The Impact of the New CHAMP and GRACE Gravity Models on the Measurement of the General Relativistic Lense–Thirring Effect

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Summary. Here we wish to discuss the improvements obtainable in the measurement of the general relativistic Lense–Thirring effect with the LAGEOS and LAGEOS II satellites, in terms of reliability of the evaluation of the systematic error and reduction of its magnitude, due to the new CHAMP and GRACE Earth gravity models.

Key words: General Theory of Relativity, Lense-Thirring effect, LAGEOS-LAGEOS II orbits, Earth gravity models

1 Introduction

The linearized weak–field and slow–motion approximation of the General Theory of Relativity (GTR) [1] is characterized by the condition $g_{\mu\nu} \sim \eta_{\mu\nu} + h_{\mu\nu}$ where $g_{\mu\nu}$ is the curved spacetime metric tensor, $\eta_{\mu\nu}$ is the Minkowski metric tensor of the flat spacetime of Special Relativity and the $h_{\mu\nu}$ are small corrections such that $|h_{\mu\nu}| \ll 1$. Until now, many of its predictions, for the motion of light rays and test masses have been tested, in the Solar System, with a variety of techniques to an accuracy level of the order of 0.1% [2]. It is not so for the gravitomagnetic Lense–Thirring effect due to its extreme smallness. It can be thought of as a consequence of a gravitational spin–spin coupling.

If we consider the motion of a spinning particle in the gravitational field of a central body of mass M and proper angular momentum $\boldsymbol{J}$, it turns out that the spin $\boldsymbol{s}$ of the orbiting particle undergoes a tiny precessional motion [3]. The most famous experiment devoted to the measurement, among other things, of such gravitomagnetic effect in the gravitational field of Earth is the Stanford University GP–B mission [4] which should fly in 2004.

If the whole orbit of a test particle in its geodesic motion around M is considered as a sort of giant gyroscope, its orbital angular momentum $\boldsymbol{\ell}$ undergoes the Lense–Thirring precession, so that the longitude of the ascending node Ω and the argument of pericentre ω of the orbit of the test particle are affected by tiny secular precessions $\dot{\Omega}_{\rm LT}$, $\dot{\omega}_{\rm LT}$ [5, 1]

$$\dot{\Omega}_{\rm LT} = \frac{2GJ}{c^2a^3(1-e^2)^{\frac{3}{2}}},\ \dot{\omega}_{\rm LT} = -\frac{6GJ\cos i}{c^2a^3(1-e^2)^{\frac{3}{2}}}, \quad (1)$$

where a, e and i are the semimajor axis, the eccentricity and the inclination, respectively, of the orbit, c is the speed of light and G is the Newtonian gravitational constant.

Up to now, the only attempts to detect the Lense–Thirring effect on the orbit of test particles in the gravitational field of Earth are due to Ciufolini and coworkers [6] who analysed the laser data of the existing LAGEOS and LAGEOS II satellites over time spans of some years. The observable is a suitable combination of the orbital residuals of the nodes of LAGEOS and LAGEOS II and the perigee of LAGEOS II according to an idea exposed in [7]

$$\delta\dot{\Omega}^{\rm L} + c_1\delta\dot{\Omega}^{\rm L\ II} + c_2\delta\dot{\omega}^{\rm L\ II} \sim 60.2\mu_{\rm LT},\ c_1 \sim 0.295,\ c_2 \sim -0.35, \quad (2)$$

where the superscripts L and L II refer to LAGEOS and LAGEOS II, respectively. The quantity $\mu_{\rm LT}$ is the solved–for least square parameter which is 0 in Newtonian mechanics and 1 in GTR. The Lense-Thirring signature, entirely adsorbed in the residuals of $\dot{\Omega}$ and $\dot{\omega}$ because the gravitomagnetic force has been purposely set equal to zero in the force models, is a linear trend with a slope of 60.2 milliarcseconds per year (mas yr^{-1} in the following). The standard, statistical error is evaluated as 2%. The claimed total accuracy, including various sources of systematic errors, is of the order of $20 - 30\%$.

