Limits to Success. The Iron Law of Verhulst

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Summary. In this chapter we develop the point of view that Verhulst is a major initiator of systems thinking. His logistic equation is a system archetype, i.e. a simple system built with few feedback loops. In the Fifth Discipline [19] Peter Senge calls this particular archetype "Limits to Success". It can also be called the "Iron law of Verhulst", expressing that trees can never grow to heaven. In a deeper analysis this equation illustrates the shifting loop dominance, one of the basic principles of system dynamics. The basic message is that the combination of some few archetypes, like the logistic growth, can afford valuable insight into many complex systems such as the economy, environment, organisations, etc. This fruitful concept is illustrated by a simple model in behavioural finance describing the equity price evolution, and based on the interplay of three main growth archetypes: "Limits to Success", "Tragedy of the Commons", and "Balancing Loop with Delay".

1 Introduction

Chaos theory is said to have been founded by the 1-D logistic equation. This is certainly true although, as it is well known, the merit of discovering chaos in the discrete formulation of this formula may be given to May [16] in 1976, more than one century later. In its original continuous format the logistic equation is unable to generate chaos. This is a consequence of the Poincaré–Bendixon theorem, which says that there is no chaos on the line, or on the plane, thus at least 3-D is needed. In this chapter we develop the point of view that Verhulst, more directly, started "systems thinking" applicable to complex systems. There is clearly a straight line between Verhulst's germane ideas and the feedback-centred thinking of System Dynamics (SD), developed by J.W. Forrester [6, 7] in the 1960's, and used by the early Club of Rome in its famous book Limits to Growth [17]. What Verhulst's equation simply says, is that there is shifting loop dominance between two feedback loops (FBL): a positive FBL initiates growth; it is brought into balance by a negative FBL with growing importance, incorporating the limits to growth in a finite world. The association of FBL's of different polarities and the shifting dominance between them is indeed the central thought of SD to model complex reality in population dynamics, ecology, economy, organisations, etc. These ideas have been later translated into management recipes by Peter M. Senge in his famous book *The Fifth Discipline* [19]. Simple archetypes are presented there as elementary building blocks, pervasive in all organizational problems. All archetypes result in the association of one to three FBL's with different polarities. Senge argues that most dynamic patterns can be reproduced from the association of some of them.

In Sect. 2 we develop some basic concepts of systems thinking from this perspective. We use as a starting point the logistic equation as an important growth archetype in SD. In Sect. 3 we present two other growth archetypes, "Tragedy of the Commons", and "Balancing Loop with Delay", developed along similar lines to Verhulst's logistic equation. In Sect. 4 we present a simple behavioural model of stock-price evolution by combining the basic mechanisms imbedded in these archetypes. Three families of investors are interacting on the equity market: fundamentalists, opportunists and long-term traders. This model comprises at least three stocks, and, therefore, chaotic dynamics is possible, contrary to the case of the continuous 1-D logistic equation. A conclusion relative to systems thinking and its links to the Iron Law of Verhulst is given in Sect. 5.

2 The Logistic Equation, a Prototype of Systems Thinking

Figure 1 reproduces a possible influence diagram of the logistic equation of Verhulst in the very framework in which it was originally published, i.e., population dynamics. It represents a one-stock, two-flow System-Dynamics (SD) model of the evolution of a deer population; the latter is submitted to a food availability constraint. The only stock is represented by a rectangular reservoir, according to the tradition introduced by J.W. Forrester, the initiator of SD, in the early sixties of the last century.

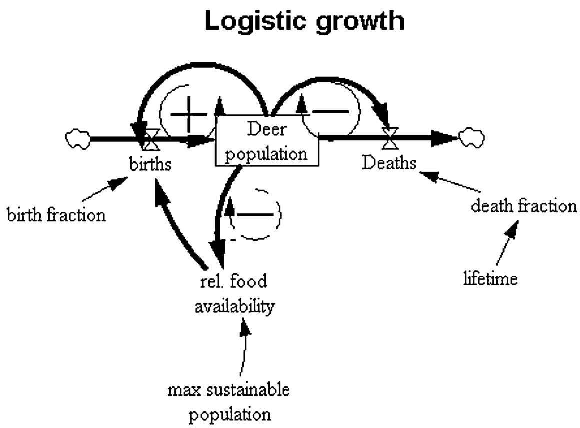


Fig. 1. The influence diagram of the logistic growth of a deer population

Calling P the population, its logistic growth is represented by the Verhulst equation in a modernized form, and slightly modified to explicitly include the deer death rate:

$$\frac{\mathrm{d}P}{\mathrm{d}t} = rP\left(1 - \frac{P}{K}\right) - DP\ . \tag{1}$$

According to the usage in ecology, r represents the fractional growth rate corresponding to the r-strategy in a biotope, and K the limiting population size at maturity, corresponding to the K-strategy; D is the fractional death rate per unit of time, such that D =1/Lifetime of deer.

Figure 2 shows the evolution of the population and of the two flows, "Births" and "Deaths". At logistic equilibrium the two flows become equal, so that the net flow vanishes. Figure 3 is the representation in the phase plane (deer population, net growth rate). The equation of the 1-D flow on the r.h.s. of (1) is a parabola. All this is of course well known. The influence diagrams and the computations originate from the SD-code VENSIM ® [23].

Let us spend some more time examining the two feedback loops (FBL) in Fig. 1. The positive FBL in the influence diagram represents the growth process. The induced growth pattern is exponential; it corresponds to the r-strategy.

Except for the natural death rate, the only negative influence is between "deer population" and "relative food availability": both variables move in opposite directions. Assuming that less food means less non-lethal births

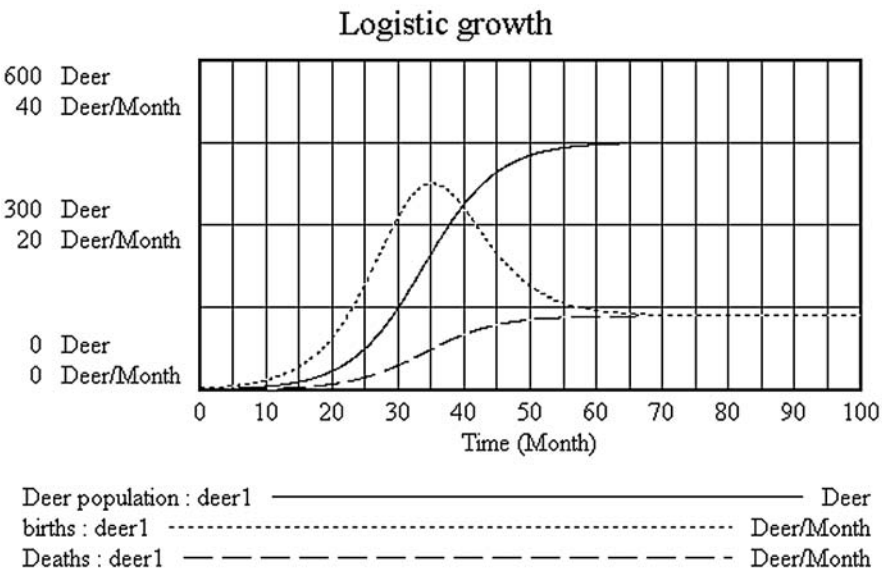


Fig. 2. The evolution of the stock and of the two flows in the logistic-growth model of Fig. 1

