# Energy Balance Relations for Validation of Gravity Field Models and Orbit Determinations Applied to the CHAMP Mission

Anno Löcher and Karl Heinz Ilk

Institute of Theoretical Geodesy, University Bonn, Germany, *ilk@theor.geod.uni-bonn.de*

**Summary.** An extended Jacobi integral describes the energy balance of the motion of a satellite referred to a terrestrial reference frame along its orbit. In addition to its classical form inertia forces and non-conservative force function contributions are included. If force function models and observed satellite orbits are consistent the energy balance sums up to a constant. Deviations from it can be caused either by orbit errors or by insufficiencies in the force function models. Therefore, the energy integral offers itself as a validation tool for consistency checks of force functions and orbit determination results. A basic question is the separation of the various sources of inconsistency. In this paper the theoretical foundation of the validation procedure is presented. It is shown that the validation method can be used to detect deficiencies in the orbit modeling and in the gravity field recovery results. Examples are presented how to separate the various causes of inconsistency. Applications to the results of the CHAMP mission demonstrate the procedure.

**Key words:** Gravity field validation, Satellite orbit validation, Energy balance, Jacobi integral, CHAMP, Satellite-to-satellite-tracking.

## 1 Introduction

The satellite missions CHAMP, GRACE and in future GOCE will provide unprecedented views of the Earth's gravity field and its changes with time. The gravity field recovery results based on three-years observations of CHAMP and the first results derived from satellite-to-satellite tracking of GRACE seem to confirm the expectations in precision and consistency of the gravity field. A basic problem related to these high-precise gravity field results is a proper validation because no comparable information exists. The frequently applied validation and verification procedures based on comparisons with existing models in well-determined regions, with altimetric geoids, with GPS-levelling results or based on orbit predictions may not fulfil the demands of a rigorous validation. All these procedures are susceptible for a broad spectrum of additional error sources based on the processing procedures and may, therefore, camouflage inconsistencies of kinematically observed orbits and the dynamical modelling of the satellite's motion. A kind of "absolute" criterion for the proof of consistency of observed orbit and the dynamical model of satellite's motion is the energy integral along the orbit, which has been applied in satellite geodesy already in 1969 by Reigber (1969). If the various energy constituents do not sum up to a constant then either the orbit is incorrect or the force function models are wrong. An advantage is the point-wise application along the orbit, avoiding accumulations of errors or instability effects. The size of the constant is only of secondary importance, while the structure of the deviations from the constant may give hints to specific force function or orbit determination deficiencies. This is based on the fact that the energy exchange relations caused by the various force function components show typical properties which may open the possibility to separate the different sources of inconsistency. A sophisticated validation procedure is important not only to control the success of global gravity field recovery result; it can be used as well for checking the possibility of a focus to regional gravity field features. The inhomogeneous structure of the gravity field will produce inhomogeneous signals in the satellite's motion or in the high-low satellite-to-satellite connections of CHAMP and the GPS-satellites. A consequence is that regions with rough gravity field features are not adequately modelled by a series of spherical harmonics. The validation procedure can be used to check whether regional improvements of the gravity field may be successful – this can be applied a-priori and a-posteriori.

## 2 Energy integral along a satellite orbit

It is well-known that the classical energy balance which describes the exchange of kinetic and potential energy is valid only in case of conservative forces. The energy function $\hat{E}(t)$ reads, referred to an inertial system:

$$\hat{E}(t) = \frac{1}{2}\frac{\mathbf{P}^2}{M} - MV(\mathbf{R}) + \hat{E}_0 = const \,, \tag{1}$$

with the linear momentum (mass $M$, velocity $\mathbf{v}$) $\mathbf{P} = M\mathbf{v}$, a gravitational potential $V(\mathbf{R})$ fixed with respect to the inertial system and the energy constant $\hat{E}_0$. In case of a uniformly rotating Earth (rotation vector $\mathbf{\Omega}$) the gravitational force function is not conservative, and therefore, the Jacobi integral has to be applied instead of the simple energy balance relation along the orbits (e.g. Löcher, 2003):

