Gravity Field Recovery by Analysis of Short Arcs of CHAMP

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Summary: The gravity field recovery strategy presented here enables the global recovery of the gravity field combined with a regional focus on geographical areas with rough gravity field features in a consistent way. The global gravity field is modeled by a series of spherical harmonics while the regional gravity field features are represented by space localizing base functions of harmonic spline type. The physical model of the orbit analysis technique is based on Newton's equation of motion, formulated as a boundary value problem in form of an integral equation of Fredholm type. The observation equations are established for short arcs of approximately 30 minutes length. The procedure can be applied either globally or regionally to selected geographical regions. For a regional application the coverage with short arcs should be slightly larger than the recovery region itself to prevent the solution from geographical truncation effects. A proper combination and weighting of the normal equations of every arc combined with a tailored regularization allows a stable solution for the field parameters. This procedure can be adapted to the roughness of the regional gravity field features, the discretization of the gravity field and the sampling rate of the observations. A global gravity field solution ITG-Champ01E has been derived based on kinematic orbits covering 360 days from March 2002 to March 2003. Regional gravity field solution have been determined for selected regions with rugged gravity field features.

Key words: Gravity field modeling – Regional gravity field recovery – CHAMP – Satellite-to-satellite-tracking – ITG-Champ01E

1 Introduction

The solution strategy presented here enables the global recovery of the gravity field combined with regional refinements in geographical areas with rough gravity field features in a consistent way. The physical model is based on Newton's equation of motion applied to short arcs of approximately 30 minutes and formulated as integral equations of Fredholm type. The integrands contain the reference and residual gravity fields and specific disturbing forces. The reference gravity field representing the low and medium frequency gravity field features are expressed by a series of spherical harmonics complete up to an appropriate degree, while the regional parts are represented by space localizing base functions of harmonic spline type. Especially for a regional refinement of the gravity field it is important to proof in an a-priori step whether there are residual gravity field signals in the kinematically determined orbits caused by rough gravity field features which are not modelled by spherical harmonics. In a post-processing step, the regionally refined gravity field is validated again by using the orbits. The validation procedure is based on the computation of an extended Jacobi integral along the satellite orbits and described in detail in Ilk and Löcher (2003).

2 Mathematical Model

The mathematical model of the gravity field analysis technique is based on Newton's equation of motion (Schneider, 1967),

$$\ddot{\mathbf{r}}(t) = \mathbf{f}(t;\mathbf{r},\dot{\mathbf{r}};\mathbf{x})\,, \tag{1}$$

formulated as a boundary value problem,

$$\mathbf{r}(t) - (1-\tau)\mathbf{r}_A - \tau\mathbf{r}_B = -T^2 \int_{\tau'=0}^{1} K(\tau,\tau')\,\mathbf{f}(t;\mathbf{r},\dot{\mathbf{r}},\mathbf{x})\,d\tau', \tag{2}$$

with the integral kernel

$$K(\tau,\tau') = \begin{cases} \tau(1-\tau'), & \tau \le \tau', \\ \tau'(1-\tau), & \tau' \le \tau, \end{cases} \tag{3}$$

satisfying the boundary values

$$\mathbf{r}_A := \mathbf{r}(t_A), \quad \mathbf{r}_B := \mathbf{r}(t_B), \quad t_A < t_B\,. \tag{4}$$

The specific force function,

$$\mathbf{f}(\tau';\mathbf{r},\dot{\mathbf{r}},\mathbf{x}) = \mathbf{f}_d(\tau';\mathbf{r},\dot{\mathbf{r}}) + \nabla V(\tau';\mathbf{r};\mathbf{x}) + \nabla T(\tau';\mathbf{r};\Delta\mathbf{x}). \tag{5}$$

can be separated in a disturbance part $\mathbf{f}_d$, which represents the non-conservative disturbing forces, in a reference part ∇V, representing the global gravity field features and in an anomalous part ∇T, modelling the corrections to the gravity field parameters. The mathematical model applied in this investigation is given by equation (2) together with the force function (5) (Ilk et al., 1995). The geocentric positions $\mathbf{r}(t)$ of the arcs over the analysis area represent the observations. The unknowns are the corrections $\Delta\mathbf{x}$ to the field parameters $\mathbf{x}$. These are in case of a global gravity field recovery corrections to the coefficients of a spherical harmonics expansion of the gravitational potential $V(\mathbf{r}_P)$ or in case of a regional recovery the parameters of space localizing base functions modelling the anomalous potential $T(\mathbf{r}_P)$:

$$T(\mathbf{r}_P) = \sum_{i=1}^{I} a_i\, \varphi(\mathbf{r}_P, \mathbf{r}_{Q_i}) \tag{6}$$

with the field parameters a_i arranged in a column matrix $\Delta\mathbf{x} := (a_i, i = 1,...,I)^T$ and the base functions,

$$\varphi(\mathbf{r}_P, \mathbf{r}_{Q_i}) = \sum_{n=0}^{\infty} k_n \left(\frac{R_E}{r}\right)^{n+1} P_n(\mathbf{r}_P, \mathbf{r}_{Q_i})\,. \tag{7}$$

