Comparison of Different Stochastic Orbit Modeling Techniques

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Summary. Reduced-dynamic orbit determination for spaceborne GPS receivers is a method promising highest accuracy of the estimated LEO trajectories. We compare the performance of different pseudo-stochastic orbit parametrizations (instantaneous velocity changes and piecewise constant accelerations) and probe the range between dynamic and heavily reduced dynamic orbits. Internal indicators like formal accuracies of orbit positions, comparisons with orbits computed at the Technical University of Munich (TUM), and validations with SLR measurements are used to assess the quality of the estimated orbits. For piecewise constant accelerations comparisons between the estimated and the measured accelerations from the STAR accelerometer allow for an additional and independent validation of the estimated orbits.

Key words: Spaceborne GPS Receivers, Reduced-Dynamic Orbit Determination, Pseudo-Stochastic Orbit Modeling, Accelerometer Data

1 Introduction

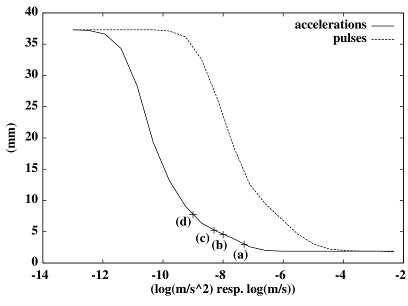
Since the launch of CHAMP on July 15, 2000, the uninterrupted GPS tracking technique of low Earth orbiters (LEOs) has proved to be a reliable method for high quality precise orbit determination (POD). This article focuses on results achieved with a pseudo-stochastic orbit modeling in reduced-dynamic LEO POD, because this technique plays a key role if highest precision is demanded (see [3]). However, the challenging low altitude in the case of CHAMP requires an efficient and flexible pseudo-stochastic orbit model due to the rather large number of parameters involved.

Our approach of LEO POD is based on undifferenced GPS phase tracking data, whereas the GPS satellite orbits and high-rate clock corrections are introduced as known. This leads to a very efficient procedure for estimating LEO orbital parameters, LEO receiver clock corrections, and real-valued phase ambiguities as the only unknowns in a least-squares adjustment. The orbital parameters estimated are the six osculating elements, three constant accelerations in radial, along-track and cross-track directions acting over the whole orbital arc, and the so-called pseudo-stochastic parameters, which are in this article either instantaneous velocity changes (pulses) or piecewise constant accelerations (see [1]). These parameters are called pseudo-stochastic as they are characterized by an expectation value of zero and an a priori variance, which constrains the estimates not allowing them to deviate too much from zero.

2 Internal orbit quality assessment

Pseudo-stochastic parameters define the degree of "strength" reduction of the dynamic laws by allowing for a stochastic component in the equations of motion. Fig. 1 (left) shows for day 141/01 the variation of the postfit RMS as a function of equal constraints in three orthogonal directions for both types of pseudo-stochastic parameters set up every 6 minutes using the gravity field model EIGEN-2 ([2]). The similar dependency signifies that both parametrizations may be considered to some extent as equivalent. In both cases looser constraints (heavily reduced dynamic orbits) obviously allow for a better fit, which, however, does not necessarily guarantee the best orbit quality.

It might be more instructive to analyze formal accuracies of the orbit positions rather than postfit RMS values to shed light on the orbit quality. Several differently constrained solutions are subsequently highlighted, based either on accelerations, which are labeled with lower case letters (a $\leftrightarrow 5 \cdot 10^{-8} m/s^2$, b $\leftrightarrow 1 \cdot 10^{-8} m/s^2$, c $\leftrightarrow 5 \cdot 10^{-9} m/s^2$, d $\leftrightarrow 1 \cdot 10^{-9} m/s^2$), or on pulses, which are labeled with capital letters (A $\leftrightarrow 1 \cdot 10^{-5} m/s$, B $\leftrightarrow 5 \cdot 10^{-6} m/s$, C $\leftrightarrow 1 \cdot 10^{-6} m/s$, D $\leftrightarrow 5 \cdot 10^{-7} m/s$). Fig. 1 (right) shows the formal accuracies of the orbit positions (3D) for all solutions based on accelerations for day 141/01. Apart from the less accurate positions at the arc boundaries, which are common to all solutions, some obvious differences between the individual solutions may be observed, like time intervals of worse formal accuracies, especially for the weakly constrained solution (a). Because only 8 satellites could be tracked simultaneously at that time, the number of good observations may be reduced significantly at certain epochs, which affects kinematic and heavily