$$\hat{E}'(t) = \frac{1}{2}\frac{\mathbf{P}'^2}{M} - \mathbf{P}' \cdot \left(\mathbf{\Omega} \times \mathbf{R}'\right) - MV(\mathbf{R}') + \hat{E}'_0 = const \,, \tag{2}$$

with the potential energy $\hat{E}'_{pot}(t) := -MV'(\mathbf{R}')$ (quantities which refer to the rotating reference system are marked by an apostrophe). The Jacobi integral has to be extended if the rotation of the Earth is considered time variable and non-conservative disturbing forces as tidal forces, Earth tides etc. and surface forces as e.g. air drag and solar radiation pressure have to be included in the balance relation. The same holds if the sum of all surface forces are measured in-situ by accelerometers. From the inertia forces, only the centrifugal force and the Eulerian force show an effect on the energy function, while the Coriolis force does not as one easily verifies. The energy contribution of the centrifugal force $\mathbf{Z}'$ reads, (Ilk and Löcher, 2003),

$$\hat{E}'_Z = -\int \mathbf{Z}' \cdot \dot{\mathbf{R}}' dt = -\frac{1}{2} \cdot \left(\mathbf{\Omega} \times \mathbf{R}'\right)^2 \,, \tag{3}$$

and the energy contribution of the Eulerian force $\mathbf{E}'$,

$$\hat{E}'_E\Big|_{t_0}^{t} = -\int_{t_0}^{t} \mathbf{E}'\cdot\dot{\mathbf{R}}\,dt = M\int_{t_0}^{t} \dot{\boldsymbol{\Omega}}\cdot\left(\mathbf{R}\times\dot{\mathbf{R}}\right)dt\,. \tag{4}$$

The contribution of an arbitrary non-conservative force $\mathbf{K}'$ to the energy function $\hat{E}'_K|_{t_0}^{t}$ by the work $A'|_{t_0}^{t}$ performed by the satellite along the orbit reads:

$$\hat{E}'_K\Big|_{t_0}^{t} = -A'\Big|_{t_0}^{t} = -\int_{t_0}^{t} \mathbf{K}'\cdot d\mathbf{R}' = -\int_{t_0}^{t} \mathbf{K}'\cdot\left(\frac{\mathbf{P}'}{M} - \boldsymbol{\Omega}\times\mathbf{R}'\right)dt\,. \tag{5}$$

With these formulae the extended (specific) energy function for CHAMP reads:

$$J_{sat} := E'(t) + E'_{akz}\Big|_{t_0}^{t} + E'_{M,S}\Big|_{t_0}^{t} + E'_{E}\Big|_{t_0}^{t} + E'_{ET}\Big|_{t_0}^{t} + E'_{OT}\Big|_{t_0}^{t} = const\,, \tag{6}$$

with the sum of kinetic, centrifugal and potential energy $E'(t)$, the surface energy $E'_{akz}|_{t_0}^{t}$, the tidal energy of Moon and Sun $E'_{M,S}|_{t_0}^{t}$, the Euler energy $E'_{E}|_{t_0}^{t}$, the Earth tides energy $E'_{ET}|_{t_0}^{t}$ and the ocean tides energy $E'_{OT}|_{t_0}^{t}$.

## 3 Energy contributions for CHAMP along PSO arcs

In this paragraph the extended Jacobi integral due to (6) shall be determined along an arc of the so-called Post-processed Science Orbits (PSO) of CHAMP. The three-dimensional accelerometer data sets as well as the observations and orbits at different production levels are provided by the CHAMP Information System and Data Centre (ISDC). The transformations between the terrestrial and celestial reference frames follow the conventions published by McCarthy (1996). For the computation of the tidal energy caused by Moon and Sun the numerical ephemeris DE405 of the Jet Propulsion Laboratory (JPL) have been used.