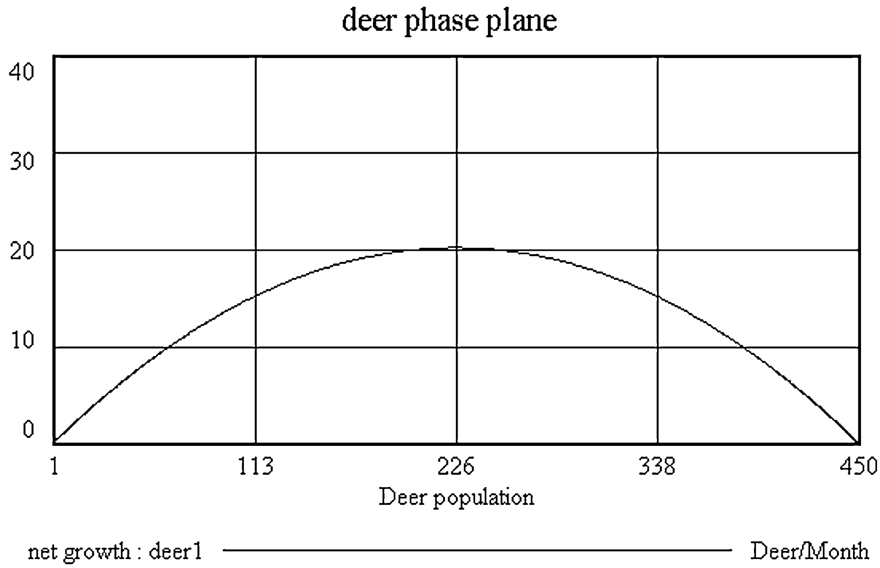


Fig. 3. The phase plane $(P, \mathrm{d}P/\mathrm{d}t)$ of the logistic equation showing the parabolic function on the r.h.s. of (1)

of fawns, a negative FBL is obtained. The induced growth pattern is goal seeking with a resulting equilibrium population size K; it corresponds to the K-strategy.

The dynamic behaviour of this simple dynamic system is dictated by "shifting loop dominance" between the two FBL's in the left part of the diagram:

- First the (+) FBL activates the r-strategy, i.e. nearly exponential growth, the (−) FBL remains weak because it is driven by the term $rP(P/K)$ in (1), which is still second-order, and nearly negligible;
- As P grows this latter term becomes larger, and progressive shifting loop dominance appears. This concept has been introduced by Forrester [6–8]. In this specific case this simply means that the weaker (−) FBL becomes increasingly active with respect to the (+) FBL. In the growth curve, an inflection point is visible when $P = K/2$;
- At equilibrium, both loops are equally active, and thus exactly in balance, and the nonlinear process of shifting loop dominance is then complete to realise the asymptotic equilibrium at $P = K$.

Shifting loop dominance is the central idea of FBL-thinking, and thus of SD [8]. The properties of nonlinear systems are changing in the phase space. Some loops are dominant, or simply active, while some other ones are dormant, or practically inactive. So that there are no universal properties any more, contrary to what happens in linear systems.

Even if all FBL's are present from the beginning in the influence diagram of the model, much different behaviour can be observed by numerical integration as the relative strengths of several FBL's change along the way. This explains why nonlinear systems often show counterintuitive behaviours as already stressed by Forrester in his Urban Dynamics [7]. This complexity can be observed with only few FBL's, but it increases when there are many possible combinations of interacting FBL's present in the model. Given n FBL's there are $n(n+1)/2$ FBL pairs to be compared. A larger system can have hundreds, or thousands FBL's!

This counterintuitive behaviour is a different concept from deterministic chaos. It has to do with the co-existence of many possible attractors of different nature (strange attractors are just one family). Another complication arises because of the possible bifurcations when parameters in the system (like the birth fraction) change value. This further increases the unpredictability and in fact the complexity of the system behaviour.

The 1-D logistic equation is unable to generate chaos, when the integration is done properly. This is because of the Poincaré–Bendixon theorem, which states that there can be no chaos on the line or the plane (see for example [9], Chap. 5.8, on stability properties in nonlinear systems). Chaos is thus only potentially observable in nonlinear systems with three stocks and more.

In the 1-stock case, chaos will only be observed as the result of an improper choice of the integration time step, and in this case it is thus a mere mathematical artefact (see [15]). Equation (1) indeed needs first to be numerically integrated, with introducing of a discrete time step. The Euler integration scheme in time t can be written as follows:

$$P(t+\Delta t) = P(t)\left[1 + r\left(1 - \frac{P(t)}{K}\right)\Delta t\right] . \tag{2}$$

Assume that the initial condition is such that $0 < P(t=0) < K$. Because for all finite t, the exact solution of (1) is such that $P(t) < K$, if Δt is small enough, $P(t)$ will be increasing from $P(0)$ without ever exceeding K, except when $P(t)$ comes very close to K from underneath. One should then observe that for small enough Δt's:

$$P(t+\Delta t) > P(t) > 0 \text{ when } K - P(t) > \epsilon > 0 , \tag{3}$$

where $\epsilon < \Delta t$ is a very small number. Numerically, for t sufficiently large $P(t)$ will slightly exceed K, so that the flow of the r.h.s. becomes negative; $P(t)$ will then gently oscillate with hardly observable amplitude around K. It can be intuitively understood that for larger Δt steps, oscillations will become of larger amplitude; once situations arise wherein $P(t)$ becomes significantly larger than K, overshoots of larger amplitude then occur, making $P(t)$ swinging hence and forth passing the K-value; the place where the population size P crosses the horizontal line at the boundary value K then changes at each period. Chaos arises when the set of crossing points becomes infinite. This

