω be the intervention response magnitude. Furthermore, let γ be the pre-intervention series length relative to the total series length. For a step model and an impulse decay model ω equals ω_0. For a step decay model ω equals $\omega_0/(1-\delta_1)$. For a linear model $\omega = \mathrm{m}\omega_0/(1-\gamma)$ where m is the number of post-intervention observations. It is indicated that a minimum level of change, relative to the process standard deviation, ω/σ_Z, of about 0.5 can be detected for the step model. For the linear model the minimum level of ω/σ_Z that can be detected is at about 0.75, and 1.0 for the impulse decay model. For the step decay model a higher minimum level is indicated. Below these values intervention effects cannot be detected with reasonable sample sizes.

Additional to these conclusions Lettenmaier et al. (1978) suggest:

1. If the fitted parameter values for a hypothesized intervention model are not significant, a simpler model should be fitted (e.g., a step model instead of a step–decay model);
2. Seasonality should be removed from the data. However, seasonal differencing will lead to substantial loss of power. Therefore alternative methods of deseasonalization are recommended (for instance differencing against a control time series, or removing seasonal means);
3. If the process variance varies seasonally, homoscedasticity (i.e., constant variance) should be achieved by an appropriate Box–Cox transformation;
4. Data should be collected uniformly, i.e., at constant time-intervals. However, Kalman filter methods for modelling irregularly spaced time series, as proposed by Bierkens et al. (1999), could be extended to intervention analysis.

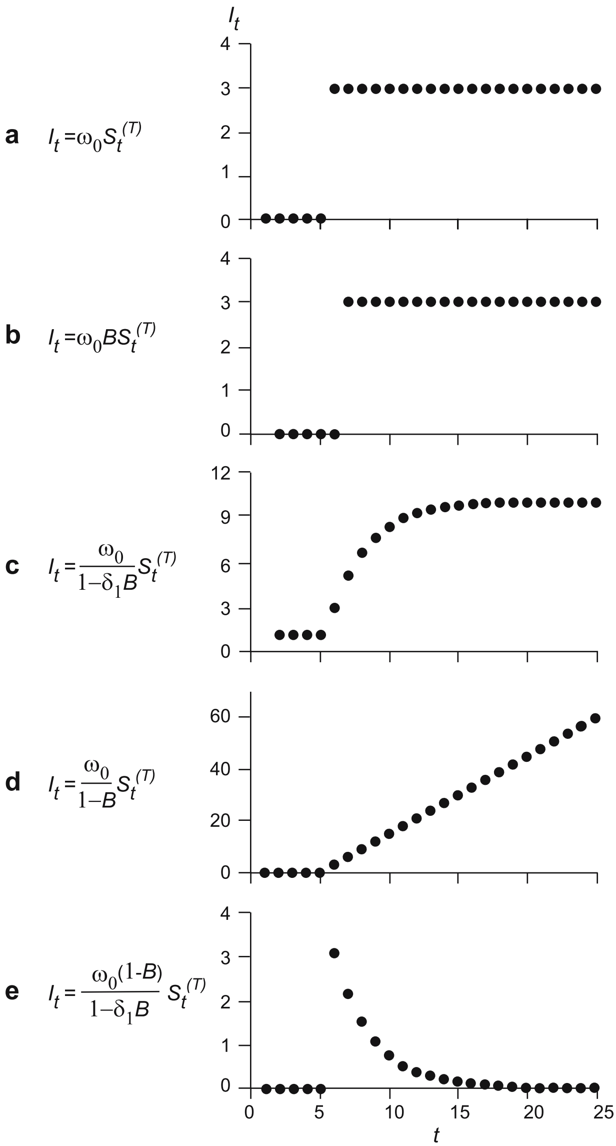

Fig. 13.10. Responses to a step intervention. $T = 5$, $S_t^{(T)} = 0$ for $t < T$, $S_t^{(T)} = 1$ for $t \geq T$, $\omega_0 = 3$, $\delta_1 = 0.7$, $b = 1$. **a**: step model, **b**: delayed step model, **c**: step decay model, **d**: linear model, **e**: impulse decay model.