# Time-Variable Gravity Seen by Satellite Missions: On its Sampling and its Parametrization

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**Summary.** Temporal variations of the gravity field may act either as a signal or as a source of noise for the current satellite gravity missions. This depends, to some extent, on the parametrization of the gravity field solution. We discuss qualitatively how temporal variations affect satellite gravity products and how their effects may be controlled by an adequate parametrization. We describe a mechanism how unparametrized temporal variations may alias into orbit-parallel spatial patterns of a gravity field solution. While the effect is too small to corrupt static gravity field models like EIGEN-2 or EIGEN-3p it may be a concern for studies on time-variable gravity from consecutive GRACE period solutions. Moreover, time-varying errors in non-gravity parameters such as CHAMP accelerometer corrections may, due to correlations with gravity parameters, cause similar effects as geophysical variations. These issues suggest that an adequate parametrization of the gravity field as a function of space and time needs further study. Eigenvalue analyses of solution normal matrices may be a useful tool for these studies.

**Key words:** time-variable gravity, aliasing, parametrization, accelerometer calibration parameters, EIGEN, GRACE

## 1 Aliasing of temporal variations into spatial variations

Consider the determination of a mean gravity signal (e.g., the total gravity field or its difference from another model) from CHAMP, GRACE or GOCE mission data over a certain solution period. During the solution period, there are temporal gravity variations which are not corrected or parametrized. Their possible effect on the mean solution is schematically demonstrated in Figure 1:

We start from a temporal variation (Fig. 1, top) which is large-scale in time and in space. It is observed along the gravity mission's free orbit (Fig. 1, center, where we assume for simplicity that the disturbing potential $T$ is a direct observable). The sampling along tracks close in space may be distant in time and may thus differ by the meantime evolution. Then, in the mean signal solution adjusted from all observations, orbit-parallel spatial patterns like those in Figure 1 (bottom) will appear. These patterns will have only a

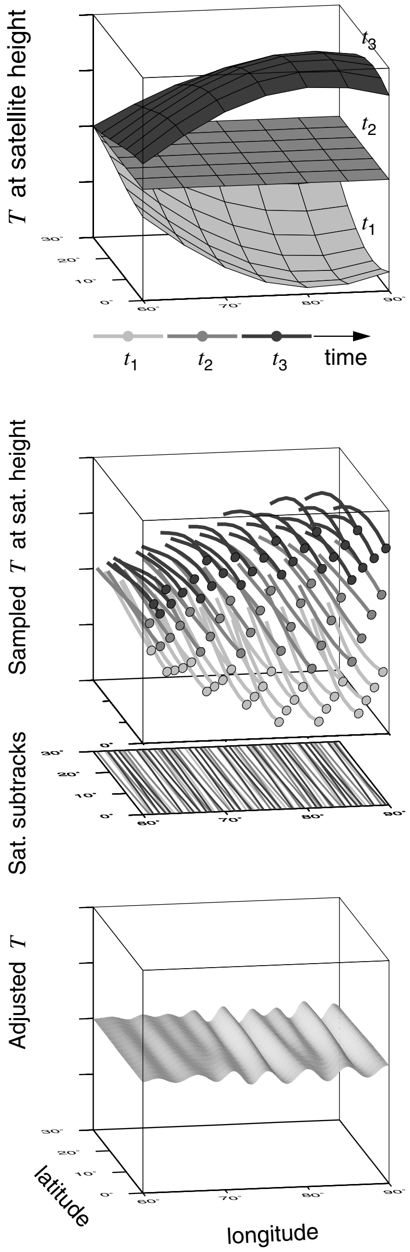


**Fig. 1.** Aliasing of temporal variations into spatial variations of the adjusted mean gravity field

part of the variance of the temporal variations since another part enters the adjustment residuals. But the features will have considerable variance at high spherical harmonic degrees even if the temporal variations are only at low degrees. Our theoretical consideration is supported by numerical simulations for GRACE [2] which obtained orbit-parallel features as an effect of temporal variations.

We call the described effect an aliasing effect: Due to, basically, an incomplete sampling one kind of signal (temporal variations) is misinterpreted as a different kind of signal (spatial variations).

A spectral view on the relative amplitudes is given in Figure 2. The temporal variations are small relative to the searched mean signal. (Fig. 2 uses the ratio $10^{-4}$ which may apply between intra-annual to inter-annual variations and the total gravity field.) Nevertheless, the low degree temporal variations may be larger than the high degree mean signal. The adjustment assigns a part of the temporal variance to the mean signal. This might dominate over the actual signal at high degrees.