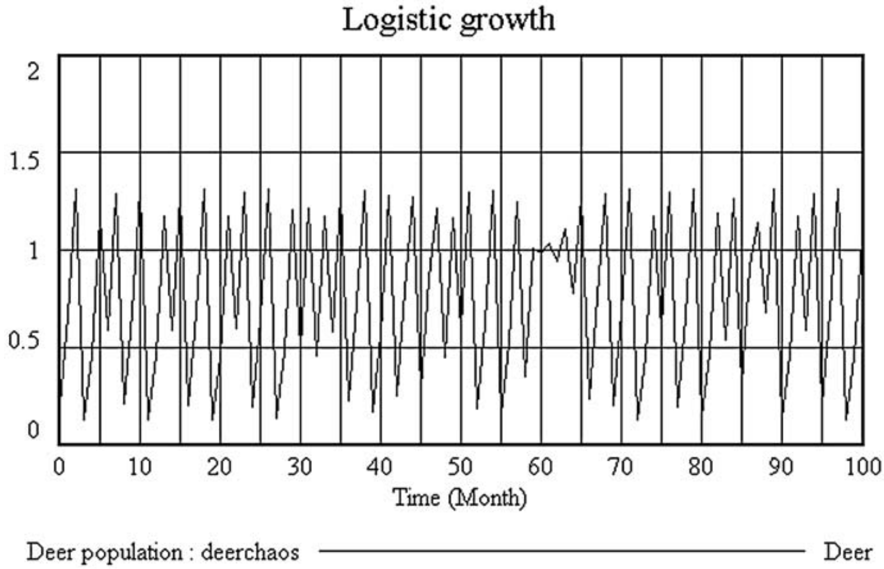


Fig. 4. When the time step is too large, the integration of the logistic equation with the Euler scheme generates similar pattern as in the logistic mapping, including chaos

situation is shown in Fig. 4. It is observed that, contrary to expectations from the continuous (2), P swings widely above the $K = 1$ boundary value of the population. Similar evolutions appear to the logistic map when the growth parameter increases. Several authors have established a correspondence between this latter parameter, and the time step, obtaining herewith the bifurcation diagrams in function of Δt. This discussion does not need to be reproduced here (see for example a review paper in [13]).

3 Archetypes

System Dynamics (SD) is a quantitative simulation technique; many authors in many different fields, such as economics and finance, organisation, environment, macroeconomics, etc. use it. A recent handbook is Sterman [22]. Soft modelling with SD is also a possibility. This approach limits the elaboration of models to the first qualitative step of establishing the influence diagram, and analysing the feedback-loop (FBL) structure to deduce some consequences for the system and to derive possible improvement strategies. This approach has some merit, though it is sometimes of limited predictive value: as mentioned before, systems often behave in a counterintuitive way due to the complex feedback interactions, and numerical simulations are necessary to test the actual behaviour patterns. Peter M. Senge is the author of *The Fifth discipline* [19]. His main message is that Systems Thinking is indispensable for understanding and curing organizational problems. Unfortunately the human mind has difficulty in abandoning linear thinking, which was well adapted to local conditions of human societies in the past, but becomes far less adapted to global societies today. Senge further argues that, in numerous cases, simple systems often consisting of two to three basic feedback loops (FBL) provide a sufficient insight on what is going wrong in the enterprise. These elementary systems, called archetypes, thus can be assembled as building blocks for modelling more complex situations. According to Senge's convictions most situations of crisis are reducible to a small number of archetypes. His book enumerates ten main archetypes. Additional ones have been developed in later books of Senge [20] on the basis of the work of Kim [12]. The most important archetypes are centred on three main growth patterns:

1. Logistic growth of (1) is described as combining exponential growth embodied in a (+) FBL, and goal-seeking growth, embodied in a (−) FBL. It represents the "Limits to Success" archetype in Senge's book. The interplay between the two FBL's leads to the described shifting loop dominance, as has been illustrated in the basic Verhulst model. The archetype is shown in a more general way in Fig. 5: the whole model rests on the assumption that some resource is limited and becomes inadequate at some point. A more business-oriented case is shown in Fig. 6, called "Doctor's Practice" [18]. It illustrates the interplay between on the one hand the growth process of the (+) FBL, around the mouth-to-mouth publicity of satisfied patients, and, on the other hand, the constraints of the time resource. The latter is impeding the further growth because of the (−) FBL related to the diminishing acceptability of time spent in the waiting room.

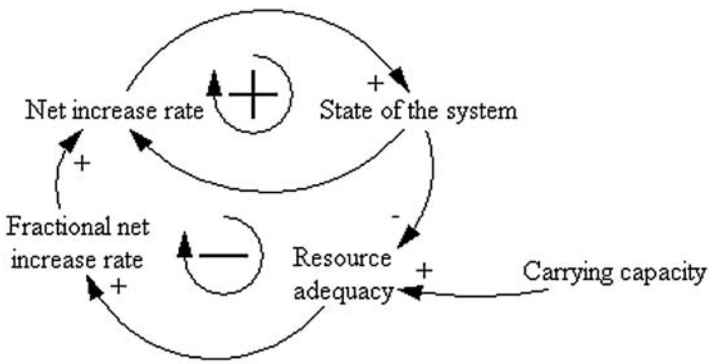


Fig. 5. The archetype "Limits to Success" as a generic model of Verhulst's Iron Law (according to [22])

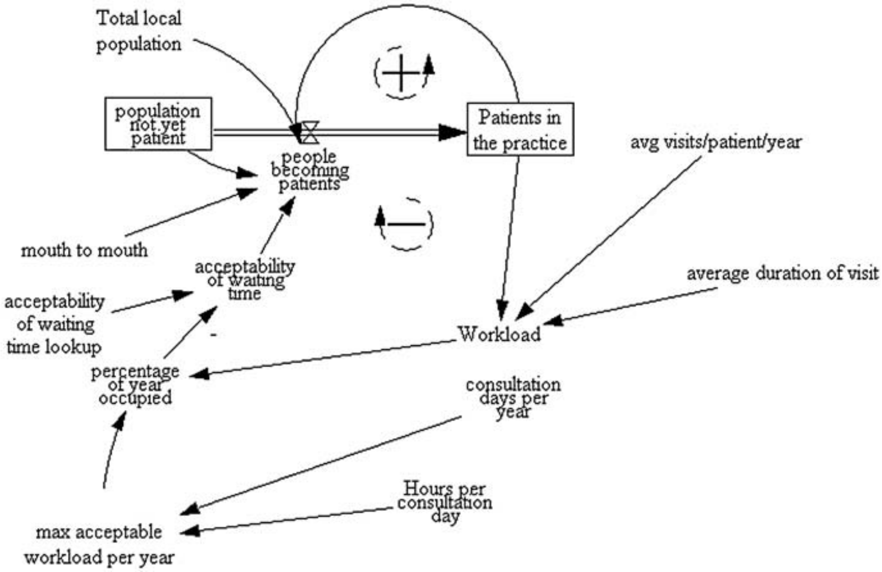


Fig. 6. The logistic growth in the doctor's surgery, as a further illustration of Verhulst's Iron Law. The limited resource is here the time that the doctor can devote to his patients (according to [18])

2. Overshoot and collapse growth appears in a second archetype "Tragedy of the Commons", according to the economist Garrett Hardin [10]. This type of growth is quite pervasive in complex systems (traffic congestion, exhaustion of depletable resources, collapsing of biotopes, etc.). It is obtained from Verhulst's logistic growth by adding just one more (−) FBL, as shown in Fig. 7. In the first archetype of logistic growth in Fig. 5, the resource is in some way an external parameter to the model, embodied in the constant K in (1). The second (−) FBL on the right of the drawing now includes the limiting resource in the model. It corresponds to an erosion mechanism. The growth goal K, instead of being constant, will now be suddenly and often unexpectedly be collapsing through the internal nonlinear forces in the system. In human systems, the erosion is caused by the inadequate use of a common good or resource (highway, oil, etc.) in the egoistic search for individual advantage.
   The model in Fig. 8 has two stocks, i.e. two ordinary differential equations, for representing both state variables, in this example deer population P and vegetation level V. These equations look as follows:

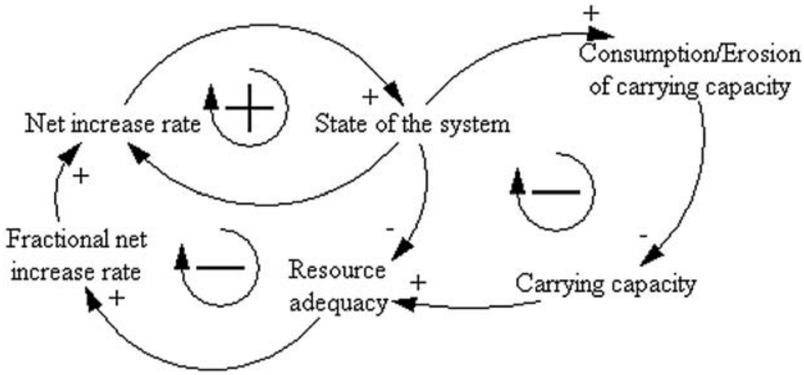
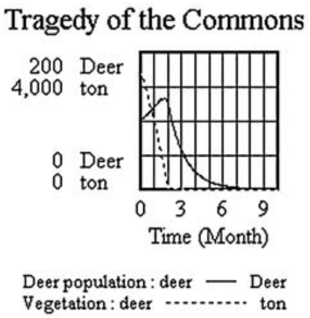


Fig. 7. The extension of the logistic model to a two-stock model representing the erosion of the food resource in the archetype "Tragedy of the Commons"



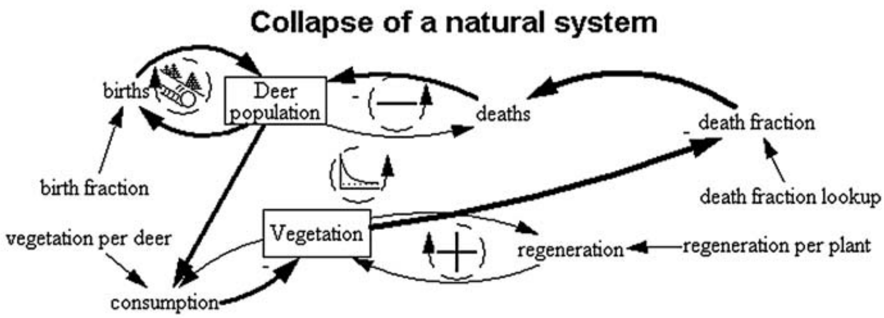


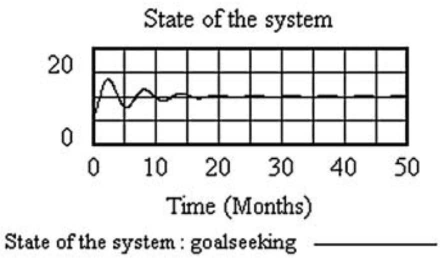
Fig. 8. The extension of the deer model of Fig. 1, including the resource *Vegetation* into the model. The deer population collapses when the food resource is eroded away

$$\frac{\mathrm{d}P}{\mathrm{d}t} = rP\left(1 - \frac{P}{K}\right) - D(V)P \tag{4}$$

$$\frac{\mathrm{d}V}{\mathrm{d}t} = sV\left(1 - \frac{V}{L}\right) - CPV\ . \tag{5}$$

The variables and parameters in (4), representing the deer-population dynamics, have the same meaning as in (1). $D(V)$, the death flow, is a declining nonlinear function of its argument V to be represented by a lookup table. It is of course equal to the natural death rate when the food is abundant, and it grows to 100 % mortality when food is disappearing. In (5), representing the vegetation dynamics, s and L are constants, and they correspond to r and K in (4). The parameter C represents the specific consumption of food per deer and time period.

3. The third archetype is called in Senge's original work "Balancing Loop with Delay". It is basically a goal-seeking loop. Delays may be present at several stages: when information is collected or processed to take action, or before action leads to a change in the state of the system. The generic archetype is shown in Fig. 9. All loops are negative, because each information delay corresponds to one or several one-stock systems with



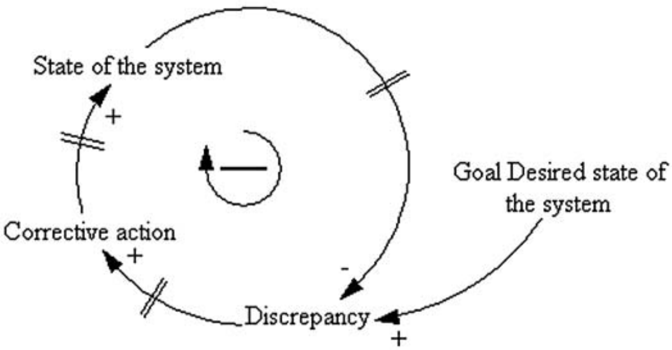


Fig. 9. The archetype "Balancing Loop with Delay" in which an information signal within a negative goal seeking FBL is submitted to delays causing overshooting and oscillations

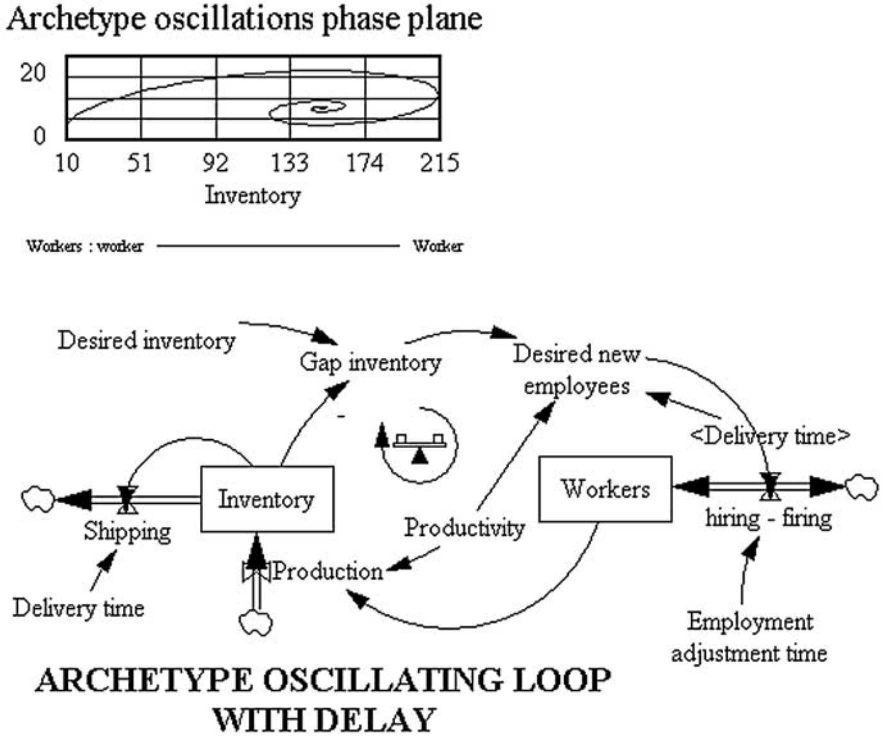


Fig. 10. In this archetype the oscillations are caused by the information delay between the inventory state and the manpower hiring