The main sources of systematic errors in this experiment are

- the unavoidable aliasing effect due to the mismodelling in the classical secular precessions induced on Ω and ω by the even zonal coefficients J_l of the multipolar expansion of geopotential
- the non–gravitational perturbations affecting especially the perigee of LAGEOS II [8, 9]. Their impact on the proposed measurement is difficult to be reliably assessed [10]

It turns out that the mismodelled classical precessions due to the first two even zonal harmonics of geopotential J_2 and J_4 are the most insidious source of error for the Lense–Thirring measurement with LAGEOS and LAGEOS II. The combination (2) is insensitive just to J_2 and J_4. According to the full covariance matrix of the EGM96 gravity model [11], the error due to the remaining uncancelled even zonal harmonics amounts to almost 13% [12]. However, if the correlations among the even zonal harmonic coefficients are neglected and the variance matrix is used in a Root–Sum–Square fashion[1], the error due to the even zonal harmonics of geopotential amounts to 46.6% [12]. With this estimate and the evaluations of [8, 9] for the impact of the non–gravitational perturbations the total error in the LAGEOS–LAGEOS II Lense–Thirring experiment would be of the order of 50%. If the sum of the

[1] Such approach is considered more realistic by some authors [10] because nothing assures that the correlations among the even zonal harmonics of the covariance matrix of the EGM96 model, which has been obtained during a multidecadal time span, would be the same during an arbitrary past or future time span of a few years as that used in the LAGEOS–LAGEOS II Lense–Thirring experiment.

absolute values of the individual errors is assumed, an upper bound of 83% for the systematic error due to the even zonal harmonics of geopotential is obtained; then, the total error in the LAGEOS–LAGEOS II Lense–Thirring experiment would become of the order of 100%. This evaluations agree with those released in [10].

The originally proposed LAGEOS III/LARES mission [13] consists of the launch of a LAGEOS–type satellite–the LARES–with the same orbit of LAGEOS except for the inclination i of its orbit, which should be supplementary to that of LAGEOS, and the eccentricity e, which should be one order of magnitude larger in order to perform other tests of post–Newtonian gravity [14, 15]. The choice of the particular value of the inclination for LARES is motivated by the fact that in this way, by using as observable the sum of the nodes of LAGEOS and LARES, it should be possible to cancel out to a very high level all the contributions of the even zonal harmonics of geopotential, which depends on $\cos i$, and add up the Lense–Thirring precessions which, instead, are independent of i. The use of the nodes would allow to reduce greatly the impact of the non–gravitational perturbations to which such Keplerian orbital elements are rather insensitive [8, 9].

In [16] an alternative observable based on the combination of the residuals of the nodes of LAGEOS, LAGEOS II and LARES and the perigee of LAGEOS II and LARES has been proposed. It would allow to cancel out the first four even zonal harmonics so that the error due to the remaining even zonal harmonics of geopotential would be rather insensitive both to the unavoidable orbital injection errors in the LARES inclination and to the correlations among the even zonal harmonic coefficients. It would amount to 0.02%–0.1% only [16, 17] (EGM96 full covariance and variance RSS calculations).

In regard to the present status of the LARES project, unfortunately, up to now, although its very low cost with respect to other much more complex and expensive space–based missions, it has not yet been approved by any national space agency or scientific institution.

2 The impact of the CHAMP and GRACE Earth gravity models

From the previous considerations it could be argued that, in order to have a rather precise and reliable estimate of the total systematic error in the measurement of the Lense–Thirring effect with the existing LAGEOS satellites it would be better to reduce the impact of geopotential in the error budget and/or discard the perigee of LAGEOS II which is very difficult to handle and is a relevant source of uncertainty due to its great sensitivity to many non–gravitational perturbations.

The forthcoming more accurate Earth gravity models from CHAMP [18] and, especially, GRACE [19] will yield an opportunity to realize both these goals, at least to a certain extent.

In order to evaluate quantitatively the opportunities offered by the new terrestrial gravity models we have preliminarily used the recently released EIGEN2 gravity model [20].

With regard to the combination (2), it turns out that the systematic error due to the even zonal harmonics of geopotential, according to the full covariance matrix of EIGEN2 up to degree $l = 70$, amounts to 7%, while if the diagonal part only is adopted it becomes 9% (RSS calculation). Of course, even if the LAGEOS and LAGEOS II data had been reprocessed with the EIGEN2 model, the problems posed by the correct evaluation of the impact of the non–gravitational perturbations on the perigee of LAGEOS II would still persist.