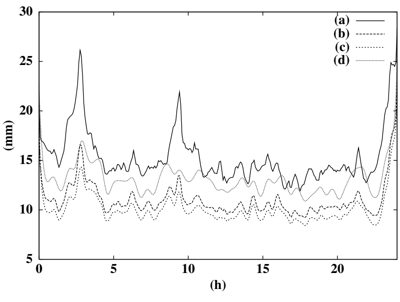


Fig. 1. Postfit RMS for day 141/01 as a function of differently constrained pseudo-stochastic parameters set up every 6 minutes using EIGEN-2 (left). Formal accuracies of orbit positions (3D) for differently constrained solutions (see text) (right).

reduced dynamic orbits like solution (a) very much. In contrast the "almost" dynamic solution (d) shows a very smooth (but also not optimal) accuracy curve which is barely affected by a poor geometry. The optimal choice may be found somewhere in-between depending on several factors, like the number of pseudo-stochastic parameters set up, the number of successfully tracked satellites, and the data quality. Detailed investigations for the CHAMP orbit comparison campaign (days 140/01 - 150/01) showed that solution (c) is close to the optimum when using the same constraints in three orthogonal directions (see [1]).

Fig. 2 (left) shows the formal accuracies of the orbit positions (3D) for the solutions (a), (b), and (c) for day 198/02 using the gravity field model EIGEN-2. Apart from the improved accuracy level (8.3 mm compared to 12.2 mm in Fig. 1 (right)) it is evident that even solution (a) shows a significantly improved performance compared to day 141/01 because it takes most benefit from the better tracking conditions (more than 8 satellites) of the CHAMP BlackJack receiver. This leads to an optimal constraining slightly favouring "more" kinematic orbits like solution (b). The differences between both solutions (c) and (b) (7.9 mm RMS of plain orbit differences) are small, however. This fact is illustrated by Fig. 2 (right) showing for a time window of about two revolution periods the actual differences in along-track direction with periodic deviations mostly below the 1.5 cm level. The solid curve in the same figure shows the differences between the solutions (B) and (b) in the same direction. Analyzing formal accuracies, plain orbit differences (0.9 mm RMS), and external comparisons (see next section) shows that the two solutions (B) and (b) must be considered as almost identical. But the sharp cusps every 6 minutes also illustrate the subtle differences between the two pseudo-stochastic parametrizations. Therefore the estimation of accelerations seems to be slightly preferable to get smooth (differentiable) orbits.

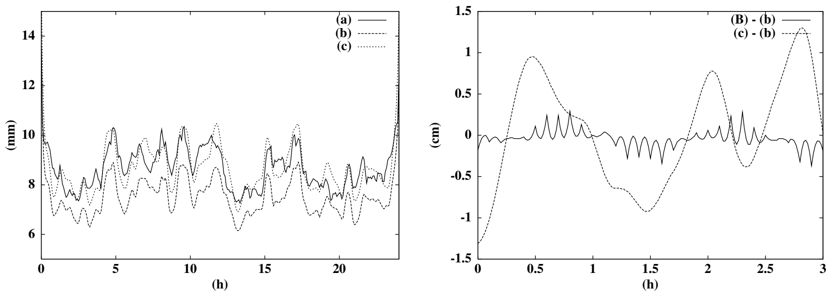


Fig. 2. Formal accuracies of orbit positions (3D) for day 198/02 for differently constrained solutions (see text) using EIGEN-2 (left). Along-track orbit differences between solutions (c) and (b) resp. (B) and (b) (see text) (right).

