On the Contribution of CHAMP to Temporal Gravity Field Variation Studies

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Summary. This study investigates the effect of temporal gravity field variability on CHAMP and other geodetic satellites. The sensitivity of these satellites to the lower order and degree harmonics is presented along with the dominant tidal periodicities. Lumped harmonics from analyses of CHAMP are discussed with a singular value decomposition identifying the dominant combinations. Temporal variability in lower order and degree harmonics are presented in studies with and without CHAMP. These results are compared against those derived from geophysical data.

Key words: Gravity, Lumped harmonics, Temporal variability

1 Sensitivity analyses of CHAMP and other geodetic satellites

The sensitivity of a particular satellite to the lower degree and order geopotential coefficients can be quantified by using analytical orbit theory (Kaula, 1966). This sensitivity is a combination of the orbital inclination and the orbital height given the accentuation of gravity field effects with altitude. For example, on consideration of the even and odd zonal harmonics

$$d\sigma^e/dt = \Sigma {\sigma_l}^e, \sigma = \Omega, \omega, M \tag{1}$$

$$d\sigma^o/dt = \Sigma {\sigma_l}^o cos\omega, \sigma = \omega, e \tag{2}$$

where the superscript, e or o, denotes even or odd zonals respectively. Utilising mean orbital elements and rates of change of the angular arguments the sensitivity of five geodetic satellites, Lageos I and II, Starlette, Ajisai and Stella, as well as CHAMP, are summarised in Tables 1 and 2. The tables illustrate the effect of a small change in the zonal harmonics, J_n on the respective Keplerian elements. The tables show the insensitivity of the Lageos I and II satellites ($ht{\approx}6000km$) to the higher degree harmonics. In contrast, Ajisai ($ht{\approx}1200km$), Stella and Starlette ($ht{\approx}800km$) and CHAMP ($ht{\approx}400-450km$) are sensitive to degree 30 and beyond. It is the different sensitivities that facilitates the separation of the lower degree and order temporal variability from multisatellite analyses.

Another important consideration for temporal field studies are the periodicities of the dominant solid earth and ocean tides. In particular, any

	Starlette		Ajisai		Stella		LageosI		LageosII		Champ	
deg	u^e	Ω^e	u^e	Ω^e	u^e	Ω^e	u^e	Ω^e	u^e	Ω^e	u^e	Ω^e
2	49.4	-47.8	38.0	-37.7	-72.6	12.1	-6.6	4.2	6.0	-7.7	-94.8	-4.5
4	61.1	-1.9	42.8	-1.9	-43.3	16.9	0.5	1.5	3.9	-0.6	-76.2	-7.4
6	-11.1	26.5	-5.7	15.9	-22.3	17.8	0.6	0.3	0.1	0.5	-60.9	-9.4
8	-34.0	7.1	-19.1	4.2	-7.2	16.2	0.3	0.0	-0.3	0.1	-52.1	-10.6
10	-2.7	-13.8	-2.0	-6.0	2.7	13.3	0.0	0.0	0.0	0.0	-33.1	-11.3
12	19.7	-7.4	-7.7	-3.2	8.5	10.0	0.0	0.0	-0.1	0.0	-35.1	-11.6
14	6.4	5.9	2.7	1.9	61.0	6.7	0.0	0.0	-0.1	0.0	-28.3	-11.5
16	-9.6	5.7	-2.8	1.8	11.3	3.9	0.0	0.0	0.0	0.0	-22.5	-11.2
18	-6.1	-2.0	-1.7	-0.4	10.1	1.7	0.0	0.0	0.0	0.0	-17.6	-10.6
20	4.0	-3.8	0.8	-0.9	8.2	0.1	0.0	0.0	0.0	0.0	-13.4	-10.0
22	4.5	0.2	1.0	0.0	6.1	-0.9	0.0	0.0	0.0	0.0	-9.9	-9.3
24	-1.2	2.3	-0.2	0.4	4.1	-1.4	0.0	0.0	0.0	0.0	-5.1	-8.5
26	-2.9	0.5	-0.4	0.1	2.3	-1.5	0.0	0.0	0.0	0.0	-4.7	-7.7
28	0.0	-1.2	0.0	-0.1	1.0	-1.5	0.0	0.0	0.0	0.0	-2.8	-6.9
30	1.7	-0.6	0.1	-0.1	0.0	-1.2	0.0	0.0	0.0	0.0	-1.1	-6.1

Table 1. Sensitivity of satellite orbits to even zonal variations (per unit ΔJn x 1.0d-11) (milliarcseconds/year); $u^e = M^e + \omega^e$

mis-modelling of tidal constituent giving rise to a signal at the annual or semi-annual frequency will be aliased into those periods and possibility misinterpreted as mass redistribution. Table 3 presents the theoretical amplitudes and periodicities due to the second degree solid earth tide, with $k2 = 0.3$. Most of the dominant tidal amplitudes are long-periodic but apart from the annual and semi-annual zonal solar tide, should not aliase recovery of signatures at the seasonal to annual frequencies.

