A Comparison of Various Procedures for Global Gravity Field Recovery from CHAMP Orbits

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Summary. We compare selected techniques for recovering the global gravity field from precisely determined kinematic CHAMP orbits. The first method derives the second derivatives by use of an interpolation polynomial. The second procedure is based on Newton's equation of motion, formulated and solved as a boundary value problem in time equivalent to a corresponding integral equation of Fredholm type. It is applied to short arcs of the CHAMP orbits. The third method is based on the energy balance principle. We implement the analysis of in-situ potential differences following Jekeli's formulation. The normal equations from the three approaches are solved using Tikhonov–type regularization, where the regularization parameter is computed according to a variance component estimation procedure. The results are compared with the recent satellite-only model EIGEN2 and the first GRACE model GGM01s. All methods provide solutions of the gravity field which represent significant improvements with respect to the reference model EGM96 below degree 50. The quality of the solutions differs only slightly.

Key words: CHAMP, gravity field recovery, boundary value problem, polynomial interpolation, energy balance approach

1 Introduction

Various groups have introduced different approaches to determine the global gravity field from precisely determined kinematic CHAMP orbits. We compare three of those techniques. The first method derives the second derivatives by use of an interpolation polynomial. The second procedure is formulated as a boundary value problem in time. It is applied to short arcs of the CHAMP orbits. The third method is based on the energy balance principle. Apart from the different observation models the same procedure has been used for the calculation of all three methods. This includes the same data set and the same way of solving the normal equations using Tikhonov–type regularization, where the regularization parameter is computed according to a variance component estimation procedure.

2 Data settings

The global gravity field recovery presented here is based on kinematic orbits of CHAMP with a sampling rate of 30 seconds provided by M. Rothacher and

D. Svehla from the FESG of the Technical University Munich. The orbits cover a time period of approximately 100 days. These orbits are processed following the zero-differencing strategy, see (Svehla D, Rothacher M, 2003) and were provided with variance-covariance information per data point. In a preprocessing step we have removed all kinematic positions to which sigma's larger than 5cm in either x, y or z were assigned. Furthermore, only data segments of at leat 2.5h have been selected. After this, the used data corresponds to about 52 days.

EGM96 has been used as a reference field in the following denoted by V. The disturbing potential T is modelled by a spherical harmonics expansion up to degree and order $L = 75$. The unknown coefficients $\Delta c_{nm}, \Delta s_{nm}$ can be estimated in an least squares adjustment.

The satellite's motion is also influenced by disturbing forces $\mathbf{f}$. We model the direct attraction by sun and moon from JPL DE ephemeris, the solid earth tides following the IERS conventions, and we implement an ocean tide model. CHAMP's STAR accelerometer measures the non-conservative forces like air drag and solar radiation pressure. Due to the spurious behavior of the accelerometer, bias parameters are estimated in all three models.

3 Observation equations

3.1 Polynomial differentiation

The functional model of the observations is based on Newton's equation of motion

$$\ddot{\mathbf{r}}(t) = \nabla V + \nabla T + \mathbf{f}. \tag{1}$$

The idea of this method is to approximate the orbit by an interpolation polynomial, in this case a Gregory-Newton n-point scheme (Austen et al., 2002)

$$\mathbf{r}(t) \approx \mathbf{r}_A + \sum_{i=1}^{n-1} \sum_{k=0}^{i} (-1)^{i+k} \binom{q}{i} \mathbf{r}_{k+1} \tag{2}$$

with

$$q = (t - t_1)/(t_2 - t_1).$$

To obtain the accelerations, we have to differentiate the interpolation scheme twice with respect to time t

$$\ddot{\mathbf{r}}(t) \approx \mathbf{r}_A + \sum_{i=1}^{n-1} \sum_{k=0}^{i} (-1)^{i+k} \binom{q}{i}'' \mathbf{r}_{k+1} \tag{3}$$

The accelerations are computed at the centre of the n-point scheme to get the smallest interpolation error. These approximated accelerations are used as pseudo observations for model (1). As they are linear combinations of positions, a full apriori variance-covariance matrix can be computed by linear error-propagation from given covariances per position.