an outgoing flow (see [22], Chap. 11). This archetype is typical for the existence of business cycles. An example is shown in Fig. 10, representing a manpower-management problem; it consists in a goal seeking loop, which is itself a first-order delay, embedded in a two-stock system: inventory and manpower, as follows:

$$\frac{\mathrm{d}S}{\mathrm{d}t} = \frac{\mathrm{Goal} - S}{T_{\mathrm{adj}}} \,, \tag{6}$$

where S is the stock due to achieve the goal, and T_{adj} is the time constant necessary for the goal-adjustment process; it also represents the time delay constant. Equation (6) is the equation of a linear proportional controller (e.g., a thermostat) used in engineered devices to bring the state variable (e.g., the room temperature) to a desired goal. More complex nonlinear controllers, used in engineering, can be developed for the same purpose.

4 Modelling a Bubble on the Stock Market

In this section we discuss the modelling of the development and crash of a speculative bubble on the equity market (EM). The recent history of the high-tech bubble mainly in the years 1997 to 2003 provides a good example.

The archetypes presented in the previous section are the starting basis for modelling, because of their characteristic growth patterns. When a financial bubble first builds up, exponential growth is observed; later on temporary plateaux appear reminding of logistic growth equilibrium; there are also pseudo-random oscillations reminding of the "Balancing Loop with Delay"; finally crashes resemble the patterns in the Tragedy of the Commons.

Financial crashes were qualitatively modelled as cusp catastrophes by Zeeman [24]. R.H. Day [4] was one of the first authors who intensively worked in quantitative non-equilibrium models inspired from chaos theory in discrete nonlinear systems. Following ideas of Shiller [21], Day postulates two families of investors, smart and ordinary investors. The formers are called α-investors; the latter are called β-investors. α-investors use quantitative valuations from fundamental analysis, they are basically goal-seekers and they stabilise the market. Their investment profile as a function of the price has a reverse shape, because they are contrarians. β-investors, by contrast, remain in phase with the price trend by using simple investment rules: they overreact to sudden price moves or to fads, creating volatility. Day combines both investors' profiles to define iterative 1-D mappings of the stock price: $p(t+1) = f[p(t)]$. The patterns he observes show phases of high volatility, betraying the existence of deterministic chaos like in the logistic mapping. Unfortunately, with those models it is much more difficult to generate more representative evolutions typical for EM, like bullish or bearish behaviours, bubble formation and crashes, etc.

The idea developed by Kunsch et al. [14] is to use a continuous model with at least three stocks, in order to have the possibility to observe chaos, and a number of FBL's able to generate representative and more realistic EM signatures. The objective of considering at least three stocks is easy to achieve by considering several investors' families and information delays. Each first-order delay requires one stock. Additional budget stocks represent the financial constraints of investors.

Of course an important literature exists dealing with nonlinear dynamic modelling of the EM (see for example [3], [2] from [11]), or with artificial stock markets [1]. The ambition is not to present an up-to-date review of these models generally placed in the field of behavioural finance. Rather it is planned here to show how archetypes, inspired from Verhulst's ancient contribution, are still a source of inspiration for complexity modelling.

The universe in the EM model presented here is very simple. There are only two assets: a risky asset quoted at a variable homogeneous price P (it could represent a common equity index like Eurostoxx 50), and a risk-free asset, e.g., a high-rating bond. This universe is frozen for a given simulation run. This means first that the total number of equity shares n is fixed in all scenarios. Second, all economic parameters of the model are constant, including the growth rate of the fundamental value, and the risk-free rate. Some fixed constraints are imposed on the available budget of the investors and their borrowing capacity.

Three homogeneous groups of investors are considered, instead of two in Day's model. They are called α-, βS-, and βL-investors, where "S" stays for short-term, and "L" stays for long-term. Influence diagrams can be drawn to represent each investor's behaviour. Several important feedback loops are identified. They assist the understanding of basic behavioural rules developed in the investors' minds. Some characteristics are important to understand. Negative loops assist the goal-seeking approach of fundamentalists. α-investors therefore help stabilizing the stock prices. By contrast positive loops, activated by short-term traders (βS-investors) are responsible for amplifying perturbations or rumours. Sometimes such a loop can act as a virtuous circle, in case it triggers a desired growth effect in prices thanks to long-term strategies of βL-investors. Sometimes the loop acts as a vicious circle, because it amplifies the market volatility, or it triggers crashes. In the EM model, the two roles will be played in turn.

The three families of investors are now described in more detail; it is shown in each case in which way they are representative of the previously introduced archetypes. Note beforehand that this model is very simplified because many variables are considered as exogenous parameters, to be held constant: the relative proportions of the different investor types, and the risk-free interest rate among others. These assumptions could be removed at the cost of higher complexity (e.g. including these parameters as model variables into additional FBL's), but that would be beyond the scope of the present work.

The same presentation is adopted as in [14].

Note first that the total equity price P is split up into the three components representing the contributions of investors from different groups:

$$\begin{aligned} P(t) &= \frac{1}{n}\left(M_{\alpha} + M_{\beta S} + M_{\beta L}\right) \\ &= P_{\alpha} + P_{\beta S} + P_{\beta L}\ . \end{aligned} \tag{7}$$

$M_{\alpha}, M_{\beta S}, M_{\beta L}$ represent the amounts of money invested by the three investor types in the EM; n is the constant number of shares; $P_{\alpha}, P_{\beta S}, P_{\beta L}$ represent the three components of the total price P, attributed to the three investor types.