2 CHAMP Normal Equations: Singular Value Decomposition

Utilising the precise positioning from GFZ rapid science orbits for CHAMP and SLR tracking to LageosI/II, Starlette, Stella and Ajisai we have recovered the gravity field up to degree and order 10 over 15 day arcs along with the other parameters of Table 4. In all orbit computations the a priori gravity field was GGM01C with ocean tidal model CSR3.0. For CHAMP, k0(L) denotes local accelerometer bias, k1(G) global accelerometer scale factors, th(L) local thruster accelerations with L indicating a daily solution and G a 15day solution. For other satellites, Cd denotes daily drag coefficients, Cr a global solar radiation coefficient and atacc along track accelerations every ($5day$) for Lageos.

Normal equations for the gravity field component over each 15day period were combined with weights according to perceived accuracy of J_2 namely,

	Starlette		Ajisai		Stella		LageosI		LageosII		Champ	
deg	e^o	ω^0	e^o	ω^0	e^o	ω^0	e^o	ω^0	e^o	ω^0	e^o	ω^0
3	-13.3	325.8	-9.7	4847.8	15.5	-377.6	0.6	-72.6	-1.1	40.7	22.1	-2757.3
5	-13.8	337.5	-9.1	4571.6	12.0	-291.2	-0.2	19.0	-0.6	21.7	23.6	-2950.4
7	7.0	-173.3	3.7	-1851.8	6.5	-158.1	-0.1	15.7	0.0	-1.0	21.9	-2735.9
9	10.8	-265.2	5.4	-2717.2	1.7	-41.3	-0.1	5.2	0.1	-2.0	19.3	-2410.7
11	-1.7	42.1	-0.5	246.4	-1.7	43.2	-0.0	1.0	0.0	-0.2	16.5	-2063.1
13	-6.9	172.2	-2.6	1309.9	-3.8	94.4	0.0	0.0	0.0	0.1	13.8	-1727.3
15	-0.8	18.0	-0.4	193.8	-4.7	117.2	0.0	-0.1	0.0	0.0	11.3	-1417.9
17	3.9	-98.4	1.1	-544.2	-4.7	118.8	0.0	0.0	0.0	0.0	9.1	-1140.6
19	1.4	-35.6	0.4	-214.0	-4.2	106.4	0.0	0.0	0.0	0.0	7.2	-897.2
21	-1.9	49.4	-0.4	192.3	-3.4	86.5	0.0	0.0	0.0	0.0	5.5	-686.9
23	-1.3	33.7	-0.3	140.0	-2.5	63.8	0.0	0.0	0.0	0.0	4.1	-507.9
25	0.8	-20.8	0.1	-52.4	-1.6	42.1	0.0	0.0	0.0	0.0	2.9	-357.7
27	1.0	-25.3	0.2	-74.2	-0.9	23.2	0.0	0.0	0.0	0.0	1.9	-233.4
29	-0.2	5.9	-0.0	5.8	-0.3	8.5	0.0	0.0	0.0	0.0	1.1	-132.1
31	-0.6	16.6	-0.1	34.1	0.1	-2.1	0.0	0.0	0.0	0.0	0.4	-51.0

Table 2. Sensitivity of satellite orbits to odd zonal variations (per unit ΔJn x 1.0d-11) (milliarcseconds/year)