3 External orbit quality assessment

In order to assess the overall quality of the estimated orbits, detailed comparisons between our solutions and reduced-dynamic orbits computed at the Technical University of Munich (TUM) (see [3]) were carried out for a time interval of one week. Fig. 3 (left) shows the RMS of plain orbit differences for GPS week 1175 using pulses. The three left bars of each group represent the daily RMS of the comparison for the solutions (A), (B), and (C) w.r.t. (TUM). The three right bars ((A'), (B'), and (C')) represent the results w.r.t. (TUM) for solutions with pulses set up every 15 minutes instead of every 6 minutes. The right figure shows the analogue results for the solutions (a), (b), (c), (a'), (b'), and (c') (i.e., based on accelerations) w.r.t. (TUM).

The best agreement may be achieved for solutions (A) resp. (a) (2.29 cm resp. 2.35 cm mean RMS) implying that the solution (TUM) might be slightly "more" kinematic than our favourite solutions (B) resp. (b). The fact that the comparison favours the solutions (A) and (B) rather than (a) and (b) is simply because the solution (TUM) is generated with pulses as well (see [3]). It is, nevertheless, remarkable that solutions of similarly good quality may be obtained using accelerations with constraints and numbers of parameters varied over a broad range, whereas the solutions based on pulses tend to deviate more rapidly from the solution (TUM) when varying the parameter space in a similar way. The comparatively poor agreement for all solutions of day 195 w.r.t. (TUM) is caused by intentionally not taking into account the attitude information from the star sensors in order to visualize the impact on the orbits (approximately 1.3 cm on the RMS level).

SLR residuals were computed for our solutions for GPS Week 1175 as well. Fig. 4 (above) shows the daily SLR RMS for the different solutions. Most residuals (94%) are below the 6 cm limit without any significant SLR bias. An identical overall RMS of 3.43 cm is achieved for the two best solutions (b) and (B) using a screening threshold of 0.5 m. This RMS level could be

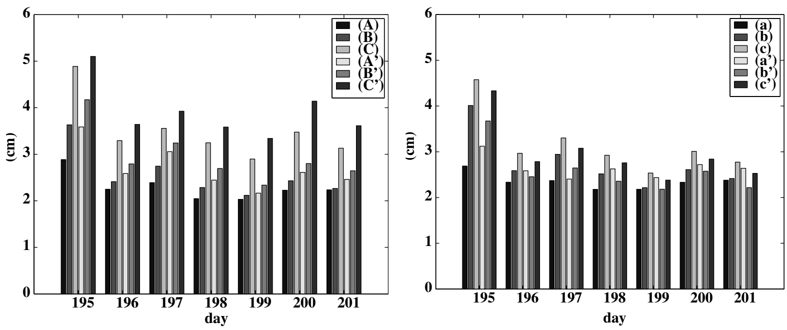


Fig. 3. Daily RMS of plain orbit differences for GPS week 1175 for different solutions (left: pulses, right: accelerations, see text) w.r.t. reduced-dynamic orbits from TUM.

