CHAMP and Resonances

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Summary. The technique of using the evolution of a satellite orbit through resonance to determine the values of appropriate *lumped* geopotential harmonic coefficients has recently been revived, and applied to the triple passage of the Champ orbit through 31:2 resonance. Preliminary results for four pairs of coefficients have been derived rapidly, without using the most precise data (which will be forthcoming). The values obtained are compared with those derivable from various global gravity models (to obtain which, vast amounts of data had to be analysed), and the comparison indicates that the resonance technique remains a competitive one.

Key words: gravity field, Champ, resonances, lumped coefficients

1 Introduction

About 30 years ago, a new orbit technique was developed at the (then) RAE (Royal Aerospace Establishment), at Farnborough in England, by means of which certain linear combinations of the geopotential (tesseral) harmonic coefficients, known as *lumped harmonics*, could be evaluated much more accurately than the values of the individual harmonics in the global models then available. The basis of the technique was the recognition that, due to orbital acceleration from the satellite's descent through the atmosphere (taking from months to years), significant resonances between the orbital motion and the Earth's rotation would in due course be encountered. The effective duration of such encounters would vary with the *order* of the resonance and the atmospheric density, but would normally be of a few months at most.

Rapid improvements in the accuracy and scope of the global models, towards the end of the century, had two effects: to confirm the accuracy of the early resonance results, in particular for 15th order resonance, but (in addition) to suggest that it was no longer possible for the resonance technique to generate superior results. More recently, however, it has appeared that, by use of more accurate orbital data and more sophisticated software, a revival of the technique would be justified, and efforts in this direction have been made in the UK, USA and Czech Republic. These efforts are concentrating on Champ, which, after passing through 46:3 and 77:5 resonances, has now passed through 31:2 three times (as a result of orbit manoeuvres). This triple passage has provided a unique opportunity for testing progress on the technique's revival, and a preliminary result is presented here.

2 Some detail

Considerable background material may be obtained from, in particular, the book of King-Hele (1992), which includes the historical development at the RAE, and a recent paper by three of the present authors (Klokočník et al., 2003), which relates to the technique's revival. The essence of the technique involves the concept of the resonant variable, Φ, defined in terms of the usual orbit elements and the sidereal angle, S, by

$$\Phi = \alpha(\omega + M) + \beta(\Omega - S) \tag{1}$$

here β and α are the pair of co-prime integers that define the particular resonance, written as either $\beta{:}\alpha$ or β/α, whilst Ω, ω and M are the usual orbit elements specifying epochal positions for the ascending node, the perigee and the satellite itself.

We proceed in terms of the element I (inclination), since the technique is most productively applied to this element. The *resonant* rate of change of *mean* I, for given $\beta{:}\alpha$, is expressible as a Fourier sum, the prototype of which is a term in $\gamma\Phi - q\omega$; in practice we are concerned with the *basic* term ($\gamma = 1$, $q = 0$), *overtone* terms ($\gamma > 1$, $q = 0$) and *sideband* terms ($\gamma = 1$, $|q| > 0$), usually at most one overtone ($\gamma = 2$) and two sidebands ($q = \pm 1$). The coefficient of a given term consists in the product of a particular lumped harmonic with functions (standardized) of I and e (eccentricity), the e-functions being of order $e^{|q|}$ (*cf* Gooding and King-Hele, 1989; Klokočník, 1983).

Each Fourier coefficient also involves a linear combination of the relevant tesseral harmonics, $C_{l,m}$ and $S_{l,m}$, for a fixed value of $m = \gamma\beta$; here l, in each combination, in principle takes (all) alternate values, from either m or $m + 1$ as its minimum value. The concept of *lumping* now follows, since we can define C_m (similarly S_m) via the sum of the effects of the relevant series; we can (as is usual) normalize these on the basis that C_m would be exactly equal to the true $C_{l(min),m}$ if all subsequent $C_{l,m}$ were zero.