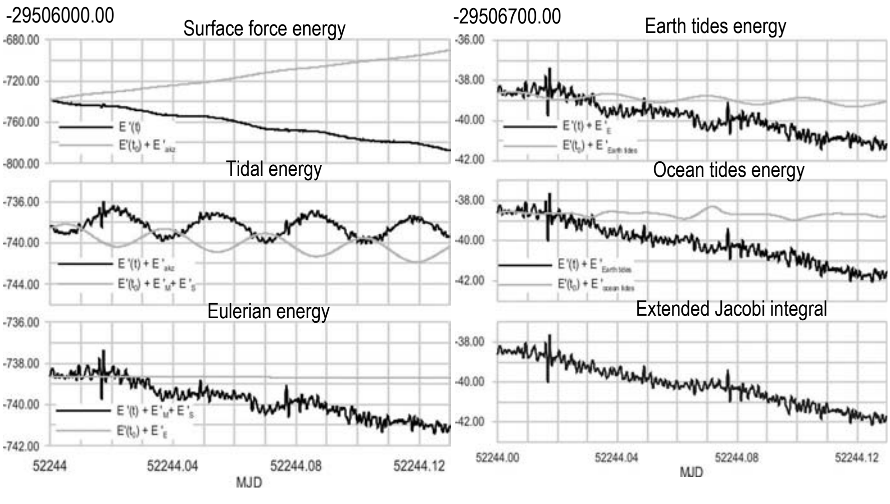


**Fig. 1.** Selected energy contributions ( $m^2/s^2$ ) to the extended Jacobi integral for a two-revolution PSO arc of CHAMP (1.12.2001).

The accelerometer measurements to determine the surface force energy for the CHAMP orbit have been processed according to the rules of the CHAMP data format (Förste et al., 2001). The energy contributions caused by the Earth tides as well as by the ocean tides have been based on the models as published by McCarthy (1996). As gravity field model the recent satellite-only gravity field solution EIGEN-2 has been used for this test (Reigber et al., 2003). The energy contributions to the extended Jacobi integral in case of a two-revolution arc are shown in Fig. 1 to demonstrate the sizes of these various effects. The kinetic and the potential energy as well as the centrifugal energy are not shown here; they are the largest by far. In case of consistency of orbit and force function the bottom graph at the right hand side should be a constant. The secular trend shown in this graph could be caused by an insufficient calibration of the accelerometer data. If a linear trend has been removed then a deviation from a constant in the size of 0,26 $m^2/s^2$ remains, which corresponds to approximately 3 $cm$ in the positions.

Especially the gravity field will have an important influence on the energy constant. Fig. 2 shows the inconsistency effects of Eigen-2 plotted along the subsatellite tracks for a 10-days arc. Similar effects occur 6 months later; the differences are shown in Fig. 3. Only orbit-dependent effects are visible while field-dependent ones cancel out. These examples clearly demonstrates the possibility to separate

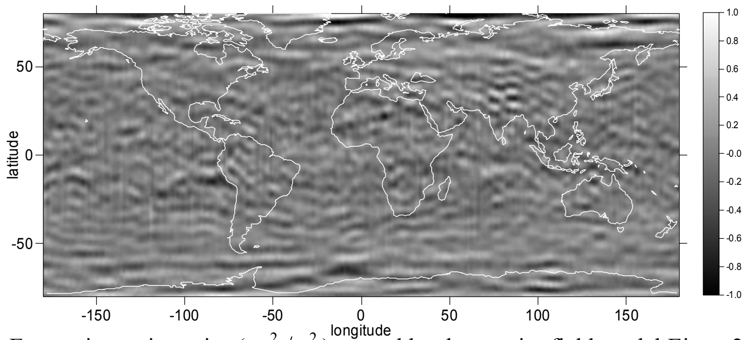


**Fig. 2.** Energy inconsistencies ($m^2/s^2$) caused by the gravity field model Eigen-2, plotted along the subsatellite tracks of CHAMP for a period of 10 days.