3.2 Solving a boundary value problem in the time domain

This functional model is based on Newton's equation of motion as well, but formulated as a boundary value problem (Schneider, 1967),

$$\mathbf{r}(\tau) - (1-\tau)\mathbf{r}_A - \tau\mathbf{r}_B = -(t_B - t_A)^2 \int_0^1 K(\tau,\tau')\mathbf{g}(\tau',\mathbf{r},\dot{\mathbf{r}})\,d\tau', \qquad (4)$$

with the integral kernel

$$K(\tau,\tau') = \begin{cases} \tau'(1-\tau) & \text{for } 0 \le \tau' \le \tau \\ \tau(1-\tau') & \text{for } \tau \le \tau' \le 1 \end{cases}$$

satisfying the boundary values

$$\mathbf{r}_A := \mathbf{r}(t_A), \quad \mathbf{r}_B := \mathbf{r}(t_B). \qquad (5)$$

The function $\mathbf{g}$ contains all forces acting on the satellite's acceleration:

$$\mathbf{g} = \nabla V + \nabla T + \mathbf{f}. \qquad (6)$$

Equation (4) is applied to short arcs after discretization in time (Ilk et al., 2003) and (P. Ditmer and A. van Eck van der Sluis, 2003). The linear combinations of three positions are used as pseudo observations. A full apriori variance-covariance matrix per arc is computed by linear error-propagation from given covariances per position.

3.3 Energy balance approach

The theory of the energy balance approach has been applied frequently, e.g. (Jekeli, 1999), (Gerlach et al., 2003), (Howe et al., 2003) or (Ilk and Loecher, 2003). We use the following formulation, which is based on expressing all quantities of interest in an inertial coordinate system:

$$E_{kin}(t) + V(t) + T(t) - R(t) - \int_{t_0}^{t} \mathbf{f}\cdot\dot{\mathbf{r}}\,d\tau = E_0 = const \qquad (7)$$

with the kinetic energy

$$E_{kin} = \frac{1}{2}|\dot{\mathbf{r}}|^2, \qquad (8)$$

and the potential rotation term

$$R(t) = \int_{t_0}^{t} \frac{\partial(V+T)}{\partial t}\,dt \approx -\omega_e(r_1\dot{r}_2 - r_2\dot{r}_1), \qquad (9)$$

which approximates the potential contribution of the rotating earth in inertial space. The satellite's velocity $\dot{\mathbf{r}}(t)$ needed for the kinetic energy is computed similar to equation (3). As the energy does not depend linearly on velocities and positions, error propagation is more difficult and not implemented yet.

4 Robust parameter estimation

For all three techniques normal equations were computed per arc and accumulated. To make the solution robust against less accurate periods of the orbit, a variance component estimation (VCE) procedure is used. This can be done efficiently by re–weighting every orbital arc individually in an iterative Monte Carlo approach (MCVCE), see (Kusche 2003).

It is known that gravity field determination from satellite data poses an ill-posed problem. Downward continuation generally amplifies the measurement noise. To stabilize the systems of normal equations and overcome the ill-posedness of the problem a Kaula regularisation starting from degree $L = 40$ is applied, which requires a properly selected regularization parameter. Linking the regularization parameter to the variance of the gravity field parameters offers the possibility to determine it efficiently by means of MCVCE as well.

5 Results

All three methods were applied to kinematic CHAMP data of altogether about 51 days, and provide gravity solutions which we believe represent significant improvements with respect to the reference model EGM96 below degree 50. Above degree 50, these solutions are mainly determined by regularization and therefore biased towards the reference model. The time–wise BVP method and the polynomial differentiation appear as slightly superior to the energy balance method, which we believe is due to the neglected apriori variance information. Generally our solutions are almost free of spurious ‘stripe patterns’, due to the apparently high homogeneity of the data quality in combination with the MCVCE statistical technique.

