I

Introduction

I.1 General Considerations

Materials are of a discrete nature, since they are made of atoms and molecules, in the case of liquids and gases, or, in the case of solid materials, also of fibres, crystals, granules, associations of different materials, etc. The physical interactions between these constituent elements determine the behaviour of the materials. Of the different facets of a material's behaviour, *rheological behaviour* is needed for the Mechanics of Materials. It may be defined as the way the material deforms under the action of forces.

The influence of those interactions on macroscopic material behaviour is studied by sciences like the Physics of Solid State, and has mostly been clarified, at least from a qualitative point of view. However, due to the extreme complexity of the phenomena that influence material behaviour, the quantitative description based on these elementary interactions is still a relatively young field of scientific activity. For this reason, the deductive quantification of the rheological behaviour of materials has only been successfully applied to some*composite materials* – associations of two or more materials – whose rheological behaviour may be deduced from the behaviour of the individual materials, in the cases where the precise layout of each material is known, such as plastics reinforced with glass or carbon fibres, or reinforced concrete.

In all other materials rheological behaviour is idealized by means of physical or mathematical models which reproduce the most important features observed in experimental tests. This is the so-called *phenomenological approach.*

From these considerations we conclude that, in Mechanics of Materials, a phenomenological approach must almost always be used to quantify the rheological behaviour of a solid, a liquid or a gas. Furthermore, as the consideration of the discontinuities that are always present in the internal structure of materials (for example the interface between two crystals or two granules, micro-cracks, etc.), substantially increases the degree of complexity of the problem, we assume, whenever possible, that the material is *continuous.*

From a mathematical point of view, the hypothesis of continuity may be expressed by stating that the functions which describe the forces inside the material, the displacements, the deformations, etc., are continuous functions of space and time.

From a physical point of view, this hypothesis corresponds to assuming that the macroscopically observed material behaviour does not change with the dimensions of the piece of material considered, especially when they tend to zero. This is equivalent to accepting that the material is a mass of points with zero dimensions and all with the same properties.

The validity of this hypothesis is fundamentally related to the size of the smallest geometrical dimension that must be analysed, as compared with the maximum dimension of the discontinuities actually present in the material.

Thus, in a liquid, the maximum dimension of the discontinuities is the size of a molecule, which is almost always much smaller than the smallest geometrical dimension that must be analysed. This is why, in liquids, the hypothesis of continuity may almost always be used without restrictions.

On the other side, in solid materials, the validity of this hypothesis must be analysed more carefully. In fact, although in a metal the size of the crystals is usually much smaller than the smallest geometrical dimension that must be analysed, in other materials like concrete, for example, the minimum dimension that must be analysed is often of the same order of magnitude as the maximum size of the discontinuities, which may be represented by the maximum dimension of the aggregates or by the distance between cracks.

In gases, the maximum dimension of the discontinuities may be represented by the distance between molecules. Thus, in very rarefied gases the hypothesis of continuity may not be acceptable.

In the theory expounded in the first part of this book the validity of the hypothesis of continuity is always accepted. This allows the material behaviour to be defined independently of the geometrical dimensions of the solid body of the liquid mass under consideration. For this reason, the matters studied here are integrated into Continuum Mechanics.

I.2 Fundamental Definitions

In the Theory of Structures, actions on the structural elements are defined as everything which may cause forces inside the material, deformations, accelerations, etc., or change its mechanical properties or its internal structure. In accordance with this definition, examples of actions are the forces acting on a body, the imposed displacements, the temperature variations, the chemical aggressions, the time (in the sense that is causes aging and that it is involved in viscous deformations), etc. In the theory expounded here we consider mainly the effects of applied forces, imposed displacements and temperature.

Some basic concepts are used frequently throughout this book, so it is worthwhile defining them at the beginning. Thus, we define:

- *Internal force* – Force exerted by a part of a body or of a liquid mass on another part. These forces may act on imaginary surfaces defined in the interior of the material, or on its mass. Examples of the first kind are axial and shear forces and bending and torsional moments which act on the cross-sections of slender members (bars). Examples of the second kind would be gravitational attraction or electromagnetic forces between two parts of the body. However, the second kind does not play a significant role in the current applications of the Mechanics of Materials to Engineering problems, and so the designation *internal force* usually corresponds to the first kind (internal surface forces).
- *External forces* – Forces exerted by external entities on a solid body or liquid mass. The forces may also be sub-divided into *surface external forces* and *mass external forces*. The corresponding definitions are:
  - *External surface forces* – External forces acting on the boundary surface of a body. Examples of these include the weight of non-structural parts of a building, equipment, etc., acting on its structure, wind loads on a building, a bridge, or other Civil Engineering structure, aerodynamic pressures in the fuselage and wings of a plane, hydrostatic pressure on the upstream face of a dam or on a ship hull, the reaction forces on the supports of a structure, etc.
  - *External mass forces* – External forces acting on the mass of a solid body or liquid. Examples of external mass forces are: the weight of the material a structure is made of (earth gravity force), the inertial forces caused by an earthquake or by other kinds of accelerations, such as impact, vibrations, traction, braking and curve acceleration in vehicles and planes, and external electromagnetic forces.
- *Rigid body motion* – displacement of the points of a body which do not change the distances between the points inside the body.
- *Deformation* – Variation of the distance between any two points inside the solid body or the liquid mass.

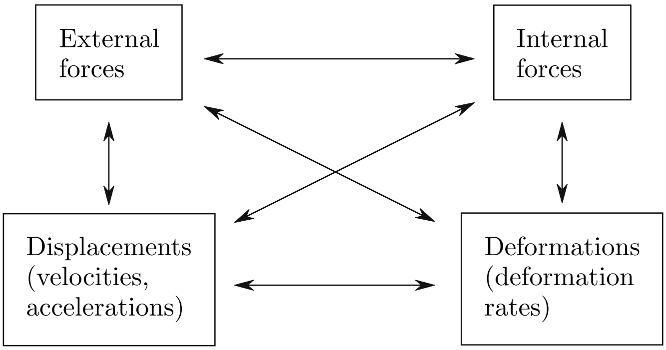
These definitions are general and valid independently of assuming that the material is continuous or not. In the case of continuous materials two other very useful concepts may be defined:

- *Stress* – Physical entity which allows the definition of internal forces in a way that is independent of the dimensions and geometry of a solid body or a liquid mass. There are several definitions for stress. The simplest one is used in this book, which states that stress is the internal force per surface unit.
- *Strain* – Physical entity which allows the definition of deformations in a way that is independent of the dimensions and geometry of a solid body or a liquid mass. As with stress, there also are several definitions for strain. The simplest one states that strain is the variation of the distance between two points divided by the original distance (longitudinal strain), or half the

variation of a right angle caused by the deformation (shearing strain). This strain definition is used throughout this book.

I.3 Subdivisions of the Mechanics of Materials

The Mechanics of Materials aims to find relations between the four main physical entities defined above (external and internal forces, displacements and deformations). Schematically, we may state that, in a solid body which is deformed as a consequence of the action of external forces, or in a flowing liquid under the action gravity, inertial, or other external forces, the following relations may be established



When the validity of the hypothesis of continuity is accepted, these relations may be grouped into three distinct sets

$$\text{force} \overset{①}{\longleftrightarrow} \text{stress} \overset{③}{\longleftrightarrow} \text{strain} \overset{②}{\longleftrightarrow} \text{displacement}$$

1 – *Force-stress relations* – Group of relations based on force equilibrium conditions. Defines the mathematical entity which describes the stress – the stress tensor – and relates its components with the external forces. This set of relations defines the *theory of stresses*. This theory is completely independent of the properties of the material the body is made of, except that the continuity hypothesis must be acceptable (otherwise stress could not be defined).

2 – *Displacement-strain relations* – Group of relations based on kinematic compatibility conditions. Defines the strain tensor and relates its components to the functions describing the displacement of the points of the body. This set of relations defines the *theory of strain*. It is also independent of the rheological behaviour of material. In the form explained in more detail in Chap. III, the theory of strain is only valid if the deformations and the rotations are small enough to be treated as infinitesimal quantities.

3 – *Constitutive law* – Defines the rheological behaviour of the material, that is, it establishes the relations between the stress and strain tensors. As mentioned above, the material rheology is determined by the complex physical phenomena that occur in the internal structure of the material, at the level of atom, molecule, crystal, etc. Since, as a consequence of this complexity, the material behaviour still cannot be quantified by deductive means, a phenomenological approach, based on experimental observation, must used in the definition of the constitutive law. To this end, given forces are applied to a specimen of the material and the corresponding deformations are measured, or vice versa. These experimentally obtained force-displacement relations are then used to characterize the rheological behaviour of the material.

The constitutive law is the potentially most complex element in the chain that links forces to displacements, since it may be conditioned by several factors, like plasticity, viscosity, anisotropy, non-linear behaviour, etc. For this reason, the definition of adequate constitutive laws to describe the rheological behaviour of materials is one of the most extensive research fields inside Solid Mechanics.