Gravity Model TUM-2Sp Based on the Energy Balance Approach and Kinematic CHAMP Orbits

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Summary. We have used one year of CHAMP data for deriving a gravity field model based on the energy balance approach. In order to avoid the use of any a priori gravity information, purely kinematic orbits have been computed from GPS measurements only. Subsequently velocities have been derived from these kinematic positions by two different methods, namely smoothing splines and Newton-Gregory interpolation. Using the principle of energy conservation, the satellite's positions and velocities are transformed into gravitational potential. CHAMP onboard micro-accelerometry is used to correct for surface forces. For spherical harmonic analysis the so-called direct approach has been implemented using the full normal equation matrix. The model, called TUM2Sp, was found to be a more accurate gravity field than EIGEN-2 model.

Key words: gravity field, energy integral, kinematic orbit

1 Introduction

Due to the BlackJack GPS receivers onboard of the CHAMP satellite [Reigber et al. (1999)], continuous satellite tracking became feasible. Simultaneously, using the micro-accelerometer onboard, the non-gravitational accelerations acting on the satellite are measured. These two instruments enable the application of the energy integral for gravity recovery from the CHAMP satellite [cf. Jekeli (1999), or Visser et al. (2003)],

$$E = \frac{1}{2}\,\dot{\boldsymbol{x}}^2 - V - \frac{1}{2}\,(\boldsymbol{\omega} \times \boldsymbol{x})^2 - \int_{\boldsymbol{x}} \boldsymbol{a_e} \cdot \mathrm{d}\boldsymbol{x} - \int_{\boldsymbol{x}} \boldsymbol{a_{ng}} \cdot \mathrm{d}\boldsymbol{x} \qquad (1)$$

The left hand side of equation (1) is the observed energy along the path of the satellite, and should be a constant, the Hamiltonian in a conservative force field. The Hamiltonian is the sum of the gravitational potential, V, and

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kinetic energy, $\frac{1}{2}\,\dot{\boldsymbol{x}}^2$. Employing a rotating Earth-fixed coordinate system the rotation in it also consumes energy, which is accounted for by the centrifugal term of equation (1) ($\frac{1}{2}\,(\boldsymbol{\omega} \times \boldsymbol{x})^2$). Further energy variations occur due to non-conservative forces. Variations due to external gravitational forces (i.e. direct, solid Earth, pole and ocean tides) are included in the acceleration $\boldsymbol{a_e}$, while non-gravitational forces acting on the satellite are contained in the $\boldsymbol{a_{ng}}$ vector.

The directly observed variables by CHAMP are the positions, $\boldsymbol{x}$, and the non-gravitational accelerations, $\boldsymbol{a_{ng}}$. The velocities, $\dot{\boldsymbol{x}}$, were derived from positions, while the remaining input data are from other sources (Earth rotation parameters from IERS, planetary ephemerides from JPL, named DE405, and ocean tide model from UT-CSR).

2 Method

As shown in Gerlach et al. (2003) the disturbing potential derived from (reduced-)dynamic orbits by the energy integral method is strongly influenced by the gravity model used for orbit determination. Therefore we stick to a purely kinematic solution in the present study (for kinematic POD see Švehla and Rothacher (2003a) and Švehla and Rothacher (2003b)). The flowchart in Figure 1 illustrates the determination of the gravity field using the energy integral, starting with a kinematic orbit.

In case of a kinematic orbit we face the problem that only positions and no velocities are determined. As we need velocities, compare equation 1, these must be derived from the kinematic positions numerically. Assuming the kinematic orbit being noisier than the (reduced-)dynamic one, in our first solution we attempted to reduce the noise of the kinematic positions by applying a smoothing on the position residuals (i.e. kinematic minus reduced-dynamic positions). Unavoidably the smoothing in the high frequencies also affected the gravity signal part to the same degree. This method was applied for the

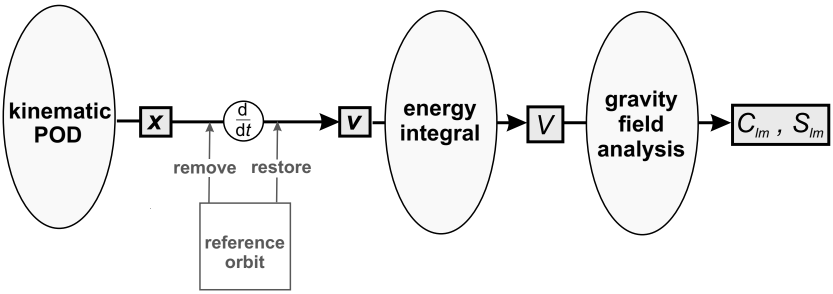

Fig. 1. Flowchart of the performed CHAMP gravity inversion in the present study.

