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A Semi-Discretized Heat Transfer Model for Optimal Cooling of Steel Profiles

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Summary. Several generalized state-space models arising from a semi-discretization of a controlled heat transfer process for optimal cooling of steel profiles are presented. The model orders differ due to different levels of refinement applied to the computational mesh.

19.1 The Model Equations

We consider the problem of optimal cooling of steel profiles. This problem arises in a rolling mill when different steps in the production process require different temperatures of the raw material. To achieve a high production rate, economical interests suggest to reduce the temperature as fast as possible to the required level before entering the next production phase. At the same time, the cooling process, which is realized by spraying cooling fluids on the surface, has to be controlled so that material properties, such as durability or porosity, achieve given quality standards. Large gradients in the temperature distributions of the steel profile may lead to unwanted deformations, brittleness, loss of rigidity, and other undesirable material properties. It is therefore the engineers goal to have a preferably even temperature distribution. For a picture of a such cooling plant see Figure 19.1.

The scientific challenge here is to give the engineers a tool to pre-calculate different control laws yielding different temperature distributions in order to decide which cooling strategy to choose.

We can only briefly introduce the model here; for details we refer to [Saa03] or [BS04]. We assume an infinitely long steel profile so that we may restrict ourselves to a 2D model. Exploiting the symmetry of the workpiece, the computational domain $\Omega \subset \mathbb{R}^2$ is chosen as the half of a cross section of the rail profile. The heat distribution is modeled by the instationary linear heat equation on Ω:

$$\begin{aligned} c\varrho\partial_t x(t,\xi) - \lambda\Delta x(t,\xi) &= 0 && \text{in } \mathbb{R}_{>0}\times\Omega,\\ x(0,\xi) &= x_0(\xi) && \text{in } \Omega,\\ \lambda\partial_\nu x(t,\xi) &= g_i && \text{on } \mathbb{R}_{>0}\times\Gamma_i, \partial\Omega = \bigcup_i \Gamma_i, \end{aligned} \tag{19.1}$$

where x is the temperature distribution ($x \in H^1([0,\infty], X)$ with $X := H^1(\Omega)$ being the state space), c the specific heat capacity, λ the heat conductivity and ϱ the density of the rail profile. We split the boundary into several parts Γ_i on which we have different boundary functions g_i, allowing us to vary the controls on different parts of the surface. By ν we denote the outer normal of the boundary.

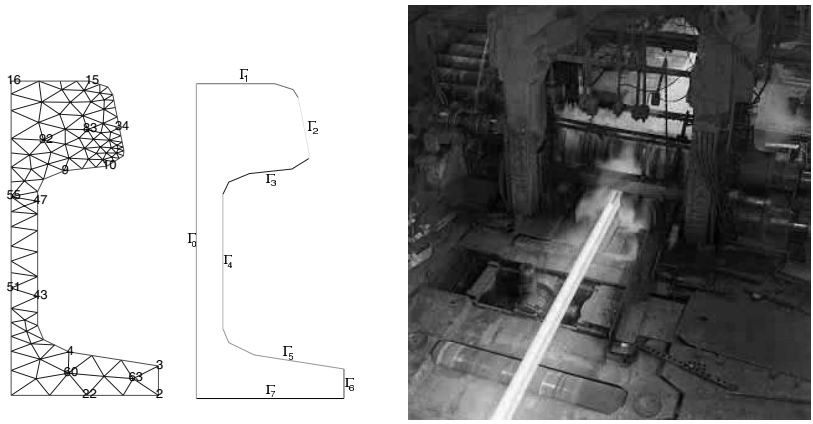

Fig. 19.1. Initial mesh, partitioning of the boundary, and a picture of a cooling plant.

We want to establish the control by a feedback law, i.e., we define the boundary functions g_i to be functions of the state x and the control u_i, where $(u_i)_i =: u = Fy$ for a linear operator F which is chosen such that the cost functional

$$\mathcal{J}(x_0,u) := \int_0^\infty (Qy,y)_Y + (Ru,u)_U dt, \qquad \text{with } y = Cx \tag{19.2}$$

is minimized. Here, Q and R are linear selfadjoint operators on the output space Y and the control space U with $Q \geq 0$, $R > 0$, and $C \in \mathcal{L}(X,Y)$.

The variational formulation of (19.1) with $g_i(t,\xi) = q_i(u_i - x(\xi,t))$ leads to:

$$(\partial_t x, v) = -\int_\Omega \alpha\nabla x\nabla v dx + \sum_k \left(q_k u_k \int_{\Gamma_k} \frac{1}{c\varrho} v \; d\sigma - \int_{\Gamma_k} \frac{q_k}{c\varrho} xv \; d\sigma \right) \tag{19.3}$$

for all $v \in \mathcal{C}_0^\infty(\Omega)$. Here the u_k are the exterior (cooling fluid) temperatures used as the controls, q_k are constant heat transfer coefficients (i.e. parameters

for the spraying intensity of the cooling nozzles) and $\alpha := \frac{\lambda}{c\varrho}$. Note that $q_0 = 0$ yields the Neumann isolation boundary condition on the artificial inner boundary on the symmetry axis.

In view of (19.3), we can now apply a standard Galerkin approach for discretizing the heat transfer model in space, resulting a first-order ordinary differential equation. This is described in the following section.

19.2 The Discretized Mathematical Model

For the discretization we use the ALBERTA-1.2 fem-toolbox (see [SS00] for details). We applied linear Lagrange elements and used a projection method for the curved boundaries. The initial mesh (see Figure 19.1. on the left) was produced by MATLABs `pdetool` which implements a Delaunay triangulation algorithm. The finer discretizations were produce by global mesh refinement using a bisection refinement method.

The discrete LQR problem is then: minimize (19.2) with respect to

$$\begin{aligned} E\dot{x}(t) &= Ax(t) + Bu(t), \text{ with } t > 0, \ x(0) = x_0, \\ y(t) &= Cx(t). \end{aligned} \tag{19.4}$$

This benchmark includes four different mesh resolutions. The best approximation error of the finite element discretization that one can expect (under suitable smoothness assumptions on the solution) is of order $O(h^2)$ where h is the maximum edge size in the corresponding mesh. This order should be matched in a model reduction approach. The following table lists some relevant quantities for the provided models.

matrix dimension	non-zeros in A	non-zeros in E	maximum mesh width (h)
1357	8985	8997	$5.5280\ 10^{-2}$
5177	35185	35241	$2.7640\ 10^{-2}$
20209	139233	139473	$1.3820\ 10^{-2}$
79841	553921	554913	$6.9100\ 10^{-3}$

Note that A is negative definite while E is positive definite, so that the resulting linear time-invariant system is stable.

The data sets are named `rail_`(problem dimension)`_C60.`(matrix name). Here C60 is a specific output matrix which is defined to minimize the temperature in the node numbered 60 (see Figure 19.1) and to keep temperature gradients small. The latter task is taken into account by the inclusion of temperature differences between specific points in the interior and reference points on the boundary, e.g. temperature difference between nodes 83 and 34. Again refer to Figure 19.1. for the nodes used. The definitions of other output matrices that we tested can be found in [Saa03].

The problem resides at temperatures of approximately 1000℃ down to about 500-700℃ depending on calculation time. The state values are scaled to 1000℃ being equivalent to 1.000. This results in a scaling of the time line with factor 100, meaning that calculated times have to be divided by 100 to get the real time in seconds.

Acknowledgments

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