Note that this dominance depends on the actual mission observable's spectral characteristics. The situations differ for the different gravity missions and even for the different CHAMP processing approaches. Moreover, describing the observable as a function of space provides only a rough scheme. In fact, the dependence is on the orbit, too,—especially for the "classical" CHAMP processing approach where, roughly speaking, orbit perturbations are observed.

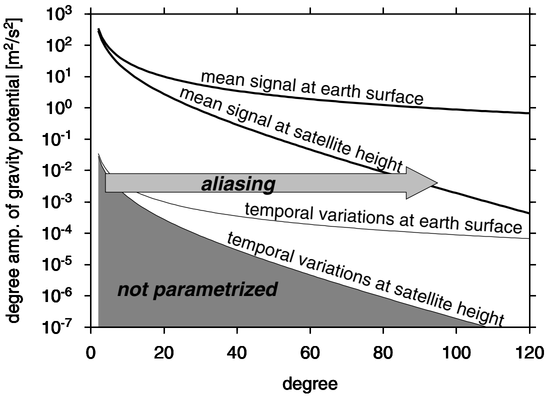


**Fig. 2.** Aliasing of temporal variations into spatial variations: spectral view

## 2 Relevance for current gravity field products

EIGEN-2 [6] and EIGEN-3p [5], the latest CHAMP models by the GeoForschungsZentrum Potsdam (GFZ), fully resolve the gravity field up to around degree $l = 40$ and $l = 65$, respectively. However, their computational resolution extends to $l = 120$ and partly to $l = 140$. The high degree parts of both models consist of distinct orbit-parallel patterns (Figure 3) which dominate above about $l = 60$ for EIGEN-2, and $l = 80$ for EIGEN-3p. They resemble the patterns predicted in the previous section. This suggests the hypothesis that these patterns are an aliasing effect of temporal gravity field variations. However, a quantitative assessment rejects this hypothesis.

For this assessment we simulate orbit perturbations which roughly represent the CHAMP observables in GFZ's processing. According to the hypothesis, the high-degree patterns entered the EIGEN models to explain a part of the orbit perturbations that was in fact induced by temporal variations. Consequently, the perturbations induced by temporal variations must be larger than those induced by the EIGEN patterns. The intra-annual to inter-annual variations reported in the literature (e.g., [7]) have magnitudes below $10^{-4}$ times the static field's magnitude. Hence, we generate random signals that obey $10^{-4}$ times Kaula's rule and simulate their effect on a CHAMP-like orbit. The perturbations are on the decimeter level. In contrast, the EIGEN-2 spectral part above $l = 70$ (and as well, the EGM96 part above $l = 70$) induces perturbations on the meter level. Hence, geophysical temporal variations are too small to induce the EIGEN model high-degree patterns. More general, spatial variations induced by the aliasing mechanism of Section 1 seem negligible compared to the total field's actual spatial variations in the respective spectral band. We expect that this result also holds for other CHAMP processing approaches and for the GRACE and GOCE missions, as their observables are more sensitive to high degrees which makes the curves in Figure 2 more flat.

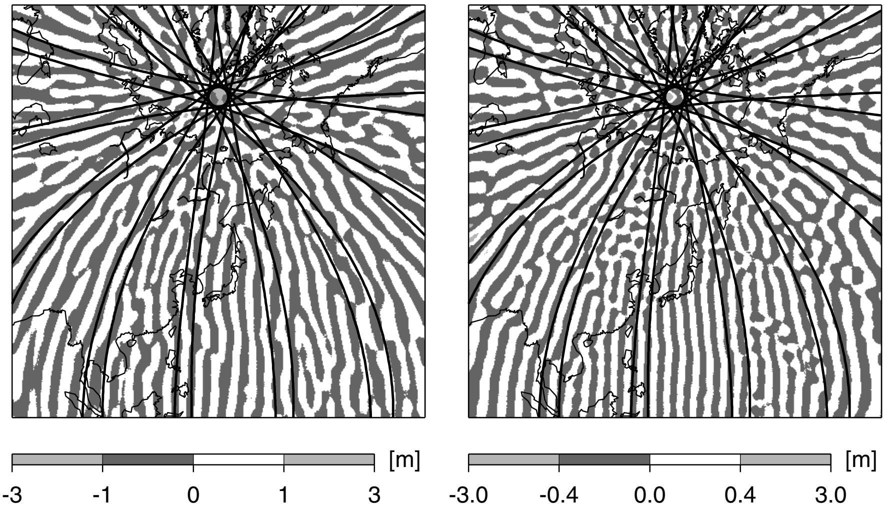


**Fig. 3.** Geoid contribution from (**left**) the EIGEN-2 spectral part above degree 70; (**right**) the EIGEN-3p part above degree 90. A 1-day CHAMP subtrack is added

The above "negative result" concerns the effect of geophysical temporal variations on static gravity field solutions. However, in two other contexts the aliasing effect of Section 1 may, indeed, play a role.