We cannot (without results from many satellites, at different orbital inclinations) separate the individual $C_{l,m}$ and $S_{l,m}$ from determinations of C_m and S_m, but we *can* proceed in the opposite direction, by starting from a particular Earth model and comparing our values of C_m and S_m with the values implied by the model. Possible models include (pre-Champ) EGM96 and TEG4 (both US), Grim5-S1 and -C1 (European), the recent Champ-only models Eigen2 and 2ee, IAPG (Nice 2003) and PGS7772p24. This is the second main topic of this paper; but first we give the results (still to be regarded as preliminary) on which the comparison is based.

3 Data and results

We based our analysis on the so-called two-line element sets (*TLE*s) for Champ, which have become a universal and classic way of disseminating orbital data rapidly; we hope to analyse the potentially much more accurate 30-sec state vectors later. TLE accuracy we assess at about 0.00006 deg for the 'angular' elements, such as I (equivalent to about 7 meters in position, when projected onto the orbit), which reflects extremely well on the improvements made in TLE generation over the years, bearing in mind that the width of field available for the angular elements allows only 4 decimal places!

The essence of resonance analysis (of Champ I's, as we now assume) is the least-squares fitting of selected pairs (C_m and S_m) of harmonics, together with a few other parameters as necessary, to the daily TLEs, over a period long enough to extract maximum information from resonance passage. Before fitting, the TLE values of I are, as far as possible, cleared of known perturbations – in particular the direct lunisolar attraction, the long-period effects of the Earth's zonal harmonics (though uniquely very small for I), the effects of the upper atmosphere, and the rotation of the adopted reference axes themselves, due to precession and nutation. At the accuracy level now required, tidal effects (indirect lunisolar attraction) are also important, but suitable software for analytical modelling was not at our disposal, so the effects were removed empirically via additional fitted parameters.

Fig. 1 indicates the variation of the Champ inclination as it passed through the three significant resonances. It shows at once why we are currently presenting results for 31:2 (the change in I was equivalent to more than 100 meters).

The independent approaches of the UK, US and Czech authors differed in nontrivial respects, of which details are not given here. In brief, the (original) UK approach (Gooding, 1971) at each stage uses the most recent TLEs (the complete

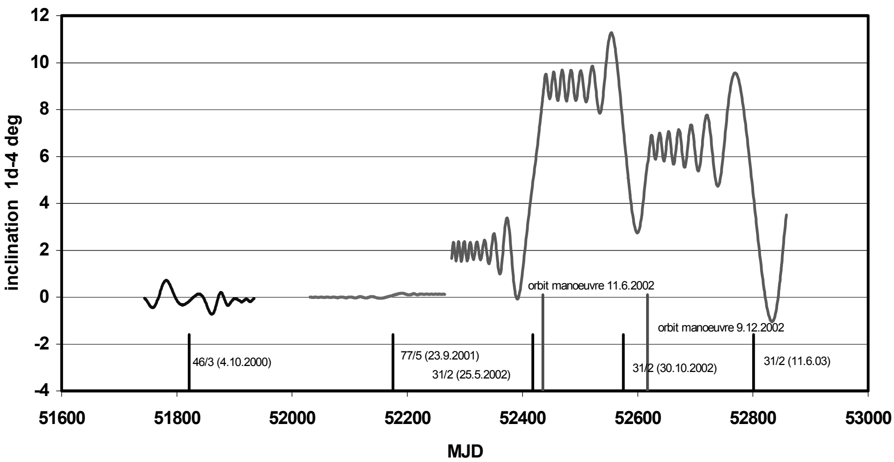

Fig. 1. Variation of inclination through three resonances (GRIM5 C1-based with arbitrary origin for each integration).

set) in computing the 'known' perturbations of I, but the US approach (Wagner, 1973) is a unified one in which the computation of these perturbations (and all the orbit arguments as well) is governed by a single orbit, assumed valid over perhaps several months; and the Czech approach (Kostelecký, 1984) applies a 'weighted numerical integration' technique after non-resonant perturbations have been removed. (When relevant, it is the first of the three approaches that should be assumed, since the results now to be presented were obtained by the first author's computer program.)