A different approach could be followed by taking the drastic decision of canceling out only the first even zonal harmonic of geopotential by discarding at all the perigee of LAGEOS II. The hope is that the resulting gravitational error is reasonably small so to get a net gain in the error budget thanks to the fact that the nodes of LAGEOS and LAGEOS II exhibit a very good behavior with respect to the non–gravitational perturbations. Indeed, they are far less sensitive to their tricky features than the perigee of LAGEOS II. Moreover, they can be easily and accurately measured, so that also the formal, statistical error should be reduced. A possible observable is[2]

$$\delta\dot{\Omega}^{\rm L} + c_1\delta\dot{\Omega}^{\rm L\ II} \sim 48.2\mu_{\rm LT},\ c_1 \sim 0.546. \tag{3}$$

According to the full covariance matrix of EIGEN2 up to degree $l = 70$, the systematic error due to the even zonal harmonics from $l = 4$ to $l = 70$ amounts to 8.5 mas yr^{-1} yielding a 17.8% percent error, while if the diagonal part only is adopted it becomes[3] 22% (RSS calculation). EGM96 would not allow to adopt (3) because its full covariance matrix up to degree $l = 70$ yields an error of 47.8% while the error according to its diagonal part only amounts even to 104% (RSS calculation). Note also that the combination (3) preserves one of the most important features of the combination (2): indeed, it allows to cancel out the very insidious 18.6-year tidal perturbation which is a $l = 2,\ m = 0$ constituent with a period of 18.6 years due to the Moon's node and nominal amplitudes of the order of 10^3 mas on the nodes of LAGEOS and LAGEOS II [21]. On the other hand, the impact of the non–gravitational perturbations on the combination (3) over a time span of, say, 7 years can be quantified in just 0.1 mas yr^{-1}, yielding a 0.3% percent error. The results of Table 2 and Table 3 of [16] have been used. It is also important to notice that, thanks to the fact that the periods of many gravitational and non–gravitational time–dependent perturbations acting on the nodes of the LAGEOS satellites are rather short, a reanalysis of the LAGEOS and LAGEOS II data over just a few

[2] A similar approach seems to be suggested in [19], although without quantitative details.

[3] The even zonal harmonics are much more mutually uncorrelated in EIGEN2 than in EGM96.

years could be performed. This is not so for the combination (2) because some of the gravitational [21] and non–gravitational [8] perturbations affecting the perigee of LAGEOS II have periods of many years. Then, with a little time–consuming reanalysis of the nodes only of the existing LAGEOS and LAGEOS II satellites with the EIGEN2 data it would at once be possible to obtain a more accurate and reliable measurement of the Lense–Thirring effect, avoiding the problem of the uncertainties related to the use of the perigee of LAGEOS II.

Very recently the first preliminary Earth gravity models including some data from GRACE have been released; among them the GGM01C model[4], which combines the Center for Space Research (CSR) TEG4 model with data from GRACE, seems to be very promising. Indeed, the error due to geopotential in the combination (2), evaluated by using the variance matrix only (RSS calculation), amounts to 2.2% (with an upper bound of 3.1% obtained from the sum of the absolute values of the individual terms). Instead, the combination (3) would be affected at almost 14% level (RSS calculation), with an upper bound of almost 18% from the sum of the absolute values of the individual errors. However, it should be pointed out that extensive calibration tests have still to be performed with the GGM01C model.

3 Conclusions

When more robust and complete terrestrial gravity models from CHAMP and GRACE will be available in the near future the combination (3) could hopefully allow for a measurement of the Lense–Thirring effect with a total systematic error, mainly due to geopotential, of some percent over a time span of a few years without the uncertainties related to the evaluation of the impact of the non–gravitational perturbations acting upon the perigee of LAGEOS II. On the other hand, the obtainable accuracy with the combination (2), whose error due to geopotential is smaller than that of (3), is strongly related to improvements in the evaluation of the non–gravitational part of the error budget and to the use of time spans of many years.

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