6 Conclusions and Outlook

We have applied three non-conventional methods for recovering the Earth’s gravity field from kinematic CHAMP orbits. All three methods can improve our knowledge of the gravity field from a very short time period, which is undoubtedly due to the high quality of the kinematic orbits. There are only slight differences between the three solutions. It remains to investigate whether these methods can supersede the traditional method of integrating the variational equations associated to the satellite’s motion. A final statement cannot be given, as research in this matter has not been completed yet. There are possible improvements in methodology especially concerning the energy balance approach. First of all we plan to implement correct error propagation. In addition to this, other differentiation procedures like spline smoothing might improve the crucial velocity derivation step.

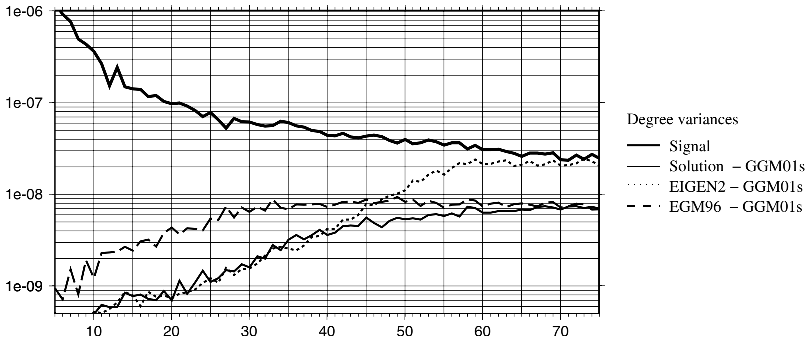

Fig. 1. Solution computed with BVP method.

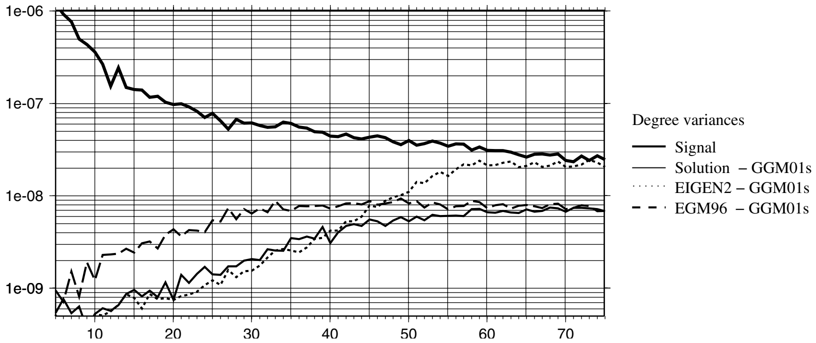

Fig. 2. Solution computed with polynom method.

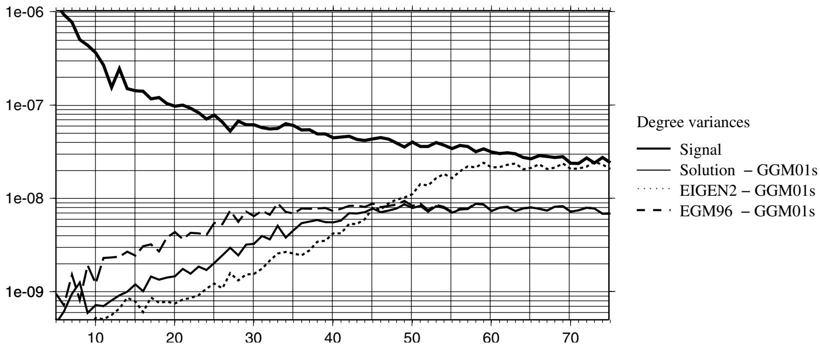

Fig. 3. Solution computed with energy method.

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