Our first analysis (Klokočník et al., 2003) was of the 46:3 resonance, where difficulties in extracting good values of lumped coefficients arose from the combination of a particularly small basic effect for the inclination of the Champ orbit (compare 46:3 with 31:2 in Fig. 1) with large sideband effects. The high order of the 77:5 resonance made any attempt to analyse this even more daunting, so (as already noted) we deal here only with the 31:2 resonance. At first it seemed that three separate analyses would be necessary, one for each of the three stages separated by the two manoeuvres. It was then realized that (thanks in part to using always the latest TLE set) a single fit should be possible, so long as two additional parameters were fitted (empirically), namely, values for the effective discontinuities in I due to the manoeuvres.

In total, 558 TLE sets were used, starting from Jan 26, 2002 (MJD52300). There were a few gaps in the otherwise daily data, including (naturally enough) around the manoeuvres. For convenience, these gaps were dealt with by interpolation in the TLEs themselves; and empirical values of –0.000266 and –0.000278 deg were found for the 'effective discontinuities'. A total of 20 formal parameters were fitted, including 2 for an overall linear effect (normally essential

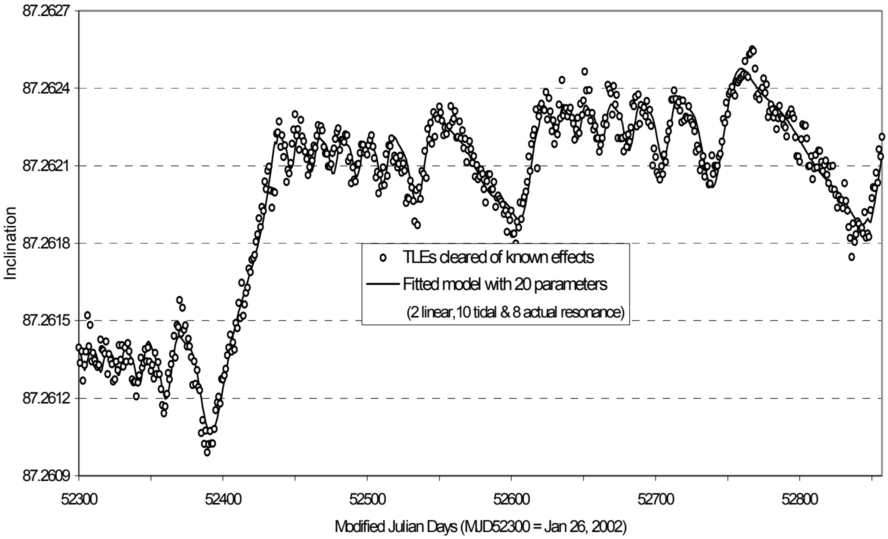

Fig. 2. Observed inclination (adjusted two-line elements), together with fitted curve.

in this approach) and 10 to cover five empirical periods for tidal effects; that left 8 parameters for the actual resonance.

Results for the basic (C, S) pair are (–15.05±0.58, –6.40±0.51), with the usual scaling factor of 10^{-9} implied; and for the 'first overtone' are (4.00±0.22, 2.20±0.33). For the only significant pair of sidebands, the results were: for $q = 1$, (–0.44±1.42, –8.61±1.14); and for q = –1, (0.68±3.07, 5.98±2.65). It is obvious that the sideband results, particularly for $q = -1$, are less accurate than the others, but in a way they are surprisingly good, since the $e^{|q|}$ factor degrades the sideband resonance analysis for I. For analysis of e, however (which we are not able to present yet), it is for $q = 0$, and NOT $|q| = 1$, that results are degraded, so analyses for e and I potentially complement each other. This is why it is normal to do both, making an appropriately weighted combination of the two sets of results.

Finally, the usual *a posteriori* estimate of *rms* was made, based on the 558 residuals and the number of *degrees of freedom*; the result was 0.00006 deg, whence our assessed accuracy at the beginning of this Section. Fig. 2 displays (as points) the *observed* values, as cleared of known perturbations (and the effective discontinuities as above), and (by the curve) the *fitted* evolution of I.