Lageos I (1.2), LageosII (1.0), Starlette (0.8), Ajisai (0.8), Stella (0.6) and CHAMP (0.1). On eliminating the contribution of the local parameters the geopotential normal equations $N\underline{x} = \underline{b}$ can be written using Singular Value Decomposition (SVD) as $N = QWP^T$, $W\underline{x}^{'} = \underline{b}^{'}$ where $\underline{x}^{'} = P^T\underline{x}$ and $\underline{b}^{'} = Q^{-1}\underline{b}$. W is a diagonal weight matrix with covariance for $\underline{x}^{'}$ being $C = W^{-1}$. SVD identifies the dominant linear combinations for each 15day arc. Thus, for example, for Lageos 1 (MJD 50864-50879) the dominant combination in a 4x4 gravity field recovery was the lumped harmonic

$$J_2^{'} = 1.000J_2 + 0.369J_4 - 0.068J_3 - 0.036C_{2,1} + ... \quad (3)$$

The 1σ sd was $\sigma_{J_2^{'}} = 2.6e-11$. Equation (3) is to be compared with the theoretical value from $d\Omega/dt$ in Table 1, namely

$$J_2^{'} = 1.000J_2 + 0.371J_4 + 0.0795J_6 + ... \quad (4)$$

In contrast, solving for a 10 by 10 field from CHAMP the 46^{th} ranking lumped harmonic (1σ sd=3.8e-11) for MJD 50249-50264 is

$$J_2^{'} = 1.000J_2 + 1.613J_4 + 2.023J_6 + 2.285J_8 + 2.433J_{10} + 1.622C_{4,3} - 1.619C_{2,2} \quad (5)$$

which can be compared against the theoretical value of $d\Omega/dt$ in Table 1, namely

$$J_2^{'} = 1.000J_2 + 1.630J_4 + 2.060J_6 + 2.336J_8 + 2.490J_{10} + ... \quad (6)$$

Tide	Theoretical Amplitude (arc secs)			Period (days
	ΔI	$\Delta\omega$	$\Delta\Omega$	
055.545	0.0000	1.0934	11.3904	6798.375
056.554 Sa	0.0000	0.0106	0.1101	365.260
057.555 Ssa	0.0000	0.0328	0.3415	182.621
145.555 O1	0.0084	0.1742	-0.0330	13.470
163.555 P1	0.0443	0.7821	-0.1417	153.586
165.555 K1	0.8394	-0.0399	0.9475	966.000
165.565	-0.1324	0.4678	-0.2434	1125.996
166.554 χ1	-0.0041	-0.1374	0.0286	587.344
255.555 M2	-0.4210	0.1419	0.6008	13.285
272.556 T2	-0.0833	0.1804	0.0826	97.239
273.555 S2	-1.9390	5.6909	1.5701	132.517
274.554	0.0254	-0.1165	-0.0107	207.968
275.545	-0.0229	0.2260	-0.0192	450.961
275.555 K2	1.9153	-20.2475	1.9255	483.000
275.565	-0.6131	6.9751	-0.7337	519.940
275.575	-0.0720	0.8871	-0.1023	562.998
276.554	-0.0473	-1.5431	0.4381	1498.387

Table 3. CHAMP tidal amplitudes and periodicities

Sat	data	data period	Parameters	Arcs
Ajisai	SLR	50859-52424	$\underline{x}$, $\dot{\underline{x}}$, Cd, Cr	5d
CHAMP	x,y,z	52049-52829	$\underline{x}$, $\dot{\underline{x}}$, k0(L), k1(G), th(L)	1d
Lageos1	SLR	50864-52424	$\underline{x}$, $\dot{\underline{x}}$, Cr, atacc	15d
Lageos2	SLR	50864-52424	$\underline{x}$, $\dot{\underline{x}}$, Cr, atacc	15d
Starllete	SLR	50904-52424	$\underline{x}$, $\dot{\underline{x}}$, Cd, Cr	5d
Stella	SLR	50904-52424	$\underline{x}$, $\dot{\underline{x}}$, Cd, Cr	5d

Table 4. Satellite data for gravity field variability

The SVD shows that the dominant combinations are long-periodic for the 5 geodetic satellites while the dominant lumped harmonics for CHAMP involve the sectorial harmonics.

3 The temporal gravity field: satellite solutions and geophysical data

Temporal gravity field solutions were recovered every 15day utilising SLR for the five geodetic satellites with and without CHAMP and from CHAMP by itself with applied constraints of 0.6e-10 to each amplitude. The amplitude and phase of the annual signal for a field to degree and order 4 (recovered

order	combinations
1	S2,2; S4,2; S6,2; S8,2
2	C2,2; C4,2; C6,2; C8,2
3	Ist order Cs and Ss
4	S4,4; S6,4; S8,4; S10,4
5	C4,4; C6,4; C8,4; C10,4
6	Ist order Cs and Ss
7	S3,2; S5,2; S7,2; S9,2
8	C3,2; C5,2; C7,2; C9,2
9	S6,6; S8,6; S10,6
10	C6,6; C8,6; C10,6