The coefficients k_n are the degree variances of the gravity field spectrum to be determined,

$$k_n = \sum_{m=0}^{M} \left(\Delta\bar{c}_{nm}^2 + \Delta\bar{s}_{nm}^2\right). \tag{8}$$

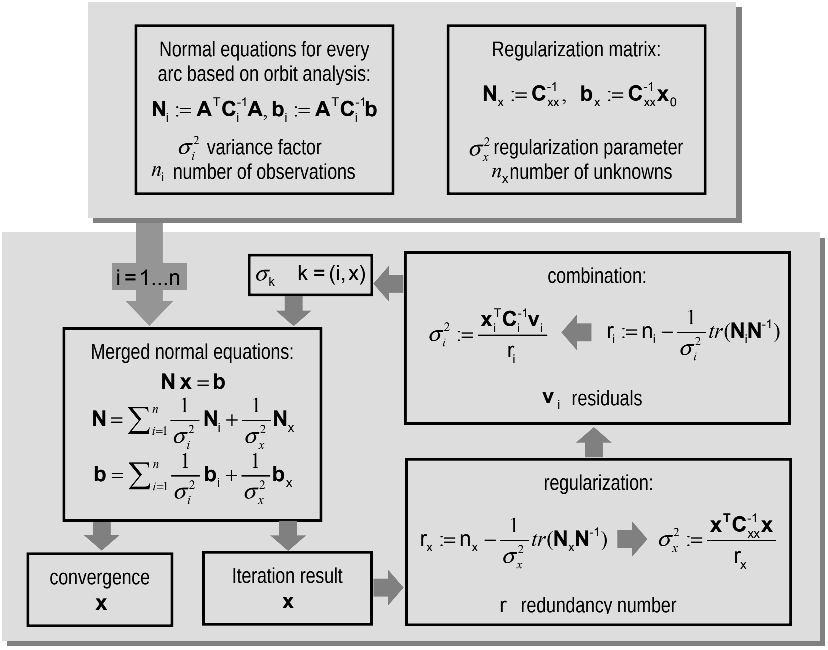

Fig. 1. Computation procedure including the determination of variance factors for every arc and the regularization parameter for the gravity field recovery.

R_E is the mean equator radius of the Earth, r the distance of a field point from the geo-centre and $P_n(\mathbf{r}_P, \mathbf{r}_{Q_i})$ are the Legendre's polynomials depending on the spherical distance between a field point P and the nodal points Q_i of the set of base functions. With this definition the base functions $\varphi(\mathbf{r}_P, \mathbf{r}_{Q_i})$ can be interpreted as isotropic and homogeneous harmonic spline functions (Freeden et al., 1998). The nodal points Q_i are defined on a grid generated by a uniform subdivision of an icosahedron of twenty equal-area spherical triangles. In this way the global pattern of spline nodal points shows approximately uniform nodal point distances. Every short arc of approximately 30 minutes builds a normal equation. All normal equations are combined by estimating a variance factor for every arc as well as an accelerometer bias (Koch and Kusche, 2002). The regularization has been restricted to all potential coefficients from degree 40 upwards and the regularization parameter has been determined during the iteration as sketched in Fig. 1.

3 Global gravity field recovery based on kinematic orbits

The global gravity field recovery presented here is based on kinematic orbits of CHAMP with a sampling rate of 30 seconds provided by M. Rothacher and D. Svehla from the FESG of the Technical University Munich (Svehla and Rothacher, 2003). The orbits cover a time period of approximately 360 days from

March 2002 to March 2003. The three-dimensional accelerometer data are provided by the CHAMP Information System and Data Centre (ISDC). The transformations between the terrestrial and celestial reference frames follow the conventions published by McCarthy (1996). For the computation of the tidal forces caused by Moon and Sun the numerical ephemeris DE405 of the Jet Propulsion Laboratory (JPL) have been used. The accelerometer measurements to determine the surface forces for the CHAMP orbit have been processed according to the rules of the CHAMP data format (Förste et al., 2001). The force functions caused by the Earth tides as well as by the ocean tides have been based on the models as published by McCarthy (1996). As reference frames ITRF2000 and ICRF2000 are used as well as the corresponding rotations according to the IERS conventions. The one-year orbit has been split up into 17000 short arcs with in total 2400000 observations. Then the procedure summarized in Fig. 1 has been applied to determine the 5772 unknown corrections to the potential coefficients of the reference gravity field (EGM96 up to degree 75).

spectral range	0...40	0...50	0...60	0...65	0...70
GGM01s – ITG-Champ01E	3,5	7,4	14,3	18,8	21,8
GGM01s – Eigen-2	7,6	20,7	47,3	57,5	68,0
GGM01s – Eigen-3p	3,9	7,6	23,1	32,0	42,1

Table 1. 1°x1°-grid comparisons of ITG-Champ01E, Eigen-2 and Eigen-3p with GGM01s: rms of undulation differences (cm).