4 Comparisons

Are there external (independent) data of equivalent or better quality than the result just given, which could therefore validate it? Or alternatively, is this resonant result for Champ significantly better than those derived from general geopotential models, and could thus serve to calibrate *them*? The lumped harmonics (C_{31}, S_{31}) from I (Champ) are the following linear sums of geopotential harmonics ($C_{l,m}$ and $S_{l,m}$):

$$\begin{aligned} C_{31}, S_{31} &= 1.0000(C_{32,31}, S_{32,31}) + 0.9096(C_{34,31}, S_{34,31}) + 0.7405(C_{36,31}, S_{36,31}) \\ &\quad + \ldots = -15.05\pm0.58, -6.40\pm0.41 \end{aligned} \tag{2}$$

the directly measured result given in the previous section.

In the 1970's and 1980's, most of the resonant results were derived for orbits not used in comprehensive satellite-geopotential solutions. As a result, the resonant lumped harmonics for these orbits were generally superior (had much lower sd-estimates) than those computed for them from the comprehensive models, so they served as calibrating markers for them (eg., Wagner and Lerch, 1978). For Champ, however, there are already a number of high-degree geopotential models that have been computed from its GPS data, used roughly every 30 seconds for up to 6 months. These models are all complete to 120x120, with terms as high as 140,140. What are the lumped harmonics for this (31,2) resonance computed from them?

Table 1 gives these values from the above series, with projections of the covariance matrix for two of these Champ-only models, together with the series for a recent high-degree field computed from Grace-intersatellite tracking on a nearby orbit (altitude ~480 km, $I = 89.02$ degrees), as well as for the pre-Champ combination model Grim5-C1 (120x120; Gruber et al., 2000).

C_{31}	S_{31}	Models	Data
		Champ's own 31:2 resonance	
-15.05±0.58	–6.40±0.41	Analysis here	Champ TLEs (2002-3)
		Comprehensive pre-Champ	
–15.71	–8.54	EGM96	satellites + surface gravity
–16.47±1.68	–7.33±1.56	Grim5-C1	29 satellites + surface gravity
		Comprehensive Champ only	
–16.91±0.45	–9.43±0.37	Eigen 2	GPS ~2 cm Phases
–16.41±0.40	–8.73±0.40	PGS7772p24	GPS ~2 cm Phases
–16.61	–10.75	IAPG(Nice 2003)	Geopotential Anomalies
		Comprehensive with Champ data	
–15.80	–10.03	Eigen 1S	Grim5-S1 + Champ + SLR
		Comprehensive Grace only	
–16.53	–9.36	GGM01S	1 µm/sec range rates (111 days)

Table 1. Lumped harmonics for Champ-type orbit (in 10^{-9} units, with standard deviations when known) (altitude = 393 km, inclination = 87.27 deg, eccentricity = 0.003)

Note the generally good agreement of all these independent results. (Among the Champ-only models, the data spans were wholly independent.)

Attesting to the method's efficiency, we also note that the precision of the Champ 31:2 resonance is roughly equal to that for the complete high-degree Champ-only models, while employing only a few hundred observations of (mean-)I, compared with more than a million GPS phases for the latter.

Formally, the resonance and Champ-only values all calibrate the less accurate Grim5-C1 values to within about 1-sd of the latter. Comparing values from the other four independent high-degree models with the resonance coefficients, we note that both C and S from resonance are numerically smaller, the discrepancy in S being the more serious in terms of the stated precisions. In the resonance solution, the extra empirical parameters, especially those in the longer period tides, may be absorbing part of the resonant signal in I.

5 Conclusions

The variation of Champ's orbital inclination has been analysed over the period of a year and a half that covers three passages through 31:2 resonance with the geopotential. This has resulted in values of certain lumped harmonics that are in excellent agreement with those that can be inferred from comprehensive geopotential models. The latter are based on vast amounts of very precise tracking, followed by highly elaborate analysis, whereas our results have been obtained just from the (mean) orbital elements of Champ that continue to be issued daily.

This work is a preliminary stage of a programme in which it is hoped that more accurate resonance results can be obtained from the more precise state vectors being derived for Champ. If possible, accurate lumped harmonics will also be obtained for the higher order resonances (46th and 77th), through which the orbit passed before reaching 31st order.

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