Table 5. Dominant CHAMP combinations in order of significance from SVD: 10 x 10 field, MJD 52409-52424

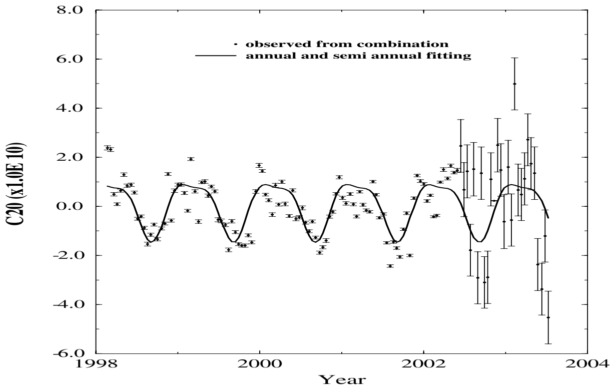

Fig. 1. Annual and semi-annual variation in $C_{2,0}$

from a 6 by 6 solution) are compared against geophysical data in Table 6. The latter comprised annual and semi-annual variations inferred from CDAS-1 atmospheric pressure data from Jan 1989 to Mar 2002; ocean mass redistribution from the Ocean Circulation and Climate Model (OCCAM) (Webb et al., 1998) for Jan 1992 to Apr 1996 and land hydrology from VIC (Nijssen et al., 2001) for 1980 to 1993. The CHAMP only results required the application of the constraint to avoid excessively amplitudes. The disparity between CHAMP and the SLR results is not unacceptable although the results can be overinterpreted. For, example Fig.1 plots the 15day SLR solutions from 1998 to mid 2002 followed by the CHAMP only solution to mid 2003. The satellite data has been fitted by annual and semi-annual sinusoids. The strength of the multisatellite solution prior to 2002 is evident in the consistency of fit and the low error bars. For the CHAMP only solution, the current methodolgy gives rise to both larger variability and larger error bars.

n,m	$C_{n,m}$				$S_{n,m}$			
	Geophy	SLR	CHAMP	SLR&CH	Geophy	SLR	CHAMP	SLR&CH
2,0	1.19/294	0.70/291	0.30/316	0.66/288				
2,1	0.28/321	0.37/334	0.75/ 32	0.45/ 9	0.49/351	0.71/ 5	0.59/ 97	0.15/307
2,2	0.28/152	0.32/ 15	0.43/269	0.25/ 95	0.25/ 37	0.72/ 43	0.60/ 69	0.36/ 96
3,0	0.98/204	1.79/286	1.43/312	1.04/295				
3,1	0.36/309	2.52/239	2.21/152	0.58/ 89	0.80/314	0.98/ 33	1.45/321	0.80/277
3,2	0.29/147	0.12/ 30	0.21/308	0.51/ 7	0.53/298	0.11/ 18	1.00/279	0.36/283
3,3	0.40/ 91	1.02/277	0.32/153	0.71/333	0.38/256	0.18/257	0.79/259	1.51/251
4,0	0.01/174	0.35/ 17	0.25/305	0.17/ 84				
4,1	0.27/255	0.50/156	0.31/160	1.14/177	0.38/256	0.16/116	0.15/113	0.16/ 39
4,2	0.22/134	0.10/104	0.73/221	0.22/155	0.68/298	0.78/272	0.70/228	0.43/242
4,3	0.20/ 61	0.96/166	0.65/100	0.23/190	0.46/311	0.49/192	0.32/ 24	0.56/142
4,4	0.55/ 91	0.86/149	0.70/ 81	0.67/105	0.18/307	0.11/ 42	0.05/355	0.38/227

Table 6. Annual variation (amp/phase) in normalized hramonics from mass distribution of atmosphere (CDAS-1), ocean (OCCAM) and hydrology (VIC); amplitude A (*1.e-10) and phase P (deg) defined by $Acos(2\pi(t-t_0)/365.25+P)$

With the method adopted in this study, there is no evidence as yet that CHAMP can facilitate temporal gravity field recovery due to both the sensitivity to higher degree and order harmonics and the possibility that gravity field signal is absorbed within the solution vector for CHAMP in Table 4. Other possibilities include the use of geophysical data to constrain the harmonics but care must be exercised as global hydrology is poorly defined.

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