4.1 Family of α-Investors

α-investors are "smart investors" behaving in a similar way to the rational goal-seekers assumed in Day's model. Their sole aim is to achieve convergence

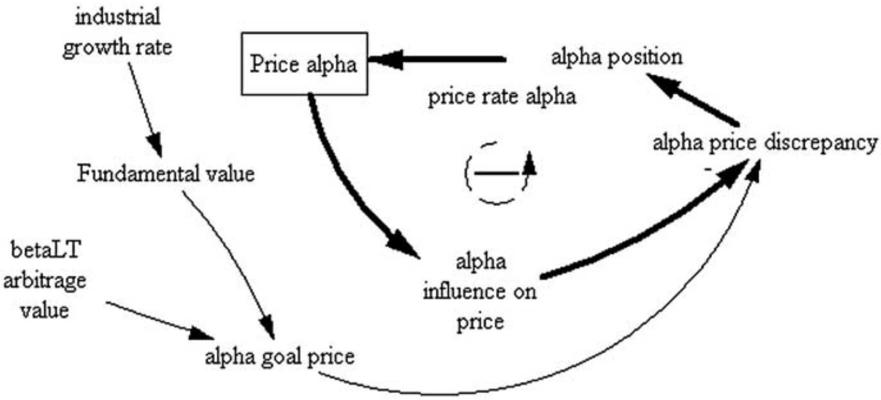


Fig. 11. Negative Feedback loop of α-investors (goal-seeking behaviour)

towards a current goal price G_α for the stock price P. It is why the unique feedback loop visible in Fig. 11 is a negative goal-seeking FBL. The price component P_α obeys to a similar equation to (6):

$$\frac{\mathrm{d}P_\alpha}{\mathrm{d}t} = \frac{G_\alpha - P}{T_\alpha} \tag{8}$$

As said above, more complex goal-seeking formulations can be adopted. Because there are possibly information delays in the price adjustment, damped oscillations caused by slight overshooting above the goal price may be observed.

In our model, the assumed current goal price G_α is the sum of two terms: fundamental value g_α, and risk premium $A_{\alpha,\beta\mathrm{L}}$ resulting from the investing behaviour of βL-investors, to be described later:

$$G_\alpha = g_\alpha + A_{\alpha,\beta\mathrm{L}} \tag{9}$$

– The fundamental value g_α results from fundamental analysis, e.g., Dividend Discount Model (DDM). Deterministic dividends are assumed here, because stochastic changes do not bring more understanding on causal mechanisms. Dividends are growing with the given constant industry growth rate: g_α is growing at the same rate.
– The risk premium depends on the arbitraging behaviour of βL-investors between the stock return and the risk-free rate; this is explained below. In case of a positive gap, they invest more money into the EM, creating herewith a price increase $\Delta P_{\beta L}$, i.e. a risk premium above the fundamental value. In this case α-investors also adjust their long-term expectations, and they follow the observed positive trend over the fundamental value. In practice, the premium is incorporated into the goal price by α-investors only up to a certain point; this occurs with a time delay τ. In the model it

is assumed that, in a bullish market mainly driven by the premium term, α-investors will cap their goal price by a maximum arbitrage value $\Delta g_{\alpha}^{\max}$. The latter corresponds to an acceptable risk level. Thus the actual risk premium is given by the following equation:

$$A_{\alpha,\beta\mathrm{L}} = \min\left(\Delta g_{\alpha}^{\max}, \mathrm{delay}_{\tau}[\max(0, \Delta P_{\beta\mathrm{L}})]\right) . \tag{10}$$

In the model α-investors do not experience any liquidity constraints. This is a reasonable assumption, as they stop anticipating further price growth, as the risk premium above the fundamental value becomes exceedingly large.

In conclusion, α-investors behave according to the goal-seeking part in the logistic equation (Verhulst's Iron Law). Because the goal is changing under the effect of βL-investor strategies, there is a need for information collecting: damped oscillations due to overshoots may be observed, as in the archetype "Balancing Loop with Delay".

4.2 Families of β-Investors

The β-investors are "ordinary investors" in Shiller's sense [21]. They are not entirely rational with respect to the use of information coming from the market. They use different approaches to process the information, from rules of thumb to advanced technical analysis. An important aspect is the time horizon of anticipation, covering a continuum between short-term to long-term. The model only considers two extreme cases in a continuum: βS-investors have a short-time horizon (S); βL-investors have a long-term horizon (L). Also proportions of the two types are kept constant. More sophisticated models may consider intermediate investors' profiles or varying proportions within the model.

Family of βS-Investors

βS-investors are opportunistic traders who are following immediate price movements; they buy in case of a price increase, and they sell when the price is going down. Therefore they destabilise the goal-seeking efforts of α-investors, who are contrarians, and they cause permanent noise. The presence of a positive feedback loop, visible in the right part of Fig. 12, confirms the existence of this destabilizing investment approach. The driver in this loop is the first derivative of the price, initiating a vicious circle of growth or decay. A negative loop is visible in the left part of the diagram. It becomes active as the available budget drops to zero, forcing βS-investors to limit their stock position or even to liquidate part of their portfolio. The dynamic equation of βS-investors has been assumed to be the following:

$$\frac{\mathrm{d}P_{\beta\mathrm{S}}}{\mathrm{d}t} = S_{\beta\mathrm{S}}\, f\left(\frac{\mathrm{d}\bar{P}}{\mathrm{d}t}, P\right) - R_{\beta\mathrm{S}}(B_{\beta\mathrm{S}}) , \tag{11}$$

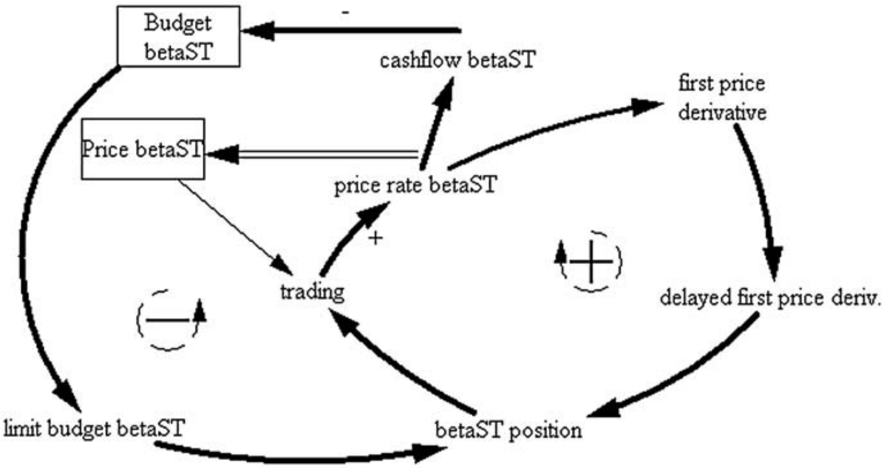


Fig. 12. Feedback structure of short-term traders, i.e. βS-investors. The budget acts as a control mechanism; the presence of information delays causes oscillations. There is also a partial correspondence between this diagram and the archetype "Tragedy of the Commons"

where $S_{\beta S} > 0$ represents the strength of βS-investors on the market (assumed to be constant). The function $f(.,.)$ depends in a nonlinear way on the stock price P and the smoothed value of its first derivative $\mathrm{d}\bar{P}/\mathrm{d}t$. Its sign is the same as the sign of the latter, indicating that βS-investors are trend-followers modifying their positions according to increasing or declining prices. A smoothed signal is calculated as an information delay as in (6), so that this may be the cause of oscillating behaviour, as in the archetype "Balancing Loop with Delay". Overshoots may also be observed, which bring the budget to negative values. In such situations the βS-investors have to liquidate part of their portfolios. This appears in the last term on the r.h.s. of (11): $R_{\beta S}(B_{\beta S})$ represents the reimbursement rate to bring the budget $B_{\beta S}$ back to balance, in case it becomes negative. In conclusion, the delay mechanism and the budget constraints make that βS-investors are a source of instability and create pseudo-random oscillations in the search for price equilibrium on the EM.