TUM-1S model [Gerlach et al. (2003)]. In this case, in a remove-restore solution, smoothing cubic splines were fitted to the position residuals, then the derivative was taken analytically, and added to the reduced-dynamic velocities. TUM-1S made use of half a year of CHAMP data, which has been extended for this study to one year. In addition, some minor improvements in modelling have been introduced. The extended solution is referred as 'cs44', and processing steps of it are displayed on the flowchart (Figure 1) with the reference orbit step included.

Since for derivation of the kinematic velocities in remove-restore way reduced-dynamic orbits were used (which are known to be dependent on a priori gravity field), this solution seems to be affected by a priori gravity information. Prohibiting any possible dependence on the a priori gravity field, kinematic velocities have now been derived in a reference-field-free manner. For this we implicitly assume that the measurements are free of systematic errors, so the one year of data exhibits random distribution of the noise (white noise). For this solution we are approximating the kinematic positions by a simple interpolation method, namely Newton-Gregory interpolation, and the derivatives are computed analytically. The interpolation is applied on the pure kinematic positions. This solution leads to the 'cs45' coefficient set, which is named TUM-2Sp officially now. The processing sequence is illustrated in flowchart (Figure 1) without the remove-restore step for velocity determination.

3 Gravity Recovery

The differences of the aforementioned three estimations for a CHAMP-only gravity field are summarized in Table 1.

The pseudo-observable for the spherical harmonic analysis is the gravitational potential of the Earth derived from equation (1). The unknown potential coefficients, C_{nm} and S_{nm}, were determined (from the spherical harmonic expansion of the potential) by least squares adjustment. The observations have been weighted equally.

The spherical harmonic coefficients were solved up to degree and order 100. As a consequence of the sampling of 30 seconds of the kinematic CHAMP orbit, which corresponds approximately to gravity information up to degree and order 90, and the numerical differentiation to obtain kinematic velocities,

Table 1. Gravity models discussed in this study.

data set	length of data	kinematic velocity	reference orbit
TUM-1S	1/2 year	smoothing splines	reduced-dynamic
cs44	1 year	smoothing splines	reduced-dynamic
cs45	1 year	Newton-Gregory interpolation	none

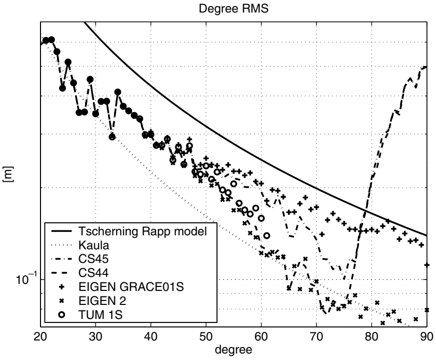

Fig. 2. Degree RMS of the geoid height.

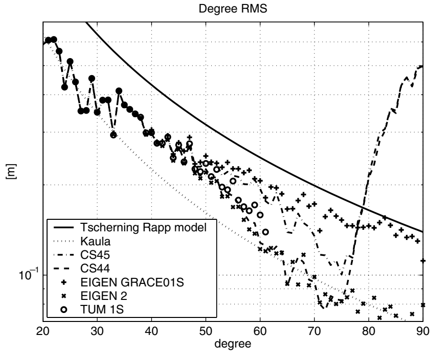

Fig. 3. RMS differences of geoid height differences at certain distances.

Table 2. Comparison between geoid heights from GPS/levelling and global potential models (low-pass filter above degree/order 60) in [cm].

data set	EIGEN-2	TUM-1S	CS44	CS45	EIGEN-GRACE01S
USA (5168 points)	60.2	64.1	56.4	47.1	41.5
Europe (180 points)	59.3	56.4	55.7	33.1	19.4
Australia (197 points)	67.4	63.3	63.8	52.7	50.3
Japan (837 points)	69.5	65.5	66.7	54.8	51.5

we found disturbing potential signal up to about degree and order 60. In the higher degrees the noise is strongly dominating (cf. Figure 2). According to this, the presented sets of coefficients were truncated at degree 60.