GPS data set	points	filled up with EGM96 from n=37			filled up with EGM96 from n=73		
		min	max	wrms	min	max	wrms
USA	5168	-122,2	179,4	43,4	-129,4	183,5	44,0
Canada	1931	-131,8	174,2	40,1	-112,4	131,8	38,5
EUVN (Europe)	186	-142,4	168,3	42,4	-88,4	143,8	36,3
BKG (Germany)	575	-114,8	73,9	29,6	-95,8	63,4	20,5

Table 2. Global gravity field recovery: rms of geoid undulation differences (cm).

The result of the global recovery has been checked by determining the rms of the differences of 1°x1° grids of point geoid undulations between our solution ITG-Champ01E and the recent GRACE solution GGM01s (CSR, 2003) for various spectral bands (Table 1). For comparison the same tests have been performed for the gravity field models Eigen-2 (Reigber et al., 2003) and Eigen-3p (Reigber et al., 2004). Our model ITG-Champ01E is biased in the higher degrees by the reference gravity field EGM96 caused by a regularization from degree n=40 upwards. But another gravity field recovery solution with no a-priori information and no regularization at all confirmed the results shown in Tab. 1. While the results in the low-frequent spectral band are similar to Eigen-3p there are slight improvements in precision in the spectral band between degree 40 and 60 compared to alternative recovery solutions. Additional test computations have been performed by F.

Barthelmes from GFZ with the GFZ standard evaluation procedure. Tab. 2 shows selected results from this evaluation test, the comparisons with geoid undulations derived from GPS and levelling measurements.

4 Regional gravity field recovery

To demonstrate the regional recovery three regions with rough gravity field features have been selected (Fig. 2): South East Asia, South America and Europe. An additional strip of 10° around the recovery areas have been considered to prevent the solution from geographical truncation effects. The parameterisation of the residual field was based on harmonic spline functions as defined in equation (7); the corresponding recovery parameters are shown in Tab. 3. The mean distance between the base function nodal points amounts approximately 130 *km*. The same data set as in case of the global gravity field recovery has been used also for the regional recoveries. Instead of EGM96 Eigen-2 has been used as reference field up to degree 120. The regional recovery results are shown in Tab. 4.

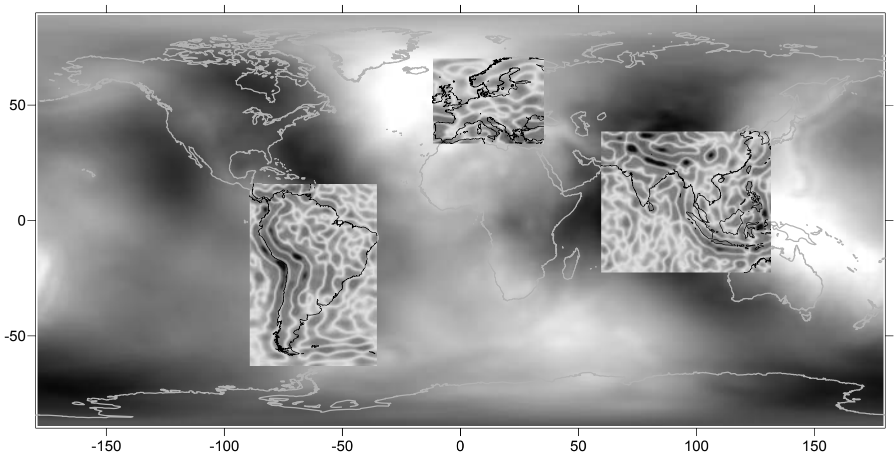

Fig. 2. Gravity field recovery for South America, Europe and South East Asia.

region	South East Asia	South America	Europe
orbits	3192	2340	1922
observations	317000	260000	135000
unknowns	4064	3046	1172

Table 3. Parameters for the regional gravity field recovery.

region	South East Asia	South America	Europe
ITG – GGM01s	92,2	85,5	65,4
Eigen-2 – GGM01s	141,2	114,7	106,0
ITG – Eigen-2	104,7	101,3	88,9

Table 4. 1°x1°-grid comparisons of ITG solutions and Eigen-2 with GGM01s in the spectral band 36…120: rms of undulation differences (cm).

5 Conclusions

The use of short arcs for global gravity field recovery as well as for regional gravity field refinements is an adequate alternative recovery technique based on high-quality kinematic orbits as performed by the FESG Munich. Despite the fact that the CHAMP mission is designed to recover first of all the long and medium wavelength features it could be shown that also regional refinements are possible with surprising accuracy.

Acknowledgements: We are grateful for providing the data by the ISDC of the GeoForschungsZentrum (GFZ) Potsdam and for the kinematical orbits by D. Svehla and M. Rothacher from the FESG at the Technical University Munich. We also thank F. Barthelmes for his computations with the GFZ standard evaluation procedure. Thanks to T. Gruber and D. Svehla as well from the Technical University Munich for additional evaluation computations. The support by BMBF (Bundesministerium für Bildung und Forschung) and DFG (Deutsche Forschungsgemeinschaft) within the frame of the Geotechnologien-Programm is gratefully acknowledged.

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