17

Boundary Condition Independent Thermal Model

Evgenii B. Rudnyi and Jan G. Korvink

Institute for Microsystem Technology, Albert Ludwig University
Georges Köhler Allee 103, 79110 Freiburg, Germany
{rudnyi,korvink}@imtek.uni-freiburg.de

Summary. A benchmark for the heat transfer problem with variable film coefficients is presented. It can be used to apply parametric model reduction algorithms to a linear first-order problem.

17.1 Modeling

One of important requirements for a compact thermal model is that it should be boundary condition independent. This means that a chip producer does not know conditions under which the chip will be used and hence the chip compact thermal model must allow an engineer to research on how the change in the environment influences the chip temperature. The chip benchmarks representing boundary condition independent requirements are described in [Las01].

Let us briefly describe the problem mathematically. The thermal problem can be modeled by the heat transfer partial differential equation

$$\nabla \cdot (\kappa(\mathbf{r}) \nabla T(\mathbf{r}, t)) + Q(\mathbf{r}, t) - \rho(\mathbf{r}) C_p(\mathbf{r}) \frac{\partial T(\mathbf{r}, t)}{\partial t} = 0 \tag{17.1}$$

with $\mathbf{r}$ is the position, t is the time, κ is the thermal conductivity of the material, C_p is the specific heat capacity, ρ is the mass density, Q is the heat generation rate, and T is the unknown temperature distribution to be determined. The heat exchange through device interfaces is usually modeled by convection boundary conditions

$$q = h_i (T - T_{bulk}) \tag{17.2}$$

where q is the heat flow through a given point, h_i is the film coefficient to describe the heat exchange for the i-th interface, T is the local temperature at this point and T_{bulk} is the bulk temperature in the neighboring phase (in most cases $T_{bulk} = 0$).

After the discretization of Equations (17.1) and (17.2) one obtains a system of ordinary differential equations as follows

$$E\dot{x} = (A - \sum_i h_i A_i)x + Bu \tag{17.3}$$

where E, A are the device system matrices, A_i is the matrix resulting from the discretization of Equation (17.2) for the i-th interface, x is the vector with unknown temperatures.

In terms of Equation (17.3), the engineering requirements specified above read as follows. A chip producer specifies the system matrices but the film coefficient, h_i, is controlled later on by another engineer. As such, any reduced model to be useful should preserve h_i in the symbolic form. This problem can be mathematically expressed as parametric model reduction [WMGG99, GKN03, DSC04].

Unfortunately, the benchmark from [Las01] is not available in the computer readable format. For research purposes, we have modified a microthruster benchmark [LRK04] (see Figure 17.1). In the context of the present work, the model is as a generic example of a device with a single heat source when the generated heat dissipates through the device to the surroundings. The exchange between surrounding and the device is modeled by convection boundary conditions with different film coefficients at the top, h_{top}, bottom, h_{bottom}, and the side, h_{side}. From this viewpoint, it is quite similar to a chip model used as a benchmark in [Las01]. The goal of parametric model reduction in this case is to preserve h_{top}, h_{bottom}, and h_{side} in the reduced model in the symbolic form.

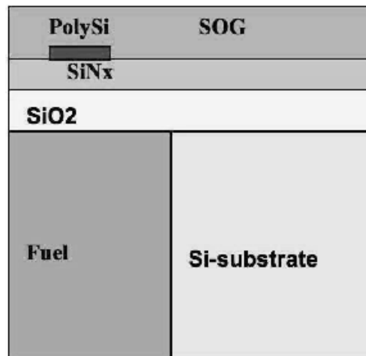

Fig. 17.1. A 2D-axisymmetrical model of the micro-thruster unit (not scaled). The axis of the symmetry on the left side. A heater is shown by a red spot.

17.2 Discretization

We have used a 2D-axisymmetric microthruster model (T2DAL in [LRK04]). The model has been made in ANSYS and system matrices have been extracted by means of mor4ansys [RK04]. The benchmark contains a constant load vector. The input function equal to one corresponds to the constant input power of 15 mW.

The linear ordinary differential equations of the first order are written as:

$$\begin{aligned} E\dot{x} &= (A - h_{top}A_{top} - h_{bottom}A_{bottom} - h_{side}A_{side})x + Bu \\ y &= Cx \end{aligned} \tag{17.4}$$

where E and A are the symmetric sparse system matrices (heat capacity and heat conductivity matrix), B is the load vector, C is the output matrix, A_{top}, A_{bottom}, and A_{side} are the diagonal matrices from the discretization of the convection boundary conditions and x is the vector of unknown temperatures.

The numerical values of film coefficients can be from 1 to 10^9. Typical important sets of film coefficients can be found in [Las01]. The allowable approximation error is 5 % [Las01].

The benchmark has been used in [FRK04a, FRK04b] where the problem is also described in more detail.

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