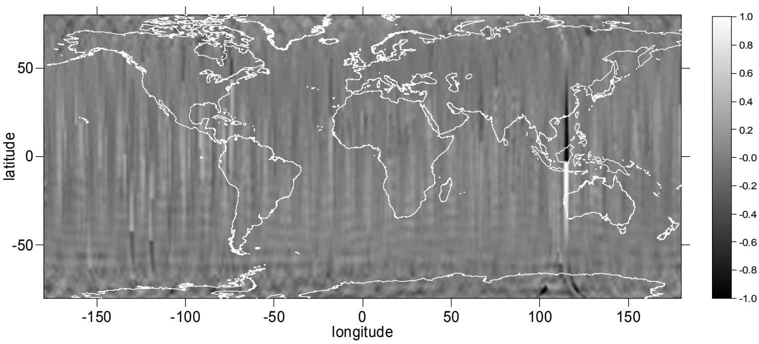


**Fig. 3.** Energy inconsistency differences ($m^2/s^2$) for a period of 10 days with a time lag of 6 months referred to the situation shown in Fig. 2.

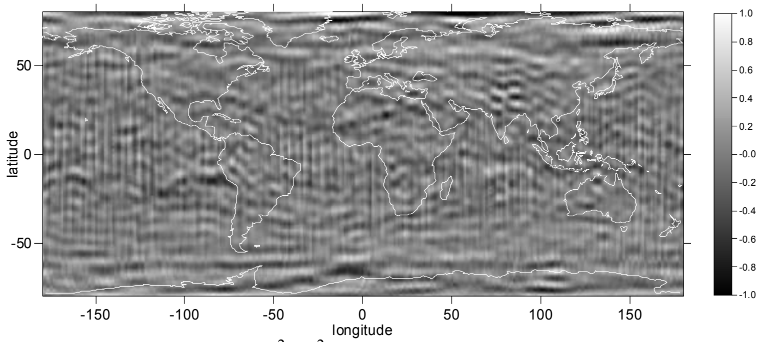


**Fig. 4.** Energy inconsistencies ( $m^2 / s^2$ ) as shown in Fig. 2, but without ocean tides, plotted along the subsatellite tracks for a 10 days arc of CHAMP.

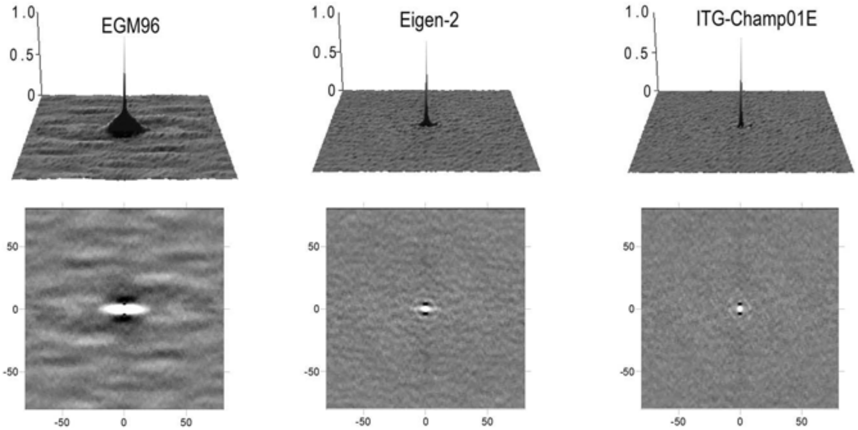


**Fig. 5.** Correlograms of the energy inconsistencies caused by the gravity field models EGM96, Eigen-2, and ITG-01s for kinematic orbits of CHAMP (60 days).

force function and orbit determination inconsistency effects. Even certain field function constituents leave a specific pattern in the deviation of the energy integral from a constant as Fig. 4 demonstrates: in this example the ocean tides are skipped in formula (6). Typical ocean related inconsistency effects can be observed.