Family of βL-Investors

βL-investors rather have a long-term perspective. They permanently compare the long-term stock return and the risk-free interest rate (i_{rate}, assumed to be constant in this simple model). In case of a positive spread, in favour of risky asset positions, they invest additional money, curbing on the growth of the stock price. Therefore a positive feedback loop is visible in the upper part of the diagram in Fig. 13. It is driven by the positive return spread between

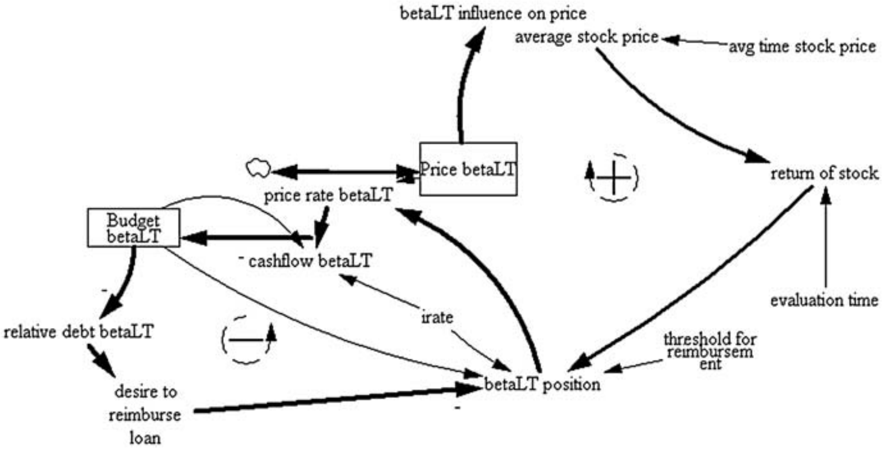


Fig. 13. Feedback structure of long-term traders, i.e. βL-investors. The budget fuels growth up to a certain point just before collapse. There is a clear correspondence between this diagram and the archetype "Tragedy of the Commons"

risky and risk-free assets, creating a risk premium. As indicated in (8) and (9), α-investors will adjust in part their goal price to follow the growing price trend caused by βL-investors. In contrast to βS-investors, βL-investors have a borrowing capacity. They invest the borrowed money reinforcing herewith the growing trend, and transform it progressively into a vicious circle. Of course at some point there is shifting-loop dominance in favour of negative FBL's like in the archetypes "Limits to Success", and the "Tragedy of the Commons". Such a loop is visible in the lower part of the diagram in Fig. 1: it relates to the available money resources. βL-investors have an initial budget and borrowing capacity up to a given permissible debt level. In any case, their willingness to reimburse their loans will grow with the relative level of their debt expressed as a percentage of the value of their stock position. As long as some borrowing capacity remains, βL-investors further strengthen their stock positions. Above some debt threshold, they experience an incentive to liquidate at least part of their positions. Another negative loop is not directly visible in Fig. 13, however. It finds its origin in the cap imposed by α-investors on the permitted price growth above fundamentals, according to (10). The simplified equation representing the dynamics of βL-investors is as follows:

$$\frac{\mathrm{d}P_{\beta\mathrm{L}}}{\mathrm{d}t} = S_{\beta\mathrm{L}}(r-i) - R_{\beta\mathrm{L}}\left(B_{\beta\mathrm{S}}, V_{\beta\mathrm{L}}\right) \ , \tag{12}$$

where $S_{\beta L} > 0$ represents the strength of βL-investors on the market (assumed to be constant in this simple model). r represents the smoothed stock return, and $i = i_{\mathrm{rate}}$, the risk-free interest rate. The growth pattern of the first term on the r.h.s. of (12) is thus exponential. $R_{\beta\mathrm{L}}$ represents the reimbursement rate of loans in case the current budget $B_{\beta\mathrm{L}}$ is becoming negative and exceeds the borrowing capacity, which depends on the current value $V_{\beta L}$ of the portfolio owned by βL-investors.

In conclusion βL-investors develop a growth mechanism that is similar to the archetype "Tragedy of the Commons". The growth is fuelled on a basis of an artificial money-borrowing resource and the landing can be quite sudden and hard, as some limits in the borrowing capacities are exceeded. Note that α-investors contribute to define when the bubble crash will start by capping the risk premium, as shown in (10).

4.3 Some Results of Simulation

In short some results of the EM model are presented in the form of time diagrams. The latter represent the total stock price (upper curve represented by a thick line), and its three components, indicated as P_{alpha} (heavy line), P_{betaST} (medium-heavy line), and P_{betaLT} (thin line), according to (7). There are many possible choices for the parameters in the model, but we shall limit our discussion to a few typical cases for bubble growth and crash. We again adopt the presentation from [14].

A first scenario represents a boundary situation, helpful to calibrate the model. The risk-free rate is assumed to be very high; it is then expected that the risk premium is vanishing, i.e. no βL-investors will be present on the market, and thus no bubble can appear. In this case the price will gently

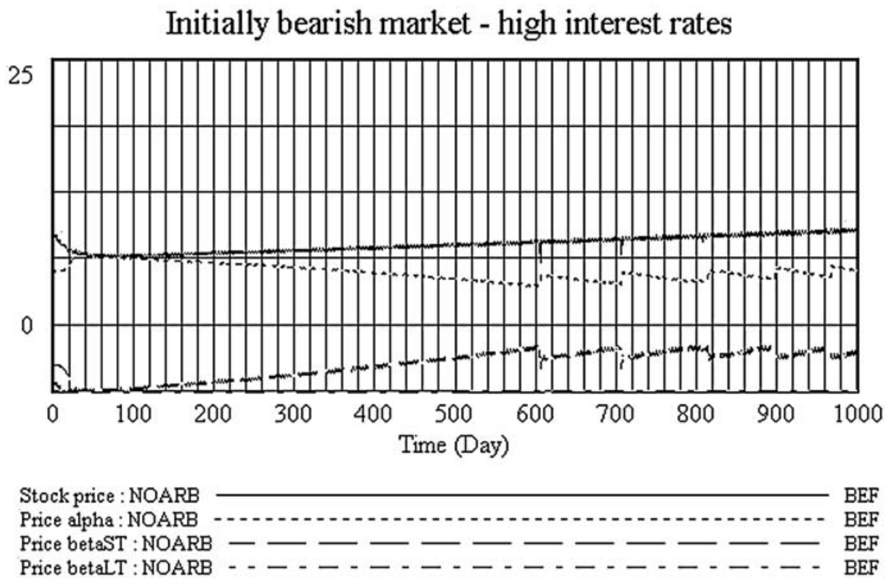


Fig. 14. (From [14]). A calibration calculation in case there is no risk premium, because of high risk-free rates

follow the industry growth rate. The price components and the total price are shown in Fig. 14 confirming our expectations. The total price rapidly comes to the fundamental value. The lower curve shows the contribution of βS-investors following the growing trends. Sometimes budgets become negative, so that, according to (11), a broken line and oscillations back to positive budget values are observed.