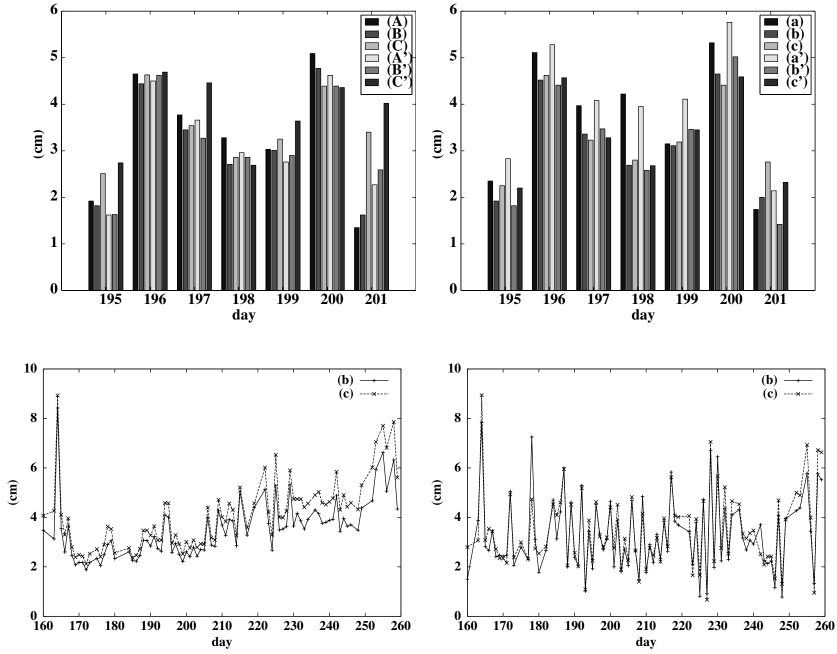


Fig. 4. Daily SLR RMS for GPS week 1175 for different solutions (above left: pulses, above right: accelerations, see text). Daily RMS of plain orbit differences for days 160/02 to 260/02 for solutions (b) and (c) w.r.t. (TUM) (below left). Daily SLR RMS for days 160/02 to 260/02 for solutions (b) and (c) (below right).

easily lowered to about 2.5 cm when applying a more restrictive screening procedure removing a few outliers.

Longer data series indicate that similar results may be achieved as for GPS week 1175. Fig. 4 (below) shows for the solutions (b) and (c) for about 100 days (data files having long observation gaps were ignored) the orbit differences w.r.t. the solution (TUM) (left) and the SLR residuals (right). Consistent with Fig. 3 (right) solution (b) agrees better with the solution (TUM) than solution (c) does. The SLR residuals also slightly favour solution (b) (3.30 cm mean of daily SLR RMS) over solution (c) (3.47 cm mean of daily SLR RMS). The observed drift in the comparison to the solution (TUM) might be due to small system inconsistencies (z-shift) which are greatly reduced after performing a Helmert transformation.

If a "good" gravity field model is available the estimated piecewise constant accelerations may be compared with the measured accelerations from the STAR accelerometer. Fig. 5 shows for a time interval of about three revolution periods the agreement (correlation: 94.8%) of the estimated piecewise constant accelerations in along-track direction with the measured accelerations (bias and scale applied) when the gravity field model EIGEN-2 is used.

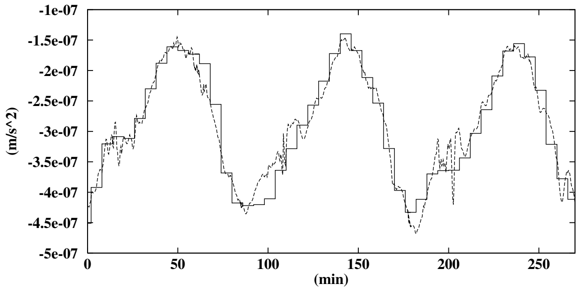


Fig. 5. Piecewise constant accelerations for day 198/02 every six minutes in along-track direction compared with accelerometer measurements (bias and scale applied) using the gravity field model EIGEN-2.

4 Summary

About 100 days of undifferenced GPS phase tracking data of the CHAMP satellite were processed to analyze the performance of different reduced-dynamic orbit parametrizations. SLR residuals proved that the orbits are accurate on an RMS level of about 3.5 cm, which was also supported by comparisons with reduced-dynamic orbits from the Technical University of Munich. Additionally, estimated piecewise constant accelerations were compared with accelerometer data showing a high correlation (94.8%) in along-track direction. Closer inspection of the agreement for the other directions and a refined piece-wise linear parametrization for the estimated accelerations could allow for a retrieval of accelerometer bias and scale parameters.

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