## 4 Application to kinematic orbits of CHAMP

PSO are based on a dynamical gravity field model and, therefore, are rather smooth. Kinematic orbits are much more critical, because they do not contain any information on dynamical models and are based exclusively on satellite observations. The present tests are based on a pure kinematic orbit determination of CHAMP provided by M. Rothacher and D. Svehla from the FESG of the Technical University Munich (Svehla and Rothacher, 2003). The critical point of the application of the energy validation approach to kinematical orbits are the velocities of CHAMP. They have to be derived from the ephemeris of the kinematically determined satellite positions. In our approach, the kinematical positions with a sampling rate of 30 sec have been filtered twice, based on the variance-covariance matrices of the positions. Those positions with an rms larger than 8 *cm* are sorted out a-priori. Then the velocities are determined based on an interpolation polynomial of degree 6. Those velocities which result in an rms for the kinetic energy larger than $5\,m^2/s^2$ are deleted as well. The energy inconsistencies are superimposed by noise which hampers the detection of systematic effects. But correlograms clearly uncover systematic effects which show the inconsistencies of orbits and field functions (Fig. 5). The gravity field EGM96 shows large inconsistencies while in case of Eigen-2 (Reigber et al., 2003) only small inconsistencies can be detected. In case of the new gravity field model ITG-Champ01E, derived by the Institute of Theoretical Geodesy, University Bonn (Ilk et al., 2004), the consistency of field function and orbits seems to be realized up to a high degree.

## 5 Conclusions

The results of this investigation clearly demonstrate that the energy integral is a sensitive tool for consistency validation of orbits and force functions. It seems that it is even possible to separate different force function contributions. A critical aspect is the determination of the velocity based on kinematically derived positions.

*Acknowledgements:* We are grateful for providing the data by the ISDC of the GeoForschungsZentrum Potsdam (GFZ) and for the kinematical orbits by D. Svehla and M. Rothacher from the FESG at the Technical University Munich. The support by BMBF and DFG within the frame of the Geotechnologien-Programm is gratefully acknowledged.

## References

Förste Ch, Schwintzer P, Reigber Ch (2001) The CHAMP Data Format. Internal publication, GFZ, http://op.gfz-potsdam.de/champ/docs_CHAMP.

Ilk KH, Mayer-Gürr T, Feuchtinger M (2004) Gravity Field Recovery by Analysis of Short Arcs of CHAMP. This issue.

Ilk KH, Löcher A (2003) The Use of Energy Balance Relations for Validation of Gravity Field Models and Orbit Determination Results. presented at the Gen. Ass. of the IUGG 2003, Sapporo, Japan.

Löcher A (2003) Untersuchungen zur Energieerhaltung bei der Satellitenbewegung am Beispiel der CHAMP-Mission. Diploma thesis at the University Bonn.

McCarthy DD (ed), (1996) IERS Conventions 1996. Central Bureau of IERS – Observatoire de Paris, Paris.

Reigber Ch (1969) Zur Bestimmung des Gravitationsfeldes der Erde aus Satellitenbeobachtungen. DGK, Reihe C, Heft Nr. 137, München.

Reigber Ch, Schwintzer P, Neumayer K-H, Barthelmes F, König R, Förste Ch, Balmino G, Biancale R, Lemoine J-M, Loyer S, Bruinsma S, Perosanz F, Fayard T (2003) The CHAMP-only Earth Gravity Field Model Eigen-2. Adv Space Res *31(8):* 1883-1888.

Svehla D, Rothacher M (2003) Kinematic Precise Orbit Determination for Gravity Field Determination. presented at the Gen. Ass. of the IUGG 2003, Sapporo, Japan.