The following figures show two situations, in which the risk-free interest rate is low, so that βL-investors can arbitrage. In Fig. 15 the market is initially bearish. The return spread is negative, i.e. in favour of risk-free investment. For quite a long time, the market is in near-equilibrium at the goal price set by α-investors; the growth rate is equal to the industrial growth rate (7% p.a.). However, the steady industrial growth brings about a fresh-born wave of βL-investors. As a result the goal price also shifts up. βL-investors soon find their limits. The borrowing capacity is reached. At this point, βL-investors have to rapidly liquidate the largest part of their stock portfolio in order to bring down their loan debt to an acceptable level. This reimbursement constraint has the same effect on the price as a reflecting barrier. The price bounces back creating chaotic ups and downs of the price in search for a new equilibrium value. βS-investors amplify the appeared volatility. The price volatility becomes so large that at some point the long-term return drops below the risk-free rate. βL-investors disappear from the scene after a crash of limited amplitude. The market moves to a new equilibrium following the natural trend of fundamental values.

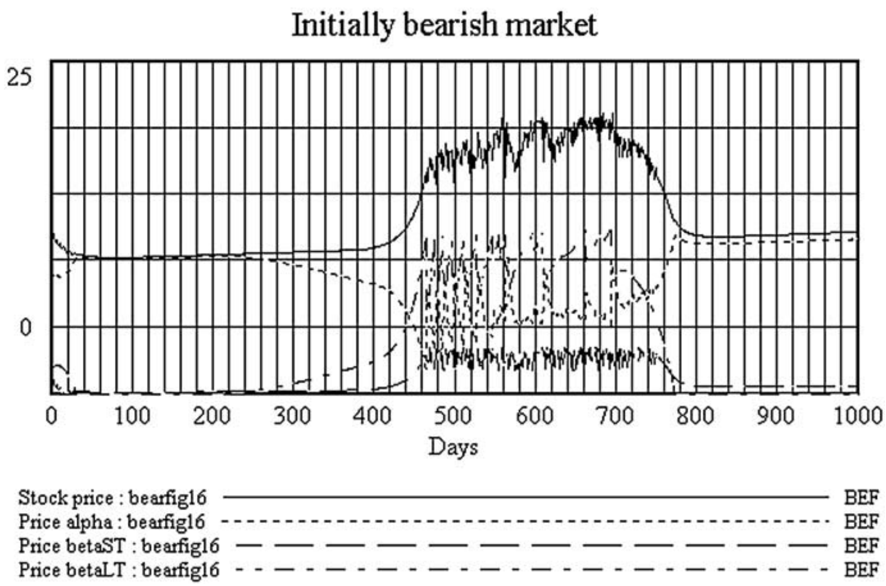


Fig. 15. (from [14]). The case of low interest rates, and an initially bearish market

In Fig. 16 the same assumptions as in the previous figure are used for simulating an initially bullish market. A burst in price sets up immediately; it is accompanied by high volatility. The bouncing back of the price against the debt barrier induces still more volatility than in the previous case. This turbulent behaviour cannot maintain itself very long. The market moves to its fundamental equilibrium as in the previous figure. After a while, a new price upsurge is observed with still more volatility than previously in the growing phase. It lasts for quite some time, exhibiting swings of considerable amplitude. As before, a crash brings back the price to its natural equilibrium. When pursuing the computation, regular replicas with similar shapes are periodically observed.

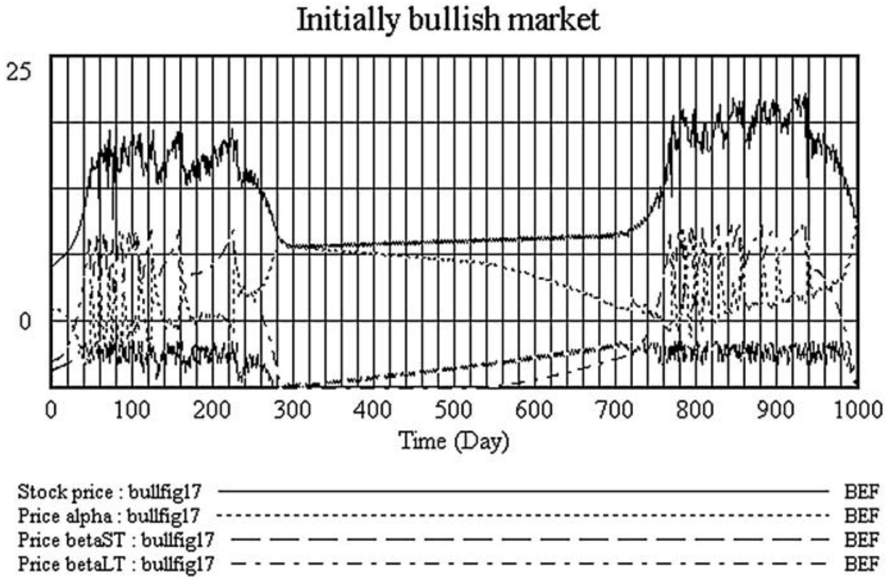


Fig. 16. (from [14]) The case of low interest rates, and an initially bullish market

The more detailed paper contains additional runs with other choices of parameters, and a comparison of simulations with real observations on the EM. The readers are referred to this paper for more details. As a source of inspiration for more advanced models, Fig. 17 presents, without any further comments, the evolution of the Eurostoxx 50 index between October 1994 and September 2004 during the lifetime of the recent high-tech bubble.

Fig. 17. The evolution of the DJ Eurostoxx 50 from October 1994 to September 2004 (from the website wallstreet-online)

5 Conclusions

The author has attempted in this chapter to present Verhulst's contribution in a somewhat different light: discussions generally go about the links between the logistic equation and chaos theory. It is argued here that in some way Verhulst was a pioneer of nonlinear system theory. Systems thinking is today becoming a necessity for survival: the world globalization forces us to think in terms of causality networks rather that in terms of isolated cause-effect links [5]. Verhulst's ancient contribution thus remains modern, and it is still needed.

With the logistic growth, Verhulst introduced for the first time in history an influence diagram, in which two feedback loops are competing for dominance. This simple system teaches us that exponential growth is impossible in the natural world, because constraints on resources must be taken into consideration. The Iron Law of Verhulst remains an important message, which is unfortunately not yet accepted by all. Without this insight it is impossible to start a reflection on how to remove the "Limits to Success", which are today threatening the very long-term existence of mankind in the limited spaceship earth.

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