The GRACE mission is designed to determine temporal variations of the gravity field, in addition to its static part. A standard approach is to analyze a sequence of, e.g., monthly period solutions. Then, the relevant signal is the small difference between two period means which may be not larger than unparametrized variations within the solution periods. Hence, aliasing of unparametrized variations may be essential.

Together with the gravity field parameters, non-gravity parameters are estimated for the gravity field solution. In particular, calibration parameters for the CHAMP onboard accelerometer are estimated in certain time intervals [6, 5]. Errors in such parameters cause a wrong assignment of satellite accelerations to either gravitational or non-gravitational forces and may thus correlate with errors in the solved gravity field. By such correlations, time-varying errors of non-gravity parameters may appear like temporal gravity field variations. These apparent variations could, in turn, alias into spatial patterns as described in Section 1. Indeed, for EIGEN-3p the accelerometer corrections were parametrized in shorter time intervals than for EIGEN-2, and the orbit-parallel features in EIGEN-3p are reduced compared to EIGEN-2. This supports the suggestion that the parametrization of non-gravity parameters has an influence on the small-scale structure of the gravity models.

## 3 Discussion on gravity field parametrization

Owing to their dedicated design and unprecedented accuracy, the satellite gravity missions CHAMP, GRACE and GOCE are able to sense gravity field variations not only with space but also with time. Temporal variations may thus be monitored, in particular by GRACE. But they are also an additional error source. Irrespective of the correction for some temporal signals within the mission data processing [1], the missions solve for a basically new kind of signal. The discrete sampling of a mission can not fully resolve this signal, so that ambiguities between different kinds of variations are an inherent problem. But reducing the solution space to only spatial variations up to a certain spherical harmonic degree, as it is done in the "traditional" static gravity field modelling, may now mean an underparametrization which unnecessarily promotes the aliasing effect discussed before. The resulting task is to parametrize the gravity field as a function of both space and time. It is more complex and may involve a trade-off between temporal and spatial resolution.

A guideline is to retrieve as much geophysical variance as possible. In this context it is simple but not optimal to fix the temporal resolution at, e.g., one month over all spatial scales. Instead, different spatial components should be determined with different temporal resolutions according to their temporal variability on the one hand and their temporal resolvability by the mission on the other hand. For example, Perosanz's and others' approach [4] to choose a degree-dependent temporal resolution of the Stokes coefficients accounts for the fact that, roughly, for low-degree coefficients the geophysical variability and, as well, CHAMP's sensitivity are higher than for high degrees.

For a further refinement, with the goal of determining every spatial component with its highest possible temporal resolution, an eigenvalue analysis of gravity field solutions' normal matrices can be used to find an adapted parametrization: A solution's eigenvectors are linear combinations of Stokes coefficients and form a new basis. The factors for these eigenvectors are a new set of gravity field parameters. Their—uncorrelated—errors are obtained from the related eigenvalues. These errors, together with the expected geophysical variability of the parameter, may indicate its appropriate temporal resolution.

An eigenvalue analysis of the normal matrices (here, before the reduction of non-gravity parameters) can also reveal error correlations between gravity and non-gravity parameters. Note that the error covariance matrix contains correlations only between individual parameter pairs. Even if they are low, linear combinations of gravity parameters may still be highly correlated with linear combinations of non-gravity parameters ([3], ch. 12). The eigenvectors now contain both gravity and non-gravity parameters and the eigenvalues indicate how well these linear combinations are determined. Briefly, a badly determined eigenvector that contains significant proportions of both gravity and non-gravity parameters indicates a considerable error correlation between the involved combinations of gravity and non-gravity parameters.

## 4 Conclusions

We have described a mechanism how unparametrized temporal variations may alias into orbit-parallel spatial patterns in gravity field solutions. The effect is too small to corrupt solutions of the static gravity field, and in particular to explain high degree patterns of the EIGEN-2 and EIGEN-3p models. The mechanism might, however, affect analyses of temporal variations from series of GRACE period solutions. Furthermore, if errors in non-gravity (e.g., accelerometer) parameters correlate with gravity field parameters, time-varying errors of the non-gravity parameters can be seen as apparent gravity field variations and their effect may be similar as an aliasing of geophysical variations.

These insights raise the question of an adequate parametrization of the gravity field as a function of space and time. Certainly, different spatial components should be solved with different temporal resolutions according to their temporal variability and to their resolvability by the mission. For further studies, an eigenvalue decomposition of a mission's normal matrices may be a useful tool. It may give a decomposition of the spatial field to components with uncorrelated errors, and give an indication of their adequate temporal resolutions. It can also reveal error correlations between non-gravity and gravity parameters in order to find adapted parametrizations of both.

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