4 Results

We ended up with three different gravity fields described in Table 1. These models were compared to GPS/leveling data over different regions (USA, Australia, Europe, Japan). The RMS values of the derived geoid height differences are listed in Table 2. The Table shows that EIGEN-GRACE01S model [Reigber et al. (2003b)] provides always the best solution, and cs45 model gets close to that, while the other 3 models are on similar level with a best performance of the cs44 model. EIGEN-2 [Reigber et al. (2003a)] is only superior to TUM-1S over the USA.

Table 3 shows the RMS differences of gravity anomalies over land and ocean and the Arctic based on gravimetric and altimetric data sets (NIMA [Lemoine et al. (1998)] and AGP [Kenyon and Forsberg (2002)]) with respect to the five models considered in Table 2.

Table 3. Comparison between gravity anomalies from terrestrial/altimetric data and the considered set of five models (low-pass filter at degree/order 60) in [mGal].

data set	EIGEN-2	TUM-1S	CS44	CS45	EIGEN-GRACE01S
Land (gravimetry)	13.92	13.78	13.72	13.27	13.25
Ocean (altimetry)	6.61	6.58	6.50	6.12	5.87
Arctic (gravimetry)	14.67	14.55	14.54	14.17	14.14

Figure 3 shows RMS values of the differences of geoid height differences arranged according to various distances [Gruber (2001)]. The figure can be interpreted as a kind of resolution dependency of the geoid errors, even though there is no theoretically exact correspondence to degrees of spherical harmonics. This figure makes use only of the US data. In Figure 3 the EIGEN-2 is superior to TUM-1S - compare also Table 2. In other regions TUM-1S has slightly lower RMS values than EIGEN-2 (Table 2 and Table 3). CS44 set is slightly better than EIGEN-2 model, due to the extended one-year data set. The cs45 set performed better than EIGEN-2 model at all the degrees, and exhibits characteristic similarities with the EIGEN-GRACE01S model.

5 Conclusion

Based on the tests in the previous section we conclude that (1) EIGEN-2, TUM-1S and cs44 show similar error characteristics (cf. Figure 3); (2) solution cs45 and the first GRACE model (EIGEN-GRACE01S) show no error increase with decreasing distance (Figure 3). This suggest that (a) the white noise characteristic of the CHAMP kinematic orbits leads to an improved spatial resolution of the gravity model; (b) the gravity model used for reduced-dynamic orbit determination affects the gravity estimation when one makes use of the reduced-dynamic orbit as a reference for kinematic velocity estimation – therefore (reduced-)dynamic orbits should not be involved in gravity modelling.

As for the accuracy of the models, we should keep in mind that EIGEN-2, TUM-1S, cs44, cs45 and EIGEN-GRACE01S models are all significant improvement above pre-CHAMP models. This proves that with CHAMP we enter into a new era of gravity field modelling. The sequence EIGEN-2, TUM-1S, cs44, cs45, EIGEN-GRACE01S corresponds in increasing order with this accuracy. The TUM-1S and the EIGEN-2 models seem to be on a similar accuracy level, according to the results of the various tests; cs44 is slightly better due to the extension of the processed data to one year. The accuracy has significantly improved for cs45 due to change of the processing method from smoothing to interpolation without any use of a reference orbit. EIGEN-GRACE01S model reflects the refinements of low-low SST over high-low SST.

Some of the above mentioned models are available via internet. Visit *http://step.iapg.verm.tu-muenchen.de/iapg/forschen.html* for the TUM-1S and the TUM-2Sp models (the latter is referred as the cs45 set in this paper).

Acknowledgement. This work is supported by the German Ministry of Education and Research, Geotechnology program, Grant No. 03F0329A. The Australian GPS-levelling data have been provided by Geoscience Australia. The Japanese GPS-levelling data have been provided by the Geodetic Division of the Geographical Survey Institute, Tsukuba. Their contributions are gratefully acknowledged.

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